Int J Adv Manuf Technol (2015) 79:2031–2040
DOI 10.1007/s00170-015-6925-5

ORIGINAL ARTICLE

Effect of measurement error on phase II monitoring of simple linear profiles

R. Noorossana · Y. Zerehsaz

Received: 26 October 2012 / Accepted: 18 February 2015 / Published online: 14 March 2015
© Springer-Verlag London 2015

Abstract In this paper, we study the effect of classical additive measurement error model on the most commonly used control charts for monitoring simple linear profiles with random explanatory variable. We showed that the in-control and out-of-control performances of these methods are significantly affected when measurement error is present in the explanatory variable. The average run length criterion is applied to represent the performance of the methods. For instance, we can clearly see that even a small amount of measurement error introduced in the explanatory variable of a profile can increase the false alarm rate about 55 %. With the same negligible amount of error, the out-of-control average run length of the exponentially weighted moving average chart (EWMA)-3 method in detecting a moderate-size shift in the slope parameter increases 15 %. Two different strategies on the basis of control limit modification have been proposed to compensate for the undesired effect of measurement error. Results indicate that the proposed strategies substantially offset the poor performances of the control charts resulted from the presence of measurement error.

Keywords Additive model · Measurement error · Statistical process control · Phase II profile monitoring

1 Introduction

In many practical applications of statistical process control, monitoring functional or profile data is of great interest. There are several papers in the context of phase II monitoring of simple linear profiles. To name a few, a Hotelling T^2 control chart is proposed by Kang and Albin [1] to simultaneously monitor the profile parameters. A combined exponentially weighted moving average chart (EWMA)\R chart is also used to monitor the average residuals. The EWMA-3 method suggested by Kim et al. [2] has proved to be a simple and efficient method for monitoring the parameters of simple linear profiles. For the phase I control of simple linear profiles, an F-test method using indicator variables is offered by Mahmoud and Woodall [3]. Mahmoud et al. [4] proposed a change point approach based on likelihood ratio test for phase I monitoring of simple linear profiles. A change point approach is developed by Yeh and Zerehsaz [5] for phase I control of linear profiles with individual observations.

The problem of monitoring profiles with random predictors is thoroughly studied by Noorossana et al. [6]. The results indicate that EWMA/R control chart and Hotelling T^2 statistic are not affected by the randomness of the explanatory variables. Performance of the EWMA-3 control scheme, however, is affected in comparison to the fixed-predictor case.

Establishing a reliable measurement system analysis is indispensable in each process dealing with measurements. Despite any rigorous measurement system and depending on the nature of the process, some of the measurement procedures suggest a certain amount of error. Although sometimes this measurement error may seem to be negligible, it may significantly affect performance; hence, it should not be ignored without further considerations. In the profile monitoring context with random predictors, it is quite possible that the explanatory variables are measured with error. Although different issues related to profile monitoring are discussed in the literature, the effect of measurement error in the independent

R. Noorossana · Y. Zerehsaz (✉)
Industrial Engineering Department, Islamic Azad University—South Tehran Branch, Tehran, Iran
e-mail: y.zerehsaz@gmail.com

variable has not been studied yet. It should be noticed that if the commonly used methods in phase II simple linear profile monitoring are applied when measurement error is present, then misleading results can be expected. Different models can be used to represent measurement error [7, 8].

The classical additive measurement error model is one of the most frequently used models as discussed by Carroll et al. [7]. In this model, the error component is assumed to be added to the true variable x. This model can be defined as

$$w = Ax + B + u \tag{1}$$

where A and B are constants and u is the measurement error term with zero mean and usually a constant variance σ_u^2 [7].

It is assumed that u is independent of x and follows a normal distribution. A usual measurement error model given by Eq. (2) is a special case of Eq. (1) with A=1 and B=0.

$$w = x + u \tag{2}$$

In statistical process control literature, the measurement error problem is extensively studied by many researchers. The model presented in Eq. (2) is used by Bennet [9] to study performance of the usual Shewhart $\overline{X}$ chart in the presence of measurement error. The performance of $\overline{X}-R$ control chart in the presence of measurement error is examined by Kanazuka [10], and it is concluded that power of control chart in detecting mean shifts is diminished when using the error-prone observations. The detection performance of $\overline{X}-S$ control chart when measurement error is not negligible is investigated by Mittag [11] and Mittag and Stemman [12]. Linna and Woodall [13] used the model defined in Eq. (1) to study the effect of measurement error on control charts.

The behavior of EWMA chart used for monitoring a variable with measurement error under the model in Eq. (1) is studied by Maravelakis et al. [14]. Linna et al. [15] addressed the effect of measurement error on the monitoring of multivariate control charts. It can be concluded that the power of multivariate control charts in detecting process shifts in the underlying process parameters is significantly affected when measurement error is present.

This paper discusses the effect of measurement error on the most commonly used methods for phase II monitoring of simple linear profiles with random explanatory variable. The in-control and out-of-control performances of the methods are assessed when the explanatory variable is subject to measurement error. The commonly used control schemes are modified based on two different strategies.

The next section provides an overview on simple linear regression with error-contaminated predictor variable. A brief review on several methods of simple linear profile monitoring techniques is given in Sect. 3. Furthermore, the charting performance of the aforementioned approaches is evaluated in terms of average run length (ARL) criterion in the presence of measurement error in this section. In Sect. 4, we propose an approach to eliminate the adverse effects of measurement error on the commonly used control charts. Section 5 suggests another strategy to improve the performance of the control charts and to estimate the true unobserved explanatory variable. Our concluding remarks are provided in the final section.

2 Simple linear profile and measurement error

Consider a simple linear profile defined as

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \qquad i = 1, 2n \tag{3}$$

where β_0 and β_1 are the intercept and slope coefficients, respectively, and ε_i follows a normal distribution with mean zero and variance σ^2. Suppose that x_i's are independently and identically distributed normal random variables with mean μ_x and variance σ_x^2. The ordinary least square (OLS) method can be used to estimate the parameters of the regression [16, 17].

In practice, there are certain situations where we are not able to observe the exact value of the independent variable due to measurement error. Therefore, an error-prone variable denoted by w is observed and used instead of the true variable x. Suppose that the relationship between the true variable x and the observed variable w can be represented by model in Eq. (2). In this case, the variability of the observed variable can be computed as $\sigma_x^2+\sigma_u^2$. It is noteworthy that the random error terms ε_i's and the measurement errors u_i's are usually assumed to be independent. The error-prone variable follows a normal distribution with the same mean of the true variable x and variance $\sigma_x^2+\sigma_u^2$.

Rewriting Eq. (3) gives us

$$y_i = \beta_0 + \beta_1(w_i - u_i) + \varepsilon_i = \beta_0 + \beta_1 w_i + \varepsilon_i - \beta_1 u_i \qquad i = 1, 2\ldots n \tag{4}$$

In the first glance, Eq. (4) may appear to be an ordinary linear regression model with as the random error term. However, w_i's are correlated with the random error term [8]. Since one of the essential assumptions of linear regression models is violated, the OLS estimators might not have the same properties as before. The next section provides a brief discussion on the main issues caused by having measurement error in the predictors.

2.1 Ordinary least squares method and measurement error

When the OLS method is used to fit a regression line in the existence of measurement error, the parameters will be estimated with bias. In other words, we have

$$E\left[\widehat{\beta}_1\right] = \left(\frac{\sigma_x^2}{\sigma_x^2+\sigma_u^2}\right)\beta_1\, E\left[\widehat{\beta}_0\right] = \beta_0+\beta_1\mu_x\left(1-\frac{\sigma_x^2}{\sigma_x^2+\sigma_u^2}\right)$$

where is $\lambda = \left(\frac{\sigma_x^2}{\sigma_x^2+\sigma_u^2}\right)$ called the reliability ratio [8]. In fact, the correlation between the independent variable and the error term in Eq. (4) leads to the bias in the two estimators. Since the pair of variables follow a bivariate normal distribution, the conditional distribution of the estimators given w_i is normal. The conditional mean and variance of the estimators are defined as

$$E\left[\widehat{\beta}_1\middle|w\right] = \lambda\beta_1 \operatorname{var}\left[\widehat{\beta}_1\middle|w\right] = \frac{\sigma_y^2-\sigma_{wy}\lambda\beta_1}{\sum_{i=1}^{n}(w_i-\overline{w})^2} \tag{5}$$

$$E\left[\widehat{\beta}_0\middle|w\right] = \beta_0+\beta_1\mu_x(1-\lambda) \text{ and } \operatorname{var}\left[\widehat{\beta}_0\middle|w\right] = \frac{\sigma^2+\lambda\sigma_u^2\beta_1^2}{n}+\overline{w}^2\operatorname{var}\left[\widehat{\beta}_1\middle|w\right] \tag{6}$$

where $\sigma_y^2=\sigma^2+\sigma_x^2\beta_1^2, \sigma_{wy}=\sigma_x^2\beta_1$ is the covariance between the observed variable w_i and the response variable y and $\overline{w}$ is the sample mean of the observed variable. The conditional covariance between the estimators is given by

$$\operatorname{cov}\left[\widehat{\beta}_0,\widehat{\beta}_1\middle|w\right] = -\overline{w}\operatorname{var}\left[\widehat{\beta}_1\middle|w\right] \tag{7}$$

Mean squared error is no longer an unbiased estimator of in this case. That is, we have

$$E\left[\text{MSE}\middle|w\right] = E[\text{MSE}] = \sigma^2+\lambda\sigma_u^2\beta_1^2 \tag{8}$$

3 Effect of measurement error on the performance of EWMA-3, EWMA/R, and T^2 control charts

In this section, the performance of common techniques for monitoring simple linear profiles is assessed using in-control and out-of-control ARL criteria. A brief illustration about the methods is given subsequently.

3.1 Description of the methods

Kang and Albin [1] introduced two control schemes with the intention to monitoring simple linear profiles in phase II. The first one is the EWMA/R method. The EWMA control statistic is given by

$$EWMA(j) = \theta\overline{e}_j+(1-\theta)EWMA(j-1) \qquad j = 1,\ 2$$

where $0<\theta\leq 1$ is the smoothing constant, $\text{EWMA}_0=0$, and $\bar{e}_j$ is the j^{th} average regression residuals. The lower and upper control limits for EWMA control chart can by written as

$$\text{LCL} = -L\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \quad \text{UCL} = -L\sigma\sqrt{\frac{\theta}{(2-\theta)n}} \tag{9}$$

where L is a positive constant which yields a desired in-control ARL. The R-chart in this method is proposed to monitor the process variance. The range statistic is defined as

$$R_j = \max(e_j)-\min(e_j) \quad j = 1,2\ldots$$

This chart will alarm as soon as statistic falls outside the control limits.

$$\text{UCL}_R = \sigma(d_2+Ld_3) \qquad \text{LCL}_R = \sigma(d_2-Ld_3) \tag{10}$$

where d_2 and d_3 are constants which depend on the sample size n and can be obtained from references such as by Montgomery [18]. The other method is a multivariate control scheme using Hotelling T^2 statistic defined as

$$T_J^2 = \left(u_j'-u\right)^T\sum^{-1}\left(u_j'-u\right) j = 1,\ 2$$

where $\mathbf{u}_\mathbf{j}' = \left[\widehat{\beta}_{0j},\widehat{\beta}_{1j}\right]$ is the vector of parameter estimators, u is the vector of regression parameters, and $\sum$ is the variance-covariance matrix of estimators given by

$$\sum = \begin{bmatrix} \sigma^2_{\widehat{\beta}_0} & \sigma_{\widehat{\beta}_0\widehat{\beta}_1} \\ \sigma_{\widehat{\beta}_0\widehat{\beta}_1} & \sigma^2_{\widehat{\beta}_1} \end{bmatrix} \tag{11}$$

The upper control limit for this chart is given by

$$\text{UCL} = \chi^2_{2,\alpha} \tag{12}$$

where $\chi^2_{2,\alpha}$ is the 100 $(1-\alpha)$ percentile of the chi-square disribution with two degrees of freedom. Among the common methods in profile monitoring literature, one of the most efficient control charts has been suggested by Kim et al. [2].

As discussed earlier, Noorossana et al. [6] investigated the effect of random predictors on phase II profile monitoring. Obviously, in both EWMA/R and T^2 control charts, the repercussions are negligible; however, for the EWMA-3 method, the authors have suggested some alterations. In a simple linear regression if the predictor centralized, the estimators will be independent. This gives the idea of setting up separate control charts for regression parameters [2]. In other words, we can write

$$y_i = \beta_0^{'} + \beta_1^{'} x_i^{*} + \varepsilon_i \quad i = 1, 2 \ldots, n \tag{13}$$

where $\beta_1^{'}=\beta_1$, $\beta_0^{'} = \beta_0 + \beta_1 \overline{x}$, and $x_i^{\rightarrow} = x_i - \overline{x}$. The charting statistics are, then, defined as

$$\begin{aligned} &\mathrm{EWMA}_I(j) = \theta\widehat{\beta}_{0j}^{'} + (1-\theta)\mathrm{EWMA}_I(j-1) && j = 1, 2 \\ &\mathrm{EWMA}_S(j) = \theta \mathrm{ST}_{\widehat{\beta}_1}(j) + (1-\theta)\mathrm{EWMA}_S(j-1) && j = 1, 2 \\ &\mathrm{EWMA}_N(j) = \max\left\{\theta\left(\mathrm{MSE}_j - \sigma_0^2\right) + (1-\theta)\mathrm{EWMA}_N(j-1), 0\right\} && j = 1, 2 \end{aligned} \tag{14}$$

where $ST_{\widehat{\beta}_1}(j) = \frac{\widehat{\beta_{1j}} - \beta_1}{\sqrt{\frac{\sigma^2}{Sxx_j}}}$, $EWMA_I(0)=\beta_0^{'}$, $EWMA_S(0)=0$, $EWMA_N(0)=0$, and σ_0^2 are the in-control error variances and θ is the smoothing constant. Also, $\widehat{\beta}_{0j} = \overline{y}$ and $\widehat{\beta}_{1j} = \frac{Sxy}{Sxx}$. Based on these results, the corresponding control limits are determined as follows

$$\begin{aligned} &\mathrm{UCL}_I = \beta_0 + \beta_1\mu_x + L_I\sqrt{\frac{\theta}{2-\theta}\frac{\sigma^2 + \sigma_x^2\beta_1^2}{n}} \\ &\mathrm{LCL}_I = \beta_0 + \beta_1\mu_x - L_I\sqrt{\frac{\theta}{2-\theta}\frac{\sigma^2 + \sigma_x^2\beta_1^2}{n}} \\ &\mathrm{UCL}_S = +L_S\sqrt{\frac{\theta}{2-\theta}} \\ &\mathrm{LCL}_S = -L_S\sqrt{\frac{\theta}{2-\theta}} \\ &\mathrm{UCL} = L_N\sqrt{\frac{\theta}{2-\theta}\mathrm{var}\left[\mathrm{MSE}_j\right]} \end{aligned} \tag{15}$$

where L_I, L_S, and L_N are positive constants selected to give specific in-control ARLs and $\mathrm{var}\left[\mathrm{MSE}_j\right] = \frac{2\sigma_0^4}{n-2}$

The EWMA-3 approach is based on the simultaneous use of EWMA_I, EWMA_S, and EWMA_N control charts..

3.2 Effect of measurement error on in-control ARLs

A simulation study is conducted to evaluate the performance of the methods under the presence of measurement error. The response variables are generated using the simple linear profile

$$y_i = 3 + 2x_i + \varepsilon_i \quad i = 1, 2 \ldots, 4$$

where the regressor x_i is normally and independently distributed with mean μ_x=0 and variance $\sigma_x^2 = \frac{5}{3}$, and ε_i's are assumed to be normal random variables with mean μ=0 and variance σ_ε^2=σ^2=1. The error-prone variable w_i is defined based on Eq. (2). The simulation is repeated 10,000 times, and the in-control ARLs are computed for a range of measurement error standard deviation σ_u. The ARL_0 in the absence of measurement error is set equal to 200 for each control scheme. The smoothing constant θ is set equal to 0.2 in the simulations. Figures 1, 2, and 3 show the in-control ARL values against different values of standard deviation σ_u and different sample sizes n for EWMA-3, EWMA/R, and T^2 control charts, respectively. As we can see, the false-alarm rate increases as the measurement error standard deviation increases.

In the EWMA-3 control chart, it is obvious that the intercept estimator can be expressed by $\widehat{\beta}_0^{'} = \overline{y}$. As stated before, is not under the influence of measurement error. Hence, neither the EWMA_I statistic nor the related control limits is affected. In contrast, the slope estimator may be substantially influenced by the presence of measurement error. In this case, the expected value and variance of the EWMA_S statistic may be rewritten as

$$E[\mathrm{EWMA}_S(j)] = \frac{-(1-\lambda)\beta_1}{\sqrt{\frac{\sigma^2}{Sww}}}\mathrm{var}[\mathrm{EWMAS}(j)] \tag{16}$$

$$= \frac{\left(\sigma_y^2 - \sigma_{wy}\lambda\beta_1\right)}{\sigma^2}\frac{\theta}{2-\theta} \quad j = 1, 2$$

where $ST_{\widehat{\beta}_1}(j) = \frac{\widehat{\beta_{1j}} - \beta_1}{\sqrt{\frac{\sigma^2}{Sww}}}$. Clearly, the expected value of EWMA_S statistic drifts from zero to $\frac{-(1-\lambda)\beta_1}{\sqrt{\frac{\sigma^2}{Sww}}}$. It is obvious that the measurement error introduces an unrealistic mean shift in the EWMA^S statistic. In addition, the variability of EWMA^S statistic in comparison to the error-free case becomes larger. Therefore, this chart would provide more false alarms. With the same analogy, it can be concluded that EWMA^N control chart performs poorly under measurement error. Therefore, the overall performance of EWMA-3 approach is deteriorated in the in-control state. This result can be extended to EWMA/R chart. It can be showed that

$$E[EWMA(j)] = 0\,\mathrm{var}[EWMA(j)] = \frac{\theta}{2-\theta}\frac{\left(\sigma^2 + \beta_1^2\sigma_u^2\right)}{n} \quad j = 1, 2 \tag{17}$$

and

$$E\left[R_j\right] = \left(\sigma^2 + \beta_1^2\sigma_u^2\right)d_2\,\mathrm{var}\left[R_j\right] = \left(\sigma^2 + \beta_1^2\sigma_u^2\right)d_3 \quad j = 1, 2 \tag{18}$$

The increased variance of EWMA statistic forces the chart to alarm more frequently leading to deteriorated in-control performance of the EWMA/R control charts.

So far, we have assumed that the j^{th} random sample collected over time includes four pairs of observations (x_i,y_{ij}). In other words, a sample of size 4 is taken to calculate the j^{th} chart statistic every time. Since, in practice, one may use different sample sizes, we have investigated the effect of sample size n on the performances of control charts. Revisiting Figs. 1, 2, and 3 reveals that, in general, all the EWMA-3, EWMA/R, and T^2 control charts produce more false alarms when sample size becomes larger. It can be shown that the mean and

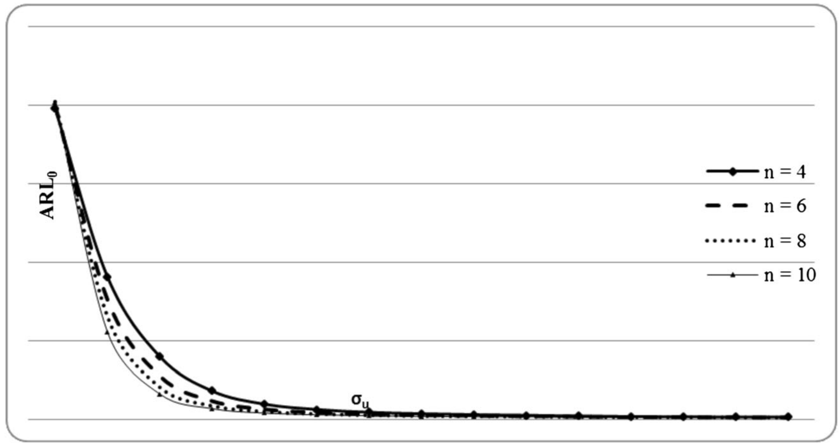

Fig. 1 Effect of measurement error on the in-control ARL of EWMA-3 control chart with various sample sizes n

variance of the EWMA$_N$ statistic are 0 and $2\left(\sigma^2+\lambda\sigma_u^2\beta_1\right)\frac{2}{n-2\frac{\theta}{(2-\theta)}}$, respectively. Increasing the sample size obviously reduces the variability of the EWMA$_N$ statistic, which in turn leads to a more effective detection power. As discussed earlier, measurement error imposes unrealistic shift in the parameters. Due to this fact, the improved detection power reveals itself by having small ARL$_0$ values. It is not difficult to see that the mean and variance of the EWMA statistic can be defined as 0 and $\frac{\theta}{2-\theta}\frac{\left(\sigma^2+\beta_1^2\sigma_u^2\right)}{n}$. Similarly, the variability of EWMA decreases by increasing the sample size leading to more frequent false alarms. Owing to the fact that Hotelling T^2 control chart has the least detection power compared to EWMA-3 and EWMA/R charts, this chart is not very sensitive to the sample size. That is why we do not observe a significant change in the performance of this method.

3.3 Effect of measurement error on the out-of-control ARLs

Another metric commonly used to assess the performance of control charts is the out-of-control ARL or ARL$_1$. An appropriate control chart should yield small ARL$_1$ values when process shifts to an out-of-control condition. In this study, all the control limits are adjusted to yield in-control ARL of approximately 200. Without loss of generality, the standard deviation of measurement error component is set to 0.2. The key point to choose such a small value for the measurement error variance is to demonstrate the adverse results of ignoring even insignificant measurement error value. Table 1 gives the ARL values for the intercept shifts in units of σ_0. Since the EWMA$_I$ chart is unaffected by measurement error, the detection power of EWMA-3 chart remains unchanged. The other two methods alarm more slowly in comparison to the case when no measurement error exists. However, there is not a significant difference between the ARL values in moderate to large shift sizes. Table 2 shows the ARL measures for the slope shifts. Although parameter value is small, there is a remarkable deterioration in the ARL performance of the methods. Table 3 shows that small departures from in-control process standard deviation are detected at a lower rate compared to the ordinary situation. In general, if measurement errors are overlooked, misleading results should be expected.

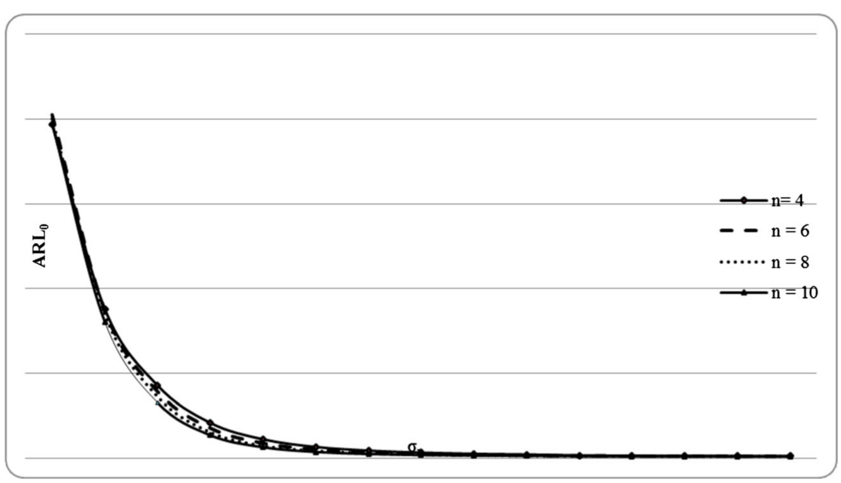

Fig. 2 Effect of measurement error on the in-control ARL of EWMA/R control chart with various sample sizes n

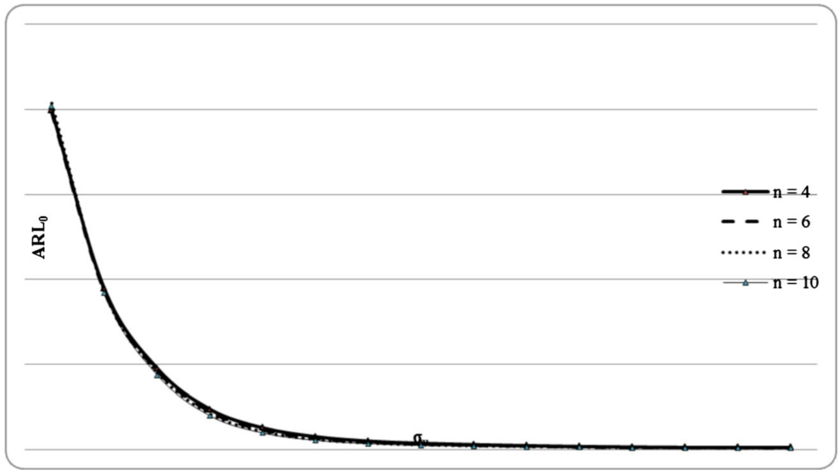

Fig. 3 Effect of measurement error on the in-control ARL of T^2 control chart with various sample sizes n

4 Adjusting the control charts to account for measurement error

Consider the simple linear profile in Eq. (5) which uses the classical additive independent measurement error under the simplified assumption of known reliability ratio λ. In this section, we try to provide remedial measures to lower the effect of measurement error on the three commonly used control charts.

4.1 EWMA-3 control chart

As previously stated, the EWMA_I chart is unaffected by the measurement error. Hence, no modification is required for this chart. However, the other two charts require proper modification. The EWMA_S statistic is modified as

$$EWMA_S(j) = \theta ST_{\widehat{\beta}_1}(j) + (1-\theta)EWMA_S(j-1) \quad j = 1, 2 \quad (19)$$

where $ST_{\widehat{\beta}_1}(j) = \frac{\widehat{\beta}_{1j}-\lambda\beta_1}{\sqrt{\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww}}}$ and $EWMA_S(0)=0$. It also follows that

$$\text{UCL}_S = +L_S\sqrt{\frac{\theta}{2-\theta}} \text{LCL}_S = -L_S\sqrt{\frac{\theta}{2-\theta}}$$

The EWMA_N statistic can be modified to

$$EWMA_N(j) = \max\left\{\theta\left(\text{MSE}_j-\left(\sigma^2+\lambda\sigma_u^2\beta_1\right)\right) + (1-\theta)EWMA_N(j-1), 0\right\} j = 1, 2 \quad (20)$$

The chart will alarm an out-of-control condition if the EWMA_N statistic exceeds the upper control limit defined by

$$\text{UCL}_N = L_N\sqrt{\frac{2\left(\sigma^2+\lambda\sigma_u^2\beta_1\right)^2}{n-2}\frac{\theta}{(2-\theta)}} \quad (21)$$

where

$$EWMA_N(0) = 0$$

Table 1 Effect of measurement error on the out-of-control ARLs when intercept shifts from β_O to $\beta_O+\tau\sigma$

		τ										
Chart	σ_u	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
EWMA-3	0	197.6	155.2	91.8	50.6	29.6	19.2	13.7	10.3	8.3	6.8	5.9
	0.2	199.8	154.1	90.1	51.3	28.6	19.2	13.5	10.1	8.1	6.9	5.2
EWMA/R	0	198.9	65.6	17.7	8.4	5.8	4.5	3.8	2.4	2.2	1.9	1.7
	0.2	199.1	72.7	19.9	9.2	5.8	4.3	3.4	2.9	2.5	2.2	2
T^2	0	197.3	138.5	64.3	28.7	13.1	6.3	4.6	2.5	1.9	1.5	1.1
	0.2	197.1	138.2	69.7	33.3	16	8.5	5	3.1	2.2	1.7	1.4

Table 2 Effect of measurement error on the out-of-control ARLs when slope parameter shifts from β_1 to $\beta_1+\beta\sigma$

		β										
Chart	σ_u	0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
EWMA-3	0	197.0	168.3	117.9	78.7	51.9	35.2	24.8	18.4	14.5	11.6	9.5
	0.2	201.4	187.9	139.9	89.4	59.7	40.9	28.6	21.4	16.6	13.2	10.9
EWMA/R	0	198.1	118.1	42.6	19.6	11.3	7.7	5.8	4.8	3.9	3.5	3
	0.2	202.4	127.4	49.6	22.2	12.9	8.6	6.5	5.1	4.3	3.7	3.3
T^2	0	198.7	166	105.8	61.8	32.4	19.8	13.1	7.9	5.1	3.9	3
	0.2	196.2	172.2	119.5	74.6	43.9	26.9	16.6	10.7	7.2	5.1	3.8

4.2 EWMA/R approach

The EWMA control limits may be modified as

$$\mathrm{UCL} = L\sqrt{\frac{\theta\left(\sigma^2+\sigma_u^2\beta_1^2\right)}{(2-\theta)n}}\quad \mathrm{LCL} = -L\sqrt{\frac{\theta\left(\sigma^2+\sigma_u^2\beta_1^2\right)}{(2-\theta)n}} \tag{22}$$

An out-of-control signal will be given as soon as R statistic falls outside the following control limits

$$\mathrm{UCL} = \sqrt{\left(\sigma^2+\sigma_u^2\beta_1^2\right)}(d_2+Ld_3) \tag{23}$$
$$\mathrm{UCL} = \sqrt{\left(\sigma^2+\sigma_u^2\beta_1^2\right)}(d_2-Ld_3)$$

4.3 T^2 control scheme

Similar to the EWMA-3 control chart, in this approach, the T^2 statistic must be modified. As a result, we have

$$T_J^2 = \left(u_j^{'}-u\right)^T\sum^{-1}\left(u_j^{'}-u\right)\qquad j=1,2$$

where $\mathbf{u} = [\beta_0+\beta_1\mu_x(1-\lambda)\ ,\lambda\beta_1]^T$, and

$$\Sigma = \begin{bmatrix} \frac{\left(\sigma^2+\lambda\sigma_u^2\beta_1\right)}{n}+\overline{w}^2\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww} & -\overline{w}\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww} \\ -\overline{w}\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww} & \frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww} \end{bmatrix} \tag{24}$$

The upper control limit for this chart is given by

$$\mathrm{UCL} = \chi^2_{2,\alpha}$$

5 Setting up the control mechanisms based on an optimal estimator of the true variable *x*

After applying the above-mentioned modifications, the in-control ARL will be adjusted to a desired quantity identical to the ordinary case where no measurement error exists. Although the performance of EWMA-3, EWMA/R, and T^2 control charts has been improved to a certain extent, the control chart modification method does not provide an unbiased estimator for the true predictor variable x. To obtain the true estimator for x variable, we can minimize the mean squared error criteria. As Carroll et al. [7] mentioned, the best linear prediction can be given by

$$wblp_i = (1-\lambda)\mu_x+\lambda w_i\qquad i = 1,2\ldots n \tag{25}$$

Table 3 Effect of measurement error on the out-of-control ARLs when process standard deviation shifts from σ to $\xi\sigma$

		ξ										
Chart	σ_u	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
EWMA-3	0	198.2	34.7	12.3	6.6	4.5	3.4	2.8	2.3	2.1	1.9	1.7
	0.2	199.3	46	17	9.3	6.2	4.7	3.8	3.2	2.9	2.6	2.3
EWMA/R	0	196.9	33.5	11.8	5.9	3.8	2.8	2.2	1.8	1.6	1.5	1.4
	0.2	197.2	40.9	14.7	7.3	4.5	3.2	2.5	2.1	1.8	1.6	1.5
T^2	0	200.2	39.1	15.5	7.6	5	4	2.9	2.3	2.1	2.1	1.6
	0.2	198.3	46.4	18.1	9.6	6.1	4.4	3.4	2.8	2.4	2.1	1.9

According to Eq. (2), we have $\mu_x=\mu_w$. The MSE for this variable is given as

$$\mathrm{MSE}_{wblp}=\lambda\sigma_u^2 \tag{26}$$

Obviously, the mean squared error for the observed variable w can be defined as

$$\mathrm{MSE}_w=\sigma_u^2 \tag{27}$$

It is reasonable, thus, to use *wblp* to determine the unobserved x variable. Another scenario to improve control chart performance is to use *wblp* variable as a substitute for the observed variable w since the OLS method yields unbiased estimators. The main reasons for constructing the control charts based on *wblp* variable can be summarized as

1. This variable gives us an unbiased estimated of the true, unobserved variable. This may be valuable in some practical situations where practitioners are interested in garnering information about the explanatory variable.
2. As we will discuss in more depth, some of the statistics built upon *wblp* have less variability. Decreasing the variability gives higher detection power since the potential shifts in the parameters seem larger when the variance of the statistic is smaller.

Since the OLS estimators of the regression of y on *wblp* are unbiased estimators, the $ST_{\widehat{\beta}_1}$ statistic can be expressed as

$$ST_{\widehat{\beta}_1}(j)=\frac{\widehat{\beta}_{1j}-\beta_1}{\sqrt{\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{\lambda^2 Sww}}}\qquad j=1,\ 2 \tag{28}$$

Superficially, one may conclude that since the variability of slope estimator increases from $\sqrt{\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww}}$ to $\sqrt{\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{\lambda^2 Sww}}$, the performance of EWMA-3 control scheme is deteriorated considerably. Suppose that, in Eq. (18), the slope parameter shifts from β_1 to $\beta_1+\beta\sigma$. Given that variable is used, the mean of EWMA_S statistic is calculated as

$$E\left[EWMA_S\middle|wblp\right]=\frac{\beta\sigma}{\sqrt{\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{\lambda^2 Sww}}}=\frac{\lambda\beta\sigma}{\sqrt{\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww}}} \tag{29}$$

The expected value of EWMA_S statistic when using w variable is given by

$$E\left[EWMA_S\middle|w\right]=\frac{\lambda\beta\sigma}{\sqrt{\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww}}} \tag{30}$$

When slope shifts, the mean of EWMA_S statistic shifts from zero to $\frac{\lambda\beta\sigma}{\sqrt{\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{Sww}}}$. This implies that the detection power of this chart is equivalent to the situation where we establish the EWMA_S chart based on the observed variable w. Since none of the components of EWMA-3 control chart changes when employing *wblp*, it is reasonable to expect that the charting performance of the EWMA-3 control chart would not be affected when we construct this approach using *wblp*. Hence, we did not use this variable to establish the EWMA-3 method.

The EWMA/R control strategy can be also constructed using *wblp* variable. It is not hard to show that the control limits for the EWMA chart are changed to

$$\mathrm{UCL}=L\sigma\sqrt{\frac{\theta\left(\sigma^2+\beta_1^2\sigma_X^2(1-\lambda)\right)}{(2-\theta)n}}$$
$$\mathrm{LCL}=-L\sigma\sqrt{\frac{\theta\left(\sigma^2+\beta_1^2\sigma_X^2(1-\lambda)\right)}{(2-\theta)n}} \tag{31}$$

In this case, the upper and lower control limits associated with R statistic are represented by

$$\mathrm{UCL}=\left(\sigma^2+\beta_1^2\sigma_X^2(1-\lambda)\right)(d_2+lld_3)\,\mathrm{LCL}$$
$$=\left(\sigma^2+\beta_1^2\sigma_X^2(1-\lambda)\right)(d_2-lld_3) \tag{32}$$

Table 4 ARL comparisons when intercept shifts from β_0 to $\beta_0+\tau\sigma$

		τ										
Control scheme	Estimating strategy	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
EWMA-3	Observed variable	198.6	154.9	89.9	50.5	30	19	13.7	10.4	8.3	6.9	5.8
EWMA/R	Observed variable	199.8	145.4	76.4	38.1	21.7	14	10.1	7.6	6.2	5.3	4.6
	wblp	198.4	132.6	59	26.7	15.2	10.1	7.4	5.8	4.8	4.1	3.6
T^2	Observed variable	202.2	182.2	136.5	94.8	63.3	41	27.5	18.7	13	9.1	6.7
	wblp	201.3	181.1	135.7	94.3	62.8	40.8	26.9	18.6	12.5	8.9	6.2

Table 5 ARL comparisons when slope shifts from β_1 to $\beta_1+\beta\sigma$

		β										
Control scheme	Estimating strategy	0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
EWMA-3	Observed variable	197.6	154.4	119.1	78.1	51.9	36.5	26.7	19.8	15.9	12.8	10.7
EWMA/R	Observed variable	196.7	178.1	125.9	83.5	53.6	34.8	26.7	19.9	15.7	12.9	10.2
	wblp	196.2	153.2	94.2	56.5	33.5	22.3	15.6	11.9	9.4	7.8	6.5
T^2	Observed variable	202.6	175.7	141	108.4	81.7	60.4	44.8	33.4	25.4	19.4	14.9
	wblp	200.5	176.1	140.8	107.9	81.2	60.6	44.1	33.1	25.2	19.2	14.1

Since the variances of EWMA and R statistics are smaller compared to those when applying error-prone variable w, the detection power of this chart must be improved. Hence, for the EWMA/R chart, we have been able to improve the performance using *wblp* variable.

Finally, the T^2 statistic is given as

$$T_J^2 = \left(u_j^{'}-u\right)^T \sum^{-1} \left(u_j^{'}-u\right) j = 1, 2$$

where $\mathbf{u} = [\beta_0, \beta_1]^T$ $\mathbf{u} = [\beta_0, \beta_1]^T$ and

$$\Sigma = \begin{bmatrix} \frac{(\sigma^2+\lambda\sigma_u^2\beta_1)}{n} + \overline{wblp}^2 \frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{\lambda^2 Sww} & -\overline{wblp}\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{\lambda^2 Sww} \\ -\overline{wblp}\frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{\lambda^2 Sww} & \frac{\left(\sigma_y^2-\sigma_{wy}\lambda\beta_1\right)}{\lambda^2 Sww} \end{bmatrix} \quad (33)$$

where $\overline{wblp}$ is the sample mean of *wblp* variable.

5.1 ARL performance comparison of the reconstructed charts using *wblp* variable

To ascertain whether establishing the control charts using *wblp* is effective or not, we have conducted a set of simulations. In this set, we tried to compare the performance of two types of charts. The first type is the charts suggested in Sect. 4 where all the charts were modified to include measurement error using the observed variable w. The second type is the charts set up using *wblp* variable. Here, we assume that the standard deviation of the measurement error is 1, i.e., $\sigma_u^2=1$. Table 4 presents the ARL values with respect to the intercept shifts. The EWMA/R chart acts better than the other methods since the best linear predictor of true variable x is applied. The deteriorated behavior of EWMA-3 charts to detect the intercept shifts is due to the variance inflation related to the intercept estimator in the random-x case. There are two main features that should be considered in Table 5. First, the *wblp*-based EWMA/R chart outperforms the competitive methods in discovering the slope parameter shifts. Second, the superiority of EWMA-3 approach is observable when error-contaminated data is used. Table 6 gives the ARLs when standard deviation shifts away from the in-control condition. Clearly, the EWMA-3 and *wblp*-based EWMA/R charts perform equally.

6 Conclusions

It is very clear that when measurement error is ignored, undesired or even serious results should be expected in terms of nonconforming products or services. Our study here shows that measurement error affects both the in-control and out-of-control performance of the commonly used phase II profile monitoring methods. To improve performances of the methods discussed in this study, we modified the control limits. Taking such an action, we have been able to equip the control charts against the measurement error. This implies that the practical ARL_0 will be exactly the same as the

Table 6 ARL comparisons when standard deviation shifts from σ to $\xi\sigma$

		ξ										
Control scheme	Estimating strategy	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
EWMA-3	Observed variable	198.9	102.3	54.5	32.6	20.3	13.7	9.8	7.5	6	5	4.2
EWMA/R	Observed variable	199.2	122.7	78.4	50.7	33.2	22.4	16.3	12.1	9.2	7.1	5.8
	wblp	198.7	102.7	56.7	33.8	20.5	13.8	9.9	7.1	5.5	4.5	3.7
T^2	Observed variable	197.1	116.3	71.5	44.1	29.4	20.1	14.7	11	8.6	7	5.7
	wblp	198.9	115.1	71.8	44	29.1	19.9	13.9	11.4	8.1	6.8	5.2

nominal, desired one. Since the suggested charts do not offer a suitable estimate for the true x variable, an unbiased estimator referred to as *wblp* was used. This variable helps the detection power of the EWMA/R method to improve. As quick detection is an important property of control charts, we aim at acquiring better detection power by applying *wblp* variable in establishing the control charts. It is straightforward to show that the EWMA-3 and T^2 control mechanisms do not improve using *wblp*. However, the EWMA/R chart seems to significantly improve when it is built upon *wblp*. A small-size shift in the intercept of a profile can be detected 22 % quicker when we take advantage of *wblp* variable instead of observed variable w.

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Kang L, Albin SL (2000) On-line monitoring when the process yields a linear profile. J Qual Technol 32:418–426
2. Kim K, Mahmoud MA, Woodall WH (2003) On the monitoring of linear profiles. J Qual Technol 35:317–328
3. Mahmoud MA, Woodall WH (2004) Phase I analysis of linear profiles with calibration applications. Technometrics 46:380–391
4. Mahmoud Mahmoud A, Parker Peter A, Woodall William H, Hawkins Douglas M (2007) A change point method for linear profile data. Qual Reliab Engng Int 23:247–268
5. Yeh AB, Zerehsaz Y (2013) Phase I control of simple linear profiles with individual observations. Qual Reliab Engng Int 29:829–840
6. Noorossana, R., Fatemi A., Zerehsaz, Y., (2014) Phase II monitoring of linear profiles with random explanatory variable. To appear in the J Advance Manufac Technol
7. Carroll Raymond J., David Ruppert, Leonard A. Stefanski, Ciprian M. Crainiceanu, (2006) Measurement error in nonlinear models, 2nd edition, Chapman & Hall/CRC Taylor and Francis Group, ISBN-10: 1-58488-633-1, ISBN-13: 978-1-58488-633-4
8. Fuller, W. A. (1987) Measurement error models, 1st edition, New York: Wiley, ISBN-13: 978-0470095713 ISBN-10: 0470095717
9. Bennett CA (1954) Effect of measurement error on chemical process control. Indust Qual Contrl 10:17–20
10. Kanazuka T (1986) The effects of measurement error on the power of -R charts. J Qual Technol 18:91–95
11. Mittag, H.-J. (1995) Measurement error effects on control chart performance. ASQC4 9th Annual Quality Congress Proceedings 66–73
12. Mittag H-J, Stemann D (1998) Gauge imprecision effect on the performance of the X-S control chart. J Appl Stat 25:307–317
13. Linna KW, Woodall WH (2001) Effect of measurement error on Shewhart control charts. J Qual Technol 33:213–222
14. Maravelakis PE, Panaretos J, Psarakis S (2004) EWMA chart and measurement error. J Appl Stat 31:445–455
15. Linna KW, Woodall WH, Busby KL (2001) The performance of multivariate control charts in the presence of measurement error. J Qual Technol 33:349–355
16. Montgomery D. C., (2013) Introduction to linear regression analysis, 5th edition, Wiley, ISBN Number: 978-0-470-54281-1
17. Montgomery D. C., (2013). Introduction to statistical quality control, 7th edition, Wiley, ISBN Number: 0471656313, 9780471656319, 978-0471656319
18. Neter John, Michael H. Kutner, William Wasserman, (2004) Applied linear regression models, 2nd edition, McGraw-Hill/Irwin, ISBN-13: 978-0073014661 ISBN-10: 0073014664