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ORIGINAL ARTICLE

Multi-criteria decision making techniques for compliant polishing tool selection

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Abstract Precision devices require surface finish of a few nanometers. The choice of compliant coated abrasive tools used in manufacturing such devices is discussed including a method for selecting a suitable one for a given component using decision-making techniques. Multiple conflicting criteria such as surface roughness and the polishing time influence the selection of appropriate compliant polishing tool. Hence, multi-criteria decision making methods (MCDMs) are implemented to rank the suitability of different polishing processes for a given workpiece geometry. New criteria such as compliance and surface integrity are introduced for such selection. In order to differentiate the level of complexity involved in these MCDMs, a traditional analytical hierarchy process (AHP) and a Fuzzy VIšekriterijumsko KOmpromisno Resenje (multi-criteria optimization and compromise solution, VIKOR) method are chosen. This study illustrates the capability of these two MCDMs as a polishing process selection tool using the linguistic information through a case study. From the decision makers' inputs, rankings of polishing tools were obtained and compared using these two methods. New factors such as compliance are seen to affect the choice significantly. The approach discussed in this work could be used for developing an intelligent decision-making system for choosing polishing tools with respect to the given conditions.

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1 Introduction

Polishing is one of the finishing processes during the final stages of the manufacturing processes, usually followed by other conventional machining operations such as milling, turning or grinding. Many industries such as aerospace, automobile, bio-medical and other manufacturing industries which rely on polished dies demand a mirror-like surface with minimal number of internal defects and less deviation from the desired geometric profile [1, 2]. It is well recorded that most of the mechanical failures are due to fatigue loading. One of the ways to improve the fatigue life is to have a mirror-like surface to decrease fatigue crack initiation life [3]. In a nut shell, polishing is an important process for parts which largely requires higher precision, dimensional accuracy and good surface finish [4–6]. These desired characteristics of polishing such as high precision and better surface finish were initially obtained by manual and labour-intensive processes. The outcome of the polished surface depends on the skill and experience of the operator. Another added problem in manual polishing is the lack of control over the material removed, which affects the desired dimensional accuracy [7]. Hence, automated robotic polishing is suggested to eliminate the problems of manual polishing with reduced process time, cost and improved quality consistency by controlling the tool end effectors [8, 9].

Classification of different polishing process commonly used for research and industrial applications are given in Fig. 1. Often, the selection of polishing process from such large varieties depends upon various parameters such as initial and final surface roughness, geometry of the workpiece, polishing time, cost

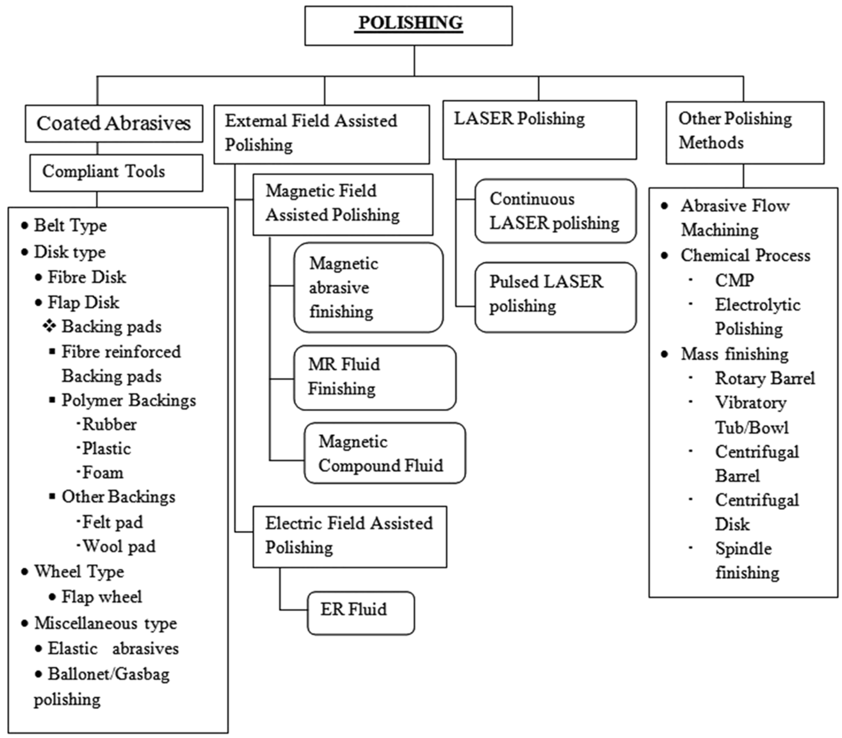


Fig. 1 Classification of different types of polishing processes

of the tool, complexity involved in machine setup etc., [10]. Although it is not possible to completely remove the ambiguity involved in polishing process selection, decision-making methods reduce the vagueness involved in it. Out of all the polishing tools and processes discussed in Fig. 1, compliant polishing tools are widely employed in metal polishing industries owing to their cost, variety of tools available for polishing workpieces of any dimensions, and lesser complex when compared to other polishing processes listed in Fig. 1. Hence, in this study, the primary focus is on compliant polishing tools.

Decision-making algorithms [11–13] is a topic which remains unexplored with respect to polishing process selection, as these algorithms helps in selecting and ranking the processes. Since there are many polishing tools available in market, it would lead to confusion for the operator to choose an appropriate tool for polishing a component. Hence these multi-criteria decision-making methods (MCDMs) would come in handy to choose an efficient tool removing the ambiguity associated in choosing the polishing tool. Singh et al. [14] ranked the five finishing process such as internal lapping, honing, magnetic assisted finishing, abrasive flow machining and ultrasonic machining using fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method. But in the earlier study, only the general polishing processes were ranked using only fuzzy TOPSIS but in this study, the application of two MCDMs viz. analytical hierarchy process (AHP) and fuzzy VIšekriterijumsko KOmpromisno Resenje (VIKOR) were compared and studied with respect to the selection of compliant polishing tools which was not investigated by researchers earlier, is the significance of this study. Fuzzy VIKOR method is considered more advantageous than the TOPSIS method because it is used widely in problems involving conflicting criteria such as surface finish, surface integrity, cost and time [15, 16].

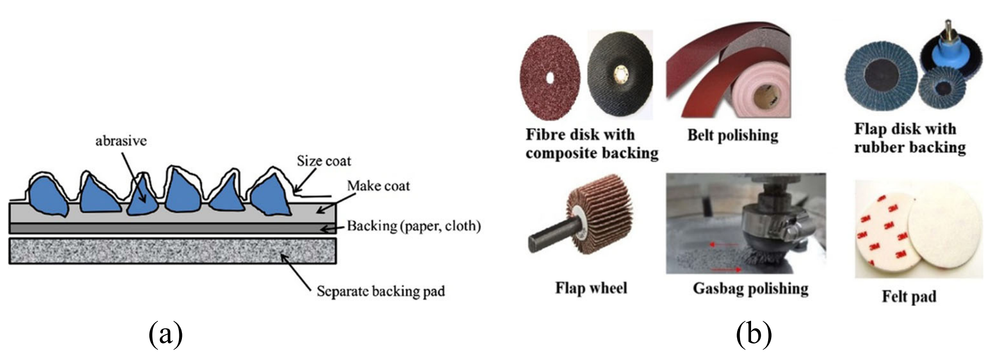


Fig. 2 **a** Typical elements of coated abrasives [18]. **b** Commercially available different coated abrasives [28, 36]

Table 1 Surface roughness achieved with different compliant tools

Compliant polishing tools	Surface finish (μm)
Belt polishing (coarser grits), fibre and flap discs, flap wheels, bob polishing	Initial polishing (R_a>1) Removing machining marks [19]
Belt polishing (finer grits), rubber pad	Intermediate polishing (R_a=1 to 0.1) [20]
Gasbag/ballonet polishing, wool pad, felt pad, elastic abrasives	Fine polishing (R_a=0.1 to 0.01) [28]

Initially, a brief review of unexplored compliant polishing tools is given in Section 2 and an overview of decision-making algorithms for the compliant tool selection and ranking is given in Section 3 of the paper. Out of all the MCDM algorithms, two algorithms viz., (i) AHP (ii) fuzzy VIKOR methods are selected based on their increasing complexity, respectively. A case study to bring out the importance of decision-making algorithm in the selection of compliant coated abrasive polishing tool is given in Section 4. Finally, concluding comments are given in Section 5.

2 Compliant abrasive polishing

Employing a rigid abrasive tool would penetrate the material deeply, whereas a flexible tool, would abrade material only on the surface level, thereby reducing material removed and change in dimensions and further compliant tools could be used with different workpiece geometries [17]. Generally, coated abrasive tools are also referred to as flexible backed abrasive and consist of three main components viz. (i) backing sheet, (ii) adhesive bond and (iii) abrasive. The abrasive grains are fixed to the backing material using base coat as shown in Fig. 2a. Coated abrasives are described in terms of grit size, shape of abrasive material, type of base and adhesive glue [18, 19]. The different types of compliant polishing tools are shown in Fig. 2b.

Belt polishing is one of the common polishing tools used in industries to polish complex 3D parts. Axinte et al. [20] conducted belt polishing on an aerospace heat-resistant titanium component to identify the optimum parameters such as cutting speed, feed rate, depth of cut, step over, etc. Further belt polishing depends on the properties of drive or contact wheel used. Polishing tools with stiffer backing material such as fibre disc backing pad is used for rough polishing purposes such as removal of pre-machined marks. Finite element models were analysed to study the pressure distribution and force involved during polishing. The change in contact area with change in pad stiffness was also noticed [18]. Another commonly used compliant tool is the flap wheel which is widely used to polish intricate shapes and contours. Slatineanu et al. [21] conducted preliminary studies to investigate the effect of cutting speed of flap wheel, the longitudinal feed rate and the grit size of the abrasive grains used in the flap wheel on the surface finish.

In industries that requires mirror finish, coated abrasive polishing tools with soft backing pad and smaller grits are used. Such soft backing pads are made of rubber, felt or wool. Apart from these soft backing tools, gasbag polishing technique was studied by many researchers as a potential automated polishing tool for obtaining mirror-like surfaces [22–27]. Ballonet polishing tool was studied by Zhang et al. [28], where a comparative study was made between ballonet polishing and wool pad. Elastic abrasive are elastomeric balls with abrasive embedded which can easily get confined to the shape of the surface profile, thereby removing less material without changing the form [29–31]. Table 1 lists surface finish obtained from a variety of compliant tools, it should be noted that this table is only a general guide and the surface finish is strongly dependent on abrasive grit size and its nature for the particular workpiece materials.

Table 2 Fundamental intensity scale for comparison matrix [32]

Intensity	Explanation
1	Two components (attributes or alternatives) are equal with respect to the objective
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance

3 MCDM

MCDM serves as a tool in creating a reliable, mathematically justified decision which is rational and efficient. MCDM consists of a finite set of alternative processes, from which the decision makers (DMs) must rank these alternatives with respect to certain finite criteria. Armillota [29] used pairwise comparison AHP to study different rapid prototyping processes. In the same study, an enhanced parameter-based approach

Table 3 Random index numbers [32]

Number	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Table 4 Linguistic terms and corresponding fuzzy numbers

Linguistic terms	Fuzzy numbers
Very low ('VL')	(0.0, 0.0, 0.1)
Low ('L')	(0.0, 0.1, 0.3)
Medium Low ('ML')	(0.1, 0.3, 0.5)
Medium ('M')	(0.3, 0.5, 0.7)
Medium high ('MH')	(0.5, 0.7, 0.9)
High ('H')	(0.7, 0.9, 1.0)
Very high ('VH')	(0.9, 1.0, 1.0)

was used to improve the efficiency of AHP and also to decrease the uncertainty. Fuzzy-based AHP was implemented by Sun et al. [30] to study and evaluate the three types of grinding fluids. Ayag et al. [31] employed the fuzzy-based AHP to evaluate different alternatives for selecting optimal machine tools. In this work, two competing MCDMs viz., AHP and fuzzy implemented VIKOR methods are considered for a case study involving the selection of compliant polishing tools.

3.1 AHP methodology

The procedure involved in AHP is as follows. Initially, the objective of the problem, the criteria and alternatives involved in the problem are outlined. Then, pairwise comparisons are made within the criteria (for each pair of criteria combinations) and the comparison matrix is formed using an intensity scale as shown in Table 2. The priority vector and averaged weights are then calculated from the comparison matrix. Similarly, pairwise comparisons are made within alternatives for every criterion and weights are calculated for every comparison matrix formed. Then, the rankings of the alternatives are obtained from the computed overall composite weights. The consistency of the input is checked using the consistency ratio (CR) values calculated based on the Random Consistency Index (RI) numbers as shown in Table 3 [32].

The CR value should be less than 0.1 to ensure the consistency of the input values. A detailed mathematical procedure for the calculations of priority vectors, CR, CI and λ_{max} can be found in the literature [33].

Table 5 Linguistic term and corresponding fuzzy numbers for alternatives

Linguistic term	Corresponding fuzzy numbers
Very poor ('VP')	(0.0, 0.0, 1.0)
Poor ('P')	(0.0, 1.0, 3.0)
Medium poor ('MP')	(1.0, 3.0, 5.0)
Fair ('F')	(3.0, 5.0, 7.0)
Medium good ('MG')	(5.0, 7.0, 9.0)
Good ('G')	(7.0, 9.0, 10.0)
Very good ('VG')	(9.0, 10.0, 10.0)

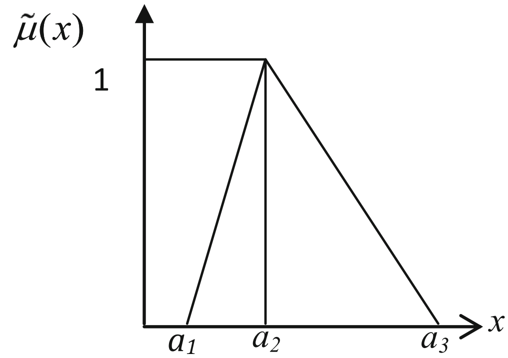


Fig. 3 A triangular fuzzy number

3.2 Fuzzy VIKOR method

VIKOR method is another MCDM which is a widely used process selection method in manufacturing and other industries. The algorithm involved fuzzy-based VIKOR is discussed in the following sections.

Step 1: Fuzzy set theory is used to represent uncertain information in mathematical terms. It is a convenient method to denote linguistic terms without ambiguity. The criteria comparison matrix is developed using the linguistic information from decision makers and converting them in terms of triangular fuzzy numbers using Eq. 1. The details of linguistic terms and the corresponding fuzzy numbers are listed in Tables 4 and 5.

The triangular function as shown in Fig. 3 can be represented as

$$\tilde{\mu}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{(x-a_1)}{(a_2-a_1)} & a_1 \leq x \geq a_2 \\ \dfrac{(a_3-x)}{(a_3-a_2)} & a_2 \leq x \geq a_3 \\ 0 & x > a_3 \end{cases} \quad (1)$$

Step 2: Calculate the aggregate fuzzy weights for each criteria from the criteria comparison matrix and form a row matrix (of

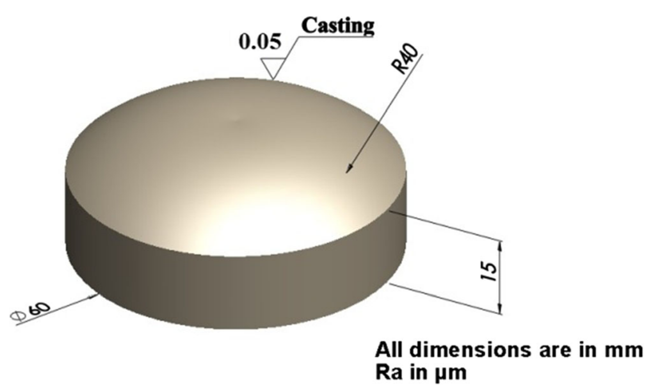


Fig. 4 CAD model of workpiece to be polished using compliant polishing tool

Table 6 Various attributes involved in coated abrasive polishing [37]

Pre polishing	Polishing	Post polishing
• Cost estimation (tool, material, other overheads, etc.)	• Pressure	• Surface roughness.
Part:	• Relative speed/spindle torque	• Surface Integrity
• Geometry of the part	• Polishing time	• SEM analysis
• Desired surface roughness	• Depth of cut	• Residual Strength due to polishing
•Material Properties	• Force	• EDX test for contamination check
Polishing tool:	• Material removal	• Tool wear
• Tool type according to surface to be polished	• Type of coolant	• Surface profile error
•Compliant/rigid tool	• Temperature maintained	
• Tool path	• Tilt angle	
• Grain size/shape		

dimension *1 x n*) for aggregated importance weight criteria where n is the total number of criteria considered in this study.
Step 3: The DMs now form an alternative comparison matrix using the linguistic terms and corresponding fuzzy numbers as listed in Table 5 [34, 35].
Step 4: Form the triangular fuzzy number decision matrix using the aggregated fuzzy ratings of m alternatives with n criteria an m x n matrix is formed, where each cell represent the aggregated fuzzy number value of ith alternative with respect to jth criteria.
Step 5: Determine the best (aspired) values and worst fuzzy values. Assuming the jth criteria representing a benefit, then the best values for setting all the criteria functions are $\{x_j^*|j=1, 2,....n\}$ and the worst values are $\{x_j^-|j=1,2,....n\}$.
Step 6: The gap $\{S_i|i=1,2,....m\}$ and $\{R_i|i=1,2,....m\}$ are calculated along with the L_p metric parameter with normalisation. The formula for the above calculation is listed below.

$$L_{pi} = \left\{ \sum_{j=1}^{n} \left[w_j \left(x_j^* - x_{ij} \right) / \left(x_j^* - x_j^- \right) \right]^p \right\}^{(1/p)} \quad 1 \le p \le \infty \tag{2}$$

$$S_i = \sum_{j=1}^{n} \left[w_j \left(x_j^* - x_{ij} \right) / \left(x_j^* - x_j^- \right) \right] \tag{3}$$

$$R_i = \max_j \left[\left(w_j - x_{ij} \right) / \left(x_j^* - x_j^- \right) \right] \tag{4}$$

where $i=1,2,\ldots,m$

Step 7: The important part of VIKOR method is the calculation of $\{Q_i|i=1,2,....m\}$ for ranking. The values are calculated by using Eq. (5) as follows:

$$\tilde{Q}_{\mathrm{i}} = \vartheta \left[\frac{(S_i - S^*)}{(S^- - S^*)} \right] + (1-\vartheta) \left[\frac{(R_i - R^*)}{(R^- - R^*)} \right] \quad i = 1, 2, \ldots, m \tag{5}$$

where $S^*=\min_i S_i$, $S^-=\max_i S_i$, $R^*=\min_i R_i$, $R^-=\max_i R_i$ and ϑ is the weight in strategy of the maximum group utility, usually 0.5.

From the $\widetilde{Q}_i$ (fuzzy values), Q_i (crisp values) are computed by using centroid method which is one of the defuzzification method. The detailed procedure for this computation can be found in the literature [34].

4 Results and discussion on compliant polishing tool case study

Consider a typical aerospace industry which requires manufacturing a convex workpiece as shown in Fig. 4. The choice of an appropriate tool for polishing the component in each stage is critical as it would directly affect the cost and manufacturing time. Complex components in high-end industries such as aerospace, photonics, sports, etc. have a combination of flat, convex and concave sections. In this case study, consider manufacturing a typical convex workpiece (convex

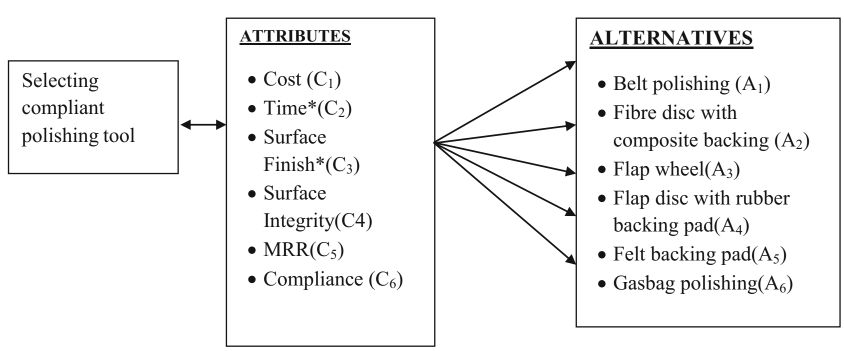


Fig. 5 Attributes and alternatives used in the selection of compliant polishing tool

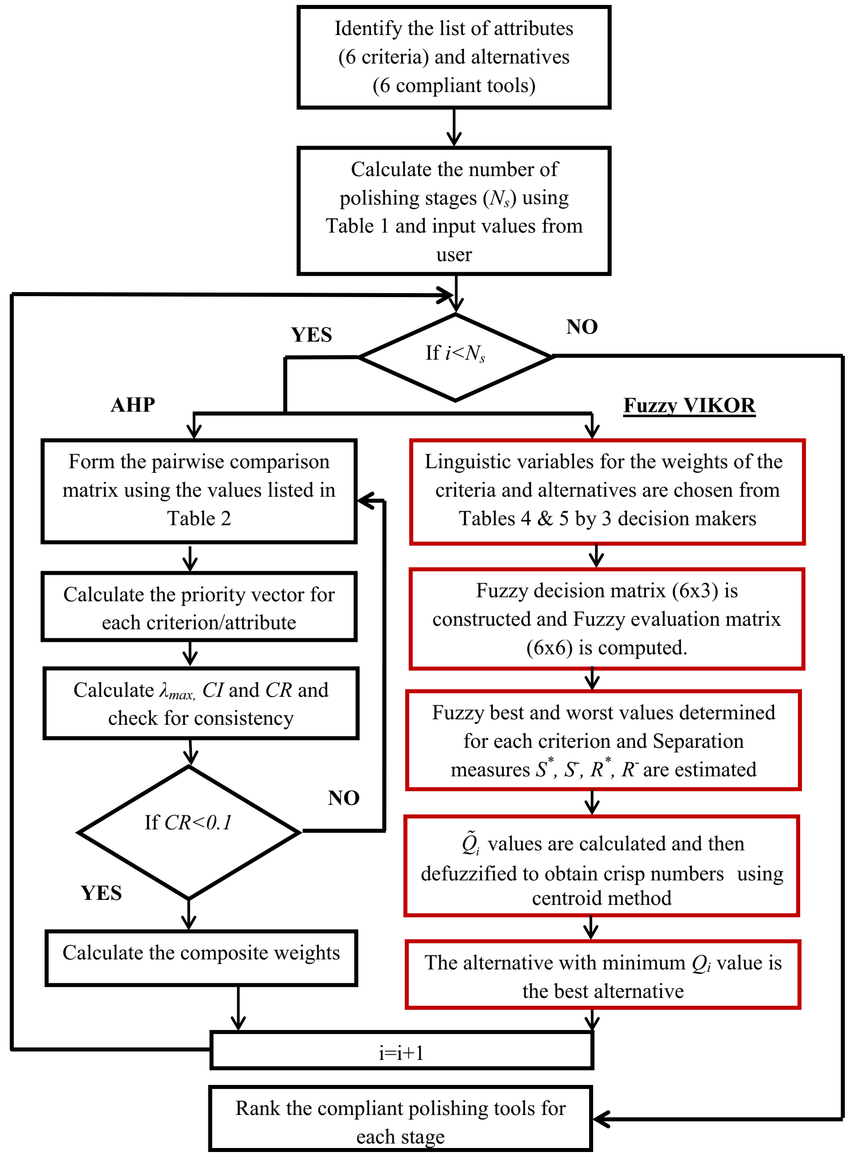


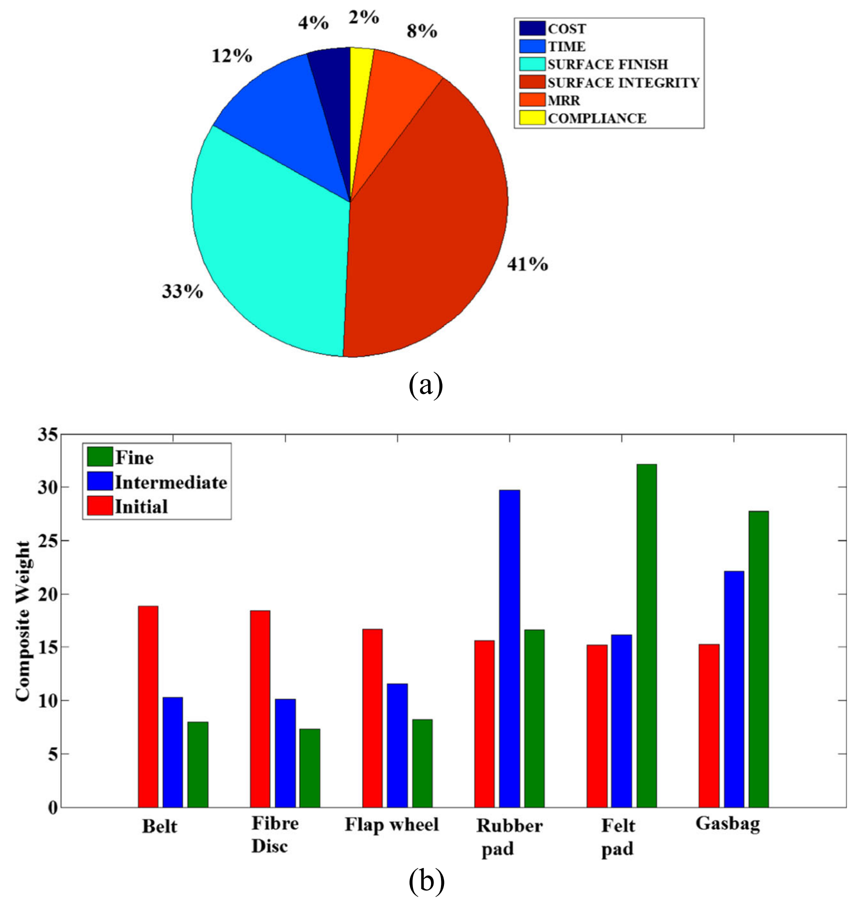
Fig. 6 AHP and fuzzy VIKOR algorithm implemented in the case study

mirror) as shown in Fig. 4. The choice of an appropriate tool for polishing the component in each stage is critical as it would affect directly the cost and manufacturing time. It should be noted that during process development, initially, the decision makers (usually three to five DMs) play a crucial role in selecting the criteria and ranking the alternatives. The decision-making methods (MCDMs) will be helpful in assisting the decision makers to choose appropriate polishing tools mathematically. The workpiece has an initial R_a of 5 μ and needs to be mirror finished with R_a of 0.05 μ. The desirable pre- and post- polishing attributes along with polishing stage operational parameters are given in Table 6.

In a broader perspective, although there are many attributes listed in Table 6, the main attributes are cost, time, surface finish, surface integrity, material removal rate and compliance. Since machining parameters like pressure, relative speed, depth of cut, force and coolant are varied for the sake of surface finish and material removal rate (MRR) of the workpiece. Hence, those attributes are not considered separately. Since the surface finish could not be obtained in a single stage, usually, they are divided into multiple stages by varying the grit size, different compliant tool, etc. So, the attributes that changes with respect to these stages are identified separately and denoted by star '*' symbol. The polishing stages are divided as discussed in Table 1. Figure 5 describes the list of alternatives and attributes considered in the case study. The description of the attributes is listed in detail in Table 9.

Using the AHP algorithm as described in the flow chart in Fig. 6, the attribute decision matrix is formed as in Table 9. The relative values of the attributes are purely judgemental based on the decision makers' intuition. The comparison matrices for the six alternatives (compliant tools) for the three polishing stages are formed as listed in Table 10. The criteria weightage for each attribute is shown in Fig. 7a. It is evident

Fig. 7 **a** Criteria weight for each attribute. **b** Weight for each polishing process in the three stages



that more importance is given to surface integrity and surface finish. The composite weights for the competing alternative processes in each polishing stage are computed as shown in Fig. 7b. Based on the composite weights, the alternatives are ranked in the order of preference.

In the fuzzy VIKOR method, the fuzzy values for the attributes weights are listed in Table 15. The attributes and alternatives are ranked using fuzzy values corresponding to the linguistic terms given by the decision makers. The fuzzy linguistic rating for alternatives with respect to fixed and changing variables for the three stages is listed in Tables 16 and 17.

After compiling the MATLAB® modules developed based on the fuzzy VIKOR algorithm, the fuzzy evaluation matrix are obtained as shown in Tables 18, 19 and 20 for the three stages. It could be observed that (i) the dimension of the matrix is 6×6 matrix denoting the six attributes (in rows) and six alternatives (in columns) (ii) only the *time* and *surface integrity* values changed in each of this matrix. Table 21 lists the values of S^*, S^-, R^* and R^- obtained from the code for each stage. Using the above values and Eq. 5, $\widetilde{Q}_i$ values are obtained and converted into crisp values. $\widetilde{Q}_i$ and corresponding crisp number are listed in Table 7.

Table 7 $\widetilde{Q}_i$, Q_i and rankings for the six compliant polishing tools

Alternatives	Initial polishing			Intermediate polishing			Fine polishing		
	$\widetilde{Q}_i$	Q_i	Rank	$\widetilde{Q}_i$	Q_i	Rank	$\widetilde{Q}_i$	Q_i	Rank
A1	(1.09, 0.65, −2.81)	0.15	2	(0.01, 0.8, −1.93)	0.21	3	(0.65, 0.96, −2.63)	0.31	5
A2	(1.04, 0.42, −2.2)	0.09	1	(0.03, 0.88, −1.52)	0.34	5	(0.67, 0.92, −2.12)	0.37	6
A3	(1.2, 0.46, −2.13)	0.15	3	(0.05, 1, −0.26)	0.63	6	(0.6,0.62, −1.94)	0.19	4
A4	(1.18, 0.29, −1.25)	0.19	4	(−0.01, 0, −1.01)	−0.17	1	(0.57, 0.37, −1.18)	0.14	3
A5	(1.25, 0.75, −0.58)	0.61	6	(0.01, 0.23, 0.11)	0.17	2	(0.49, 0, −0.63)	−0.02	2
A6	(1.25, 0.75, −1.89)	0.39	5	(0.01, 0.46, −0.54)	0.21	4	(0.51, 0.24, −1.91)	−0.07	1

Table 8 Comparison of alternative rankings obtained from AHP and Fuzzy VIKOR methods

Alternatives	Initial polishing		Intermediate polishing		Fine polishing	
	AHP	Fuzzy VIKOR	AHP	Fuzzy VIKOR	AHP	Fuzzy VIKOR
Belt polishing (A_1)	1	2	6	3	5	5
Fibre disc with composite backing (A_2)	2	1	5	5	6	6
Flap wheel (A_3)	3	3	4	6	4	4
Flap disc with rubber backing pad (A_4)	4	4	1	1	3	3
Felt backing pad (A_5)	5	6	3	2	1	2
Gasbag polishing (A_6)	6	5	2	4	2	1

The rankings of the alternatives obtained from both the fuzzy VIKOR and AHP are compared as shown in Table 8. From Tables 7 and 8 and Fig. 7b, it is noticed that for selection of polishing tool for initial polishing by AHP method, the belt polishing is highly preferred owing to the maximum weight of 19.76 and the next preferred tool is the fibre disc polishing tool with second highest weight of 19.66. Fuzzy VIKOR method leads to a similar solution with fibre disc polishing tool as the best tool for initial polishing with least Q_i value (0.09) and belt polishing as the second preferred tool with second least Q_i value (0.15). Thus for initial polishing, polishing with fibre disc or belt polishing is preferred followed by other tools such as flap wheel, rubber pad, felt and gasbag. This indicates that the tool with considerably low compliance such as fibre disc polishing tool is preferred for initial polishing compared to more compliant backing materials like rubber or felt. In the second stage of polishing, i.e. intermediate polishing, tools with rubber as packing pad is highly preferred from the AHP solution with the highest weight of 27.48. Similar result was obtained from fuzzy VIKOR method indicating rubber pad as the most preferred polishing tool with the least Q_i. Thus, it is evident that rubber backing pad, which has comparatively higher compliance than fibre disc and lower compliance than felt or gas bag, is suitable for intermediate polishing. Using the AHP method, felt pad polishing is seen to be the preferred tool with the highest composite weight of 32.58 followed by gasbag polishing with 29.03. Similar results were obtained from fuzzy VIKOR where gasbag polishing tool (−0.08) is the most preferred tool followed by felt polishing tool (−0.03). It is evident that both felt and gasbag, with relatively more compliance, are the preferred polishing tools for fine polishing. The outcome of the result suggests that compliance of the backing pad material influences the surface finish in each polishing stage apart from the usual tool parameters such as grit size, tool-workpiece contact pressure and tool velocity (rpm). It is thus clear that MCDM techniques can assist in tool selection. From the managerial point of view, the decision-making about choosing polishing tools can thus be made promptly and consistently at the conceptual stage of the process development thereby saving on cost and time and decreases iterations involved during experimental testing.

5 Conclusions

Identifying suitable compliant polishing methods for micro or nano level polishing of complicated geometries is a challenging problem. In this paper, various competing compliant abrasive-based polishing methods are briefly reviewed along with decision-making methods for their selection. The case study carried out demonstrates the applicability of two MCDMs in selecting appropriate compliant polishing tools widely used in industries for polishing workpieces varying from flat geometry to complex geometries. The two MCDMs methods, AHP and fuzzy VIKOR method, incorporated the decision makers' preferences with respect to the attributes and alternatives considered. Three polishing stages are used in the case study and the ranking of the compliant polishing tools in each of the stage was obtained. Both the methods provided good correlations in the rankings of the compliant polishing tools considered. The case study thus exhibited the applicability of the MCDMs to compliant polishing tool selection. From the results, it was noticed that the compliance of the tools played a significant role in each polishing stage. In the future study, the ranking obtained through linguistic values could be substantiated using experimental measurements. Moreover, the effect of backing pad compliance with respect to the polishing stages could be validated experimentally. One of the main limitations of the MCDM is that the ranking obtained is highly dependent on the subjective and personal judgement of the weights used by the DMs. When the pre-processing and desired surface conditions change the weights assigned to each criterion would change resulting in a different set of tool rankings. Hence, a sensitivity analysis would be an appropriate future work that can be conducted. But, the overall algorithms and steps implemented would not change. This particular case study is focused to investigate the implementation of MCDMs in compliant polishing tools with a limited number of main attributes, but it is noted that are many other attributes which can affect the selection of the tools. In the future, more attributes will be taken into consideration, supported by experimental results. In addition, an expert system could be developed that can check the tool rankings with the machined parts and can update automatically when alterations are made to the criteria.

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Appendix

Compliant polishing tool selection by AHP

Table 9 Description of attributes used in decision-making

Attributes	Symbol	Definition
Cost	C_1	Cost refers to the tool cost
Time	C_2	Time includes the machining time, tool change time, etc.
Surface finish	C_3	Surface finish refers to the R_a value
Surface integrity	C_4	Properties of the workpiece influenced by both physical and chemical effects after polishing process
Material removal rate	C_5	Material removed from the workpiece in a given time which is a function of tool force, spindle velocity, etc.
Compliance	C_6	Tool deflection for the applied force on the tool. Less compliant tool results in profile inaccuracy of the machined workpiece

Table 10 Criteria comparison matrix for different compliant polishing tool attributes

Attributes	C_1	C_2	C_3	C_4	C_5	C_6
C_1	1	0.33	0.143	0.143	0.33	3
C_2	3	1	0.2	0.33	3	5
C_3	7	5	1	0.33	7	9
C_4	7	3	3	1	5	9
C_5	3	0.33	0.143	0.2	1	5
C_6	0.33	0.2	0.111	0.111	0.2	1

Table 11 Fixed attributes comparison matrix

Cost (*PV – Priority vector)							
	A_1	A_2	A_3	A_4	A_5	A_6	PV*
A_1	1	0.33	1	0.33	9	9	16.25
A_2	3	1	3	1	9	9	31.94
A_3	1	0.33	1	0.33	7	7	14.37
A_4	3	1	3	1	7	9	31.25
A_5	0.11	0.11	0.143	0.143	1	0.2	2.25
A_6	0.11	0.11	0.143	0.111	5	1	3.94
Surface integrity							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	3	1	0.33	0.2	0.33	7.41
A_2	0.33	1	0.33	0.2	0.143	0.143	3.25
A_3	1	3	1	0.333	0.2	0.33	7.42
A_4	3	5	3	1	0.33	1	19.01
A_5	5	7	5	3	1	0.33	30.30
A_6	3	7	3	1	3	1	32.60
MRR							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	0.33	1	3	7	5	20.13
A_2	3	1	3	5	9	7	42.42
A_3	1	0.33	1	3	7	5	20.13
A_4	0.33	0.2	0.33	1	5	3	9.72
A_5	0.143	0.11	0.143	0.2	1	3	4.16
A_6	0.2	0.143	0.2	0.33	0.33	1	3.43
Compliance							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	3	1	0.33	0.2	0.33	7.78
A_2	0.33	1	0.33	0.2	0.143	0.2	3.67
A_3	1	3	1	0.333	0.2	0.33	7.79
A_4	3	5	3	1	0.33	1	19.13
A_5	5	7	5	3	1	3	42.51
A_6	3	5	3	1	0.33	1	19.13

Table 12 Attributes comparison matrix for initial polishing

Initial polishing							
Time							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	1	3	3	7	7	7.98
A_2	1	1	3	3	7	7	3.93
A_3	0.33	0.33	1	3	5	5	3.00
A_4	0.33	0.33	0.33	1	5	5	13.58
A_5	0.143	0.143	0.2	0.2	1	3	25.70
A_6	0.143	0.143	0.2	0.2	0.33	1	45.81
Surface finish							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	1	3	3	7	7	4.82
A_2	1	1	0.33	3	7	7	2.34
A_3	0.33	3	1	3	5	5	7.55
A_4	0.33	0.33	0.33	1	5	5	14.33
A_5	0.143	0.143	0.2	0.2	1	3	46.64
A_6	0.143	0.143	0.2	0.2	0.33	1	24.33

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Table 13 Attributes comparison matrix for intermediate polishing

Intermediate polishing							
Time							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	0.33	0.33	0.2	5	3	10.96
A_2	3	1	1	0.33	7	5	4.53
A_3	3	1	1	0.33	7	5	10.96
A_4	5	3	3	1	5	3	45.27
A_5	0.2	0.143	0.143	0.2	1	3	5.42
A_6	0.33	0.2	0.2	0.33	0.33	1	22.85
Surface finish							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	3	1	0.2	3	0.33	10.41
A_2	0.33	1	0.33	0.143	1	0.2	21.19
A_3	1	3	1	0.2	3	0.33	21.19
A_4	5	7	5	1	5	3	37.67
A_5	0.33	1	0.33	0.2	1	0.33	5.04
A_6	3	5	3	0.33	3	1	4.50

Table 14 Attributes comparison matrix for fine polishing

Fine polishing							
Time							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	5	3	0.33	0.2	0.143	7.98
A_2	0.2	1	3	0.2	0.143	0.11	3.93
A_3	0.33	0.33	1	0.2	0.143	0.143	3.00
A_4	3	5	5	1	0.33	0.2	13.58
A_5	5	7	7	3	1	0.33	25.70
A_6	7	9	7	5	3	1	45.81
Surface finish							
	A_1	A_2	A_3	A_4	A_5	A_6	PV
A_1	1	5	0.33	0.2	0.11	0.2	4.82
A_2	0.2	1	0.2	0.143	0.11	0.143	2.34
A_3	3	5	1	0.33	0.143	0.2	7.55
A_4	5	7	3	1	0.2	0.33	14.33
A_5	9	9	7	5	1	3	46.64
A_6	5	7	5	3	0.33	1	24.33

Table 15 Importance weight of the criteria

Attributes	D1	D2	D3
C_1	'H'	'M'	'ML'
C_2	'H'	'MH'	'M'
C_3	'VH'	'VH'	'VH'
C_4	'VH'	'VH'	'H'
C_5	'L'	'VL'	'L'
C_6	'VH'	'VH'	'H'

Table 16 Fuzzy rating of six alternatives with respect to the fixed four attributes

Cost				Surface integrity			
	D1	D2	D3		D1	D2	D3
A_1	'MP'	'F'	'F'	A_1	'F'	'F'	'MG'
A_2	'VG'	'G'	'VG'	A_2	'G'	'G'	'F'
A_3	'F'	'MP'	'MP'	A_3	'F'	'F'	'G'
A_4	'P'	'P'	'MP'	A_4	'VG'	'G'	'MG'
A_5	'P'	'VP'	'VP'	A_5	'VG'	'G'	'VG'
A_6	'P'	'VP'	'F'	A_6	'MG'	'F'	'G'
Compliance				MRR			
	D1	D2	D3		D1	D2	D3
A_1	'G'	'F'	'MG'	A_1	'G'	'F'	'MG'
A_2	'F'	'MG'	'G'	A_2	'VG'	'VG'	'VG'
A_3	'MG'	'G'	'VG'	A_3	'G'	'MG'	'F'
A_4	'VG'	'G'	'MG'	A_4	'F'	'MP'	'F'
A_5	'VG'	'VG'	'G'	A_5	'VP'	'VP'	'P'
A_6	'MG'	'G'	'VG'	A_6	'MG'	'F'	'MP'

Table 17 Fuzzy rating for 6 alternatives with respect to two alternatives for three stages

Initial polishing							
Time				Surface finish			
	D1	D2	D3		D1	D2	D3
A_1	'VG'	'VG'	'VG'	A_1	'VG'	'VG'	'MG'
A_2	'G'	'VG'	'VG'	A_2	'VG'	'VG'	'MG'
A_3	'P'	'MP'	'P'	A_3	'P'	'MG'	'P'
A_4	'P'	'MP'	'MP'	A_4	'P'	'MP'	'MP'
A_5	'VP'	'P'	'VP'	A_5	'VP'	'VP'	'P'
A_6	'VP'	'VP'	'P'	A_6	'P'	'MP'	'VP'
Intermediate polishing							
Time				Surface finish			
	D1	D2	D3		D1	D2	D3
A_1	'G'	'G'	'MG'	A_1	'G'	'G'	'MG'
A_2	'F'	'MP'	'F'	A_2	'MP'	'F'	'F'

Table 17 (continued)

A_3	'MP'	'MP'	'MP'	A_3	'F'	'MP'	'MP'
A_4	'G'	'VG'	'VG'	A_4	'VG'	'VG'	'MG'
A_5	'F'	'MG'	'G'	A_5	'F'	'MG'	'F'
A_6	'F'	'MP'	'F'	A_6	'F'	'G'	'F'
Fine polishing							
Time				Surface finish			
	D1	D2	D3		D1	D2	D3
A_1	'MP'	'MP'	'P'	A_1	'MP'	'P'	'MP'
A_2	'P'	'VP'	'P'	A_2	'VP'	'P'	'VP'
A_3	'MP'	'MP'	'MP'	A_3	'MP'	'F'	'MP'
A_4	'MP'	'F'	'MP'	A_4	'MP'	'F'	'MP'
A_5	'VG'	'MG'	'G'	A_5	'VG'	'VG'	'VG'
A_6	'MG'	'VG'	'G'	A_6	'VG'	'VG'	'G'

Table 21 S^*, S^-, R^* and R^- values for the three stages

	Initial polishing	Intermediate polishing	Fine polishing
S^*	(−0.31, 1.58, −7.77)	(−0.54, 1.13, −2.62)	(0.7, 3.63, 3.32)
S^-	(0.81, 3.1, 2.85)	(0.33, 3.25, 6.92)	(−0.48, 0.63, −5.13)
R^*	(0, 0.79, 2.5)	(0.09, 0.49, 3)	(0.17, 0.57, 2.5)
R^-	(0.66, 1, 9.5)	(0.24, 1, 9.5)	(0.66, 1, 9.5)

Table 18 Fuzzy evaluation matrix for six compliant polishing tools with respect to six criteria (attributes) in initial polishing stage

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(2.33, 4.33, 6.33)	(9, 10, 10)	(9, 10, 10)	(3.67, 5.67, 7.33)	(5, 7, 8.67)	(5, 7, 8.67)
A_2	(8.33, 9.67, 10)	(8.33, 9.67, 10)	(7.67, 9, 9.67)	(2.33, 4.33, 6.33)	(9, 10, 10)	(5, 7, 8.67)
A_3	(1.67, 3.67, 5.67)	(0.33, 1.67, 3.67)	(1.67, 3, 5)	(1.67, 3.67, 5.67)	(5, 7, 8.67)	(7, 8.67, 9.67)
A_4	(0.33, 1.67, 3.67)	(0.67, 2.33, 4.33)	(0.67, 2.33, 4.33)	(7.67, 9, 9.67)	(2.33, 4.33, 6.33)	(7, 8.67, 9.67)
A_5	(0, 0.33, 1.67)	(0, 0.33, 1.67)	(0, 0.33, 1.67)	(6.33, 8.33, 9.67)	(0, 0.33, 1.67)	(8.33, 9.67, 10)
A_6	(1, 2, 3.67)	(0, 0.33, 1.67)	(0.33, 1.33, 3)	(7.67, 9, 9.67)	(3, 5, 7)	(7, 8.67, 9.67)

Table 19 Fuzzy evaluation matrix for six compliant polishing tools with respect to six criteria (attributes) in intermediate polishing stage

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(2.33, 4.33, 6.33)	(6.33, 8.33, 9.67)	(6.33, 8.33, 9.67)	(3.67, 5.67, 7.33)	(5, 7, 8.67)	(5, 7, 8.67)
A_2	(8.33, 9.67, 10)	(2.33, 4.33, 6.33)	(2.33, 4.33, 6.33)	(2.33, 4.33, 6.33)	(9, 10, 10)	(5, 7, 8.67)
A_3	(1.67, 3.67, 5.67)	(1, 3, 5)	(1.67, 3.67, 5.67)	(1.67, 3.67, 5.67)	(5, 7, 8.67)	(7, 8.67, 9.67)
A_4	(0.33, 1.67, 3.67)	(8.33, 9.67, 10)	(7.67, 9, 9.67)	(7.67, 9, 9.67)	(2.33, 4.33, 6.33)	(7, 8.67, 9.67)
A_5	(0, 0.33, 1.67)	(5, 7, 8.67)	(3.67, 5.67, 7.67)	(6.33, 8.33, 9.67)	(0, 0.33, 1.67)	(8.33, 9.67, 10)
A_6	(1, 2, 3.67)	(2.33, 4.33, 6.33)	(4.33, 6.33, 8)	(7.67, 9, 9.67)	(3, 5, 7)	(7, 8.67, 9.67)

Table 20 Fuzzy Evaluation matrix for six compliant polishing tools with respect to six criteria (attributes) in fine polishing stage

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(2.33, 4.33, 6.33)	(0.67, 2.33, 4.33)	(0.67, 2.33, 4.33)	(3.67, 5.67, 7.33)	(5, 7, 8.67)	(5, 7, 8.67)
A_2	(8.33, 9.67, 10)	(0, 0.67, 2.33)	(0, 0.33, 1.67)	(2.33, 4.33, 6.33)	(9, 10, 10)	(5, 7, 8.67)
A_3	(1.67, 3.67, 5.67)	(1, 3, 5)	(1.67, 3.67, 5.67)	(1.67, 3.67, 5.67)	(5, 7, 8.67)	(7, 8.67, 9.67)
A_4	(0.33, 1.67, 3.67)	(1.67, 3.67, 5.67)	(1.67, 3.67, 5.67)	(7.67, 9, 9.67)	(2.33, 4.33, 6.33)	(7, 8.67, 9.67)
A_5	(0, 0.33, 1.67)	(7, 8.67, 9.67)	(9, 10, 10)	(6.33, 8.33, 9.67)	(0, 0.33, 1.67)	(8.33, 9.67, 10)
A_6	(1, 2, 3.67)	(7, 8.67, 9.67)	(8.33, 9.67, 10)	(7.67, 9, 9.67)	(3, 5, 7)	(7, 8.67, 9.67)

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