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ORIGINAL ARTICLE

Modelling and optimisation of a femtosecond laser micro-machining process for micro-hole array products

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Abstract Femtosecond laser micro-drilling has progressed remarkably in recent years to become an essential tool for micro-hole drilling in many applications where the extreme high accuracy machining is required. This paper addresses the problem of finding optimal inspection policy in a femtosecond laser micro-machining process for micro-hole array products to improve successful rate of machined parts and hence to minimise the unit manufacturing cost. A solution algorithm is proposed based on particle swarm optimisation (PSO), and the simulation is used to provide better insight into the optimal solution. The numerical experiments and verification experiments which are conducted with different schemes illustrate the applicability and efficiency of the proposed approach.

Keywords Femtosecond laser micro-machining · Micro-hole array products · Modelling · Optimisation

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1 Introduction

Due to the rapid development of micro-hole processing technologies in recent years, miniature and precise products with micro-hole array are commonly applied in many industrial applications, such as miniature mixers [1], distribution structures for nozzles in micro-jet cooling devices [2], aerostatic air bearing [3], miniature oil sprayers [4], inkjet nozzle [5] and miniature oil atomizer [6] fabrication. There are various micro-machining technologies, including micro-electro mechanical system (MEMS) process, laser micro-hole drilling, micromechanical drilling, micro-hole punching, micro-ultrasonic machining and micro-electro discharge machining (micro-EDM), to use for machining micro-holes [7]. Considering the demands on high accuracy, high aspect ratio, high quality and various processing materials, laser micro-hole drilling is the preferred choice amongst these technologies due to its relative simple, efficient and reliable machining mechanism.

Femtosecond laser drilling gets more and more attention for being economically efficient in drilling large numbers of closely located holes on micro-scale [8]. Based on femtosecond laser ablation, laser micro-drilling using computerised numerical control (CNC) motion system provides a high accuracy approach which could be applied to fabricate micro-holes in a variety of materials at very high speed without expensive masks, thus offering a quick, low cost and flexible micro-machining technology, and enables us to finish the micro-hole fabrication with high quality features.

During the laser micro-hole drilling process, one of the major aspects that concern the micro-hole array products (MHAP) is the quality of the drilled holes, which is judged by a number of different characteristics or acceptance criteria. The geometric factors include hole roundness, hole taper and variation in hole entrance diameter. The metallurgical factors include recast layer and micro-cracks. Moreover, the other factors, such as overlap between two adjacent micro-holes which may result in generating non-qualified products, also need to be taken into consideration when machining MHAP. In conventional laser micro-machining process of MHAP, micro-holes with high accuracy and high density are often very closely positioned to one another on a workpiece, and any deviation in size may adversely encroach on other neighbouring holes and then make the products defective or even scrapped. Consequently, from both economic and quality's points of view, the main aim of our work is to provide an effective policy and optimal scheme which can simultaneously decrease the diameter variation and overlap generation of micro-holes in order to enhance the successful rate of MHAP and jointly considering the criteria of minimum unit manufacturing cost.

The remainder of this article is structured as follows. Section 2 reviews the existing literature in the field of improving the micro-hole quality whilst introducing the main applications of inspection policy. Then, in Section 3, the research problem is stated in detail. We show the proposed laser micro-machining scheme for MHAP based on the inspection and rework policies. The mathematical model formulation is also presented in this section. The proposed particle swarm optimisation (PSO) algorithm to find the optimal inspection and rework policies is presented in Section 4. In Section 5, we set a number of numerical experiments of femtosecond laser micro-machining for MHAP to illustrate the advantage of the proposed model. The paper finally ends up with the conclusions in Section 6.

2 Literature review

The micro-hole quality improvement using appropriate methodologies has been a continual research endeavour. The relevant papers, which are used as references in our research, are reviewed as follows.

Ghoreishi et al. [9] employed a statistical model to analyse and compared hole taper and circularity in laser percussion drilling on stainless steel and mild steel. Kuar et al. [10] experimentally investigated the influence of laser machining parameters on the heat-affected zone thickness and phenomena of tapering during CNC-pulsed Nd:YAG laser micro-drilling of zirconium oxide (ZrO_2) and performed parametric analysis through response surface methodology (RSM). Biswas et al. [11] applied RSM to investigate the effect of different process parameters on output quality characteristics (circularity at exit and hole taper) during Nd:YAG laser micro-drilling of gamma titanium aluminide sheet of different thickness. They observed that the optimum values for circularity and hole taper may be obtained at moderate values of lamp current, pulse frequency, air pressure and at higher values of sheet thickness. Ganguly et al. [12] used GRA to determine the laser micro-drilling parameters (lamp current, pulse frequency, air pressure and pulse width) for the simultaneous optimisation of multi-quality characteristics (Ta, width of heat-affected zone (HAZ)) during the laser micro-drilling of millimetre-thick zirconium oxide ceramic. Kuar et al. [13] applied grey relation analysis combined with the Taguchi method to find the optimum parameter setting for laser micro-drilling of alumina. The experimental results for the optimal setting showed that there is considerable improvement on both quality characteristics hole taper and HAZ width in the laser micro-drilling process. Mishra and Yadava [14] established a model of laser percussion drilling using artificial neural network coupled with finite element method model, indicating that the increase of pulse width would cause an increase in HAZ due to diffusion of more heat energy on the top surface of the sample.

In another research, we have found that in many industries, inspection policy is an important approach to ensure desired quality and reduce quality-related costs. However, inspection is an inferior way of dealing with quality problems [15] and should not be used as an effective way to improve the quality, but an appropriate inspection policy results in improving the customers' satisfaction as it decreases the number of defective items sent to the market. Process quality improvement using optimisation methodologies has been a continual research effort [16]. Many researchers have discussed the problem of optimising inspection policies in different settings and applied various approaches to find the optimal policy.

Rau and Chu [17] discussed the inspection allocation problem for serial production systems considering repair, rework and scrap as three possibilities for the treatment of detected nonconforming units and developed a profit model for optimally allocating inspection stations through using a heuristic method. This problem has been further discussed in two other studies considering more complex assumptions and settings [18, 19]. Vaghefi and Sarhangian [20] developed a mathematical model to obtain the optimal inspection plan that minimises the total inspection-related costs whilst still assuring a required output quality for multistage manufacturing systems with possible misclassification errors. They applied a simulation algorithm to model the multistage manufacturing system subject to inspection and hence to estimate the resulting inspection costs. Moreover, Azadeh et al. [21] also developed a PSO algorithm to determine the optimal inspection policy with the minimum total inspection cost where the size of sample in sampling inspection stations should be optimised. Subsequently, Azadeh et al. [22] extended this approach to the case of uncertain inspection costs.

From the existing literature, we have found that the existing research mainly focuses on the optimisation of hole quality characteristics in laser micro-hole drilling process. However, very few researches have been focused on fabricating the micro-hole array with high accuracy and high density. Therefore, based on some assumptions, which are the case in most industrial environments, the research as reported in this paper aims at filling the gaps in the literature and investigates the problem of inspection and rework policy optimisation in a femtosecond laser micro-machining process for MHAP.

3 Model development

3.1 Problem statement

Any micro-hole array product invariably comprises one or more matrix arrays involving a number of micro-holes having one or multiple quality characteristics that are required to be machined within a range of specified tolerances. Each machined product is accepted only when all the characteristics meet the specified tolerance range. In this study, we consider a conventional femtosecond laser micro-machining process for $m \times n$ array consisting of Z micro-holes as illustrated in Fig. 1. Each micro-hole is drilled by femtosecond laser, and the array is completed in a hole-by-hole manner. After inspection in the last step, the qualified products are delivered to the customer, and the non-qualified products are scrapped. Under the high accuracy requirement of diameter and position of the holes, the laser micro-machining process for MHAP has the following problems:

1. High diameter variations: As the diameter variations are the same order as diameter size, it is very difficult to meet the diameter requirements of all micro-holes in a qualified MHAP simultaneously.
2. Cumulative positioning errors: Since the laser micro-machining process of MHAP is a serial multi-hole manufacturing process, the positioning errors exist in each drilled micro-hole will transfer and accumulate along the machining direction during the process. Thus, it is very hard to drill all micro-holes onto a workpiece without any overlaps.

Due to the reasons mentioned above, if a MHAP is produced or in a large batch, the successful rate is particularly low. To solve this problem, a proposed laser micro-machining scheme using a mixed policy of inspection and rework is developed. The detailed discussion for the solved scheme is shown in the next section.

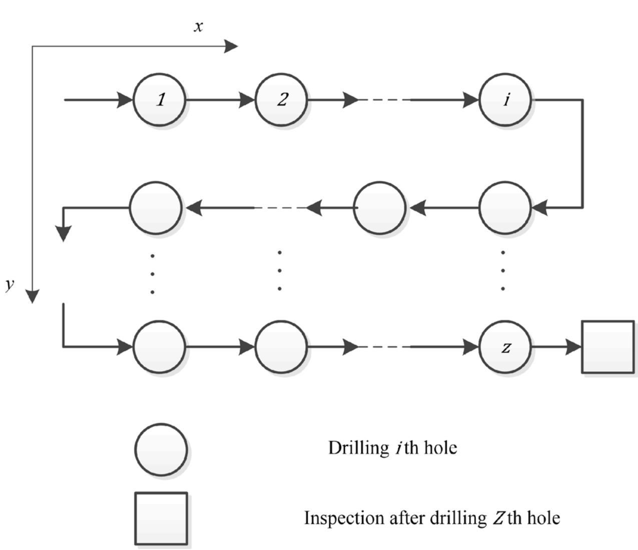

Fig. 1 The conventional laser micro-machining process of MHAP

3.2 The proposed laser micro-machining scheme

The proposed multistep laser micro-machining scheme includes the following procedures: (1) micro-hole drilling, (2) possible inspection, (3) possible rework and (4) final inspection. Initially, the first micro-hole on a workpiece is drilled by laser ablation. In the following step, inspection on the drilled micro-hole is performed if it is necessary; otherwise, the second micro-hole is drilled. Furthermore, if necessary, a rework on the drilled micro-hole is performed based on the inspection result. In the end, after all the micro-holes are drilled, a final inspection on the last micro-hole is performed. The whole procedure is summarised and shown in Fig. 2.

By implementing inspection and rework in such a process, the downstream operations are not applied to already scrapped products, which may result in saving unnecessary manufacturing cost, manufacturing time and manufacturing resources whilst improving successful rate of producing MHAP. The inspection and rework policies in multistep micro-machining processes are identified by deciding the following three parameters:

1. Whether the inspection should be performed after each micro-hole drilling step or not
2. The acceptance limits for each inspection, that is tolerance of inspection
3. Whether the rework should be applied after each inspection step or not

The criterion that an inspection should be performed is based on the optimal inspection scheme. In general, the tolerance of inspection is equal to the tolerance requirement of product dimension. According to the results of inspection, it is known that the rework should be applied or not. Moreover, in an actual manufacturing process, all the reworks of one workpiece are performed after the final inspection.

Therefore, the designer deals with the problem of finding the most efficient combination of these parameters, that is the optimal inspection and rework policies such that the number of qualified products is increased, meanwhile the unit manufacturing cost is minimised.

To better understand the proposed multistep laser micro-machining scheme, the flow chart is shown in Fig. 3.

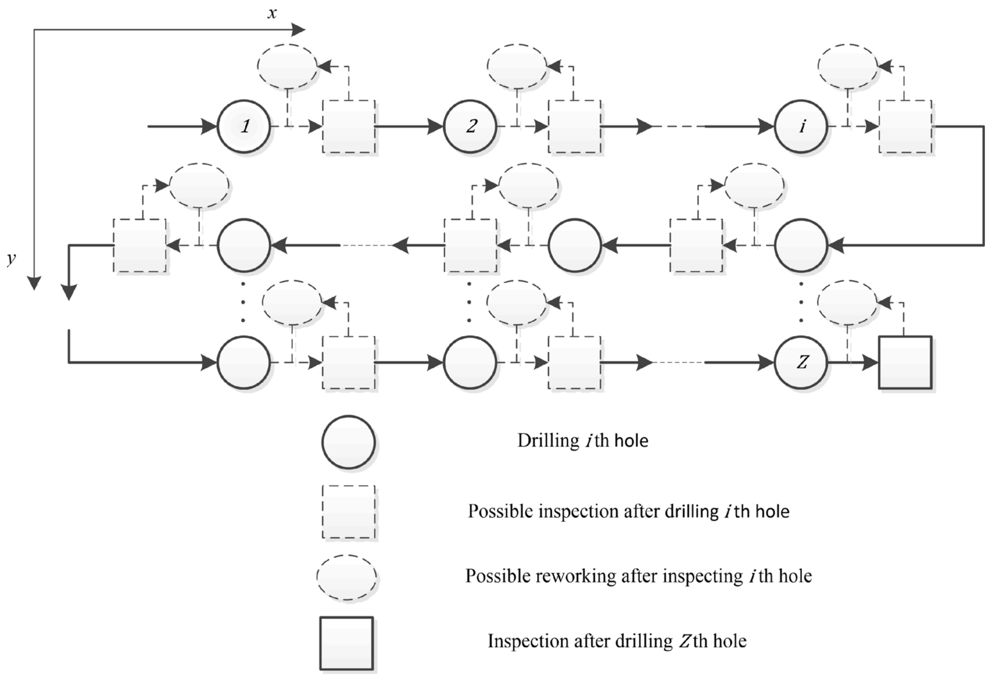

Fig. 2 The multistep laser micro-machining scheme of micro-hole array products

3.3 Formulation of the mathematical model for MHAP

3.3.1 Assumptions, parameters and notations

We assume that hole diameters and positioning errors, which are the two most important decision variables in the laser micro-machining process of MHAP, are all normal distribution.

To facilitate the analysis that follows, we use parameters and notations to summarise and formulate the mathematical model for the proposed multistep laser micro-machining scheme of MHAP as follows:

C_u	Unit manufacturing cost per hour
C_t	Total manufacturing cost
C_m	Unit manufacturing cost for a qualified MHAP
C_w	Unit procurement cost per workpiece
T_d	Drilling time per micro-hole
T_t	Inspection time per micro-hole
T_r	Rework time per micro-hole
T_t	Total manufacturing time
N_w	Number of workpieces
N_q	Number of qualified products
N_s	Number of scrapped products
N_d	Number of times of micro-hole drilling
N_t	Number of times of micro-hole inspection
N_r	Number of times of micro-hole rework
Q	Successful rate of producing MHAP
S	Size of workpiece batch
LSL	Lower limit of the micro-hole diameter specification
USL	Upper limit of the micro-hole diameter specification
Z	Number of micro-holes of a MHAP
hd	Diameter of micro-hole
hs	Spacing between two neighbouring holes
ε_x	Positioning errors along x direction
ε_y	Positioning errors along y direction

The units of measurements for dimensions, time and costs are micron (μm), hour (h) and Canadian dollar (CAD), respectively.

3.3.2 The unit manufacturing cost model

In the femtosecond laser micro-machining manufacturing process for producing a MHAP, different manufacturing strategies need different manufacturing time and hence result in different manufacturing cost. For instance, one-step manufacturing strategy needs the shortest machining and inspection time but with highest risk to get failure. For the multistep manufacturing strategy, it needs more machining and inspection time but with lower risk to get failure. Hence, to better formulate the trade-offs amongst manufacturing cost, time and successful rate, we use C_m which means unit manufacturing cost for a qualified MHAP.

The manufacturing process requires the use of N_w units of workpiece to produce N_q units of qualified products. N_s units, which are beyond the tolerance limits, are scrapped in the process. The total manufacturing time T_t and total manufacturing cost C_t are obtained by calculations. Based on the previous description for the proposed multistep laser micro-machining scheme of MHAP, we find that N_d, N_i and N_r are all closely related to N_w and Z. Hence, the

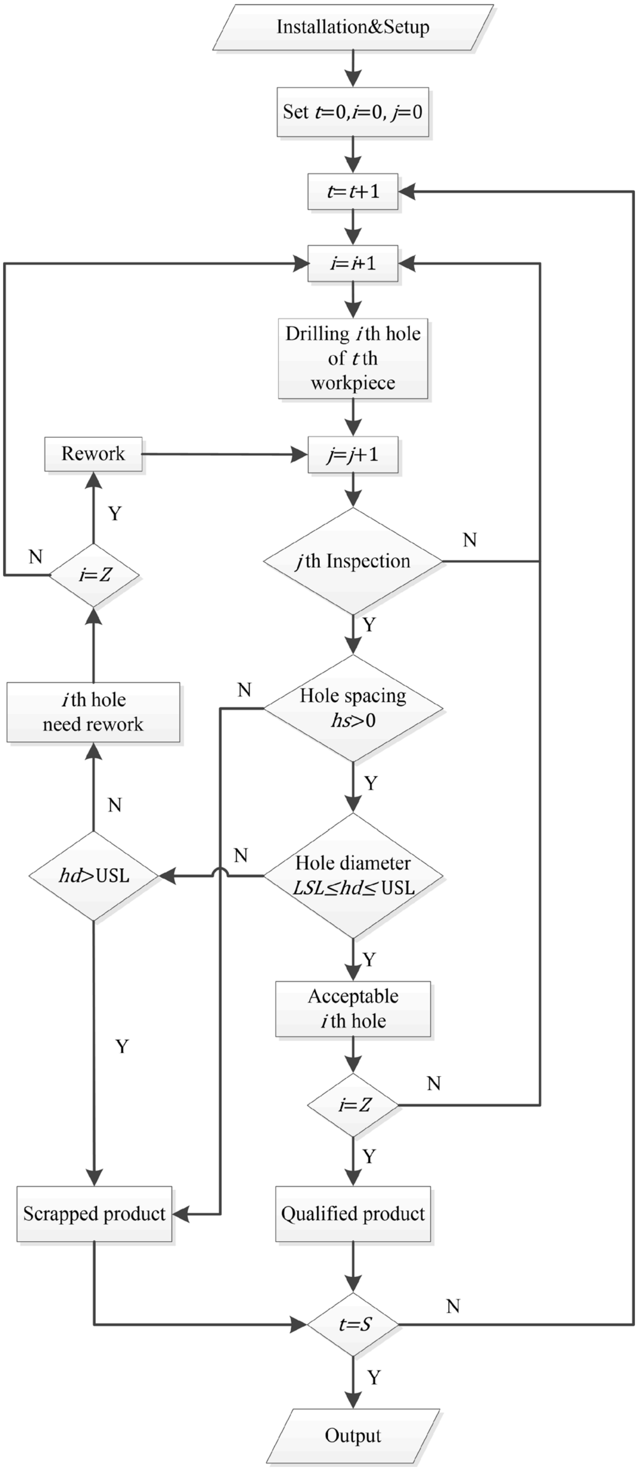

Fig. 3 The flow chart of the proposed laser micro-machining scheme for MHAP

formulation for the unit manufacturing cost model is shown as follows:

$$Q = N_q / N_w \tag{1}$$

$$N_w = N_q + N_s \tag{2}$$

$$T_t = \sum_{x=1}^{N_w} \sum_{y=1}^{Z} \left(T_d N_d(x,y) + T_i N_i(x,y) + T_r N_r(x,y) \right) \tag{3}$$

$$C_m = C_t / N_q = \left(C_w N_w + C_u T_t \right) / N_q \tag{4}$$

3.3.3 Optimisation model for unit manufacturing cost problem

We consider C_m as our objective to be minimised. The initial design requirements are considered as the constraints. The optimisation problem can be stated as follows:

$$\begin{array}{c} \text{Minimum } C_m \\ \text{subject to} \\ \text{hs} > 0 \\ \text{LSL} \leq \text{hd} \leq \text{USL} \end{array} \tag{5}$$

The expression of C_m is given by Eqs. (3) and (4).

4 Solution algorithm

Based on the proposed multistep processing scheme and optimisation model for unit manufacturing cost problem, it is clear that a laser micro-machining process with $m \times n$ matrix micro-holes offers $2^{m \times n}$ possible inspection combinations (whether to inspect or not in each micro-hole drilling process). Identifying all possible combinations and comparing their unit manufacturing cost via complete enumeration are too tedious to be feasible. Therefore, application of meta-heuristic methods will be much more efficient, because they require limited computational effort whilst yielding nearly optimal solutions. In this research project, a PSO-based algorithm has been developed to identify the optimal inspection policy.

4.1 Basic particle swarm optimisation algorithm

PSO was initially proposed by James and Russell [23] as an adaptive population-based and derivative-free optimisation method for non-linear functions with continuous variables. It was developed through simulation of social behaviours such as fish schooling and bird flocking. The PSO is recently applied in many fields because of its simple structure with few numbers of parameters, which simplifies coding of the algorithm.

The PSO provides a population-based search procedure in which the individuals, called particles, change their positions with time. Each particle is updated by means of two ‘best’ values, namely *pbest* and *gbest* in successive iterations. The *pbest* is the best solution a particle has achieved so far. The *gbest* is the best value obtained so far by any particle in the population. The quality of each particle position is then evaluated based on the objective function. To proceed from iteration t to the next iteration $t+1$, the velocity and position of a particle k are, respectively, calculated using the following equations:

$$v_{xy}^{k}(t+1) = \omega \cdot v_{xy}^{k}(t) + c_1 r_1 \left[p_{xy}^{k} - i_{xy}^{k}(t)\right] + c_2 r_2 \left[\mathrm{pg} - i_{xy}^{k}(t)\right] \quad (6)$$

$$i_{xy}^{k}(t+1) = i_{xy}^{k}(t) + v_{xy}^{k}(t+1) \quad (7)$$

where ω is the inertia weight, c_1 and c_2, called learning factors, are acceleration coefficients, t represents the iteration number and r_1 and r_2 are random numbers distributed uniformly between 0 and 1. $x(x=1,2,\ldots,m)$ and $y(y=1,2,\ldots,n)$ represent a $m \times n$ matrix of each particle.

In general, the inertia weight ω is set according to the following equation:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{\mathrm{iter}_{\max}} \cdot \mathrm{iter} \quad (8)$$

where $\mathrm{iter}_{\max}$ represents the maximum number of iterations, and iter is the current number of iterations. Moreover, $\omega_{\max}$ and $\omega_{\min}$ are the maximum and minimum weight values, respectively.

4.2 The proposed PSO-based algorithm for laser micro-machining of MHAP

It is well known that the solution space of standard particle swarm optimisation algorithm is a real space domain. However, when it comes to the problem of optimising inspection policy in a multistep manufacturing process, the combination of inspection scheme is a discrete integer space domain. Therefore, standard encoding scheme of PSO cannot be directly applied for the problem of optimising inspection policy. Herein, the important aspect in applying PSO is to find an applicable mapping between the combination of inspection policies and the position of particles in PSO. The position and velocity of particles in PSO are defined by Lv and Lu [24].

Position of particle *k* The position vector of each particle k is coded with a combination of inspection policies.

$$I_{xy}^{k} = \begin{bmatrix} i_{11}^{k} & i_{12}^{k} & & i_{1y}^{k} & & i_{1n}^{k} \\ i_{21}^{k} & i_{22}^{k} & \cdots & i_{2y}^{k} & & i_{2n}^{k} \\ \vdots & & \ddots & & \vdots & \\ i_{x1}^{k} & i_{x2}^{k} & & i_{xy}^{k} & & i_{xn}^{k} \\ i_{m1}^{k} & i_{m2}^{k} & \cdots & i_{my}^{k} & & i_{mn}^{k} \end{bmatrix}, i_{xy}^{k} \in \{1,2\}, x \in \{1,2,\cdots,m\}, y \in \{1,2,\cdots,n\}$$

where 1 and 2 of {1,2} denote inspection and no inspection, respectively, and m and n denote the size of particle k. For instance, the vector $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ shows a particle of inspection policy combinations for a 3×3 MHAP manufacturing process. In this vector, after machining first, second, fifth and ninth micro-hole which is drilled along the machining direction and path in Fig. 1, an inspection is selected and performed as the optimal policy, respectively. However, there is not inspection after any other micro-hole drilling.

Velocity of particle *k* The velocity vector of a combination of inspection policies k or particle k can be defined as a transformation to the position of combination of inspection policies. The velocity vector of particle k is represented as follows:

$$V_{xy}^{k} = \begin{bmatrix} v_{11}^{k} & v_{12}^{k} & & v_{1y}^{k} & & v_{1n}^{k} \\ v_{21}^{k} & v_{22}^{k} & \cdots & v_{2y}^{k} & & v_{2n}^{k} \\ \vdots & & \ddots & & \vdots & \\ v_{x1}^{k} & v_{x2}^{k} & & v_{xy}^{k} & & v_{xn}^{k} \\ v_{m1}^{k} & v_{m2}^{k} & \cdots & v_{my}^{k} & & v_{mn}^{k} \end{bmatrix}, v_{xy}^{k} \in \{1,2\}, x \in \{1,2,\cdots,m\}, y \in \{1,2,\cdots,n\}$$

where 1 and 2 of {1,2} denote the value of each velocity vector element, and m and n denote the size of particle k.

In this research, a PSO-based algorithm for laser micro-machining of MHAP has been developed to find the optimal inspection matrix scheme to minimise the unit manufacturing cost. The basic framework of the proposed algorithm is shown in Fig. 4, and the detailed design layout of this algorithm is given in the following sections.

4.2.1 Input module

The required input data are as follows:

1. Inspection matrix size: $m \times n$
2. Swamp size: s
3. Holes' dimension distribution (μm): $hd \in N(\mu, \sigma^2)$
4. Positioning errors' distribution (μm): $\varepsilon_x \in N(\mu_x, \sigma^2), \varepsilon_y \in N(\mu_y, {\sigma_y}^2)$

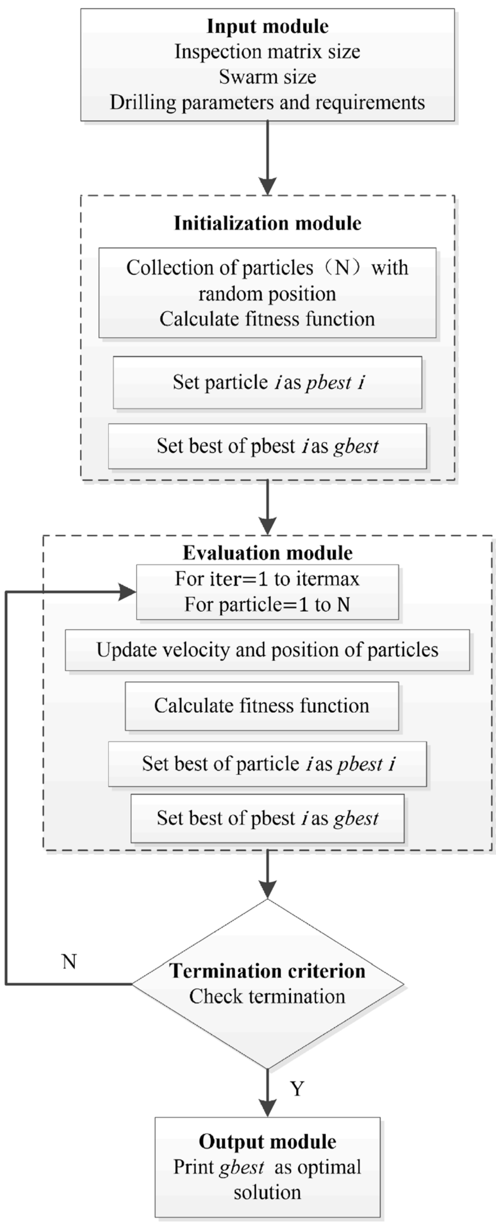

Fig. 4 The framework of the proposed PSO-based algorithm for laser micro-machining of MHAP

5. Holes' dimension tolerance requirement (μm): $hd \in [LSL\ \ USL]$
6. Unit procurement cost per workpiece (CAD): C_w
7. Drilling time per micro-hole (h): T_d
8. Inspection time per micro-hole (h): T_i
9. Rework time per micro-hole (h): T_r
10. Unit manufacturing cost per hour (CAD/h): C_u
11. Size of workpiece batch: S

To better illustrate the whole process of the algorithm, we can use a numerical example based on the following input data:

1. $m = 5,\ n = 10$
2. $s = 5$
3. $hd \in N\left(35, 2^2\right)$
4. $\varepsilon_x \in N\left(0, 0.5^2\right),\ \ \varepsilon_y \in N\left(0, 0.5^2\right)$
5. $\mathrm{LSL} = 30, \quad \mathrm{USL} = 40$
6. $C_w = 0.5$
7. $T_d = 0.01$
8. $T_i = 0.2$
9. $T_r = 0.01$
10. $C_u = 100$
11. $S = 100$

4.2.2 Initialization module

Five matrixes of inspection scheme are generated randomly in the initialisation module. Considering that there is a final inspection after drilling the last micro-hole, the last element must be set as 1 in each particle. Matrixes [1]–[5], which have five rows and ten columns, show the initial population consists of five particles.

$$\begin{bmatrix} 2 & 2 & 1 & 1 & 1 & 2 & 2 & 1 & 2 & 2 \\ 2 & 2 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 \end{bmatrix} \qquad ([1])$$

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 1 & 2 & 1 & 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 2 \\ 1 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 1 \end{bmatrix} \qquad ([2])$$

$$\begin{bmatrix} 2 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 1 & 1 & 1 & 2 & 2 & 2 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 \end{bmatrix} \quad ([3])$$

$$\begin{bmatrix} 2 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 2 \\ 2 & 2 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 1 & 2 & 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 1 & 2 & 1 & 1 & 2 & 2 & 1 \end{bmatrix} \quad ([4])$$

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 2 & 1 \\ 1 & 1 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 2 & 2 & 2 & 1 & 1 & 1 \\ 2 & 2 & 2 & 1 & 2 & 2 & 1 & 1 & 2 & 1 \end{bmatrix} \quad ([5])$$

In the initial calculation of the unit manufacturing cost, the fitness function value of particle k, $C_m(k)$, is performed by using Eqs. (3) and (4). It is shown in Table 1.

Matrix [k] is called I_{xy}^k and set as *pbest* (P_k). From Table 1, the initial minimum unit manufacturing cost is CAD905.41 corresponding to the fourth particle. Hence, the fourth particle is selected and set as *gbest* (P_g).

Table 1 Unit manufacturing cost for all particles in initial population

Particle no.	Unit manufacturing cost C_m(CAD)
1	1,011.72
2	954.43
3	1,258.62
4	905.41
5	990.64

4.2.3 Evaluation module

The main iterative process begins with this module. The particle will not move at the beginning. So, for the first iteration, the initial velocity $V_{xy}^k(t=1)=0$ and $P_k=I_{xy}^k$. r_1 and r_2 are $m\times m$ matrixes created by random integers between 1 and 2.

Inertia weight ($\omega_{\max}=0.9$, $\omega_{\min}=0.4$ and $\text{iter}_{\max}=100$), ω is updated as per Eq. (8). c_1 and c_2 are learning factors, set as 0.5. Since the value of each element in I_{xy}^k is 1 or 2, in order to satisfy the constrain that each particle cannot fly away from its range [1,2], the maximum position change $V_{\max}$ during one iteration is taken as 2.

For particle k, the above values are substituted in Eq. (6):

$$v_{xy}^k(t+1) = \omega \cdot v_{xy}^k(t) + c_1 r_1 \left[p_{xy}^k - i_{xy}^k(t)\right] + c_2 r_2 \left[\text{pg} - i_{xy}^k(t)\right] = c_2 r_2 \left[\text{pg} - i_{xy}^k(t)\right]$$

Firstly, to find $[\text{pg}-i_{xy}^k(t)]$, considering the absolute values, matrix [k] is subtracted from matrix [4]. For example, $k=1$:

$$\begin{bmatrix} 2 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 1 & 2 \\ 2 & 2 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 1 & 2 & 1 & 2 & 2 & 2 \\ 2 & 2 & 2 & 1 & 2 & 1 & 1 & 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 1 & 1 & 1 & 2 & 2 & 1 & 2 & 2 \\ 2 & 2 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \quad ([6])$$

For r_2, generating random integers from 1 to 2 develops a 5×5 matrix. The random matrix r_2 for the first iteration is shown in matrix [7]:

$$\begin{bmatrix} 1 & 2 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 2 & 1 \end{bmatrix} \quad ([7])$$

Matrix [7] and matrix [6] are multiplied:

$$\begin{bmatrix} 1 & 2 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 1 \\ 1 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 8 & 8 & 2 & 6 & 4 & 5 & 3 & 5 & 4 \\ 2 & 7 & 7 & 3 & 6 & 4 & 4 & 2 & 4 & 5 \\ 3 & 7 & 7 & 2 & 5 & 4 & 4 & 3 & 4 & 3 \\ 3 & 7 & 7 & 4 & 6 & 5 & 4 & 3 & 4 & 5 \\ 3 & 8 & 8 & 4 & 7 & 5 & 5 & 3 & 5 & 6 \end{bmatrix} \quad ([8])$$

The resultant matrix is multiplied by $c_2(c_2=0.5)$. Modulus of 2 is taken for each element value of this resultant matrix, so that all the entries in the matrix will be less than 2. After rounding the elements of the matrix to the nearest integers, $v_{xy}^k(t+1)$ for the first iteration is shown in matrix [9]. $i_{xy}^k(t+1)$ is obtained by adding $i_{xy}^k(t)$ and $v_{xy}^k(t+1)$ (see Eq. (7)). Modulus of 2 is taken for each element value of this resultant matrix. Furthermore, the last element must be set as 1. It is shown in matrix [10].

The fitness function value for matrix [10] is calculated. Similarly, all five particles are updated and calculated. Based on the above results, *pbest* (P_k) and *gbest* (P_g) are updated for the first iteration.

From the second iteration onwards, $i_{xy}^k(t+1)$ becomes $i_{xy}^k(t)$, and $v_{xy}^k(t+1)$ is displaced by $v_{xy}^k(t+1)$. The whole procedure is repeated until the termination criterion is met.

$$\begin{bmatrix} 2 & 2 & 2 & 1 & 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 1 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 1 \\ 2 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 1 & 2 & 1 & 1 & 2 & 2 & 2 & 1 \\ 2 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 1 \end{bmatrix} \tag{[9]}$$

$$\begin{bmatrix} 2 & 2 & 1 & 2 & 2 & 2 & 2 & 1 & 2 & 2 \\ 2 & 2 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 1 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 & 2 & 2 & 1 & 2 & 1 \end{bmatrix} \tag{[10]}$$

4.2.4 Termination criterion

The PSO is terminated after a predefined number of iterations. After many trials, the total number of iteration is taken as 100.

4.2.5 Output module

The optimal matrix of inspection scheme is given as the output. The minimum unit manufacturing cost is CAD702.44, and the optimal matrix is shown in matrix [11].

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \\ 2 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 1 \end{bmatrix} \tag{[11]}$$

5 Experiments and discussion

5.1 Numerical experiments

Based on the developed model, the proposed PSO algorithm for laser micro-machining of MHAP to find the optimal inspection policy is coded using MATLAB R2011a, and the computer simulation experiments are conducted on a PC with a 1.8-GHz processor and with 4 GB of RAM. To illustrate the advantage of the proposed methodology, the experiments are designed and performed in three cases as follows:

- Traditional one-step laser micro-machining scheme with final inspection
- Extreme multistep laser micro-machining scheme with full inspections
- Proposed multistep laser micro-machining scheme with optimal inspections

As mentioned in the previous sections, the experiment parameters for the above three cases are all the same. The obtained inspection matrixes for the three cases are shown in matrixes [12]–[14], respectively. The unit manufacturing cost and successful rate obtained are compared and tabulated in Table 2. The evolution performance of the proposed PSO algorithm with swarm size 5 is shown in Fig. 5.

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \end{bmatrix} \tag{[12]}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{[13]}$$

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \\ 2 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 \\ 1 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 1 \end{bmatrix} \tag{[14]}$$

Considering the randomization and variability of hole diameters and position errors generated in each iteration, the obtained optimal inspection matrix and its corresponding unit manufacturing cost are not unique. However, as far as unit manufacturing cost is concerned under any generated data, we find that the proposed multistep laser micro-machining scheme with optimal inspections outperforms the traditional

Table 2 The unit manufacturing cost and successful rate obtained for three cases

Case no.	Successful rate %	Unit manufacturing cost C_m(CAD)
1	4.85	1,554.54
2	53.73	1,504.03
3	29.14	702.44

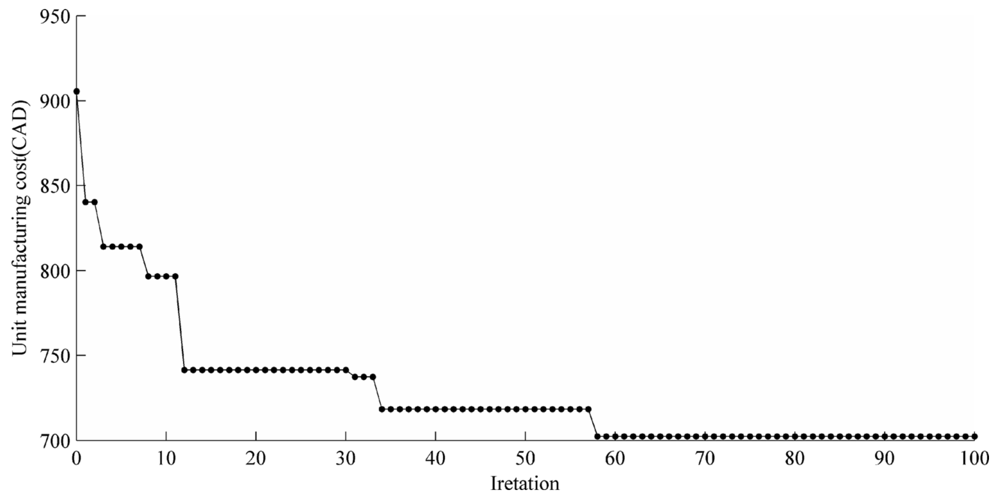

Fig. 5 Evolution performance of the proposed PSO algorithm with swarm size 5

one-step laser micro-machining scheme with final inspection and the extreme multistep laser micro-machining scheme with full inspections with more than 50 % higher, respectively. Moreover, according to the results in Table 2, it is also known that the proposed multistep laser micro-machining scheme with the optimal inspections has a higher successful rate than the traditional one-step laser micro-machining scheme with final inspection. Furthermore, the algorithm is found to be efficient for varying micro-hole array sizes.

5.2 Verification experiments

In terms of the simulation results, we choose a machining strategy which is the proposed multistep laser micro-machining scheme with optimal inspections. By this machining strategy, we made a cover for an atmosphere satellite sensor which is designed to measure the particles in the atmosphere. The cover consists of 25 matrix arrays, and each matrix array has $5 \times 10 = 50$ micro-holes. Figure 6 shows the microscopic view of one of these matrix arrays. Every hole must have a diameter of 10 ± 2 μm. By the simulation results, we successfully made this cover by one-machining process without any defects. Without the technology and simulation method as reported in this paper, this type of micro-scale part is used to being made in laboratory through many try-error processes until a satisfactory one is obtained. Consequently, it results in a time-consuming and high cost manufacturing process.

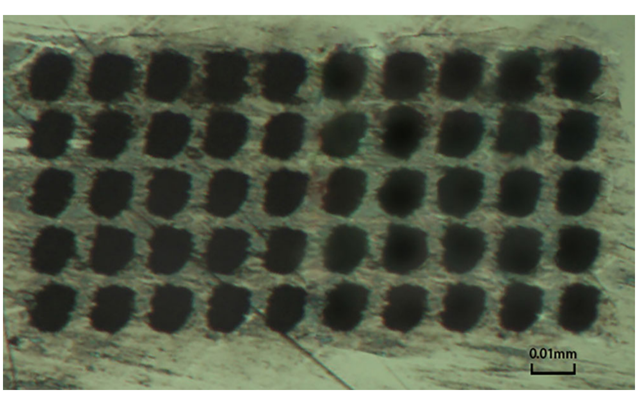

Fig. 6 Microscopic view of 5×10 micro-holes of the cover

6 Conclusions

In the research as reported in this paper, an attempt has been made to study the problem of optimising inspection policies in a femtosecond laser micro-machining process for micro-hole array products where the quality characteristic of the final product is dependent on all micro-holes. With the current capabilities through advanced laser micro-machining system and dimension measuring devices, the multistep laser micro-machining scheme with the optimal inspections is suggested in place of traditional one-step laser micro-machining scheme with final inspection. A PSO-based algorithm has been developed to minimise the unit manufacturing cost whilst enhancing the successful rate for manufacturing MHAP by reducing unnecessary micro-machining progresses.

Based on the results of the numerical experiments which are compared with the traditional one-step laser micro-machining scheme with final inspection, we find that the proposed multistep laser micro-machining scheme with the optimal inspections is able to increase the percentage of acceptable products from 4.85 to 29.14 % and reduce the unit manufacturing cost from CAD1,554.54 to CAD702.44. Through using this optimisation and simulation method, a complicated micro-machining process as described in Section 5.2 can be made in a much more efficient way than it does usually in try-errors. It also finds that the proposed methodology is effective irrespective of the size of micro-hole array products produced.

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