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ORIGINAL ARTICLE

The skipping strategy to reduce the effect of the autocorrelation on the T^2 chart's performance

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Abstract In this article, we consider the T^2 control chart for bivariate samples of size n with observations that are not only cross-correlated but also autocorrelated. The cross-covariance matrix of the sample mean vectors were derived with the assumption that the observations are described by a first-order vector autoregressive model—VAR (1). To counteract the undesired effect of autocorrelation, we build up the samples taking one item from the production line and skipping one, two, or more before selecting the next one. The skipping strategy always improves the chart's performance, except when only one variable is affected by the assignable cause, and the observations of this variable are not autocorrelated. If only one item is skipped, the average run length (ARL) reduces in more than 30 %, on average. If two items are skipped, this number increases to 40 %.

Keywords Autocorrelation · Skipping strategy · Hotelling T^2 chart · VAR (1) model

1 Introduction

In many modern processes, the multivariate observations gathered in rational subgroups of size n are not only cross-correlated but also autocorrelated. Pan and Jarret [1] describe many of these processes. A typical approach to study the performance of control charts for autocorrelated processes is to fit a time-series model. The first-order autoregressive model, AR (1), and the first-order vector autoregressive model, VAR (1), have been adopted in many studies dealing with univariate and multivariate control charts [2–14].

The autocorrelation among sample items has a serious impact on the performance of the control charts. To counteract the undesired effect of autocorrelation on the performance of the $\overline{X}$ chart, Costa and Castagliola [15] proposed a sampling strategy denoted as "s-skip." According to this strategy, samples are obtained by collecting one item from the production line and then skipping s consecutive items before selecting the next one. More recently, Franco et al. [16] investigated the skipping strategy on the economic-statistical design of the $\overline{X}$ chart used to control AR (1) processes. The practical appeal of skipping observations lies in the fact that the chart's performance enhances substantially when a few observations are skipped. Based on that, we also consider the skipping strategy to improve the performance of the T^2 chart. The deduction of the cross-covariance matrix of the sample mean vector $\left(\Gamma_{\overline{X}}\right)$ under the skipping strategy is not simple but necessary to obtain λ, the square root of the non-centrality parameter of the chi-square distribution. As the power of the T^2 chart is a function of λ, the knowledge of the cross-covariance matrix greatly facilitates the investigation of the cross-correlation and autocorrelation effects on the performance of the T^2 chart.

The combined effect of autocorrelation and cross-correlation on the performance of the T^2 chart under the skipping strategy is worthy of investigation. Some motivational examples dealing with multivariate autocorrelated processes are in Section 2. In Section 3 (and Appendix), we obtained the cross-covariance matrix of the sample mean vector $\left(\Gamma_{\overline{X}}\right)$ considering the skipping strategy and bivariate observations given by a VAR (1). In Section 4, we investigate the effects of

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autocorrelation and cross-correlation on the performance of the bivariate T^2 chart under the skipping strategy. Section 5 presents a guide to choosing the skipping parameter. In Section 6, an illustrative example is provided to show how the skipping strategy helps to reduce the negative effect of the autocorrelation. Finally, Section 7 presents the main conclusions and future research.

2 Illustrations of processes which are autocorrelated

The assumption of independent observations is not even approximately satisfied in some manufacturing process; that is, the successive observations in these processes are dependent. According to Montgomery [17], in many modern manufacturing processes, the data of the quality characteristics are autocorrelated, especially when the samples are collected according to the rational subgroup concept. Autocorrelation is usually caused by the high frequency of sampling due to the automated measurement and inspection procedures, where every quality characteristic is measured on every unit in time order of production [18].

Illustrations of processes, which are both multivariate and autocorrelated, are numerous in the production of industrial gases, silicon chips, and highly technical computer-driven products and accessories [1, 19]. For example, Jarret and Pan [19] describe the fiber optic production, where the preparation of the silicon dioxide rod, made from the condensation of silicon and oxygen gases, requires the control of some autocorrelated quality characteristics, like temperature and pressure.

In service industries, the processes are autocorrelated due to the inertia of human behaviors and cross-correlated because of the interactions among various human actions and activities. Pan and Jarret [1] give an example where the number of visits to a restaurant at a tourist attraction may be serially dependent and related to the room occupation percentage of nearby overnight residences and the cost and convenience of transportation. Furthermore, the latter factors are also autocorrelated and cross-correlated to each other. In another example, Pan and Jarret [1] describe some multivariate and autocorrelated business management problems.

Recently, Huang and Bisgaard [13] provided a motivational example dealing with a set of autocorrelated multivariate data from a ceramic furnace, where the data are observations of temperatures measured in different locations. They also provided a detailed discussion of a two-dimensional VAR (1) model, the one considered in this article.

In summary, a growing number of processes, the maintenance of the production quality requires the control of several quality characteristics. Dealing with more than one quality characteristic increases the chances of finding autocorrelated variables; consequently, the design and use of multivariate charts require the knowledge of not only the cross-correlations but also the autocorrelations among observations, and, more than that, strategies to counteract their negative effect on the chart's performance.

3 The autoregressive model and the cross-covariance matrix

The autocorrelation measures the dependence between observations of the same quality characteristic, while the cross-correlation measures the dependence between variables. The vector autoregressive model for cross and serially correlated data has been adopted in recent studies dealing with multivariate control charts [10, 11, 13]. It follows:

$$X_t - \mu = \Phi(X_{t-1} - \mu) + \varepsilon_t \qquad (1)$$

where $X_t \sim N_p(\mu, \Gamma)$ is the $(p\times 1)$ vector of observations at time t (p is the number of variables), μ is the mean vector, ε_t is an independent multivariate normal random vector with a mean vector of zeros and covariance matrix Σ_e, and Φ is a $(p\times p)$ matrix of autocorrelation parameters. According to Kalgonda and Kulkarni [8], the cross-covariance matrix of X_t has the following property: $\Gamma = \Phi\Gamma\Phi' + \Sigma_e$. After some algebra, we obtain

$$\text{Vec}\,\Gamma = \left(I_{p^2} - \Phi\otimes\Phi\right)^{-1}\text{Vec}\,\Sigma_e \qquad (2)$$

where $\otimes$ is the Kronecker product and Vec is the operator that transform a matrix into a vector by stacking its columns.

To study the effects of the auto- and cross-correlation on the performance of the T^2 chart, we consider a bivariate stationary process ($p=2$) with

$$\Phi = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \qquad (3)$$

$$\Sigma_e = \begin{pmatrix} \sigma^2_{e_X} & \sigma_{e_{XY}} \\ \sigma_{e_{XY}} & \sigma^2_{e_Y} \end{pmatrix} = \begin{pmatrix} \sigma^2_{e_X} & \rho\sigma_{e_X}\sigma_{e_Y} \\ \rho\sigma_{e_X}\sigma_{e_Y} & \sigma^2_{e_Y} \end{pmatrix} \qquad (4)$$

where ρ is the correlation of X and Y.

From (2), (3) and (4) follows:

$$\Gamma = \begin{pmatrix} \sigma^2_X & \sigma_{XY} \\ \sigma_{XY} & \sigma^2_Y \end{pmatrix} = \begin{pmatrix} (1-a^2)^{-1}\sigma^2_{e_X} & (1-ab)^{-1}\sigma_{e_{XY}} \\ (1-ab)^{-1}\sigma_{e_{XY}} & (1-b^2)^{-1}\sigma^2_{e_Y} \end{pmatrix} \qquad (5)$$

Leoni at al. [20] obtained the cross-covariance matrix $\Gamma_{\overline{X}}$ of the sample mean vector $\overline{X}$ when the sample items are collected according to the rational subgroup concept:

$$\Gamma_{\overline{X}} = \begin{pmatrix} \frac{\sigma_X^2}{n}\left[1+\frac{2}{n}\sum_{j=1}^{n-1}(n-j)\,a^j\right] & \frac{\sigma_{XY}}{n}\left[1+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,a^j+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,b^j\right] \\ \frac{\sigma_{XY}}{n}\left[1+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,a^j+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,b^j\right] & \frac{\sigma_Y^2}{n}\left[1+\frac{2}{n}\sum_{j=1}^{n-1}(n-j)\,b^j\right] \end{pmatrix} \quad (6)$$

where n is the size of the samples.

It is well known that the autocorrelation has a negative impact on the performance of the $\overline{X}$ chart; Leoni et al. [20] proved that the autocorrelations also reduce the ability of the T^2 chart to signal.

The cross-covariance matrix of the sample mean vector for samples of size n, built-up according to the s-skip strategy, is developed in Appendix:

$$\Gamma_{\overline{X}} = \begin{pmatrix} \frac{\sigma_X^2}{n}\left[1+\frac{2}{n}\sum_{j=1}^{n-1}(n-j)\,a^{(s+1)j}\right] & \frac{\sigma_{XY}}{n}\left[1+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,a^{(s+1)j}+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,b^{(s+1)j}\right] \\ \frac{\sigma_{XY}}{n}\left[1+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,a^{(s+1)j}+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,b^{(s+1)j}\right] & \frac{\sigma_Y^2}{n}\left[1+\frac{2}{n}\sum_{j=1}^{n-1}(n-j)\,b^{(s+1)j}\right] \end{pmatrix} \quad (7)$$

In the absence of autocorrelation (a=b=0), $\Gamma_{\overline{X}}$ reduces to $\Psi_{\overline{X}} = n^{-1}\Sigma_e$. As the non-centrality parameter of the T^2 distribution is function of the inverse of the cross-covariance matrix, the behavior of $\Gamma_{\overline{X}}^{-1}$ is important to study the effect of the auto- and cross-correlation and the skipping strategy on the performance of the T^2 chart. The elements θ_{ij} of $\Gamma_{\overline{X}}^{-1}$ are complex:

$$\Gamma_{\overline{X}}^{-1} = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{12} & \theta_{22} \end{pmatrix} \quad (8)$$

Table 1 brings the values of θ_{11}, θ_{22}, and θ_{12} for $\rho\in(0,0.7)$, n=4, and several combinations of a and b. The elements of the main diagonal of $\Gamma_{\overline{X}}^{-1}$ increase with ρ, the cross-correlation, and with s, the number of skipped items, but decrease with the autocorrelations, (a, b). When the variables are independent, the elements of the secondary diagonal of $\Gamma_{\overline{X}}^{-1}$ are zero, that is, θ_{12}=0. In the presence of the cross-correlation, they become negative.

4 The effect of the auto- and cross-correlation on the performance of the T^2 control chart

The Hotelling T^2 control chart is the most referenced control scheme for detecting mean shifts in multivariate

Table 1 The inverse cross-covariance matrix values; a and $b\in$(0, 0.2, 0.5, 0.7); $\rho\in$(0,0.7).

ρ		0							0.7						
s	a	0.0	0.0	0.2	0.0	0.5	0.0	0.7	0.0	0.0	0.2	0.0	0.5	0.0	0.7
	b	0.0	0.2	0.2	0.5	0.5	0.7	0.7	0.0	0.2	0.2	0.5	0.5	0.7	0.7
0	θ_{11}	4.00	4.00	2.86	4.00	1.45	4.00	0.75	7.84	7.70	5.60	6.87	2.85	5.86	1.48
	θ_{22}	4.00	2.86	2.86	1.45	1.45	0.75	0.75	7.84	5.50	5.50	2.50	2.85	1.10	1.48
	θ_{12}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−5.49	−4.51	−3.92	−2.68	−2.00	−1.43	−1.03
1	θ_{11}	4.00	4.00	3.62	4.00	2.08	4.00	1.00	7.84	7.56	7.09	6.45	4.07	5.58	1.97
	θ_{22}	4.00	3.62	3.62	2.08	2.08	1.00	1.00	7.84	6.83	7.09	3.35	4.07	1.40	1.97
	θ_{12}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−5.49	−4.93	−4.96	−2.87	−2.85	−1.48	−1.38
2	θ_{11}	4.00	4.00	3.79	4.00	2.49	4.00	1.23	7.84	7.55	7.44	6.36	4.88	5.45	2.42
	θ_{22}	4.00	3.79	3.79	2.49	2.49	1.23	1.23	7.84	7.16	7.44	3.96	4.88	1.68	2.42
	θ_{12}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−5.49	−5.05	−5.21	−3.05	−3.42	−1.56	−1.69

processes [21]. When the in-control mean vector $\mu_0=(\mu_{0X},\mu_{0Y})'$ and the cross-covariance matrix of the sample mean vector $\Gamma_{\overline{X}}$ are known, the monitoring statistic for Hotelling's T^2 control chart is given by

$$T^2 = \left(\overline{X}-\mu_0\right)' \Gamma_{\overline{X}}^{-1} \left(\overline{X}-\mu_0\right) \quad (9)$$

The terms of the cross-covariance matrix $\Gamma_{\overline{X}}$ of the sample mean vector $\overline{X}$ depend on the elements of the

Table 2 The ARL values for the T^2 control chart, $\delta_1=\delta_2$.

(*a*, *b*)		(0, 0)	(0, 0.2)			(0, 0.5)			(0, 0.7)		
s			0	1	2	0	1	2	0	1	2
δ_1	δ_2	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.0	0.0	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
ρ=0.0											
0.50	0.50	27.73	34.43	29.74	28.78	46.56	40.51	37.09	55.16	51.84	49.02
0.75	0.75	7.74	9.99	8.40	8.09	14.47	12.17	10.92	17.96	16.58	15.44
1.00	1.00	3.06	3.89	3.30	3.18	5.63	4.72	4.24	7.07	6.50	6.03
1.25	1.25	1.68	2.02	1.78	1.73	2.78	2.38	2.18	3.43	3.17	2.96
1.50	1.50	1.21	1.36	1.25	1.23	1.71	1.53	1.43	2.03	1.90	1.80
ρ=0.3											
0.50	0.50	39.83	48.07	42.20	40.96	59.46	53.24	49.45	64.48	61.80	59.30
0.75	0.75	11.92	15.06	12.79	12.33	19.80	17.16	15.61	22.03	20.84	19.74
1.00	1.00	4.62	5.87	4.97	4.79	7.86	6.74	6.10	8.83	8.30	7.83
1.25	1.25	2.34	2.89	2.49	2.41	3.79	3.28	2.99	4.24	3.99	3.77
1.50	1.50	1.51	1.77	1.58	1.54	2.21	1.95	1.81	2.43	2.31	2.20
ρ=0.7											
0.50	0.50	55.82	64.12	58.39	56.95	67.08	66.04	63.61	65.51	67.22	67.16
0.75	0.75	18.24	21.87	19.34	18.72	23.23	22.75	21.64	22.51	23.29	23.26
1.00	1.00	7.19	8.76	7.66	7.40	9.36	9.14	8.66	9.04	9.39	9.37
1.25	1.25	3.48	4.20	3.70	3.57	4.49	4.38	4.16	4.34	4.50	4.49
1.50	1.50	2.06	2.42	2.16	2.10	2.56	2.51	2.39	2.48	2.56	2.56
(*a*, *b*)		(0, 0)	(0.2, 0.2)			(0.5, 0.5)			(0.7, 0.7)		
s			0	1	2	0	1	2	0	1	2
δ_1	δ_2	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.0	0.0	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
ρ=0.0											
0.50	0.50	27.73	43.87	31.98	29.90	93.65	64.57	52.09	159.45	129.50	108.90
0.75	0.75	7.74	13.43	9.15	8.45	36.68	22.08	16.69	80.52	58.64	45.48
1.00	1.00	3.06	5.22	3.57	3.31	15.74	8.85	6.54	41.01	27.61	20.29
1.25	1.25	1.68	2.60	1.89	1.79	7.62	4.25	3.19	21.85	13.98	9.98
1.50	1.50	1.21	1.63	1.30	1.26	4.19	2.44	1.91	12.35	7.71	5.46
ρ=0.3											
0.50	0.50	39.83	60.48	45.39	42.68	118.51	85.46	70.57	187.11	156.80	135.09
0.75	0.75	11.92	20.25	14.01	12.98	51.43	32.28	24.86	103.84	78.44	62.47
1.00	1.00	4.62	8.05	5.45	5.04	23.53	13.57	10.09	56.91	39.68	29.85
1.25	1.25	2.34	3.88	2.70	2.52	11.72	6.53	4.83	31.98	21.04	15.25
1.50	1.50	1.51	2.25	1.68	1.59	6.43	3.61	2.73	18.70	11.86	8.43
ρ=0.7											
0.50	0.50	55.82	81.12	62.78	59.41	145.92	109.97	92.97	214.55	185.10	163.19
0.75	0.75	18.24	30.04	21.27	19.78	70.22	46.13	36.31	130.24	102.03	83.48
1.00	1.00	7.19	12.50	8.49	7.85	34.52	20.64	15.55	76.85	55.62	42.94
1.25	1.25	3.48	6.00	4.08	3.78	17.96	10.17	7.52	45.73	31.13	23.04
1.50	1.50	2.06	3.33	2.35	2.20	10.02	5.57	4.13	27.90	18.16	13.08

Table 3 The ARL values for the T^2 control chart, δ_1 or δ_2=0

(*a*, *b*)		(0, 0)	(0, 0.2)			(0, 0.5)			(0, 0.7)		
s			0	1	2	0	1	2	0	1	2
δ_1	δ_2	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.0	0.0	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
ρ=0.0											
0.00	0.50	67.32	95.27	75.12	71.36	163.04	126.03	108.04	230.33	201.94	180.35
0.50	0.00	67.32	67.32	67.32	67.32	67.32	67.32	67.32	67.32	67.32	67.32
0.00	1.00	9.41	16.19	11.09	10.26	42.87	26.28	20.02	90.63	67.10	52.66
1.00	0.00	9.41	9.41	9.41	9.41	9.41	9.41	9.41	9.41	9.41	9.41
0.00	1.50	2.57	4.31	2.98	2.77	13.05	7.28	5.39	35.03	23.24	16.92
1.50	0.00	2.57	2.57	2.57	2.57	2.57	2.57	2.57	2.57	2.57	2.57
ρ=0.3											
0.00	0.50	60.48	87.08	68.09	64.54	154.61	118.79	101.38	224.59	196.46	175.07
0.50	0.00	60.48	60.61	60.75	60.76	61.50	62.03	62.16	62.91	63.40	63.63
0.00	1.00	8.05	13.99	9.57	8.84	38.60	23.63	17.96	85.39	63.18	49.52
1.00	0.00	8.05	8.08	8.10	8.10	8.25	8.35	8.37	8.52	8.62	8.66
0.00	1.50	2.25	3.72	2.61	2.44	11.47	6.46	4.80	32.25	21.45	15.64
1.50	0.00	2.25	2.26	2.26	2.26	2.30	2.32	2.33	2.36	2.38	2.39
ρ=0.7											
0.00	0.50	28.53	46.04	34.59	32.41	107.78	81.44	68.10	192.22	167.20	147.73
0.50	0.00	28.53	29.27	30.07	30.10	34.34	37.40	38.16	42.47	45.26	46.61
0.00	1.00	3.15	5.55	3.91	3.63	19.93	12.57	9.57	60.27	45.09	35.34
1.00	0.00	3.15	3.24	3.34	3.34	3.87	4.28	4.39	5.01	5.43	5.64
0.00	1.50	1.23	1.70	1.37	1.31	5.36	3.35	2.61	20.16	13.90	10.30
1.50	0.00	1.23	1.24	1.26	1.26	1.36	1.44	1.46	1.58	1.67	1.72
(*a*, *b*)		(0, 0)	(0.2, 0.2)			(0.5, 0.5)			(0.7, 0.7)		
s			0	1	2	0	1	2	0	1	2
δ_1	δ_2	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.0	0.0	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
ρ=0.0											
0.00	0.50	67.32	95.27	75.12	71.36	163.04	126.03	108.04	230.33	201.94	180.35
0.50	0.00	67.32	95.27	75.12	71.36	163.04	126.03	108.04	230.33	201.94	180.35
0.00	1.00	9.41	16.19	11.09	10.26	42.87	26.28	20.02	90.63	67.10	52.66
1.00	0.00	9.41	16.19	11.09	10.26	42.87	26.28	20.02	90.63	67.10	52.66
0.00	1.50	2.57	4.31	2.98	2.77	13.05	7.28	5.39	35.03	23.24	16.92
1.50	0.00	2.57	4.31	2.98	2.77	13.05	7.28	5.39	35.03	23.24	16.92
ρ=0.3											
0.00	0.50	60.48	86.92	67.80	64.26	153.08	116.62	99.17	221.26	192.21	170.40
0.50	0.00	60.48	86.92	67.80	64.26	153.08	116.62	99.17	221.26	192.21	170.40
0.00	1.00	8.05	13.94	9.51	8.79	37.86	22.87	17.31	82.48	60.27	46.86
1.00	0.00	8.05	13.94	9.51	8.79	37.86	22.87	17.31	82.48	60.27	46.86
0.00	1.50	2.25	3.71	2.59	2.42	11.20	6.23	4.62	30.74	20.16	14.58
1.50	0.00	2.25	3.71	2.59	2.42	11.20	6.23	4.62	30.74	20.16	14.58
ρ=0.7											
0.00	0.50	28.53	45.00	32.87	30.75	95.44	66.03	53.36	161.55	131.52	110.81
0.50	0.00	28.53	45.00	32.87	30.75	95.44	66.03	53.36	161.55	131.52	110.81
0.00	1.00	3.15	5.39	3.68	3.42	16.24	9.14	6.76	42.09	28.40	20.91
1.00	0.00	3.15	5.39	3.68	3.42	16.24	9.14	6.76	42.09	28.40	20.91
0.00	1.50	1.23	1.66	1.32	1.28	4.32	2.51	1.96	12.75	7.96	5.65
1.50	0.00	1.23	1.66	1.32	1.28	4.32	2.51	1.96	12.75	7.96	5.65

Table 4 The ARL values for the T^2 control chart, $\delta_1 \neq \delta_2$

(a, b)		(0, 0)	(0, 0.2)			(0, 0.5)			(0, 0.7)		
s			0	1	2	0	1	2	0	1	2
δ_1	δ_2	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.0	0.0	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
ρ=0.0											
0.50	1.00	6.50	9.99	7.42	6.97	20.48	14.41	11.75	33.41	27.67	23.58
1.00	0.50	6.50	7.17	6.72	6.62	8.15	7.69	7.41	8.72	8.51	8.32
0.50	1.50	2.22	3.43	2.51	2.37	8.44	5.28	4.12	17.76	13.15	10.33
1.50	0.50	2.22	2.31	2.25	2.23	2.43	2.37	2.34	2.49	2.47	2.45
1.00	1.50	1.61	2.10	1.74	1.67	3.54	2.71	2.34	5.25	4.50	3.96
1.50	1.00	1.61	1.79	1.66	1.64	2.09	1.94	1.85	2.29	2.22	2.15
ρ=0.3											
0.50	1.00	8.76	14.02	10.10	9.43	29.07	20.06	16.14	44.37	36.84	31.38
1.00	0.50	8.76	9.15	8.87	8.81	9.40	9.26	9.12	9.40	9.40	9.35
0.50	1.50	2.56	4.24	2.96	2.76	11.48	6.80	5.15	24.25	17.56	13.54
1.50	0.50	2.56	2.57	2.57	2.57	2.55	2.57	2.57	2.53	2.55	2.56
1.00	1.50	2.12	2.98	2.34	2.23	5.23	3.88	3.27	7.18	6.18	5.43
1.50	1.00	2.12	2.31	2.18	2.15	2.51	2.40	2.32	2.56	2.53	2.49
ρ=0.7											
0.50	1.00	8.30	15.70	10.27	9.38	39.85	25.91	19.99	58.22	49.07	41.80
1.00	0.50	8.30	7.56	8.24	8.35	6.92	7.97	8.38	7.20	7.91	8.32
0.50	1.50	1.88	3.35	2.26	2.09	1.75	6.84	4.93	32.26	22.39	16.67
1.50	0.50	1.88	1.77	1.89	1.91	12.74	1.94	2.02	1.89	2.03	2.12
1.00	1.50	2.56	4.19	2.98	2.77	7.87	5.75	4.69	9.24	8.45	7.60
1.50	1.00	2.56	2.45	2.55	2.56	2.24	2.46	2.53	2.22	2.38	2.47
(a, b)		(0, 0)	(0.2, 0.2)			(0.5, 0.5)			(0.7, 0.7)		
s			0	1	2	0	1	2	0	1	2
δ_1	δ_2	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
0.0	0.0	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
ρ=0.0											
0.50	1.00	6.50	11.33	7.68	7.10	31.74	18.81	14.13	72.02	51.69	39.67
1.00	0.50	6.50	11.33	7.68	7.10	31.74	18.81	14.13	72.02	51.69	39.67
0.50	1.50	2.22	3.64	2.55	2.38	11.00	6.12	4.53	30.26	19.82	14.33
1.50	0.50	2.22	3.64	2.55	2.38	11.00	6.12	4.53	30.26	19.82	14.33
1.00	1.50	1.61	2.46	1.80	1.71	7.14	3.99	3.00	20.60	13.13	9.36
1.50	1.00	1.61	2.46	1.80	1.71	7.14	3.99	3.00	20.60	13.13	9.36
ρ=0.3											
0.50	1.00	8.76	15.13	10.34	9.56	40.53	24.67	18.74	86.86	63.93	49.96
1.00	0.50	8.76	15.13	10.34	9.56	40.53	24.67	18.74	86.86	63.93	49.96
0.50	1.50	2.56	4.30	2.97	2.77	13.02	7.27	5.37	34.97	23.19	16.89
1.50	0.50	2.56	4.30	2.97	2.77	13.02	7.27	5.37	34.97	23.19	16.89
1.00	1.50	2.12	3.46	2.44	2.28	10.43	5.80	4.30	28.90	18.86	13.61
1.50	1.00	2.12	3.46	2.44	2.28	10.43	5.80	4.30	28.90	18.86	13.61
ρ=0.7											
0.50	1.00	8.30	14.37	9.80	9.06	38.82	23.52	17.82	84.07	61.60	47.98
1.00	0.50	8.30	14.37	9.80	9.06	38.82	23.52	17.82	84.07	61.60	47.98
0.50	1.50	1.88	2.99	2.14	2.01	8.90	4.95	3.69	25.15	16.24	11.65
1.50	0.50	1.88	2.99	2.14	2.01	8.90	4.95	3.69	25.15	16.24	11.65
1.00	1.50	2.56	4.29	2.97	2.77	13.00	7.26	5.37	34.93	23.16	16.86
1.50	1.00	2.56	4.29	2.97	2.77	13.00	7.26	5.37	34.93	23.16	16.86

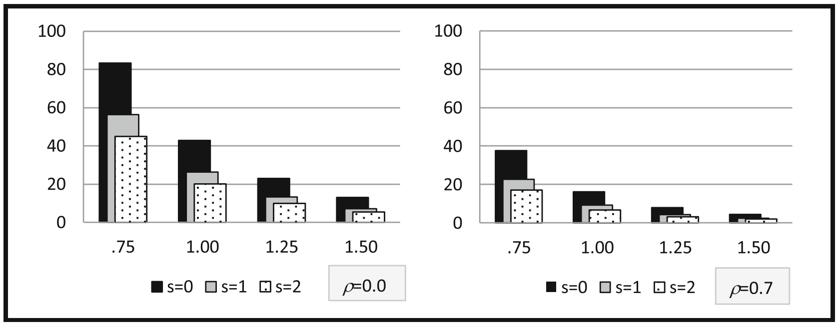

Fig. 1 The ARL values for the T^2 control chart when the variable affected by the assignable cause is the autocorrelated one: a=0.5, b=0.5, δ_1=0, $\delta_2\in$(0.75, 1.00, 1.25, 1.50)

two matrices, Φ and Σ_e, see expression (7). During the in-control period, T^2 follows a chi-square distribution with p degrees of freedom (χ^2_p). Similarly, to the work of Champ et al. [22], the upper control limit of the chart is chosen to be the $(1-\alpha)$th quantile of the chi-square distribution to achieve a desired in-control average run length (ARL) of $1/\alpha$. The chi-square approximation makes more conservative control limits than the original F distribution [23]. After the assignable cause occurrence, the mean vector changes to $\mu_1=(\mu_{1X},\mu_{1Y})'$ and the distribution of the monitoring statistic T^2 changes to a non-central chi-square distribution ($\chi^2_{(p,\lambda)}$) with non-centrality parameter $\lambda^2=\delta'\Gamma_{\overline{X}}^{-1}\delta$, where δ is the standardized mean vector shift $\delta=(\delta_X,\delta_Y)'=\left(\frac{\mu_{1X}-\mu_{0X}}{\sigma_{e_X}},\frac{\mu_{1Y}-\mu_{0Y}}{\sigma_{e_Y}}\right)'$, see Wu and Makis [24] and Franco et al. [25]. Without loss of generalization, we consider $\mu_{01}=\mu_{02}=0$. In the absence of autocorrelated variables, the non-centrality parameter is given by $\lambda^2=\delta'\Psi_{\overline{X}}^{-1}\delta$. As $\delta'\Gamma_{\overline{X}}^{-1}\delta<\delta'\Psi_{\overline{X}}^{-1}\delta$, we conclude that the autocorrelation reduces the ability of the chart in signaling an assignable cause. According to expressions (8) and (9),

$$T^2=\left(\overline{X}_1+\delta_1\right)^2\theta_{11}+\left(\overline{X}_2+\delta_2\right)^2\theta_{22}+2\left(\overline{X}_1+\delta_1\right)\left(\overline{X}_2+\delta_2\right)\theta_{12} \quad (10)$$

where $\overline{X}_1$ and $\overline{X}_2$ are the sample means of the two quality characteristics used to control the bivariate process.

To study the effects of the auto- and cross-correlation on the performance of the T^2 chart, we assume a bivariate process and samples of size n=4 build up without skipping items (s=0) and skipping one or two items, that is $s\in$(1, 2). The ARL measures the performance of the T^2 chart. When the process is in-control, the ARL measures the rate of false alarms. A chart with a larger in-control ARL (ARL_0) has a lower false alarm rate than other charts. A chart with a smaller out-of-control ARL has a better ability to detect process changes than other charts. In practice, a large sample is preliminarily taken to estimate the cross-covariance matrix Γ and, subsequently, the parameters (a, b, ρ); see Eq. 5.

The ARL value is given by

$$\text{ARL}=\left[1-\Pr\left(\chi^2_{(p,\lambda)}<\text{CL}\right)\right]^{-1} \quad (11)$$

The CL is the control limit of the T^2 chart; it is computed using Eq. 11 with an in-control ARL=370.4 and λ=0. The ARLs in Tables 2, 3, and 4 were computed with Eq. 11. We considered uncorrelated variables, ρ=0, and variables with low and high correlation, ρ=0.3 or 0.7. To investigate the effect of the autocorrelations on the overall performance of the bivariate T^2 charts, Tables 2 and 4 consider the cases in which the two variables are affected by the assignable cause (in Table 2, $\delta_1=\delta_2$, and in Table 4, $\delta_1\neq\delta_2$). Table 3 considers the cases in which only one variable

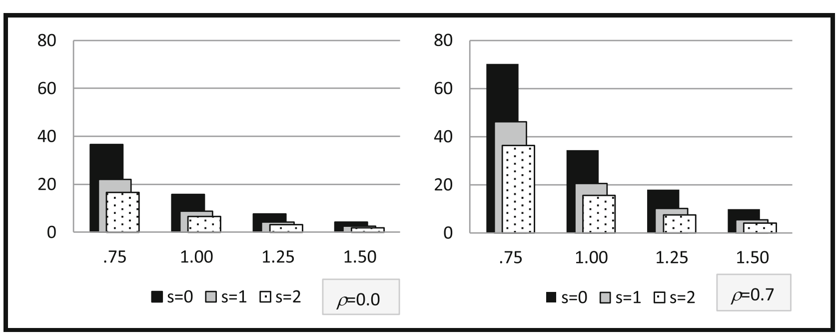

Fig. 2 The ARL values for the T^2 control chart when the variable affected by the assignable cause is the autocorrelated one: a=0.5, b=0.5, $\delta_1=\delta_2\in$(0.75, 1.00, 1.25, 1.50)

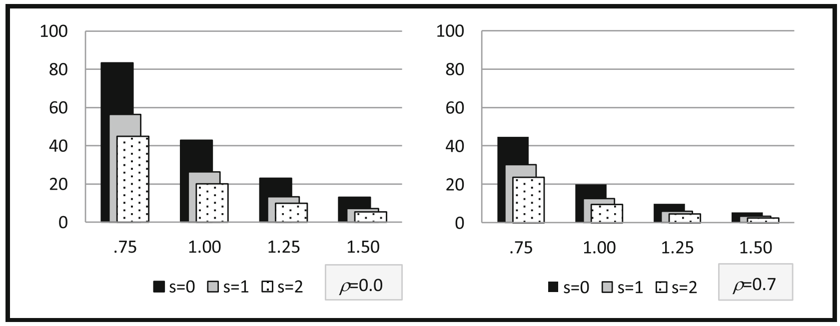

Fig. 3 The ARL values for the T^2 control chart when the variable affected by the assignable cause is the autocorrelated one: a=0, b=0.5, δ_1=0, δ_2∈(0.75, 1.00, 1.25, 1.50)

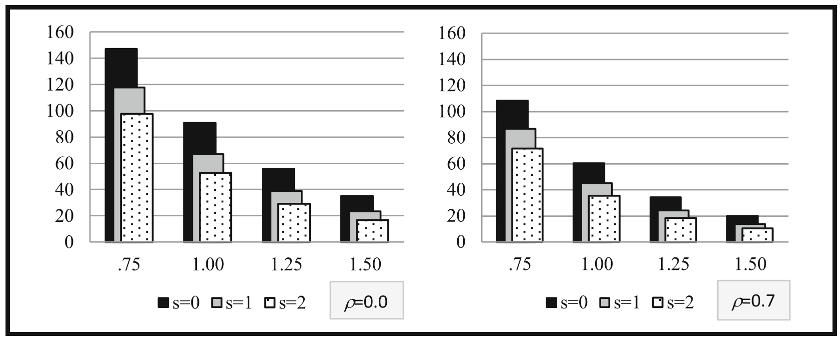

Fig. 4 The ARL values for the T^2 control chart when the variable affected by the assignable cause is the autocorrelated one: a=0, b=0.7, δ_1=0, δ_2∈(0.75, 1.00, 1.25, 1.50)

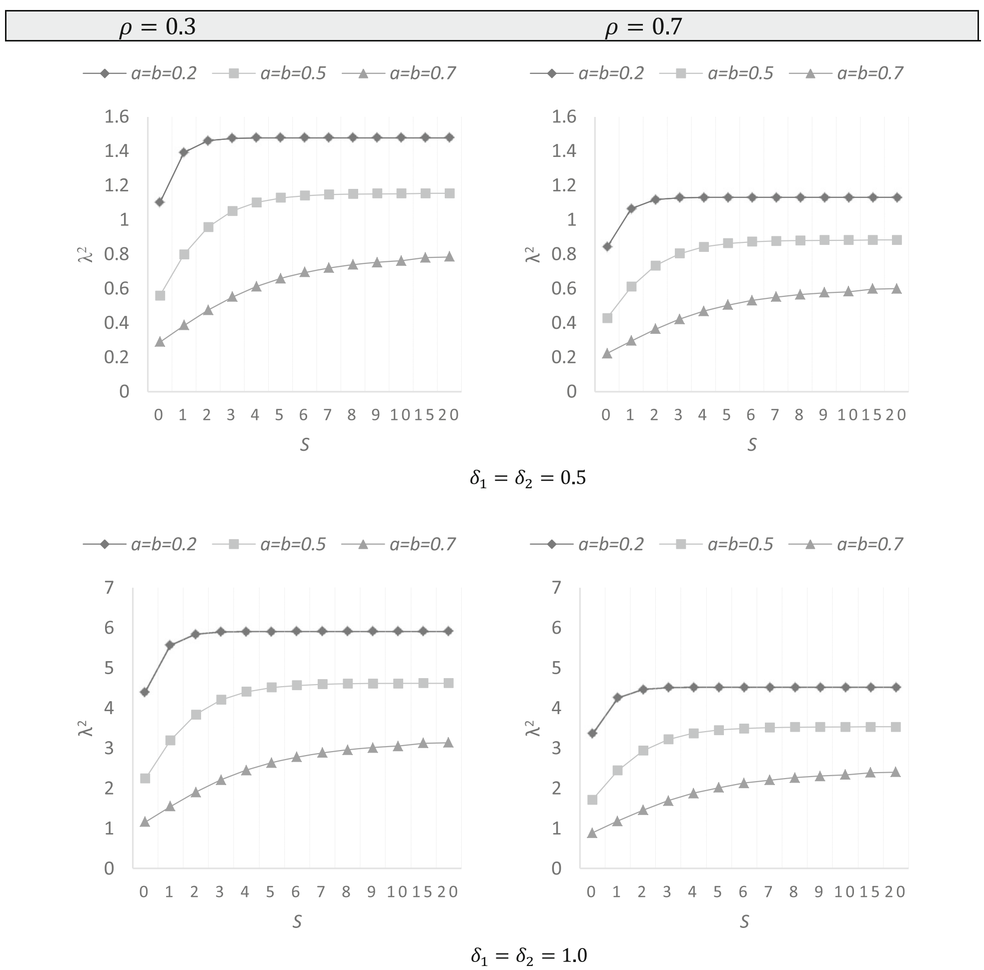

Fig. 5 The non-centrality parameter: a=b∈(0.2, 0.5, 0.7), δ_1=δ_2∈(0.5, 1.0)

is affected by the assignable cause (δ_1 or δ_2=0). It was also considered independent and autocorrelated observations, that is, a, $b\in(0, 0.2, 0.5, 0.7)$, and samples of size n=4.

In the absence of autocorrelation ($a=b=0$) or in the presence of two variables with the same level of autocorrelation ($a=b\neq0$), a higher dependence between the two variables improves the performance of the T^2 chart in signaling shifts in only one variable ($\delta_1\neq0$; $\delta_2=0$ or $\delta_1=0$; $\delta_2\neq0$). For instance, in Table 3, when the variables are independent and uncorrelated ($a=b=\rho=0$) and (δ_1, δ_2)=(0, 1.0), the T^2 chart requires, on average, 9.41 samples to signal. This number decreases to 3.15 when the variables are highly correlated ($a=b=0$, $\rho=0.7$). When the variables are autocorrelated ($a=b=0.5$, $\rho=0$) and (δ_1, δ_2)=(0, 1.0), the T^2 chart requires, on average, 42.87 samples to signal. This number decreases to 16.24 when the variables are highly correlated ($a=b=0.5$, $\rho=0.7$). In these last two cases of uncorrelated and highly correlated variables, the skipping strategy (s=2) reduces the ARLs, respectively, from 42.87 to 20.02 and from 16.24 to 6.76, see Fig. 1 and Table 3.

A contrary effect is observed when the assignable cause shifts both variables. For instance, in Table 2, when the variables are independent and uncorrelated ($a=b=\rho=0$) and (δ_1, δ_2)=(1.0, 1.0), the T^2 chart requires, on average, 3.06 samples to signal. This number increases to 7.19 when the variables are highly correlated ($a=b=0$, $\rho=0.7$). When the variables are autocorrelated ($a=b=0.5$, $\rho=0$) and (δ_1, δ_2)=(1.0, 1.0), the T^2 chart requires, on average, 15.74 samples to signal. This number increases to 34.52 when the variables are highly correlated ($a=b=0.5$, $\rho=0.7$). In these last two cases of uncorrelated and highly correlated variables, skipping strategy (s=2) reduces the ARLs, respectively, from 15.74 to 6.54 and from 34.52 to 15.55, see Fig. 2 and Table 2.

If only one variable is autocorrelated and only one variable is affected by the assignable cause ($a\neq0$; $b=$

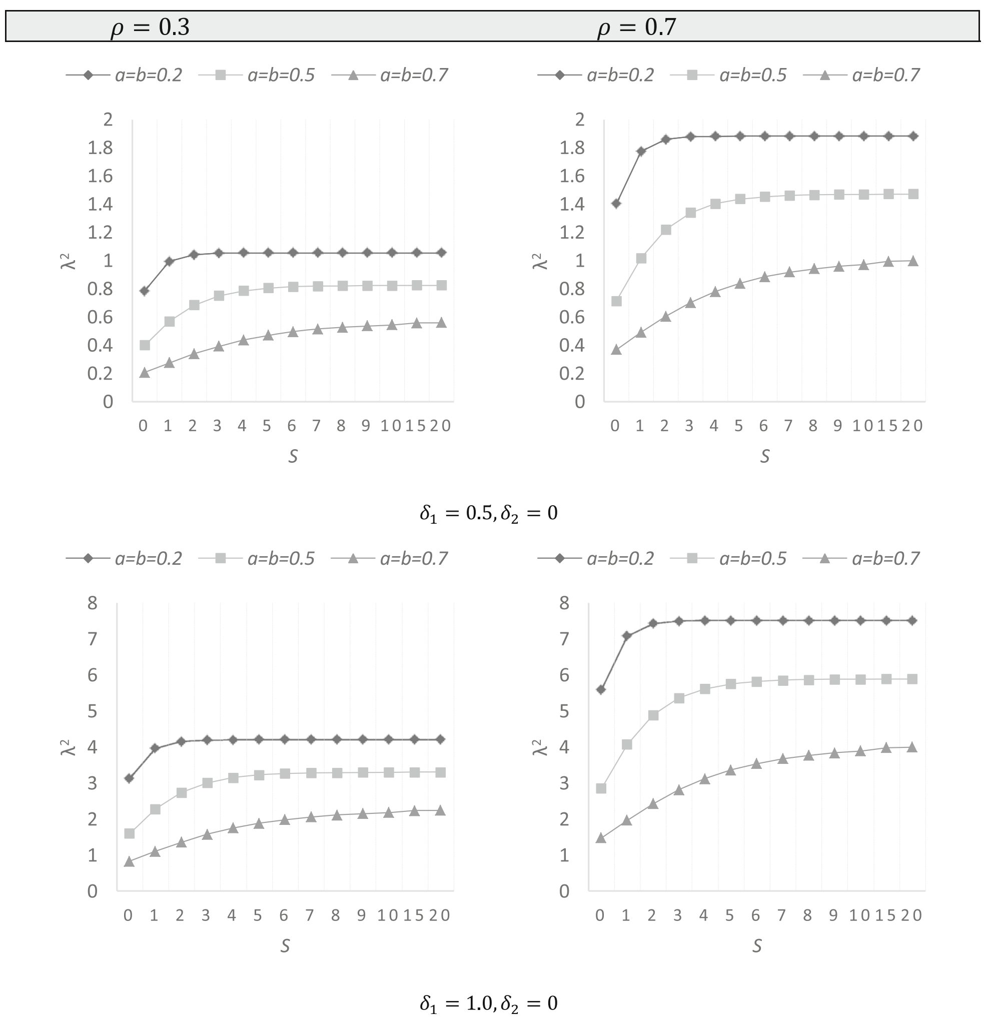

Fig. 6 The non-centrality parameter: $a=b\in(0.2, 0.5, 0.7)$, $\delta_1\in(0.5, 1.0)$, δ_2=0.0

0 or $a=0$; $b\neq0$ and $\delta_1\neq0$; $\delta_2=0$ or $\delta_1=0$; $\delta_2\neq0$), a higher dependence between the two variables improves the T^2 chart's performance. For instance, in Table 3, if the assignable cause changes the mean of the independent variable (a, b, δ_1, δ_2)=(0, 0.2, 1.0, 0) and the two variables are uncorrelated ($\rho=0$), the T^2 chart requires, on average, 9.41 samples to signal. This number decreases to 3.24 when the variables are highly correlated ($\rho=0.7$). In this case, the skipping strategy does not improve the chart's performance. When the assignable cause changes the mean of the autocorrelated variable (a, b, δ_1, δ_2)=(0, 0.2, 0, 1.0) and the two variables are uncorrelated ($\rho=0$), the T^2 chart requires, on average, 16.19 samples to signal. This number decreases to 5.55 when the variables are highly correlated ($\rho=0.7$). In this last case, applying the skipping strategy ($s=2$), the ARLs, respectively, decrease to 10.26 and 3.63, see Figs. 3 and 4 for the case that $b=0.7$.

5 The choice of the skipping parameter

The power of the T^2 chart depends on the non-centrality parameter of the chi-square distribution (λ^2), that is, as the non-centrality increases, the power of the T^2 improves. Because of that, it is interesting to investigate the way the non-centrality parameter increases with s.

Figures 5, 6, 7, and 8 present the results of this investigation for samples of size $n=4$, skipping $s=0, 1,\ldots, 20$ observations. According to Fig. 8, the skipping strategy is not recommended for cases in which only one variable is autocorrelated and the practitioner knows in advance that the autocorrelated variable is robust to the occurrence of assignable causes, that is, the mean of this variable rarely goes off-target. Excluding these cases, the non-centrality always increases with s. It is interesting to note that the non-centrality increase is significant when skipping one, two, or three observations; after that, the increase is marginal, except when the level of the autocorrelations are high (a and/or b are equal to 0.7).

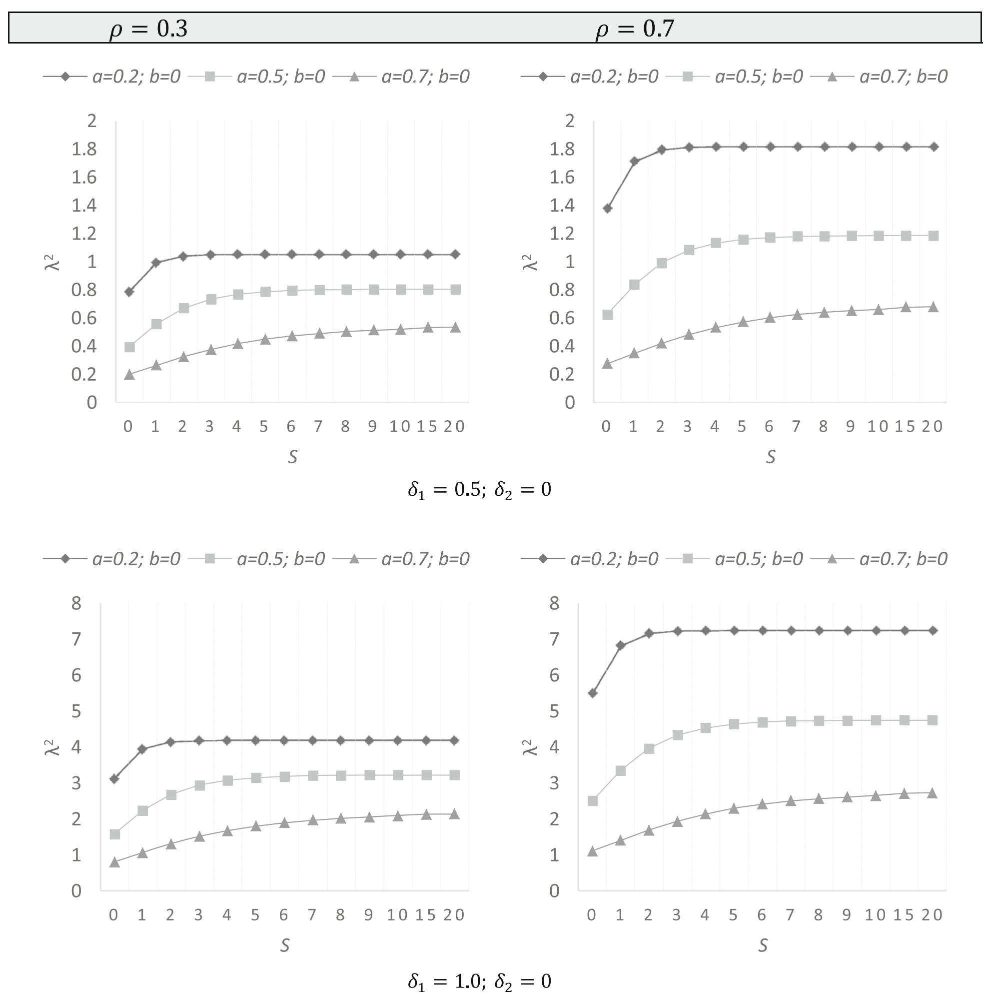

Fig. 7 The non-centrality parameter: $a\in(0.2, 0.5, 0.7)$, $b=0$, $\delta_1\in(0.5, 1.0)$, $\delta_2=0.0$

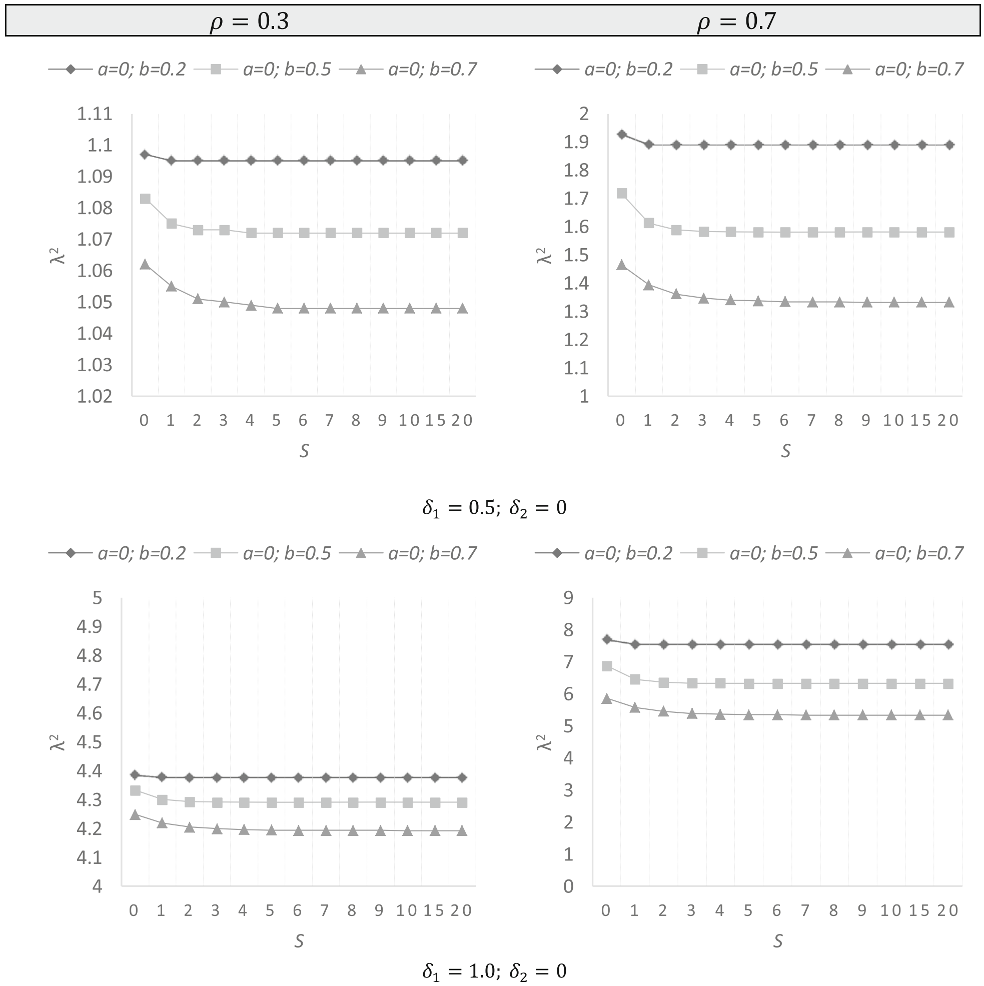

Fig. 8 The non-centrality parameter: a=0, $b\in$(0.2, 0.5, 0.7), $\delta_1\in$(0.5, 1.0), δ_2=0.0

Taking account the trade-off between the gain in chart's performance and the additional effort to select the four sample items, the skip of one or two observations seems to be the best option for variables with low level of autocorrelation (a and/or b are equal to 0.2). If the level of the autocorrelation is moderate (a and/or b are equal to 0.5), the skip of two or three observations seems to be the ideal decision. The non-centrality also increases with the cross-correlation, except when the two variables are autocorrelated and the disturbance simultaneously changes their mean position.

Table 5 The *ARL* for the T^2 chart

(a, b)		(0, 0)	(0.4, 0.4)			
s			0	1	2	3
δ_1	δ_2					
0	0.5	4.20	14.53	8.27	6.54	5.95
1	0	2.86	9.72	5.48	4.35	3.97
0.5	1	14.23	42.92	26.68	21.63	19.84
0.5	0	25.98	68.77	45.61	37.90	35.08
0	1	1.69	5.08	2.94	2.39	2.21
0.5	0.5	53.97	117.18	85.07	73.41	69.01
1	1	6.87	23.06	13.49	10.72	9.76

6 Illustrative example

Let us consider a bivariate process that has two quality characteristics to be controlled: the tensile strength and diameter of a textile fiber [17]. They are correlated to each other with ρ= 0.78 and

$$\mu_0 = (115.9, 0.0106)';\ \sum_e = \begin{pmatrix} \sigma^2_{e_X} & \sigma_{e_{XY}} \\ \sigma_{e_{XY}} & \sigma^2_{e_y} \end{pmatrix} = \begin{pmatrix} 1.23 & 0.79 \\ 0.79 & 0.83 \end{pmatrix}$$

The quality engineer has decided to use the T^2 chart with samples of size n=4. In Table 5, two scenarios are compared: the scenario where the tensile strength and diameter are both non-autocorrelated variables ($a=b=0$), and the scenario where the tensile strength and diameter are both autocorrelated variables ($a=b=0.4$). Even moderate autocorrelations reduce significantly the speed with which the T^2 chart detects changes in the process mean vector; in the first scenario, the ARL corresponding to ($\delta_1=0.5$, $\delta_2=1.0$) is 14.23; in the second scenario, this number increases more than 300 % (ARL=42.92). The skipping strategy helps to reduce the negative effect of the autocorrelation, with $s=1$, the ARL reduces to 26.69, and with $s=2$, the *ARL* reduces to 21.63.

7 Conclusions

In this paper, we considered the skipping strategy to reduce the negative effect of the autocorrelations on the performance of the T^2 control chart. The skipping strategy enhances the chart's performance when the mean of the autocorrelated variables goes off-target. The property that the cross-correlation improves the chart's performance when one of the two variables is robust to mean shifts is also observed with the skipping strategy. However, it is not recommended to apply the sampling strategy when a bivariate process has only one autocorrelated variable and this variable is robust to mean shifts. In order to preserve the rational subgroup concept, it is highly recommended to select the sample units skipping no more than three units.

It is worthwhile to note that the skipping strategy can be applied with the exponentially weight moving average, cumulative sum, adaptive, or synthetic schemes.

Conflict of interest The authors declare that they have no conflict of interest.

Appendix—the cross-covariance matrix of the sample mean vector under the skipping strategy

If the n units of the sample are collected close together in time, and the length of the sampling interval is large enough to eliminate any dependence between samples, the vector with the observations of the jth unit is described by a first-order autoregressive model:

$$X_j = \sum_{i=1}^{j} \Phi^{j-i}\varepsilon_i; j = 1, 2, \ldots, n. \quad \text{(A1)}$$

where $\varepsilon_i \sim N_p(\mu; \Sigma)$ with $\Sigma=(a_{ij})_{p\times p}$, $a_{ij}=1$ if $i=j$, $a_{ij}=\rho$ if $i\neq j$, and $\Phi=\text{diag}(a; b; \ldots)_{p\times p}$.

According to the first-order autoregressive model:

$$\text{Var}\left(X_j\right) = \left(\Phi^{j-1}\right)\Gamma\left(\Phi^{j-1}\right)' + \left(\Phi^{j-2}\right)\Sigma_e\left(\Phi^{j-2}\right)' + \ldots + \Sigma_e \quad \text{(A2)}$$

where $\Gamma=\Phi\Gamma\Phi'+\Sigma_e$. The cross-covariance matrix of the sample mean vector $\left(\Gamma_{\overline{X}}\right)$ is given by

$$\Gamma_{\overline{X}} = \text{Var}\left[\frac{X_1 + X_2 + \ldots + X_n}{n}\right] \quad \text{(A3)}$$

When the samples are built by taking one item from production line and skipping one ($s=1$), two ($s=2$), three ($s=3$), or more before selecting the next, the cross-covariance matrix of the sample mean vector $\left(\Gamma_{\overline{X}}\right)$ is given by

$$\begin{aligned}\Gamma_{\overline{X}} &= \frac{1}{n^2}\text{Var}\left[\sum_{i=0}^{n-1} X_{1+(s+1)i}\right]\\ &= \frac{1}{n^2}\left[\begin{array}{l}\left(I + \Phi^{(s+1)} + \Phi^{2(s+1)} + \ldots + \Phi^{(n-1)(s+1)}\right)\Gamma\left(I + \Phi^{(s+1)} + \Phi^{2(s+1)} + \ldots + \Phi^{(n-1)(s+1)}\right)' + \\ \left(\Phi^{s} + \Phi^{2s-1} + \ldots + \Phi^{(n-1)s-1}\right)\Sigma_e\left(\Phi^{s} + \Phi^{2s-1} + \ldots + \Phi^{(n-1)s-1}\right)' + \\ \left(\Phi^{s-1} + \Phi^{2s} + \ldots + \Phi^{(n-1)s}\right)\Sigma_e\left(\Phi^{s-1} + \Phi^{2s} + \ldots + \Phi^{(n-1)s}\right)' + \ldots + \Phi\Sigma_e\Phi' + \Sigma_e\end{array}\right]\end{aligned} \quad \text{(A4)}$$

From (A1), (A2), (A3), and (A4), it follows that

$$\Gamma_{\overline{X}} = \frac{1}{n^2}\left\{\left(\sum_{\substack{I\\s=0}}^{n-1}\Phi^{(s+1)i}\right)\Gamma\left(\sum_{\substack{I\\s=0}}^{n-1}\Phi^{(s+1)i}\right)' + \sum_{k=1}^{n-1}\left\{\sum_{j=1}^{s+1}\left[\left(\sum_{\substack{I\\s=1}}^{k}\Phi^{(s+1)i-j}\right)\Sigma_e\left(\sum_{\substack{I\\s=1}}^{k}\Phi^{(s+1)i-j}\right)'\right]\right\}\right\} \quad \text{(A5)}$$

After some algebra, for the bivariate case, Eq. (A5) leads to

$$\Gamma_{\overline{X}} = \begin{pmatrix} \frac{\sigma_X^2}{n}\left[1+\frac{2}{n}\sum_{j=1}^{n-1}(n-j)\,a^{(s+1)j}\right] & \frac{\sigma_{XY}}{n}\left[1+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,a^{(s+1)j}+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,b^{(s+1)j}\right] \\ \frac{\sigma_{XY}}{n}\left[1+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,a^{(s+1)j}+\frac{1}{n}\sum_{j=1}^{n-1}(n-j)\,b^{(s+1)j}\right] & \frac{\sigma_Y^2}{n}\left[1+\frac{2}{n}\sum_{j=1}^{n-1}(n-j)\,b^{(s+1)j}\right] \end{pmatrix} \quad \text{(A6)}$$

where $\sigma_X^2 = \frac{\sigma_{eX}^2}{1-a^2}$, $\sigma_Y^2 = \frac{\sigma_{eY}^2}{1-b^2}$, and $\sigma_{XY} = \frac{\sigma_{eXY}}{1-ab}$.

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