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ORIGINAL ARTICLE

Developing a fuzzy multivariate CUSUM control chart to monitor multinomial linguistic quality characteristics

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Abstract Currently, there are many situations in industry where simultaneous monitoring and control of two or more related quality characteristics of a product or a process becomes necessary. Independent monitoring of these kinds of quality characteristics can be very deceptive. Conventional multivariate control charts have been used to monitor and control the multivariate quality characteristics. However, these types of charts are not useful tools when the quality characteristics of a product or process are linguistic or fuzzy. Hence, fuzzy Hotelling's T^2 chart (F-T^2) and fuzzy multivariate exponentially weighted moving average (F-MEWMA) control chart have been used to monitor these processes. In this paper, a fuzzy multivariate cumulative sum (F-MCUSUM) control chart is developed by means of the fuzzy set theory. Through a numerical comparison via a simulation study, the performance of the developed control approach is investigated on the basis of the average run length (ARL) in various out-of-control scenarios. When small shifts make the process out of control, the F-MCUSUM control chart is almost two times quicker than the F-T^2 and F-MEWMA control charts in detecting shifts. The results of numerical comparison indicate better performance of developed multivariate control approach in detecting small- and medium-sized shifts in the process. A case study in food industry is utilized to show the applicability of the proposed approach and the interpretation of the out-of-control signals.

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Abbreviation

F-MCUSUM	Fuzzy multivariate cumulative sum
F-MEWMA	Fuzzy multivariate exponentially weighted moving average

1 Introduction

In empirical applications in industry, it is inevitable to monitor simultaneously two or more related quality characteristics (QCs) of a process or a product. Despite an extensive use of univariate control charts, there are many situations where the QCs have some interactions, so monitoring the process via univariate control charts can be very misleading to inefficient results. The multivariate control charts are more sensitive than univariate ones [1]. Several types of multivariate control charts, including Hotelling's χ^2 and T^2, MCUSUM, and MEWMA, have been developed in attempts to improve concurrent monitoring of correlated QCs by using the correlation structure that exists between QCs. If the correlation structure between the QCs is ignored and a set of separate univariate control charts is used instead for monitoring purposes, then inefficient and potentially misleading results could be expected [2]. MCUSUM and MEWMA control charts use the process information from both past and present samples, and therefore, compared to the T^2 control chart which uses only the current information of process, they are sensitive to small process variations.

On the other hand, in some real cases, the quality characteristics are associated with some uncertainty, fluctuations, imprecision, and vagueness. Therefore, binary classification of these quality characteristics is not suitable because the quality of a product or a process cannot change completely from perfect to imperfect. Some QCs such as appearance, taste, hardness, color, etc. cannot be expressed numerically. Thus, some intermediate assessments are required and QCs should be explained by some linguistics such as good, average, poor, etc. Unlike traditional approaches in SPC, the fuzzy set theory can put up with imprecision and uncertainties without loss of performance and effectiveness [3]. The fuzzy set theory supports subjective natural language descriptors of quality and provides a methodology for allowing them to enter into the modeling process. Measurement of quality in the context of the fuzzy set theory where conventional modeling is not applicable provides an opportunity to monitor process appropriately. A major contribution of the fuzzy set theory is its capability to represent and model linguistic data [2].

Monitoring of multivariate fuzzy QCs involves some fuzzy multivariate quality control charts which are an extension of conventional ones through fuzzy set theory applications. In this paper, a fuzzy multivariate cumulative sum control chart (F-MCUSUM) is developed by using the fuzzy set theory and conventional MCUSUM control chart in order to monitor multivariate attribute fuzzy QCs expressed in linguistic verbal words.

2 Literature review

In literature, fuzzy control charts are classified mainly into two categories: fuzzy univariate control charts and fuzzy multivariate control charts. In literature review of fuzzy univariate control charts, it can be found that Bradshaw [4] initially introduced fuzzy control charts. Then, two approaches named a probabilistic approach and a membership approach were proposed for the construction of control charts by Raz and Wang [5]. More of the developments in fuzzy univariate control charts were in the field of Shewhart-type control charts and have been done by many researchers [6–22]. A fuzzy approach for attribute control charts in a multistage process was presented by Engin et al. [23]. The degree of nonconformity on the basis of fuzzy concepts was defined by Amirzadeh et al. [24] for fuzzy attribute quality characteristics. Senturk and Erginel [25] transformed numeric control limits to fuzzy ones by using α-cuts in fuzzy $\widetilde{\overline{X}}-\widetilde{R}$ and $\widetilde{\overline{X}}-\widetilde{S}$ control charts. In the area of non-Shewhart-type control charts, the classical cumulative sum (CUSUM) chart was applied by Wang [26] for representative values of fuzzy quality data. A fuzzy multinomial process control chart with variable sample size was proposed by Pandurangan and Varadharajan [27] and its control limits were obtained by using multinomial distribution. Hou and Tong [28] proposed a fuzzy quality control chart by analyzing the rules for judging the abnormity of control chart. In the field of quality profile monitoring, Noghondarian and Ghobadi [29] used some fuzzy control charts for separately monitoring intercept and slope of fuzzy quality profiles and recommended the use of their approach for detecting medium and large shifts in the process profiles. The power of fuzzy I-MR control charts in terms of signal probability was calculated by Ghobadi and Noghondarian [30] in monitoring the fuzzy quality profiles.

In a literature review of fuzzy multivariate quality control, a neural-fuzzy model was developed for detecting a mean shift and classifying their magnitude in a multivariate process by Wang and Chen [31]. Also, correct classification percentages and some general guidelines for the proper use of their model was developed in [31]. Construction of control chart for monitoring multivariate attribute processes when data is presented in multi-dimensional linguistic form is analyzed by using fuzzy sets and probability theories in [32] and [33]. Two monitoring statistics T^2 and W^2 were developed on the basis of fuzzy and probability theories. The first is similar to the Hotelling's T^2 multivariate statistic on the basis of representative values of fuzzy sets. The W^2 statistic is a linear combination of dependent chi-square variables. A fuzzy multinomial chart was introduced by Amirzadeh et al. [34] for monitoring a multinomial process where there were more than two categories of the specifications with respect to the quality characteristic. Control limits of their chart were obtained by using the multinomial distribution and the degrees of membership assigned to the distinct categories. Then, multivariate variable control charts were evolved in a fuzzy environment where each observation in each sample is assumed to be a canonical fuzzy number [35]. Alipour and Noorossana [2] developed fuzzy multivariate exponentially weighted moving average (F-MEWMA) control chart and indicated uniformly the superior performance of the F-MEWMA control chart over Fuzzy T^2 control chart introduced by Taleb.

In recent researches carried out in the field of fuzzy multivariate control charts, all studies have been done about T^2 and MEWMA control charts, and no work has been done about MCUSUM in fuzzy space. MCUSUM control chart, like MEWMA control chart, has been shown to be more efficient in detecting small shifts in the mean of a process. In particular, the analysis of average run length (ARL) for MCUSUM control charts shows that they are better than Shewhart-type control charts when it is expected to detect small and medium shifts in the process [36]. However, the application of multivariate CUSUM control chart in a fuzzy environment was not investigated in the previous researches. Also, its performance in detecting shifts in a process, including fuzzy quality characteristics, has not been explored yet. In this paper, fuzzy multivariate cumulative sum control charts is developed for monitoring various correlated attribute QCs when expressed in linguistic verbal words. Then, a case study in food industry is presented to illustrate the application of the developed control chart and the interpretation of the out-of-control signals. Concepts and definitions needed for fuzzy multivariable control methods are described in Section 3. Design and development of F-MCUSUM control charts are also described in Section 3. In Section 4, the interpretation of an out-of-control signal on F-MCUSUM control chart is discussed. Application of a proposed chart is perused through a case study originated from [32] in Section 5. The performance of F-MCUSUM chart is investigated on the basis of ARL and is compared to F-T^2 and F-MEWMA charts in Section 6. Finally, in Section 7, the conclusions of research and suggestions for future studies are presented.

3 Fuzzy multivariate CUSUM control chart

In cumulative sum control charts, sums are accumulated, but an observation is *accumulated* only if it differs from the goal value (e.g., the estimate of the process mean or zero) by more than K units. Parameter K is named as a *reference value* [37]. The MCUSUM control chart, similar to its univariate version, utilizes a series of observations rather than simply taking the current sample used in T^2 chart. The MCUSUM is more sensitive to small shifts and, therefore, could be an effective tool for quickly identifying the change point. Monitoring QCs includes two phases: phase I and phase II. In phase I, a set of process sample data collected over time is analyzed to identify any out-of-control samples and then to remove them from the data set in order to strictly estimate the in-control process parameters for using in phase II. The main concern in phase II is to quickly detect shifts in the process parameters via online monitoring the QCs. F-MCUSUM control charts are constructed through phase I, and after chart abilities were checked, they will be utilized for online monitoring purposes in phase II [29].

3.1 The framework of fuzzy multivariate control chart

Suppose that p fuzzy attribute quality characteristics, Q_1, $Q_2,\ldots,Q_p$, are simultaneously monitored. Quality characteristic Q_j is a linguistic variable and is expressed as a set of terms $T(Q_j)$. As shown in Fig. 1, each term set is determined by k_j words q_{jh} that are identified by a triangular fuzzy set F_{jh} and membership function $\mu_{jh}(x)$, where x is a measure of the quality level. The scale of x varies from zero to one so that zero is equivalent to the best quality and one indicates the worst quality.

$$T\left(Q_j\right)=\left\{q_{j1},q_{j2},\ldots,q_{jh}\right\} \qquad \text{for} \quad h=1,2,\ldots,k_j \tag{1}$$

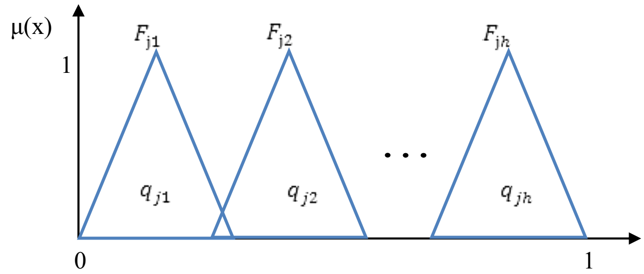

Fig. 1 Quality characteristics Q_j

Each sample contains n observations composed of multivariate QCs. The quality levels of any QCs are determined by an expert in terms of linguistic words. So, each sample can be shown as below:

$$\left\{\left\{\left(F_{11},n_{11}\right),\ldots,\left(F_{1m_1},n_{1k_1}\right)\right\};\ldots;\left\{\left(F_{p1},n_{p1}\right),\ldots,\left(F_{pk_p},n_{pk_p}\right)\right\}\right\} \tag{2}$$

n_{jh} is the number of observations in a sample that their quality characteristic Q_j has been assessed as term q_{jh} and should meet the following condition.

$$\sum_{h=1}^{K_j} n_{jh}=n \tag{3}$$

Using fuzzy mathematics, the quality characteristic Q_j converts to a fuzzy set via the following relation:

$$F_j=\frac{1}{n}\sum_{h=1}^{K_j} n_{jh}F_{jh}. \tag{4}$$

Thus, each sample can be revealed as $P\times 1$ vector $(F_1,\ldots,F_p)$, comprised p number of fuzzy set F_j indicating measure of fuzzy quality of related QC.

3.2 Developing F-MCUSUM control chart

After sample vector was determined, the representative value of each vector should be calculated on the basis of fuzzy median method. Since the internal distribution of fuzzy data may be asymmetric, fuzzy median is proposed for converting [32]. Assume that each triangular fuzzy number cab be shown as (a_{1j}, a_{2j}, a_{3j}). The representative value by using the fuzzy median transformation method is as follows.

$$Rj=\begin{cases} a_{3j}-\sqrt{\dfrac{\left(a_{3j}-a_{1j}\right)\left(a_{3j}-a_{2j}\right)}{2}} & \text{for} \quad a_{2j}<\dfrac{a_{3j}+a_{1j}}{2} \\ a_{1j}+\sqrt{\dfrac{\left(a_{3j}-a_{1j}\right)\left(a_{2j}-a_{1j}\right)}{2}} & \text{for} \quad a_{2j}>\dfrac{a_{3j}+a_{1j}}{2} \end{cases} \tag{5}$$

Afterwards, the representative values for m samples can be specified same as relation 6. The R_{ij} is the representative values of fuzzy quality measure F_{ij} in the sample i (i=1,2,…,m).

$$R = \begin{pmatrix} R_{11} & R_{12} & \dots & R_{1p} \\ R_{21} & R_{22} & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ R_{m1} & \dots & \dots & R_{mp} \end{pmatrix} \tag{6}$$

In this research, Crosier's [38] conventional statistic is employed as a basis for constructing F-MCUSUM control chart. Mean vector $\overline{R}$ and sample covariance matrix of R_{ij} are calculated by using relations 7, 8, and 9, respectively. Then, they will be replaced with mean vector μ and covariance matrix Σ of in-control QCs in Crosier's statistic.

$$\overline{R}_j = \frac{1}{m}\sum_{i=1}^{m} R_{ij} \tag{7}$$

$$S_j^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(R_{ij}-\overline{R}_{ij}\right)^2 \tag{8}$$

$$S_{jh} = \frac{1}{m-1}\sum_{i=1}^{m}\left(R_{ij}-\overline{R}_j\right)\left(R_{ik}-\overline{R}_k\right) \quad \text{for } j\neq h \tag{9}$$

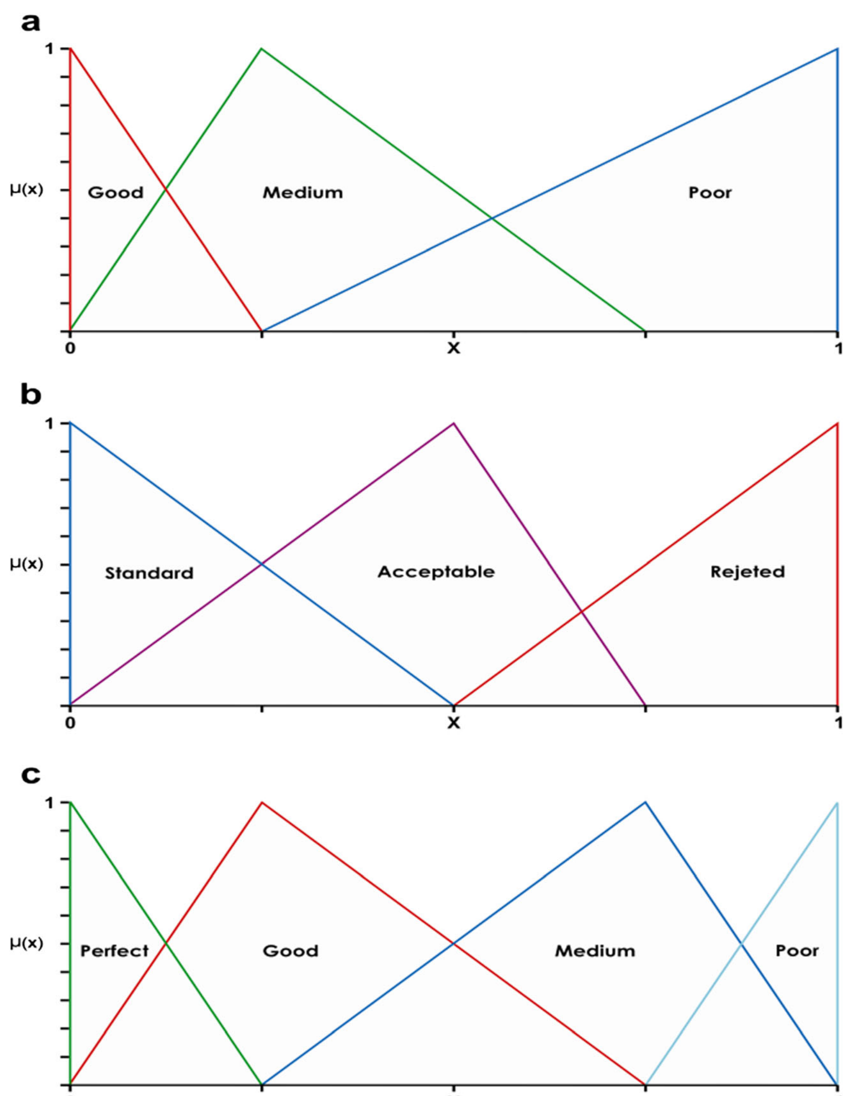

Fig. 2 Sets of membership functions related to **a** appearance, **b** color, and **c** taste

Table 1 Process data related to appearance, color, and taste of frozen food process

Sample	Appearance			Color			Taste			
	Good q_{11}	Medium q_{12}	Poor q_{13}	Standard q_{21}	Acceptable q_{22}	Rejected q_{23}	Perfect q_{31}	Good q_{32}	Medium q_{33}	Poor q_{34}
1	210	7	3	206	9	5	167	48	3	2
2	211	6	3	207	8	5	176	42	2	0
3	206	9	5	202	12	6	163	55	2	0
4	211	5	4	207	8	5	163	51	5	1
5	203	16	1	194	18	8	175	45	0	0
6	210	6	4	206	9	5	174	44	1	1
7	208	7	5	204	9	7	174	40	5	1
8	207	7	6	204	9	7	169	46	3	2
9	206	7	7	202	9	9	169	48	2	1
10	186	25	9	200	12	8	169	48	3	0
11	196	13	11	196	13	11	163	46	10	1
12	203	12	5	200	13	7	167	44	9	0
13	203	9	8	198	11	11	174	42	3	1
14	202	9	9	198	11	11	174	40	6	0
15	209	6	5	207	9	4	172	42	5	1
16	210	3	7	205	5	10	172	44	4	0
17	205	11	4	201	13	6	172	45	2	1
18	210	6	4	206	8	6	169	48	2	1
19	206	10	4	203	13	4	172	46	0	2
20	206	12	2	202	14	4	169	46	5	0

Table 2 F-MCUSUM statistic values

Sample number	R_i			$\left(\mathrm{Sum}_{i-1}+R_i-\overline{R}\right)$			C_i	$C_i \leq K$	$(1-K/C_i)$	Sum_i			F-MCUSUM$_i$
1	0.0898	0.1722	0.1410	−0.0096	−0.008	0.0059	1.4691	No	0.3346	−0.0032	−0.0027	0.0020	0.4916
2	0.0887	0.1711	0.1240	−0.0138	−0.0118	−0.009	1.8198	No	0.4629	−0.0064	−0.0055	−0.0042	0.8424
3	0.0981	0.1788	0.1383	−0.0077	−0.0069	−0.0009	0.8593	Yes	0	0.0000	0.0000	0.0000	0.0000
4	0.0907	0.1711	0.1458	−0.0087	−0.0092	0.0108	1.9912	No	0.5091	−0.0044	−0.0047	0.0055	1.0137
5	0.0935	0.1921	0.1220	−0.0104	0.0072	−0.0076	2.4444	No	0.6001	−0.0062	0.0044	−0.0045	1.4667
6	0.0918	0.1722	0.1274	−0.0138	−0.0036	−0.0122	1.9156	No	0.4897	−0.0068	−0.0018	−0.0060	0.9381
7	0.0959	0.1784	0.1337	−0.0102	−0.0036	−0.0073	1.2458	No	0.2153	−0.0022	−0.0008	−0.0016	0.2682
8	0.0990	0.1784	0.1388	−0.0026	−0.0026	0.0022	0.4691	Yes	0	0.0000	0.0000	0.0000	0.0000
9	0.1021	0.1847	0.1344	0.0027	0.0044	−0.0006	0.56	Yes	0	0.0000	0.0000	0.0000	0.0000
10	0.1280	0.1851	0.1333	0.0286	0.0048	−0.0018	3.707	No	0.7363	0.0211	0.0036	−0.0013	2.7293
11	0.1210	0.1956	0.1537	0.0427	0.0189	0.0174	4.353	No	0.7754	0.0331	0.0147	0.0135	3.3754
12	0.1014	0.1831	0.1450	0.0351	0.0176	0.0234	4.0789	No	0.7604	0.0267	0.0134	0.0178	3.1014
13	0.1073	0.1932	0.1305	0.0347	0.0264	0.0133	3.7224	No	0.7374	0.0256	0.0194	0.0098	2.7449
14	0.1104	0.1932	0.1325	0.0366	0.0325	0.0073	4.0889	No	0.7609	0.0279	0.0247	0.0056	3.1113
15	0.0949	0.1691	0.1359	0.0233	0.0136	0.0064	2.2745	No	0.5702	0.0133	0.0078	0.0037	1.2969
16	0.0977	0.1831	0.1316	0.0116	0.0106	0.0002	1.3331	No	0.2667	0.0031	0.0028	0.0000	0.3554
17	0.0972	0.1800	0.1312	0.0009	0.0026	−0.0038	0.6197	Yes	0	0.0000	0.0000	0.0000	0.0000
18	0.0918	0.1742	0.1344	−0.0076	−0.0061	−0.0006	0.7992	Yes	0	0.0000	0.0000	0.0000	0.0000
19	0.0961	0.1738	0.1308	−0.0033	−0.0064	−0.0043	1.0832	No	0.0976	−0.0003	−0.0006	−0.0004	0.1057
20	0.0922	0.1750	0.1364	−0.0075	−0.0059	0.001	0.841	Yes	0	0.0000	0.0000	0.0000	0.0000

The statistic of the F-MCUSUM control charts for monitoring multivariate processes is

$$\text{F-MCUSUM}_i = \sqrt{\text{Sum}_i \sum^{-1} \text{Sum}_i} \tag{10}$$

where Sum_i is the multivariate cumulative sum related to sample i and computes as follows:

$$\text{Sum}_i = \begin{cases} (1-K/C_i)\left(\text{Sum}_{i-1} + R_i - \overline{R}\right) & \text{if } C_i > K \\ 0 & \text{if } C_i \leq K \end{cases} \tag{11}$$

$$C_i = \sqrt{\left(\text{Sum}_{i-1} + R_i - \overline{R}\right) \sum^{-1} \left(\text{Sum}_{i-1} + R_i - \overline{R}\right)} \tag{12}$$

K is the reference value and equals to $\left(\sqrt{\delta \sum^{-1} \delta}\right)/2$ in conventional control chart. δ is a $P \times 1$ vector $(\delta_1, \ldots, \delta_p)$, comprised p value of δ_j indicating the out-of-control range of Q_j. The vector δ specifies the out-of-control range of process corresponding to $\overline{R}_{\text{out of control}} = \overline{R}_{\text{in control}} + \delta$. The values of δ is determined by decision maker due to current performance of process.

3.3 Control limit of F-MCUSUM control chart

An alarm indicating an out-of-control process is announced when $\text{F-MCUSUM}_i > H$ where H is a predefined threshold yielding the expected in-control ARL. It is understood that the statistical distribution of process statistic is required to determine the control limit. But it is very hard to guesstimate the distribution of statistic in F-MCUSUM. Therefore, by using some resampling methods such as bootstrap or jackknife, we identify the decision interval H. Detailed calculations for specifying the H through bootstrap resampling are described in Section 5.2 by using a case study.

Table 3 New samples of frozen food process

K	Appearance			Color			Taste			
	q_{11}	q_{12}	q_{13}	q_{21}	q_{22}	q_{23}	q_{31}	q_{32}	q_{33}	q_{34}
21	202	10	8	204	11	5	169	44	5	2
22	184	25	11	206	12	2	174	44	1	1
23	208	7	5	196	13	11	174	44	1	1
24	206	6	8	196	13	11	174	40	5	1
25	210	2	8	198	12	10	165	44	1	10

4 Interpretation of out-of-control signals

The interpretation of an out-of-control signal on multivariate control chart has been investigated by Mason et al. [39]. Plotting a separate univariate control chart for each QC is a conventional method to identify the responsible for an out-of-control alarm. However, the use of separate charts does not take into account the information relating to the correlation of the QCs. Taleb et al. [32] offered utilizing simultaneously multivariate and univariate control charts for this purpose. Thus, F-MCUSUM statistic should be decomposed into its components to reveal the contribution of each individual QC in a signal. If F-MCUSUM_i is the current value of the control chart statistic, then the y_j, named reduced statistic, is the value of the statistic for all process QCs except the jth one. Also, we use the criterion d_j for the absolute difference between reduced statistic and F-MCUSUM statistic. The criterion d_j is an indicator of the role of the jth QC in the overall statistic. Subsequently, QC with the biggest value of d_j is the main responsible for the out-of-control signal. In that case, samples related to the out-of-control statistics should be omitted to construct the new modified control chart.

$$d_j = \left|\text{F-MCUSUM}_i - y_j\right| \quad j = 1, \ldots, p \tag{13}$$

Additionally, it is recommended that a run chart of R_j for each QCs should be used for a better interpretation. The QC

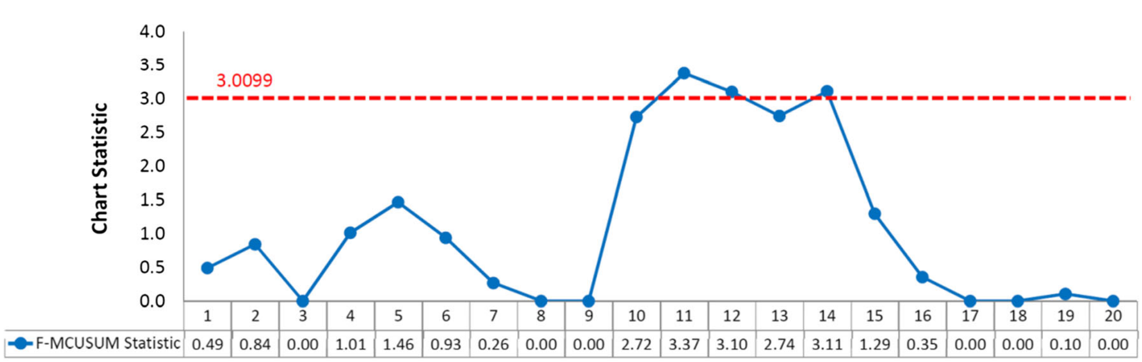

Fig. 3 Fuzzy multivariate CUSUM control chart

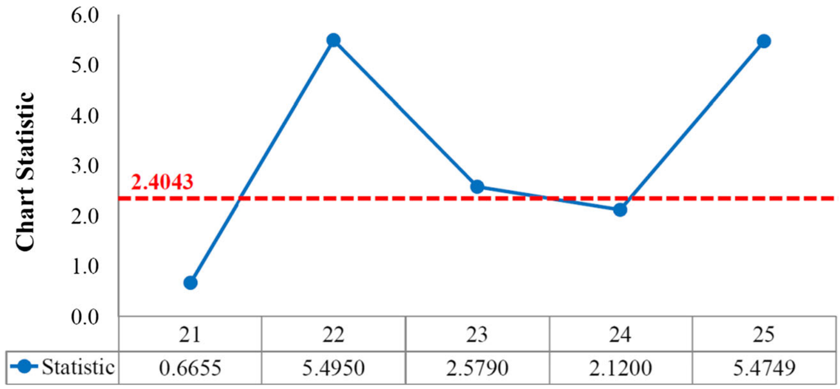

Fig. 4 Fuzzy multivariate CUSUM control chart in phase II

with the largest sway in its trend has a much more prominent role than the others in leading the process to the out-of-control status. Since inferring separately from two interpretation methods may be incomplete and lead to seductive perceptions, we suggest the combination of reduced statistics method and run chart method to be used mutually to interpret the out-of-control signals.

5 Numerical example

In this section, a case study in food industry is presented to show the application of the proposed chart and its comparison with fuzzy T^2 and F-MEWMA chart. In producing a frozen food, the appearance, color, and taste are key characteristic of product quality and should be simultaneously monitored and controlled. The corresponding linguistic term, set for the abovementioned QCs, are as follows:

$$T\ (Q_1) = \{q_{11}, q_{12}, q_{13}\} = \{\text{good, medium, poor}\}$$
$$T\ (Q_2) = \{q_{21}, q_{22}, q_{23}\} = \{\text{standard, acceptable, rejected}\}$$
$$T(Q_3) = \{q_{31}, q_{32}, q_{33}, q_{34}\} = \{\text{perfect, good, medium, poor}\}$$

Membership functions of these terms are illustrated in Fig. 2. According to Table 1, in this case study, 20 samples, each with size 220, are used for monitoring a linguistic QC vector. In each sample, n_{jh} is the number of the products with their quality characteristic Q_j assessed as a linguistic term q_{jh}.

5.1 Constructing F-MCUSUM control chart

F_j representing the fuzzy set for the quality characteristics Q_j in a given sample is determined by Eq. 4. Membership functions of F_j are calculated by using the fuzzy set theory. For example, for sample 1, the triangular membership function associated with F_{11} is obtained as follows:

$$F_{11} = \frac{1}{220}[270 \quad 7 \quad 3]\begin{bmatrix} 0 & 0 & 0.25 \\ 0 & 0.25 & 0.75 \\ 0.25 & 1 & 1 \end{bmatrix}$$
$$= [0.00341 \quad 0.02159 \quad 0.27614]$$

By transforming the triplet [0.00341 0.02159 0.27614] via fuzzy median method, we obtain the value of R_{11}=0.090. Representative value for F_j and statistic for each sample are calculated and summarized in Table 2. The mean vector of quality characteristics is shown as follows:

$$\overline{R} = \left(\overline{R}_1 \quad \overline{R}_2 \quad \overline{R}_3\right) = (0.0994 \quad 0.1802 \quad 0.1350)$$

The sample covariance matrix S can be obtained by using Eqs. 8 and 9 as given in the following:

$$S = \begin{bmatrix} 0.0001063 & 0.00006 & 0.0000226 \\ 0.00006 & 0.0000689 & 0.0000037 \\ 0.0000226 & 0.0000037 & 0.0000555 \end{bmatrix}$$

Table 4 Values of reduced statistic and criterion d_j

Sample number	Statistic	Reduced statistic			Criterion d_j		
	F-MCUSUM$_i$	y_1	y_2	y_3	d_1	d_2	d_3
21	0.6655	0.0000	0.8237	1.0412	0.6655	0.1582	0.3758
22	5.4950	0.9292	3.6819	6.3243	4.5659	1.8131	0.8293
24	2.5790	0.6132	2.6109	3.7892	1.9658	0.0319	1.2102
24	2.1200	1.5722	2.1952	3.7194	0.5477	0.0752	1.5994
25	5.4749	5.5854	3.8668	4.0596	0.1105	1.6081	1.4153

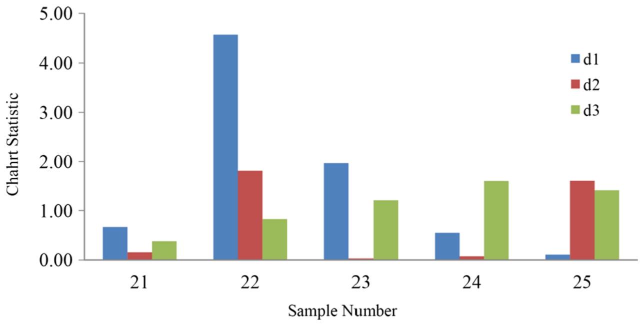

Fig. 5 Diagram of d_j for samples 21 to 25

The F-MCUSUM statistic, F-MCUSUM, is calculated by using Eqs. 8, 9, and 10 as showed in Table 2.

The reference value K in relation 11 equals to 0.9775 due to determining the value of vector δ as (0.011, 0.011, 0.011) by decision maker.

5.2 Control limit for F-MCUSUM control charts (design control chart in phase I)

Using bootstrap resampling method on the data from Table 1, 10,000 new 220-sized samples are generated through MATL AB software. After the samples' statistics were calculated and then sorted in ascending order, the upper control limit can be determined by a false alarm rate of 0.05. The results show that the upper control limit is equal to 3.0099.

It can be seen in Fig. 3 that three points state out-of-control signals. So samples related to these out-of-control statistics should be eliminated to construct the new modified control limit. Three out-of-control samples are replaced by some other samples. The new control limit is calculated. Plotting the statistics on the new chart shows an in-control process. Therefore, the resulting control chart with threshold of 2.4043 could be used for online monitoring of frozen food multivariate process in phase II.

5.3 Application of F-MCUSUM control chart in phase II

To show the ability of new F-MCUSUM control chart in understanding and checking the stability of process over time, some new observations gathered from the frozen food production process are shown in Table 3.

Then, the F-MCUSUM statistic y_i could be calculated and plotted on the control chart for new samples as illustrated in Fig. 4. If the computed statistic is less than UCL=2.4043, the process is in control; otherwise, we face an out-of-control process.

When the process yields an out-of-control signal, it should be interpreted to determine which of QCs was responsible for it. Then, samples related to the out-of-control statistics should be abandoned to construct the new modified control chart. Also, root causes due to out-of-control situations in process should be apparent, and some corrective actions should be done to eliminate the non-conformities.

5.4 Interpretation of out-of-control signals

Using the reduced statistics, F-MCUSUM statistic decomposes into its components to explore the contribution of each individual QC in an out-of-control signal. Figure 4 shows that when the samples 22, 24, and 25 were taken, the process was out of control. To determine which of the three QCs is responsible for such signals, the reduced statistics y_j and criterion d_j were calculated as shown in Table 4. Subsequently, the QC with the largest value of d_j is detected as the main responsible for the out-of-control signal. Based on Fig. 5, it can be found that the QC *appearance* has the main role in the out-of-control state of process due to having the largest absolute amount of d_j for samples 22 and 23. Both the QCs *color* and *taste* are the main responsible for the out-of-control signal for sample 25.

Also, for better identification, a run chart of R_j was plotted for each QCs as illustrated in Fig. 6. Such perceptions can be understood on the basis of the run chart. The QC appearance faces the worst situation among the others because of a big swing in its trend. So, its role in leading the process to the out-of-control status is more prominent than the two other QCs

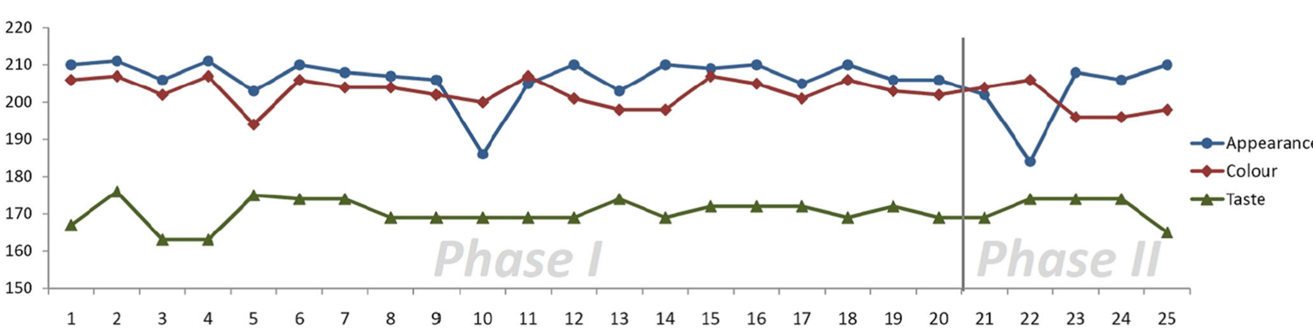

Fig. 6 Run chart for appearance, color, and taste

Table 5 Shifts in process data

Shift scenario	Appearance			Color			Taste			
	Good q_{11}	Medium q_{12}	Poor q_{13}	Standard q_{21}	Acceptable q_{22}	Rejected q_{23}	Perfect q_{31}	Good q_{32}	Medium q_{33}	Poor q_{34}
I.P.V.	0.942	0.035	0.023	0.925	0.045	0.03	0.774	0.206	0.016	0.004
Shift 1	0.942	0.035	0.023	0.905	0.065	0.03	0.754	0.226	0.016	0.004
Shift 2	0.922	0.035	0.043	0.925	0.045	0.03	0.774	0.206	0.016	0.004
Shift 3	0.942	0.035	0.023	0.925	0.045	0.03	0.674	0.306	0.016	0.004
Shift 4	0.892	0.035	0.073	0.925	0.045	0.03	0.774	0.206	0.016	0.004

IPV in-control process vector

color and taste in the samples 22 and 23. Furthermore, the QCs color and taste are the responsible factors in increasing the intensity of deterioration in process in sample 25.

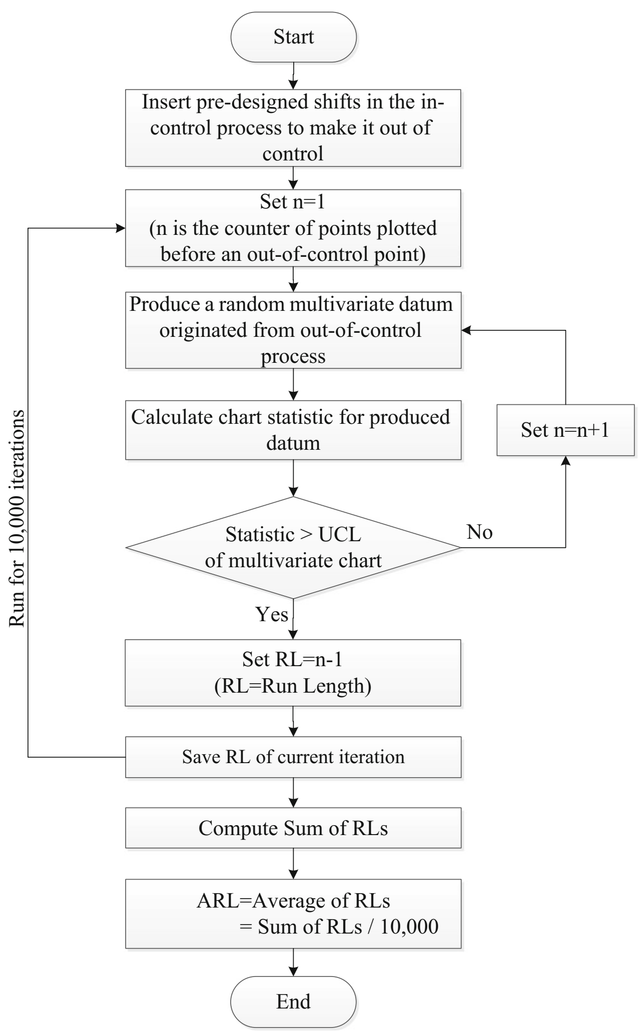

Fig. 7 Flowchart of calculating ARL through simulation

6 Performance of F-MCUSUM control chart

Four predesigned changes to the in-control process have been considered for comparative study. First, in an in-control process, the proportion of q_{jh} was calculated. The proportion of q_{jh} is the ratio of the products with their qualitative characteristics Q_j expressed in terms of linguistic term q_{jh}. Then, four intentional shifts in the process are begotten via changing in these ratios. In-control vector's proportion is shown in the first row of Table 5.

In shift 1, the ratio of products with standard color was reduced by 0.02, and instead, the ratio of products with acceptable color was decreased by the same amount. Also, the proportion of products with perfect taste has been reduced by 0.02, and instead, the proportion of products with good taste has been decreased by the same amount. Shift 1 is considered as a small shift because the bad quality proportions of QCs were not affected. In shift 2, the proportion of poor appearance, q_{13}, is increased by 0.02, and the proportion of good appearance, q_{11}, is decreased by the same amount. Shift 2 is a small shift, but the process situation became worse than the shift 1 because of affecting poor quality. Shift 3 and shift 4 can be considered as medium and high shifts in process, respectively.

An effective way to evaluate decisions regarding sample size and sampling frequency is through the average run length (ARL) of the control chart. The ARL

Table 6 Comparison of three multivariate control charts' ARL

Control chart	In control ARL_0	Shift 1 ARL_1	Shift 2 ARL_1	Shift 3 ARL_1	Shift 4 ARL_1
F-T^2	370	110.186	42.415	5.8282	0.6549
F-MEWMA	370	32.4794	10.8291	1.477	0.3941
F-MCUSUM	370	21.8442	7.2543	2.0998	1.1687

Table 7 Proportion of other charts' observations needed by F-MCUSUM

Percent of observations needed to detect	Out-of-control situation			
	Small shifts in the process		Large shifts in the process	
	Shift 1	Shift 2	Shift 3	Shift 4
F-MCUSUM to F-T^2	0.1982	0.1710	0.3603	1.7845
F-MCUSUM to F-MEWMA	0.6726	0.6699	1.4217	2.9655

is the average number of points that must be plotted before a point indicates an out-of-control condition [40]. When a process goes out of control, the main concern is to detect shifts as quickly as possible. Therefore, a control chart is the best chart if it has the least plotted points before the point indicates out-of-control state. It means that the control chart with the least ARL for out-of-control situation should have better performance.

ARL of three multivariate control charts has been computed through 10,000 simulation runs for the four proposed shift scenarios in the same process. The simulation algorithm for computing the ARL for each shift scenario is based on a flowchart illustrated in Fig. 7. Comparison of each control chart performance related to multiple shift scenarios can be done by using detailed data in Tables 6 and 7. Moreover, Fig. 8 shows plots of the ARLs of three control approaches. It can be concluded that F-MCUSUM is more sensitive and uniformly performs better than F-MEWMA and F-T^2 in detecting small and medium shifts. The ARL of three control chart decreases while shift intensity increases. It means that for large shifts, three approaches have approximately the same performance in indicating an out-of-control signal due to the negligible difference of ARL. Consequently, the F-MCUSUM control approach can be chosen to monitor multivariate linguistic quality characteristics due to its relatively better performance in fast shift detecting and using cumulative information of the current and past samples.

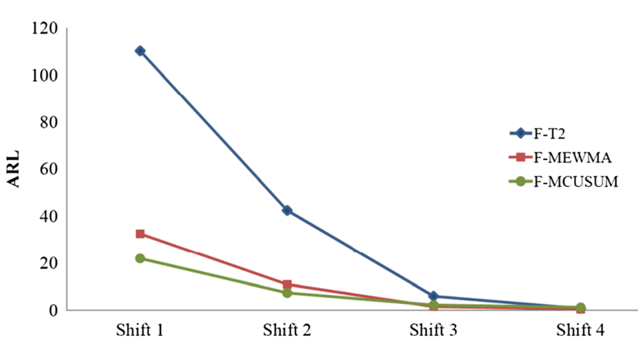

Fig. 8 Comparison of multivariate control charts' ARL

7 Conclusion

An alternative tool for monitoring linguistic correlated quality characteristics is fuzzy multivariate control chart. The conventional multivariate control chart in conjunction with fuzzy logic was considered simultaneously to develop the F-MCUSUM control chart for monitoring linguistic observations in fuzzy environment. The performance of F-MCUSUM control chart developed in this paper was compared to the fuzzy T^2 and fuzzy MEWMA control charts using the ARL criterion.

When a small shift in the process occurs such as shift 1 in Table 5 in which only one QC deteriorates a little, the F-MCUSUM control chart detects shift more quickly than F-MEWMA. Because the F-MCUSUM needs 0.6725 of observations that F-MEWMA needs to yield an out-of-control signal. Compared with F-T^2 control chart in Table 7, the F-MCUSUM needs 0.1982 of observations needed by F-T^2 for signaling out-of-control.

When a large shift happens in the process such as shift 4 in Table 5, in which all QCs of product deteriorates, the F-MCUSUM needs 1.7845 and 2.9655 of observations required by F-T^2 and F-MEWMA control charts, respectively. Therefore, the F-MCUSUM control chart detects a shift slower than the F-MEWMA and F-T^2 control charts, while large shifts make the process to show an out-of-control situation. Based on numerical results, the F-T^2 control chart has poor performance in detecting small shifts occurred in multivariate fuzzy quality characteristics over the process. The F-MCUSUM and F-MEWMA control charts are uniformly better than F-T^2 method in detecting small shifts. Note that the F-MCUSUM method gives an out-of-control signal sooner than the F-MEWMA method while detecting tiny shifts in process. The performance of three charts is approximately the same in detecting relatively few large shifts occurred by fuzzy quality characteristics in process. For the reason of utilizing cumulative information of the current and past samples, the F-MCUSUM has a better feature for monitoring multinomial linguistic quality characteristics compared to F-T^2 chart that uses only the information of current sample. Consequently, the F-MCUSUM is much more capable than F-MEWMA and F-T^2 charts to discover small and medium shifts in processes. Combination of reduced statistics method and run chart method mutually is proposed as an effective approach for interpreting the out-of-control signals. Developing some new methods for detecting the main responsible fuzzy QC for out-of-control situations is proposed as a future research.

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