#### ORIGINAL ARTICLE

## Method for measuring thermal distortion in large machine tools by means of laser multilateration

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Abstract A new methodology to measure thermal distortion in large machine tools is proposed in this paper. The advantage of this method is that a single tracking interferometer can be used to measure thermal distortion of machines with large work volumes while maintaining low enough measurement cadence and uncertainty. A multilateration scheme is conducted using a single laser tracking device positioned on top of the machine table which is automated, and for each target point, all laser stations are reached prior to moving to the next target point; then, the whole measurement cycle is repeated during the test. For measuring angular thermal distortion, precision electronic levels are located in machine ram and column top; also, temperatures are registered in key points of the machine. Experimental measurements on a large column-type milling machine are done, and the effectiveness of the proposed methodology is verified.

**Keywords** Thermal deviation · Large-scale metrology · Machine tool accuracy

#### **1** Introduction

Thermal distortion is one of the principal factors among those that limit accuracy in large machine tools [1-4] and can be of a

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magnitude equivalent to geometric errors [5, 6]. Internally generated heat and environment conditions expose machine tools to non-uniform and variable temperature distributions. The existence of gradients implies that different parts of the environment have different mean temperatures; additional complexity is created when these temperature gradients change in time. Movement of machine components or workpieces from one area to another will result in a change in the geometric distortion pattern. Machine tools are affected by temperature gradients in a variety of ways. For example, a machine with a tall vertical column will have a progressive positional deviation along the vertical axis per unit of motion if there is a vertical temperature gradient. In addition, if the vertical slide carries a long cantilever quill, the quill will undergo a transient change of length when raised or lowered. Vertical gradients also cause bending of horizontal guideways, resulting in angular and straightness error motions [7].

Different strategies can be adopted to improve the thermal behavior of a machine tool and to reduce its thermal distortion [8]:

(i) Identification and minimization of thermal sources

Main thermal sources usually are spindle motor, reduction gears and transmission pulley bearings. Other heat sources include linear actuators, electrical cabinet, cooling system radiator, and chip from machining. To minimize generated heat, some strategies can be followed such as using adequate preloads and low friction bearings. Also, static forces such as own weights can be compensated with passive means [9]; compensation of weights will lead to smaller motors; therefore, less heat will be emitted to the surroundings.

(ii) Heat flow management

To maintain thermal loads far from critical zones is an effective strategy [10]. Heat evacuation channels such as

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fins can be used, and insulation can be applied to avoid heat propagation between machine elements.

(iii) Thermal control

Spindle cooling is common in machine tools; some machine manufacturers have also started including cooling in the ram and other parts of the structure. In the past, other techniques were applied such as using controlled artificial heat sources to compensate for inactive heat sources and maintain the steady state [11–13].

- (iv) Optimization of thermal design to prevent distortions Geometry of the machine will be critical in the deformation behavior [14]. During design phase, the influence of propagated heat in the deformation of the machine should be studied. An adequate thermal design will help reducing tool center point (TCP) progressive deviation from the nominal position, for example creating symmetry in the machine structure geometry. Lately, thermal calculation modules of finite element method (FEM) software packages have become powerful enough to be applied in machine thermal design and numerical solutions can reach high accuracy if the boundary conditions are correctly defined [15]. In the past, several thermal models were developed obtaining satisfactory results [16-18]. In some cases, it can be interesting to design geometries that help to reach the steady state quickly while they do not interfere with machine performance. It is also worth making the system less sensitive to heat by a careful selection of materials, using when possible materials or material combinations with low thermal expansion coefficients [19] [20], while looking for a similar thermal expansion as that of the workpiece. A metrology frame separated from the machine structure may be used to prevent errors due to machine structure deformation.
- (v) Implementation of a thermal distortion compensation system

Different approaches can be followed when designing a thermal compensation system. Most methods are based on mathematical models that correlate thermal distortion to other variables that are easier to measure [21]; others have relied on real-time data from external measurement systems [22]. Commonly, temperature measurements at certain key positions of the machine tool have been used. Although for a period of time putting a large number of temperature sensors was the generally adopted method by researchers [23, 24], an optimized temperature sensor location is recommended, so waste of time and resources are reduced [25, 26]. Other variables have also been used such as spindle speed [27–29], strain gauges data [12], or ball bar data [30].

Different techniques can be used to assess thermal distortion in machine tools. Measurement of thermal effects is regulated by international standards using conventional measurement techniques such as laser interferometer for thermal distortion caused by moving linear axes and capacitive, inductive, or retractable contacting displacement sensors for environment testing and thermal distortion caused by rotating spindles. Zeroing all readings at the start provides the positioning drift in time [7].

The use of invariable metrological reference artifacts and non-contact displacement sensors allows the measurement of thermal distortion at the TCP in different points of the work volume. This kind of artifacts has been employed for calibration either in 2D configuration like a grid plate [31], or in 3D configuration like a spatial frame of tetrahedral form [32]. Materials with a low expansion coefficient are commonly used, and good uncertainties are achieved [33]. Gauge blocks are also used along with probes in a machine tool to measure thermal distortion [34]. Reference artifact method is worth using in shop floor environments because of its low cost, easy setup, and robustness; this makes it valuable for performing periodical characterization by machine users. On the other hand, only small workspaces can be reached and bulky metrological reference artifacts can be difficult to manipulate.

Large-scale metrology equipment such as laser tracking interferometer devices provides great flexibility and long range which are useful for machine setup, but the measurement uncertainty given by this kind of equipment is not low enough to perform thermal distortion measurements with the required uncertainty. The laser multilateration technique greatly reduces measurement uncertainty and has been applied in high-accuracy systems such as coordinate measuring machines [35] and machine tools [36]. This technique uses displacement measurements between a target and a series of fixed measuring stations, calculating by a mathematical model the spatial coordinates of the points of interest [37]. Early works in thermal distortion multilateration measurement include those by Mize and Ziegert [38]. This technique has been usually conducted by sequential multilateration with a single interferometer in which all the target points were measured from a single laser station prior to moving to the next laser station [39, 40]. The associated long measurement times require invariable ambient conditions to avoid thermal distortion influencing the measurement [41]. This makes the mentioned method not practical for thermal distortion assessment of machine tools, especially in the bigger ones where the large trajectories and usually slower movements compared to smaller machines make the measurement cycle even longer. This problem can be solved by the simultaneous use of several laser tracking equipment units [42], but this is not commonly realizable in a practical manner because of its high cost.

The above references demonstrate that both the thermal effects and the methods for measuring thermal distortion were studied in the past, but no convenient procedure for measuring thermal distortion in large machine tools was proposed. For that reason, in this paper, a new methodology for the characterization of thermal distortion in large machine tools is presented which solves many difficulties associated with this kind of measurements. Main benefit of this method is being able to measure thermal distortion of machine tools with large working volumes while maintaining a low enough measurement uncertainty.

### 2 Proposed methodology

# 2.1 Laser multilateration measurement of thermal distortion

In the proposed methodology for measuring thermal positioning deviation of the TCP, laser multilateration technique is selected as it permits to measure points at a height of several meters and provides low measurement uncertainties. A laser interferometer with tracking capability is used, and the reflector is located next to the TCP as the spindle has to rotate to heat up the machine. The reflector is sequentially positioned in the coordinates of interest that will be called henceforth *target points*. For each target point, distance measurements between the interferometer and the reflector are taken from different locations of the laser interferometer that will be called henceforth *laser stations*.

The methodology presented in this work proposes a new technique for performing the sequential laser multilateration

technique that overcomes the difficulties associated with conventional sequential multilateration. In our new proposal, laser stations are reached by automatically moving the interferometer which is mounted on top of the machine table, controlled by the CNC. All the linear interferometer measurements for each single target point are taken from all the laser stations prior to moving to the next target point. In this manner, the time needed to make the multilateration measurement of each target point is reduced proportionally to the total number of target points, therefore also reducing the amount of thermal deviation during the multilateration calculation of each target point and consequently the uncertainty of the measurements.

For measuring thermal distortion of the machine over time, the complete cycle of measuring all target points from all the laser stations is repeated periodically. The sequence of this procedure is depicted in the flowchart from Fig. 1 and represents the proposed methodology for laser multilateration measurement of thermal distortion in large machine tools.

To solve the multilateration calculation, a measurement model is used where the problem inputs are the theoretical target point coordinates and the laser interferometer linear measurements; the unknowns are the three spatial coordinates of the target points and those of the laser stations. Main factors influencing measurement uncertainty are the following:

 i) Interferometer linear measurement uncertainty: a commercial laser interferometer unit provides an uncertainty of a given value when used in laboratory conditions.





Fig. 2 Machine configuration, coordinate system, and interferometer location

This uncertainty will vary depending on the distance to the point to be measured. In addition, air temperature, pressure, and moisture variations that occur in a common shop floor will increase interferometer uncertainty.

- Machine repeatability in the positioning of laser stations and target points, considering only momentary mechanical issues and not thermal effects.
- iii) Thermal drift during the time spent in each single target point multilateration: structural thermal dynamics affect the reflector position which is located next to the tool holder in the headstock.

As explained in Draper and Smith [43], measurement uncertainty can be estimated from the variance of the measurement residuals using a Student's t-distribution.

Therefore, interferometer measurement residuals between interferometer linear measurements and distances from estimated TCP position to the estimated laser stations were used to this purpose. Following the steps described by deVicente [44], associated covariances were calculated. Then, the covariance matrix was propagated using the measurement model as explained in the work of Tarantola [45]. Following this, uncertainties of the estimated TCP positions were calculated. Finally, covariance matrix of TCP position uncertainties was propagated using an analysis model that considers the relative measurements of each target point to their initial value. Thus, the covariance matrix of the uncertainties from the thermal drift measurement of each target point with respect to its initial position is obtained, along X-, Y-, and Z-axes. Obtained uncertainty values, depending on the target point, have been estimated ranging from 0.015 to 0.020 mm with a coverage factor of k=2. This value is considered low enough for the measurement of the thermal distortion of a large machine tool, like the column-type machine studied in this work, if the linear expansion of a steel column of 7 m with a temperature variation of 2 °C is taken as a reference, which is one order of magnitude above the measurement uncertainty.

# 2.2 Additional measurements: angular thermal distortion and temperatures

This methodology proposes also using a set of precision electronic inclinometers for measuring angular thermal drift. This provides a direct measurement of the magnitude of interest that is angular thermal drift in X- and Z-axes. Wireless levels

Target point	X axis	Y axis	Z axis	
	[mm]	[mm]	[mm]	
1	0	0	-1200	
2	0	-1333	-1200	۸ <sup>Y</sup>
3	0	-2666	-1200	
4	0	-4000	-1200	0
5	0	-4000	-866	
6	0	-4000	-533	
7	0	-4000	-200	
8	0	-2666	-200	3 ● ●8
9	0	-1333	-200	
10	0	0	-200	
11	0	0	-533	
12	0	0	-866	

 Table 1
 Coordinates of target points

allow measurement in mobile elements such as the headstock and in high places such as the top of the machine column.

For measuring temperatures, thermocouples with a resolution of 0.1  $^{\circ}$ C are used. The positions of the temperature sensors are selected to optimize the understanding of the influence of the main heat sources of the machine on its thermal behavior.

#### **3** Experimental setup

To confirm proposed method, the thermal distortion measurement of a large milling machine was carried out. In Fig. 2, the configuration of studied machine and the coordinate system used according to ISO 841 [46] and ISO 230–1 [47] are

 Table 2
 Coordinates of laser stations

shown. This milling machine had a configuration of travelling column with horizontal spindle and a workpiece table controlled by the CNC with rotary motion and one additional linear motion. For the multilateration measurement, the interferometer of a commercial laser tracker unit model Leica Absolute Tracker AT901MR was used. The location of the interferometer on top of the machine table is shown in Fig. 2.

In this study, the drift in the volumetric position of the TCP due to thermal effects suffered by machine ram and column was covered. Therefore, the position of the column along longitudinal X-axis was not changed during the measurements. This methodology makes possible to include points in other X-axis positions of the column if desired; however, including more points will increase the measurement cycle time; therefore, a compromise should be met.

Laser station	U' axis [mm]	B' axis [Deg]	Drawing
1	0	153	
2	1250	153	
3	1900	218	
4	2500	153	
5	2500	87	
6	700	87	
7	0	108	

#### Fig. 3 Measurement setup



Studied machine had a range of 6.0 m in the vertical *Y*-axis and 1.4 m in the *Z*-axis. Twelve target points within the *ZY*plane were selected, distributed in a rectangle of 4.0 m in the vertical *Y*-axis and 1.0 m in the ram *Z*-axis. The entire range of *Y*- and *Z*-axes was not covered due to the limited visibility of the reflector and also because the levels were on the ram end, thus occupying certain space of the working volume as shown in Fig. 4. Details of the target point coordinates can be found on Table 1.

Due to the current machine table configuration, laser stations could only be located within a single plane. In the general case, multilateration requires four laser stations that do not lie on the same plane; however, if all laser stations must lie in the same plane, six stations are the minimum that provide a solution. In our case, seven laser stations were used to achieve a redundancy on the system of equations, which leads to a final lower measurement uncertainty. Laser station locations were chosen by numerical simulation to theoretically achieve the lowest measurement uncertainty and are shown in Table 2.

Figure 3 shows pictures of the actual machine and the laser interferometer positioned on top of the machine table in two different laser stations.

A full cycle for measuring all the 12 target points, each of them from all the seven laser stations spanned around 30 min. Restart of the cycle was manual; thus, the exact amount varied; for example, during one of the cycles, a technical problem caused a delay. The timing of the measurement cycles and the spindle thermal cycle followed during the test is shown in Table 3 and also in the figures with measurement results. Previous works [48, 29] show that usually in large-sized machines, structural deformations present long time constants of more than 1 h. As it is shown in temperature measurement results, temperature near spindle motor showed a rise curve with a time constant of 1.6 h, which means that the steady state would be reached in 2.5 h. Considering this, the authors accept that a measuring cadence of 30 min is sufficient for assessing structural thermal distortion of this kind of large machine, although faster local dynamics with time constants of only few minutes will not be detected.

 Table 3
 Measurement and thermal cycle

Cycle number	Start time (min)	End time (min)	Duration (min)	Spindle speed (rpm)
1	0	30	30	0
2	30	61	31	0
3	61	92	31	0
4	92	123	31	1400
5	123	167	44	1400
6	167	198	31	0
7	198	230	32	0
8	230	264	34	0
9	264	296	32	0
10	296	327	31	1400
11	327	359	32	1400
12	359	391	32	1400
13	391	424	33	1400
14	424	455	31	0
15	455	487	32	0
16	487	518	31	0

**Fig. 4** Precision wireless levels in ram and column



For measuring angular thermal distortion of the machine, a set of four electronic precision wireless levels was used (model Wyler BlueLevel with a resolution of 0.001 mm/m and a thermal error of 0.002 mm/m per Kelvin). Two levels were located on the ram end, measuring inclinations in *X*- and *Z*-axes, and the other two levels were located on top of the column measuring also inclinations in *X*- and *Z*-axes, as seen in Fig. 4.

For measuring temperatures, thermocouples were used with a resolution of 0.1 °C. In Fig. 5, selected sensor locations are shown.

Positions of actual temperature sensors are shown in Figs. 6 and 7 corresponding to ram and column sensor locations. Sensor numeration corresponds to that of the diagram of Fig. 5.

#### **4** Experimental results

In the following figures, results of the different measurements are presented. In Fig. 8, multilateration measurement results for the thermal linear distortion in the TCP along X-. Y-, and Z-axes are shown; the same scale and limits are shown for all axes. Due to confidentiality issues, the authors are not able to present numerical results of thermal distortion measurements; however, it can be said that thermal distortion was acceptable for the size of this machine. In Fig. 8, each line represents the drift of the position of a target point over time relative to its first measurement to remove static errors such as those resulting from own weights. The patterns that appear show the thermal behavior of the machine which is position dependent. Target point numeration is that of the Table 1. A good continuity and smooth tendencies can be seen in the obtained results. Drifts during heating ups and cooling downs are logical and related to the performed spindle thermal cycle. The dependence of the thermal distortion with the coordinates of the target point can be seen.

Finally, it was found that target points numbered 3 and 8 were at a similar *Y*-axis height,  $Y_{P3,P8}=-2666$  mm, as that of the interferometer plane which was estimated in



Fig. 6 Temperature sensors in ram



the multilateration calculation of being  $Y_{\text{interferometer}}$ = -2602 mm, and this caused an excessive measurement uncertainty (one order of magnitude above expected result value); therefore, these target points were discarded and are not shown in the results. The rest of the target points presented adequate uncertainty values, with values ranging, depending on the target point, from 0.015 to 0.020 mm with a coverage factor of k=2. This value is considered low enough for the current measurand magnitude.

In Fig. 9, the temperature measurement results are shown for the sensors located in ram and column.

Regarding temperature results, the temperature measured near the spindle motor, which is the main heat source of the machine, registered the biggest fluctuation; temperature rise curve showed a time constant of 1.6 h, which means that the steady state would be reached in 2.5 h. The results also show that the rest of the temperatures farther from the main motor followed mostly room air temperature with little influence from the spindle state, which usually is not the case in smaller machines.

In Fig. 10, results of angular thermal distortions are shown. Each line represents the drift of the inclination of a target point over time relative to its first inclination



Fig. 7 Temperature sensors in column





measurement. Target point numeration is that of the Table 1.

It can be seen in the results that the ram inclination is affected by the spindle state in both axes. Due to thermal effects, a difference in the angular thermal drift of the TCP appeared over time depending on its Z-axis coordinate, making the thermal behavior of the TCP different when the ram was in and out. This was a different effect from the own weight inclination as it changed over time, and it is thought of being derived from a bending of the ram that happened because of a temperature gradient in the ram and the restriction imposed by the guideways.

### **5** Conclusions

A new methodology for measuring thermal distortion in large machine tools by laser multilateration has been









presented. The advantage of this method is that a single tracking interferometer can be used for measuring thermal distortion in machines with large work volumes while maintaining a low enough measurement uncertainty.

Measurements of tool center point thermal distortion exclude static errors such as those resulting from own weights. The multilateration measurement cadence is sufficient for assessing structural thermal distortion of large machines; this technique will not detect local effects with fast dynamics, whose study is already present in the literature; instead, this technique is intended for assessing the holistic thermal behavior with long time constants of the structure of machines with large working volumes. Measurement uncertainty related to the method was estimated and found sufficient to this purpose. In addition, this method proposes the use of precision electronic levels for measuring thermal angular distortion and also recording temperatures in key points of the machine.

Experimental measurements were performed in a large column-type milling machine, and results showed the thermal distortion of the machine which was position dependent. In view of these results, it is concluded that that the proposed method is suitable for obtaining valid measurements of thermal distortions in large machine tools. Acknowledgments This work was framed within Research Project IMPELER supported by Spanish Centre for Industrial Technology Development CDTI, and it was developed through the collaboration of JUARISTI Machine Tool Company.

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