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ORIGINAL ARTICLE

Quality and pricing decisions in a two-echelon supply chain under multi-manufacturer competition

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Abstract In this paper, we consider the pricing and quality decisions of a single product in a two-echelon supply chain with multi-manufacturer and a single retailer. The manufacturers compete for the quality of the product and sell through a common retailer with different retail prices. The demand at the market place is dependent on both the retail price and product quality. A centralized model is developed as the benchmark case. A Stackelberg structure is assumed, where the retailer is the leader who decides the retail prices of different brands of the product produced by the manufacturers, and the manufacturers are follower who set the product quality under Cournot and Collusion policies. A special case is considered where these retail prices are the same. We compare the optimal results under two different policies, each with two pricing strategies (same and different). A numerical example demonstrates the developed models and shows that the same pricing strategy is the worst one from the supply chain's point of view while different pricing strategy is occasionally gainful from consumer's point of view.

Keywords Supply chain · Retail price · Product quality · Cournot and Collusion policy

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1 Introduction

In recent times, the role of supply chain (SC) endures some dramatic changes. Satisfying the retailer's demand is not only the prime interest of the manufacturer, but also maintaining the quality of the supplied product. The product quality can be defined as the fulfillment of customer expectations. If customer expectations are not fulfilled then the product is termed as low-quality product. Customer expectations may vary from product to product. For example, for a mechanical or electronic product, these are performance, reliability, safety, and appearance; for pharmaceutical products, physical and chemical characteristics, medicinal effect, toxicity, taste, and shelf life may be important; for a food product, expectations include taste, nutritional properties, texture, and shelf life and so on. Product quality is very important for the company. Bad quality products affect the customer's confidence, image and sales of the company. Product quality is equally important for consumers who are ready to pay high price, but in return, expect high quality. If they are not satisfied with the quality of product of a company, they purchase from the competitors. Nowadays, very good quality international products are available in the local markets. So, if the domestic companies don't improve product quality, they will struggle to survive in the market. Thus, product quality has significant impact on the life and performance of a supply chain.

There are quite a few successful firms who have focused on service and quality of their products in building brand loyalty. As for example, IBM and HP companies are famous for their customer support [27]. This reputation gives them an edge over their competitors. In electronic appliances (washer and dryer) market, Maytag and GE are competing to sell their appliances through common retailers such as Sears or BestBuy. One of the major concerns for end customers is not only how low the price is, but also how good the service that comes with the appliance he or she expects to receive. In the above examples, the manufacturers interact directly with the end customers through service channel. They can get feedback of product quality from customers. On the other hand, the retailer can also collect the feedback information from customers through returned products. However, the information collection and follow up feedback require retailer's effort. Thus, the vertical coordination takes account of the players' optimal allocation of efforts for quality improvement.

Competition between multi-quality but same product of multiple manufacturers is common in the market place. Though such competition is important, only limited research progress has been made. In this paper, we present a general analysis of oligopolistic competition in quantities between manufacturers offering multiple quality-differentiated products. We address some important issues: Firstly, when do the insights of single-product Cournot models or Collusion models carry over to a multi-quality product's world? Secondly, how is manufacturer's product line determined by the properties of demand, its selling prices, and qualities? What is impact on manufacturer's product quality when the retailer declares the same price for all qualities product?

To answer these questions, we develop a decision support framework for product quality and pricing in a two-echelon supply chain in which a retailer sells a product of different qualities (or varieties) produced by different manufacturers. In the practice, there exist various forms of competition, Cournot competition being one relatively common in the real world. GOME and Suning are two home appliances where retailers follow Cournot competition. Wal-Mart and Tesco, Carrefour and Auchan, etc. are also cases where Oligopolistic Cournot game structure applies. Another behavior pursued by the duopolistic retailers in reality is Collusion, which is a non-competitive agreement between rivals. By collaborating with each other, rival firms look to alter the local advertising effort or price of a product to their advantage. Since explicit Collusion is usually illegal, most Collusion behaviors between two retailers or manufacturer are confidential, including secret agreement, tacit agreement or price alliance, etc. For more detailed information about the application of Cournot and Collusion competition, see Wang et al. [39].

Many supply chain models have been developed on the issue of price/non-price competition, but only a few of them consider both price and non-price (quality, service, etc.) competition between duopolistic manufacturers/suppliers in upstream level or duopolistic retailers/buyers in downstream level in a two-echelon supply chain. However, in some practical supply chains, oligopolistic situation is frequently occurred in any echelon of the supply chain. The main contribution of this paper is to analyze the effects of Cournot and Collusion policies (adopted by oligopolistic manufacturers in a Stackelberg game where the retailer is the leader and manufacturers are follower) on the optimal solution as well as the channel profit of a two-echelon supply chain.

In order to focus our study on the role of quality and selling price of a product in competition between the multiple oligopolistic manufacturers in the supply chain, it is necessary to structure situations of study. There are some possible scenarios for the strategic interactions between multi-manufacturer and a monopolistic retailer: (1) The centralized case: benchmark case, (2) Retailer-Stackelberg where manufacturers pursue Cournot competition, (3) Retailer-Stackelberg where manufacturers pursue Collusion collaboration. In a market, a monopolistic manufacturer has generally the dominant power. However, in our study, oligopolistic manufacturers are assumed to have lower bargaining power. The retailer has more negotiation power due to its dominating size or customer loyalty. So, he usually sets different prices for different brands of the product. The retailer can also consider the case where different brands of the product from different manufacturers have the same retail price.

The rest of the paper is organized as follows. The next section provides a review of the related literature. Section 3 describes the notation and assumptions adopted in this paper. Section 4 describes model formulation and analysis. The exclusive form of the optimal solution and the case of unique retail price strategy (single pricing strategy) is illustrated in Section 5. To check the validity of the model, numerical solution and sensitivity analysis are carried out in Sections 6 and 7, respectively. Section 8 draws conclusions and suggests some directions for future investigations.

2 Literature review

Supply chain quality management practices in industries (automobile, electronic appliances assembly industries, and food processing industries) are significantly correlated with players' strategies which influence tangible business results and customer satisfaction levels [25]. Supply chain managers (e.g., Wal-Mart supply chain manager) check samples of products provided by the manufacturer. If the product is acceptable, considering market demand, the manager places an order on the basis of quality, and price of products (Walmart annual report [38]). Although higher quality can be a reason for higher price, it can also cause higher cost. Both quality and price influence demand and profit [4, 5]. Therefore, quality improvement and pricing decision are important for supply chain players.

In the academic literature, the effect of quality improvement has received less attention. Singer et al. [32] derived to unravel the strategic behavior regarding quality within a supplier-retailer partnership in a disposable product industry. The effect of SC relationship on quality performance has been considered by Fynes et al. [16]. Chambers et al. [10] considered the impact of variable production costs on competitive behavior in a duopoly where manufacturer compete on quality and price in a two-stage game. Chao et al. [11] discussed two contractual agreements by which product recall costs can be shared between a manufacturer and a supplier to induce quality improvement effort. Hsieh and Liu [20] examined quality improvement actions in their production processes to reduce defective items being produced. Rong et al. [31] studied quality improvement in different SC models. They applied in an illustrative case study to show (1) how the generic model can be implemented in a specific situation, (2) how the product quality can be modeled on a discrete scale, and (3) what kind of results are obtained from the model. Xie et al. [42] investigated quality investment and price decision of a make-to-order (MTO) supply chain with uncertain demand in international trade. Xie et al. [43] considered quality improvement in a given segment of the market, shared by two supplier-manufacturer supply chains which offer a given product at the same price but compete on quality. Giovanni [18] characterized advertising, pricing, and quality improvement strategies in a dynamic setting in which the demand depends on both price and goodwill. Tse and Tan [37] argued that better visibility of risk in supply chain could minimize the threat of product harm. They proposed a supply chain product quality risk management framework, integrating both the incremental calculus and marginal analysis. Wang and Li [40] discussed two issues: (1) impact of the accuracy of quality or shelf-life indicator, which underlies the pricing and sales management decisions at retailing operations, on retailing performance; (2) impact of pricing in terms of timing and frequency of discount in a selling period on retailing performance.

The above studies focus on the coordination and effect of quality maintenance for single manufacture and single retailer. In practice, a retailer can buy different quality levels product with different prices to satisfy various levels of customer demand. In many industries, competition is shifting from price to quality in specific segments of the market [17, 30]. That is, competitors adopt the same price policy but offer different qualities of product in a given market segment. Dolgui and Proth [14] showed the impact of pricing on selling volume and analyzed the associated pricing strategy. Consequently, Liu et al. [26] established the fact that a high-reputation seller is more likely to charge a lower price than a low-reputation seller. The variant pricing strategy is employed to obtain both sides' cost savings for a long-term relationship [12]. For example, in the fast food market, McDonalds and KFC compete by providing products with different designs and tastes. Also, there are competing pairs such as Coca-Cola and Pepsi-cola in soft drinks market.

The majority of studies mentioned above have considered price or product quantity as the only dimension of competition. Early research focusing on attributes such as product quality and service can be found in the economics literature [13, 27, 34]. In marketing literature, Jeuland and Shugan [21] included non-price variable such as quality and services in their model with the profit function as a linear function of service amount. Moorthy [28] examined a competition in duopoly through both price and quality. Yang and Zhou [44] considered the pricing and quantity decisions of a two-echelon system with a manufacturer who supplies a single product to two duopolistic retailers. They analyzed the effects of the duopolistic retailers' different competitive behaviors—Cournot, Collusion, and Stackelberg—on the optimal decisions of the manufacturer and the duopolistic retailers themselves. Although many works have been done on quality competition but those competitions are between two duopolistic members only. Here, we consider competition among oligopolistic members (multiple manufacturers). Also, we establish a collaborative behavior among these members (manufacturers) which form the Collusion solution in the system. A comparative study of the centralized policy and the two competition (Cournot and Collusion) policies has been carried out. A special form of general oligopolistic model, i.e., duopolistic competition, is studied. Further, the case where the retailer sets the same price for all brands of the product is also investigated.

The present work considers game theoretical approach in the manufacturer-retailer interaction. Pricing game has been studied for decades. Several recent research papers discuss the pricing Stackelberg game—Cournot and Nash—under cooperative or competitive situations. Kohli and Park [22] formulated a cooperative game and examined the negotiation process between the seller and the buyer when they bargain for the order quantity and the average unit price. Abad [1] formulated the problem of vendor-buyer coordination as a two-person cooperative game and developed the Pareto efficient and Nash bargaining solutions. Weng [41] studied a supply chain with one manufacturer and multiple identical retailers. He showed that the Stackelberg game guaranteed perfect coordination considering quantity discounts and franchise fees. Lariviere and Porteus [23] and Slikker et al. [33] studied the newsvendor problem by Game Theory approach. Nash equilibria have received a considerable attention both in theory and practice [19]. A few research works use Nash game and coordination scheme in supply chain [3, 6–9, 33]. Yu et al. [45–47] considered

Stackelberg game and its improvement in a vendor managed inventory (VMI) system for optimizing advertising, pricing, and inventory policies. They simultaneously considered pricing and order intervals as decision variables using Stackelberg game in a supply chain with one manufacturer and multiple retailers. Mukhopadhyay et al. [29] considered a duopoly market where two separate firms offer complementary goods in a leader- follower type move. Feng and Lu [15] applied the Nash–Nash solution to a two-level supply chain in which two manufactures outsource production to two exclusive suppliers or to a single supplier. Li et al. [24] examined the influence of competition among supply chain partners on product demand under Cournot and Stackelberg games environments. Tsao et al. [35] studied shelf-space allocation and trade allowance simultaneously for category-level shelf-space management under Stackelberg game framework between retailer and manufacturers. Recently, Alaei et al. [2] investigated some Stackelberg game situations in which the retailers can either compete or cooperate that lead to Cournot and Collusion behavior, respectively. In this paper, we demonstrate Stackelberg leader-follower game theory where the retailer acts as the leader and manufacturers as follower. Under this Stackelberg game, the manufacturers apply two types of decision policy—Cournot and Collusion policies.

3 Notation and assumptions

The following notation is used for developing the proposed model:

n	number of manufactures in the system.
p_i	selling price of the product supplied by the manufacturer i where $i \in \{1, 2, 3....n\}$.
x_i	quality of the product supplied by the manufacturer i where $i \in \{1, 2, 3....n\}$ and $0 \le x_i \le 1$.
d_i	basic market demand of the product supplied by the manufacturer i where $i \in \{1, 2, 3....n\}$.
$D_i(x_i, p_i)$	demand rate of the product supplied by manufacturer i at the retailer, where $i \in \{1, 2, 3....n\}$.
c_i	quality improvement cost at the manufacturer i where $i \in \{1, 2, 3....n\}$.
g_i	good will loss cost for completely impure product at the manufacturer i where $i \in \{1, 2, 3....n\}$.
c_{mi}	procurement price of unit product at the manufacturer i where $i \in \{1, 2, 3....n\}$.
ω_i	wholesale price of unit product at the manufacturer i where $i \in \{1, 2, 3....n\}$.
Π_r	average profit of the retailer.
Π_{mi}	average profit of the manufacturer i where $i \in \{1, 2, 3....n\}$.
Π	average profit of the whole system.

The following assumptions are made to develop the proposed model:

(1) The supply chain under consideration consists of n competing manufacturers and a single retailer who trades a single product with different brands or qualities.

(2) The demand rate $D_i(x_i, p_i)$ of the product of the manufacturer i at the retailer is dependent on both the product quality (x_i) and selling price (p_i) of the product, which is the extended form of [5, 42] in multi-manufacturer competitive environment. We take
$$D_i(x_i, p_i) = (d_i - a_i p_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} b_j p_j + \alpha_i x_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j x_j$$
where a_i, b_i, α_i, and β_i are positive $\left(a_i > \sum_{\substack{j=1 \\ j \neq i}}^{n} b_j, \alpha_i > \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j\right)$ constants and such that demand is always positive. The form of demand function indicates that higher quality gives higher demand and also lower selling price gives higher demand. If one manufacturer produces high-quality product, then it affects demands of other manufacturers who produce lower quality product. As a result, all manufacturers keen to produce high-quality product. If the retailer sets a high price to a product of a particular manufacturer then the manufacturer are bound to produce high-quality product. Otherwise, the product demand of the particular manufacturer decreases because of other manufacturers who produce a better quality product with lower selling price. In case of unique retail price, the demand of the product from manufacturer i is $D_i(x_i, p) = d_i - a_i p + \alpha_i x_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j x_j$. With the same price, a higher quality level always brings more consumers in the same market segment. Generally speaking, in the market for a particular product, products with "high quality, high price" are provided for high-end customers, who constitute the most price insensitive segment of the market [10]. This policy is suitable to the consumers who are not conscious about the product quality but accept lower price.

(3) Each firm has a complete knowledge about the demand conditions of its product. Also, each firm decides about its output under the assumption that the rival will not change his output.

(4) Shortages are not allowed.

4 Model formulation and analysis

We consider a supply chain where n manufacturers produce the same product with different brands and sell through a common retailer. Each brand of the product is not 100 % pure in quality. Some impurity (piracy) is mixed up with the product. Thus, the product quality (x_i) lies between 0 and 1. So $(1-x_i)$ is the impurity which is mixed up with the product. For this impurity, the manufacturer i may lose his goodwill, which incurs a cost (goodwill loss cost) at a rate g_i. We assume that the investment in qualities has a decreasing return to scale viz., the next dollar invested by the manufacturer returns less quality than the last dollar invested, i.e., it is harder (and costs more) to provide the next unit of quality than the last one. This can be reflected in the quadratic form of the cost of providing quality or services. The same quadratic equation is also used in [5, 28, 36]. The demand of each brand is dependent on its quality and selling price. Clearly, the selling price of each brand is to be decided by the retailer but the quality and wholesale price (constant) are decided by the corresponding manufacturer. At first, we derive the profit functions of the retailer and manufacturers.

The manufacturer i's selling revenue $= (\omega_i - c_{mi}) D_i(x_i, p_i)$, quality improvement cost $= c_i x_i^2$, the quadratic form suggests diminishing returns. Diminishing returns are certainly natural if this notion of quality has a significant component. Under the assumptions, any product quality increasing from, say, 87 to 89 % typically requires a greater incremental investment than does increasing from 85 to 87 % [36].

The goodwill loss cost ([42, 43]) $= (1 - x_i)g_i$. It is assumed that when products are 100 % impure, i.e, $x_i = 0$ then cost incurs at g_i rate. As the manufacturer increases the product quality gradually, the cost decreases at $(1-x_i)g_i$ rate. If the products are 100 % pure (i.e $x_i = 1$), then no cost incurs in the manufacturer's profit function.

Therefore, the manufacturer i's profit function is given by

$$\Pi_{mi}(x_i) = (\omega_i - c_{mi})D_i(x_i, p_i) - c_i x_i^2 - (1 - x_i)g_i, \quad \text{where } i \in \{1, 2, 3 \ldots n\}$$

The total profit of all manufacturers is

$$\Pi_m(x_1, x_2, x_3, \ldots x_n) = \sum_{i=1}^{n} \left[(\omega_i - c_{mi})D_i(x_i, p_i) - c_i x_i^2 - (1 - x_i)g_i \right] \tag{1}$$

The retailer profit component for the product delivered by the manufacturer i is $(p_i - \omega_i)D_i(x_i, p_i)$. Therefore, the retailer's profit is given by

$$\Pi_r(p_1, p_2, p_3, \ldots p_n) = \sum_{i=1}^{n} (p_i - \omega_i)D_i(x_i, p_i) \quad \text{where } i = \{1, 2, 3 \ldots n\}. \tag{2}$$

4.1 Centralized policy

In the centralized policy, the retailer and the manufacturers cooperatively decide the qualities and the retail prices of the product and maximize the whole system profit. From Eqs. 1 and 2, we have the decision model for the integrated system with variables $x_1, x_2, x_3, \ldots x_n$, and $p_1, p_2, p_3, \ldots p_n$.

$$\Pi(x_1, x_2, x_3, \ldots x_n; p_1, p_2, p_3, \ldots p_n) = \sum_{i=1}^{n} (p_i - c_{mi})D_i(x_i, p_i) - \sum_{i=1}^{n} c_i x_i^2 - \sum_{i=1}^{n} (1 - x_i)g_i \tag{3}$$

For the optimal solution, the necessary conditions are $\frac{\partial \Pi}{\partial p_i} = 0$ and $\frac{\partial \Pi}{\partial x_i} = 0$, $i = 1, 2, 3, \ldots n$.

$$\frac{\partial \Pi}{\partial p_i} = 0 \text{ gives } d_i - 2a_i p_i + \sum\nolimits^{n}_{\substack{j=1 \\ j \neq i}} b_j p_j + \alpha_i x_i - \sum\nolimits^{n}_{\substack{j=1 \\ j \neq i}} \beta_j x_j + c_{mi} a_i + \sum\nolimits^{n}_{\substack{j=1 \\ j \neq i}} (p_j - c_{mj}) b_i = 0.$$

and from $\frac{\partial \Pi}{\partial x_i} = 0$, we get, $(p_i - c_{mi})\alpha_i - 2c_i x_i + g_i - \sum\nolimits^{n}_{\substack{j=1 \\ j \neq i}} (p_j - c_{mj}) b_i = 0$

Therefore, we have the system of equations of the form $TX = R$, i.e.,

$$\begin{pmatrix} -2a_1 & 2b_2 & 2b_3 & \ldots & 2b_n & \alpha_1 & -\beta_2 & -\beta_3 & \ldots & -\beta_n \\ 2b_1 & -2a_2 & 2b_3 & \ldots & 2b_n & \beta_1 & \alpha_2 & -\beta_3 & \ldots & -\beta_n \\ 2b_1 & 2b_2 & -2a_3 & \ldots & 2b_n & \beta_1 & -\beta_2 & \alpha_3 & \ldots & -\beta_n \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 2b_1 & 2b_2 & 2b_3 & \ldots & -2a_n & \beta_1 & -\beta_2 & -\beta_3 & \ldots & \alpha_n \\ \alpha_1 & -\beta_1 & -\beta_1 & \ldots & -\beta_1 & -2c_1 & 0 & 0 & \ldots & 0 \\ -\beta_2 & \alpha_2 & -\beta_2 & \ldots & -\beta_2 & 0 & -2c_2 & 0 & \ldots & 0 \\ -\beta_3 & -\beta_3 & \alpha_2 & \ldots & -\beta_3 & 0 & 0 & -2c_3 & \ldots & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ -\beta_n & -\beta_n & -\beta_n & \ldots & -\alpha_n & 0 & 0 & 0 & \ldots & -2c_n \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ . \\ p_n \\ x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ . \\ A_n \\ B_1 \\ B_2 \\ B_3 \\ . \\ B_n \end{pmatrix} \tag{4}$$

where, $A_i = -\left(d_i + \alpha_i x_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j x_j + c_{mi} a_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} c_{mj} b_j \right)$, $B_i = \alpha_i c_{mi} - \beta_i \sum_{\substack{j=1 \\ j \neq i}}^{n} c_{mj} - g_i$

From Eq. 3, we have

$$\frac{\partial^2 \Pi}{\partial p_i^2} = -2a_i < 0, \ \frac{\partial^2 \Pi}{\partial p_i \partial x_i} = \alpha_i, \ \frac{\partial^2 \Pi}{\partial x_i^2} = -2c_i, \ \frac{\partial^2 \Pi}{\partial p_i \partial p_j} = b_i + b_j, \ \frac{\partial^2 \Pi}{\partial x_i \partial x_j} = 0, \ \frac{\partial^2 \Pi}{\partial p_i \partial x_j} = -\beta_j$$

For the existence of unique optimal solution, the Hessian matrix H_1 should be negative definite, i.e., all the eigen values of H_1 should be negative. Now, the Hessian matrix for these $2n$ variable is given below.

$$H_1 = \begin{pmatrix} -2a_1 & b_1+b_2 & b_1+b_3 & \ldots & b_1+b_n & \alpha_1 & -\beta_2 & -\beta_3 & \ldots & -\beta_n \\ b_2+b_1 & -2a_2 & b_2+b_3 & \ldots & b_2+b_n & -\beta_1 & \alpha_2 & -\beta_3 & \ldots & -\beta_n \\ b_3+b_1 & b_3+b_2 & -2a_3 & \ldots & b_3+b_n & -\beta_1 & -\beta_2 & \alpha_3 & \ldots & -\beta_n \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ b_n+b_1 & b_n+b_2 & b_n+b_3 & \ldots & -2a_n & -\beta_1 & -\beta_2 & -\beta_3 & \ldots & \alpha_n \\ \alpha_1 & -\beta_1 & -\beta_1 & \ldots & -\beta_1 & -2c_1 & 0 & 0 & \ldots & 0 \\ -\beta_2 & \alpha_2 & -\beta_2 & \ldots & -\beta_2 & 0 & -2c_2 & 0 & \ldots & 0 \\ -\beta_3 & -\beta_3 & \alpha_3 & \ldots & -\beta_3 & 0 & 0 & -2c_3 & \ldots & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ -\beta_n & -\beta_n & -\beta_n & \ldots & \alpha_n & 0 & 0 & 0 & \ldots & -2c_n \end{pmatrix}$$

Theorem 1 *Every eigen value λ of matrix A_{nn} satisfies the condition $|\lambda - A_{ii}| \leq \sum_{i=1}^{n} A_{ij}, \ \ i, j \in \{1, 2, 3, ...n\}$.*

Proof Suppose that λ is an eigenvalue of the matrix A. The matrix $\lambda I - A$ is strictly diagonally dominant if $|\lambda - A_{ii}| \geq \sum_{i=1}^{n} A_{ij}, \ \ i, j \in \{1, 2, 3, ...n\}$. If the above result is not true, $\lambda I - A$ is strictly diagonally dominant, which implies that $\lambda I - A$ is non-singular. But, it contradicts that λ is an eigen value. So, our result is true. □

Corollary 1 *The range of eigen value λ for any matrix A_{nn} is given by $\lambda \in \left[A_{ii} - \sum_{i=1}^{n} A_{ij}, \ A_{ii} + \sum_{i=1}^{n} A_{ij} \right]$.*

The Theorem and Corollary above suggest that every eigen value of the Hessian matrix H_1 is negative if $a_i > nb_i + \alpha_i - \sum_{j=1, j \neq i}^{n} \beta_j$ and $2c_i > \alpha_i - n\beta_i$ hold. Therefore, H_1 is negative definite. Hence, the required solution is unique.

4.2 Retailer-Stackelberg game

In this sub-section, we will develop the model with n competitive manufacturers and a retailer. We assume that the retailer is the Stackelberg leader and manufacturers are follower. The manufacturers play two different games in the upstream level. In the first one, all manufacturers find their Cournot solution, i.e., each manufacturer independently sets its brand quality by assuming rivals' brand qualities as a parameter. In the second case, manufacturers act in Collusion, i.e., they jointly design qualities of their brands in order to maximize the total profit. In both the policies, the retailer first declares his retail prices p_is for the products delivered by different manufacturers. After the retailer's declaration, the manufacturers set their product qualities in two different policies. The mathematical calculations are done in reverse way, i.e., first the product qualities set by the manufacturers are determined and then the optimal retail prices set by the retailer are found. The derivations are shown in the following two subsections.

4.2.1 Manufacturers pursue the Cournot solution

The Cournot solution is appropriate to a market with a single homogeneous product which is produced by n distinct firms or manufacturers. In the Cournot model, each firm chooses a level of output which maximizes its profits, given the output of its competitors. The Cournot equilibrium is a Nash equilibrium where each firm correctly assumes that its competitors behave optimally. As the number of firms in a market changes from one to many, the Cournot equilibrium changes from monopoly to the perfectly competitive equilibrium. All firms know the total number of firms in the market and take the output of the others as given. In this model, manufacturer i chooses his output x_i in order to maximize his profit $\Pi_{mi}(x_i)$, given the output decisions of all other manufacturers.

Let us suppose that the retailer sets the retail price p_i. Then, all the manufacturers independently decide their brand qualities under the basic behavior assumption of the Cournot solution. For mathematical analysis, we first derive the manufacturers' responses and take these responses to set their retail prices. For this, we solve $\frac{\partial \Pi_{mi}}{\partial x_i} = 0$, for $i = 1, 2, 3, ...n$. It is easy to see that $\frac{\partial^2 \Pi_{mi}}{\partial x_i^2} = -2c_i < 0$ and $\frac{\partial^2 \Pi_{mi}}{\partial x_i \partial x_j} = 0$ implying that the Hessian matrix is negative definite. Therefore, there exists a unique optimal response which is given by

$$x_i = \frac{(\omega_i - c_{mi})\alpha + g_i}{2c_i} \tag{5}$$

After getting these responses from manufacturers, the retailer sets the retail prices for all brands by equating $\frac{\partial \Pi_r}{\partial p_i} = 0$ for $i = 1, 2, 3, ...n$.

From modeling point of view, many production settings in practice involve markets that simultaneously trade multiple products or a single product with different qualities. Firstly, assume that products differ on a number of characteristics and consumers have a taste for variety and they consume a variety of brands. For example, most consumers prefer to go to a variety of restaurants, rather than eating at the same Chinese, Italian, or Mexican restaurant time and again. Secondly, consider the model with, say, horizontal product differentiation where consumers have a preference for one brand over another. For example, when a red and a blue Honda Civics are priced the same and are homogeneous in every other way, some consumers will prefer the red Civic and others the blue Civic. Thus, these different colored cars are horizontally differentiated.

4.2.2 Manufacturers pursue the Collusion solution

The Collusion solution takes place within an industry in the study of economics and market competition when rival companies cooperate for their mutual benefit. Collusion most often takes place within the market structure of oligopoly, where the decision of a few manufacturers to collude can significantly impact the market as a whole. If the manufacturers are independent then they optimize their own profits and efficiencies. As a result, loss of the whole system increases. However, if manufacturers collude to all and determine the optimal decisions then loss of sales is minimized. So, we assume that the competitive manufacturers recognize their interdependence and agree to act in union in order to maximize their total profit. The total profit of the manufacturers is

$$\Pi_m(x_1, x_2, x_3, ...x_n) = \sum_{i=1}^{n} \Pi_{mi}(x_i) \tag{6}$$

The system of equations $\frac{\partial \Pi_m}{\partial x_1} = 0$, $\frac{\partial \Pi_m}{\partial x_2} = 0$, $\frac{\partial \Pi_m}{\partial x_3} = 0$, ..., $\frac{\partial \Pi_m}{\partial x_n} = 0$ gives the optimal solution as

$$x_i = \frac{1}{2c_i}\left[\alpha_i(\omega_i - c_{mi}) - \beta_i \sum_{\substack{j=1 \\ j \neq i}}^{n} (\omega_j - c_{mj})\right] + g_i \tag{7}$$

We check that $\frac{\partial^2 \Pi_m}{\partial x_i^2} = -2c_i < 0$ and $\frac{\partial^2 \Pi_m}{\partial x_i \partial x_j} = 0$ implying that the associated Hessian matrix is negative definite and thus the solution is unique.

From the retailer's perspective, we have the profit function

$$\Pi_r(p_1, p_2, p_3, ...p_n) = \sum_{i=1}^{n} (p_i - \omega_i) D_i(x_i, p_i) \tag{8}$$

For optimality, $\frac{\partial \Pi_r}{\partial p_i} = 0$ which gives

$$-2a_i p_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} b_j p_j + b_i \sum_{j=1}^{n} p_j = -d_i - \alpha_i x_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j x_j - a_i \omega_i + b_i \sum_{\substack{j=1 \\ j \neq i}}^{n} \omega_j$$

The solution can be put in the form

$$AP = B \tag{9}$$

where

$$A = \begin{pmatrix} -2a_1 & b_2 + b_1 & b_3 + b_1 & ... & b_n + b_1 \\ b_1 + b_2 & -2a_2 & b_3 + b_2 & ... & b_n + b_2 \\ b_1 + b_3 & b_2 + b_3 & -2a_3 & ... & b_n + b_3 \\ .. & .. & .. & .. & .. \\ b_1 + b_n & b_2 + b_n & b_3 + b_n & ... & -2a_n \end{pmatrix}$$

$$P = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ . \\ . \\ . \\ p_n \end{pmatrix}, \quad B = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ . \\ . \\ . \\ B_n \end{pmatrix}$$

where

$$B_i = -d_i - \alpha_i x_i - a_i \omega_i - b_i \sum_{\substack{j=1 \\ j \neq i}}^{n} \omega_j - \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j x_j$$

Solving Eq. 9, we get $p_1, p_2, ..., p_n$ because $x_1, x_2, ..., x_n$ are known from Eqs. 5 and 7.

Now, the associated Hessian matrix is given by

$$H_2 = \begin{pmatrix} \frac{\partial^2 \Pi_r}{\partial p_1^2} & \frac{\partial^2 \Pi_r}{\partial p_1 \partial p_2} & \frac{\partial^2 \Pi_r}{\partial p_1 \partial p_3} & \cdots & \frac{\partial^2 \Pi_r}{\partial p_1 \partial p_n} \\ \frac{\partial^2 \Pi_r}{\partial p_1 \partial p_2} & \frac{\partial^2 \Pi_r}{\partial p_2^2} & \frac{\partial^2 \Pi_r}{\partial p_2 \partial p_3} & \cdots & \frac{\partial^2 \Pi_r}{\partial p_2 \partial p_n} \\ \frac{\partial^2 \Pi_r}{\partial p_1 \partial p_3} & \frac{\partial^2 \Pi_r}{\partial p_2 \partial p_3} & \frac{\partial^2 \Pi_r}{\partial p_3^2} & \cdots & \frac{\partial^2 \Pi_r}{\partial p_3 \partial p_n} \\ .. & .. & .. & .. & .. \\ \frac{\partial^2 \Pi_r}{\partial p_1 \partial p_n} & \frac{\partial^2 \Pi_r}{\partial p_2 \partial p_n} & \frac{\partial^2 \Pi_r}{\partial p_3 \partial p_n} & \cdots & \frac{\partial^2 \Pi_r}{\partial p_n^2} \end{pmatrix}$$

$$= \begin{pmatrix} -2a_1 & b_2 + b_1 & b_3 + b_1 & ... & b_n + b_1 \\ b_1 + b_2 & -2a_2 & b_3 + b_2 & ... & b_n + b_2 \\ b_1 + b_3 & b_2 + b_3 & -2a_3 & ... & b_n + b_3 \\ .. & .. & .. & .. & .. \\ b_1 + b_n & b_2 + b_n & b_3 + b_n & ... & -2a_n \end{pmatrix}$$

Similarly, Theorem 1 and Corollary 1 suggest that eigen values of the Hessian matrix H_2 are negative if $a_i > nb_i + \alpha_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j$. Then H_2 is negative definite. Hence, the required solution is unique.

5 Particular cases

5.1 2-Manufacturer system

To ease complexity of the model, we now consider the system with two manufacturers. Important results derived are given in the following Lemma and Proposition.

Lemma 1 *For the model with two manufactures, the optimal solution exists when*

$$16a_1a_2c_1c_2 - 16b_1b_2c_1c_2 - 4a_2c_2\alpha_1^2 - 4a_1c_1\alpha_2^2 + \alpha_1^2\alpha_2^2$$
$$+4b_1c_2\alpha_1\beta_1 + 4b_2c_2\alpha_1\beta_1 - 4a_1c_2\beta_1^2 + 4b_1c_1\alpha_2\beta_2$$
$$+4b_2c_1\alpha_2\beta_2 - 2\alpha_1\alpha_2\beta_1\beta_2 - 4a_2c_1\beta_2^2 + \beta_1^2\beta_2^2 > 0$$

Proof The condition follows from the associated Hessian matrix. □

Proposition 1

(i) *The optimal centralized solution is*

$$x_i = \big[\alpha_j\ (b_i g_j\alpha_i + b_j g_j\alpha_i - 2a_i g_i\alpha_j - d_i\alpha_i\alpha_j - 2a_i g_j\beta_i) + (-2a_j g_j\alpha_i + 2(b_i+b_j)g_i\alpha_j - d_j\alpha_i\alpha_j + (b_i+b_j)g_j\beta_i + d_i\alpha_j\beta_i)\beta_j + (-2a_j g_i + d_j\beta_i)\beta_j^2 + 2c_j(2a_j d_i\alpha_i - (b_i+b_j)((b_i+b_j)g_i - d_j\alpha_i + d_i\beta_i) + a_i(4a_j g_i - 2d_j\beta_i) + (b_j(b_i+b_j)\alpha_i + a_i(-2a_j\alpha_i + b_i\beta_i - b_j\beta_i))c_{mi} + (-b_i(b_i+b_j)\beta_i + a_j(b_i\alpha_i - b_j\alpha_i + 2a_i\beta_i))c_{mj}) + (\alpha_i\alpha_j - \beta_i\beta_j)\ ((a_i\alpha_j - b_j\beta_j)c_{mi} + (-b_i\alpha_j + a_j\beta_j)c_{mj})\big] \times [\ (\alpha_i\alpha_j - \beta_i\beta_j)^2 + 4(-a_i\alpha_j^2 + \beta_j((b_i+b_j)\alpha_j - a_j\beta_j))c_i - 4(a_j\alpha_i^2 + \beta_i(-(b_i+b_j)\alpha_i + a_i\beta_i) + (-4a_i a_j + (b_i+b_j)^2)c_i)\ c_j\big]^{-1} \quad \textit{where}\ \ i,j \in = \{1,2\}\ \ \textit{and}\ \ j \neq i. \tag{10}$$

$$p_i = \big[-(\alpha_i\alpha_j - \beta_i\beta_j)\ (g_i\alpha_j + g_j\beta_i + (-\alpha_i\alpha_j + \beta_i\beta_j)c_{mi}) - 2c_j(-2a_j g_i\alpha_i + \beta_i((b_i+b_j)g_i + d_j\alpha_i + d_i\beta_i) + (2a_j\alpha_i^2 + \beta_i(-(2b_i+b_j)\alpha_i + a_i\beta_i))c_{mi} + \beta_i(-a_j\alpha_i + b_i\beta_i)c_{mj}) + 2c_i(\alpha_j((b_i+b_j)g_j - d_i\alpha_j) - (2a_j g_j + d_j\alpha_j)\beta_j + (-a_i\alpha_j^2 + (2b_i+b_j)\alpha_j\beta_j - 2a_j\beta_j^2)c_{mi} + \alpha_j(-b_i\alpha_j + a_j\beta_j)c_{mj} + 2c_j(2a_j d_i + (b_i+b_j)d_j + (2a_i a_j - b_i(b_i+b_j))c_{mi} + a_j(b_i - b_j)c_{mj}))\ ] \times [\ (\alpha_i\alpha_j - \beta_i\beta_j)^2 + 4(-a_i\alpha_j^2 + \beta_j((b_i+b_j)\alpha_j - a_j\beta_j))c_i - 4(a_j\alpha_i^2 + \beta_i(-(b_i+b_j)\alpha_i + a_i\beta_i) + (-4a_i a_j + (b_i+b_j)^2)c_i)\ c_j\big]^{-1} \quad \textit{where}\ \ i,j \in = \{1,2\}\ \ \textit{and}\ \ j \neq i. \tag{11}$$

(ii) *The optimal Cournot solution is*

$$p_i^{cr} = \frac{1}{2(4a_i a_j - (b_i+b_j)^2)c_i c_j}\big[c_j\left(2a_j\alpha_i - (b_i+b_j)\beta_i\right)(g_i + w_i\alpha_i - \alpha_i c_{mi}) + c_i\big\{2c_j\left((b_i+b_j)d_j - b_i(b_i+b_j)w_i + a_j(2d_i + 2a_i w_i + b_i w_j - b_j w_j)\ \right) + \left((b_i+b_j)\alpha_j - 2a_j\beta_j\right)\left(g_j + w_j\alpha_j - \alpha_j c_{mj}\right)\ \}\ ] \quad \textit{where}\ \ i,j \in = \{1,2\}\ \ \textit{and}\ \ j \neq i. \tag{12}$$

$$x_i^{cr} = \frac{g_i + w_i\alpha_i - \alpha_i c_{mi}}{2c_i} \tag{13}$$

(iii) *The optimal Collusion solution is*

$$p_i^{cl} = \frac{1}{2(4a_i a_j - (b_i+b_j)^2)c_i c_j}\big[c_j\left(2a_j\alpha_i - (b_i+b_j)\beta_i\right)\left(g_i + w_i\alpha_i - w_j\beta_i - \alpha_i c_{mi} + \beta_i c_{mj}\right) + c_i\left\{2c_j\left((b_i+b_j)d_j - b_i(b_i+b_j)w_i + a_j(2d_i + 2a_i w_i + b_i w_j - b_j w_j)\right) + \left((b_i+b_j)\alpha_j - 2a_j\beta_j\right)\left(g_j + w_j\alpha_j - w_i\beta_j + \beta_j c_{mi} - \alpha_j c_{mj}\right)\ \right\}\ ] \tag{14}$$

$$x_i^{cl} = \frac{g_i + w_i\alpha_i - \alpha_i c_{mi} - \beta_i(w_j - c_{mj})}{2c_i} \quad \textit{where}\ \ i,j \in = \{1,2\}\ \ \textit{and}\ \ j \neq i. \tag{15}$$

Proof **(i)** Solving equations $\frac{\partial\Pi(p_1,p_2,x_1,x_2)}{\partial p_1} = 0$, $\frac{\partial\Pi(p_1,p_2,x_1,x_2)}{\partial p_2} = 0$, $\frac{\partial\Pi(p_1,p_2,x_1,x_2)}{\partial x_1} = 0$ and $\frac{\partial\Pi(p_1,p_2,x_1,x_2)}{\partial x_2} = 0$ simultaneously, we get the required results.

(ii) For Cournot solution, we first find the responses of two manufacturers by solving $\frac{\partial\Pi_{m1}}{\partial x_1} = 0$ and $\frac{\partial\Pi_{m2}}{\partial x_2} = 0$ simultaneously. Then the solution of the system of equations $\frac{\partial\Pi_r}{\partial p_1} = 0$ and $\frac{\partial\Pi_r}{\partial p_2} = 0$ gives the optimal retail prices for Cournot policy. Putting these optimal prices in the manufacturers' reactions, we get the optimal qualities.

(iii) For Collusion solution, we first find the responses of two manufacturers by solving $\frac{\partial\Pi_m}{\partial x_1} = 0$ and $\frac{\partial\Pi_m}{\partial x_2} = 0$ simultaneously. Then solving equations $\frac{\partial\Pi_r}{\partial p_1} = 0$ and $\frac{\partial\Pi_r}{\partial p_2} = 0$ simultaneously, we get the optimal retail prices which when put in the manufacturers' reactions determine the optimal qualities. □

Proposition 2 *The retail price under Cournot policy is higher than that under Collusion policy if* $\frac{2a_j\alpha_i-(b_i+b_j)\beta_i}{2a_j\beta_j-(b_i+b_j)\alpha_j} > \frac{(w_i-c_{mi})c_i\beta_j}{(w_j-c_{mj})c_j\beta_i}$.

Proof The result follows immediately as $p_i^{cr} - p_i^{cl} = \frac{\beta_j(b_i\alpha_j+b_j\alpha_j-2a_j\beta_j)c_i(w_i-c_{mi})+\beta_i(2a_j\alpha_i-(b_i+b_j)\beta_i)c_j(w_j-c_{mj})}{2(4a_i a_j-(b_i+b_j)^2)c_i c_j}$ □

Proposition 3 *The quality of the produced items is higher in Cournot policy.*

Proof We have $x_i^{cr} - x_i^{cl} = \frac{\beta_i(w_j - c_{mj})}{2c_i} > 0$, as the wholesale price is always greater than the procurement cost of a item. Hence, the proposition follows.

Proposition 3 reveals that as the competitive factor increases, the manufacturers tend to produce lower quality product. □

5.2 Same retail price for all brands of a product

Retailers sell different variations of a product from the same manufacturer at the same price. For example, in the yogurt category, all flavors of six ounce Dannon Fruit-on-the Bottom yogurt are sold at one price, while all flavors of six ounce Yoplait Original yogurt are sold at a second (uniform) price. Many real world examples are there for this same pricing strategy. For example, within a particular style, clothing is typically sold at the same price for different colors and sizes. There are, however, exceptions to this rule: while S, M, L, and XL sizes are typically the same price, many retailers charge more for XXXL and "tall" sizes. These sizes generally cost the retailer more, either because of the amount of fabric used, or because average costs are higher due to lower volumes. Similarly, although most teas are sold at uniform prices, some varieties of tea are frequently sold at a higher price. In this subsection, we modify the model developed in Section 4 when variations of a product have the same selling price. The optimal results for demand function $D_i(x_i, p) = d_i - a_i p + \alpha_i x_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j x_j$ are derived in the following proposition.

Table 1 Parameter values

$d_1 = 60;$	$d_2 = 60;$	$d_3 = 60;$	$d_4 = 60;$	$d_5 = 60;$
$a_1 = 0.52;$	$a_2 = 0.55;$	$a_3 = 0.58;$	$a_4 = 0.60;$	$a_5 = 0.62;$
$b_1 = 0.05;$	$b_2 = 0.05;$	$b_3 = 0.05;$	$b_4 = 0.05;$	$b_5 = 0.05;$
$\alpha_1 = 0.22;$	$\alpha_2 = 0.23;$	$\alpha_3 = 0.25;$	$\alpha_4 = 0.26;$	$\alpha_5 = 0.28;$
$\beta_1 = 0.03;$	$\beta_2 = 0.03;$	$\beta_3 = 0.03;$	$\beta_4 = 0.03;$	$\beta_5 = 0.03;$
$w_1 = 50;$	$w_2 = 50;$	$w_3 = 50;$	$w_4 = 50;$	$w_5 = 50;$
$c_{m1} = 13;$	$c_{m2} = 13;$	$c_{m3} = 13;$	$c_{m4} = 13;$	$c_{m5} = 13;$
$c_1 = 6;$	$c_2 = 6;$	$c_3 = 6;$	$c_4 = 6;$	$c_5 = 6;$
$g_1 = 0.8;$	$g_2 = 0.9;$	$g_3 = 0.95;$	$g_4 = 0.98;$	$g_5 = 0.99;$

Proposition 4 *(i) The optimal centralized solution is*

$$p^n = \frac{\sum_{i=1, i\neq j}^{2} \left[(\alpha_i - \beta_i)(g_i - \alpha_i c_{mi} + \beta_i c_{mj})c_j + 2c_i c_j (d_i + a_i c_{mi})\right]}{4(a_i + a_j)c_i c_j - (\alpha_i - \beta_i)^2 c_j - (\alpha_j - \beta_j)^2 c_i} \tag{16}$$

$$\begin{aligned} x_i^n = & \frac{-1}{2((\alpha_i - \beta_i)^2 c_j + c_i((\alpha_j - \beta_j)^2 - 4(a_i + a_j)c_j))} [\, 2c_j \,(\, 2a_i g_i + 2a_j g_i + d_i\alpha_i + d_j\alpha_i - d_i\beta_i - d_j\beta_i \\ & -(2a_j\alpha_i + a_i(\alpha_i + \beta_i))c_{mi} + (2a_i\beta_i + a_j(\alpha_i + \beta_i))c_{mj} \,) + (\alpha_j - \beta_j)\,(\, g_j\alpha_i - g_i\alpha_j - g_j\beta_i + g_i\beta_j \\ & +(\alpha_i\alpha_j - \beta_i\beta_j)c_{mi} + (-\alpha_i\alpha_j + \beta_i\beta_j)c_{mj} \,)\,] \end{aligned} \tag{17}$$

(ii) The optimal Cournot solution is

$$p^{crn} = \frac{\sum_{i=1, i\neq j}^{2} \left[(\alpha_i - \beta_i)(g_i + w_i\alpha_i - \alpha_i c_{mi})c_j + 2(d_i + a_i w_i)c_i c_j\right]}{4(a_i + a_j)c_i c_j}) \tag{18}$$

$$x_i^{crn} = \frac{g_i + w_i\alpha_i - \alpha_i c_{mi}}{2c_i} \tag{19}$$

(iii) The optimal Collusion solution is

$$p^{cln} = \frac{\sum_{i=1, i\neq j}^{2} \left[(\alpha_i - \beta_i)(g_i + w_i\alpha_i - \alpha_i c_{mi} - \beta_i(w_j - c_{mj}))c_j + 2(d_i + a_i w_i)c_i c_j\right]}{4(a_i + a_j)c_i c_j)} \tag{20}$$

$$x_i^{cln} = \frac{g_i + w_i\alpha_i - \alpha_i c_{mi} - \beta_i(w_j - c_{mj})}{2c_i} \quad \textit{where } \; i, j \in = \{1, 2\} \; \textit{and } \; j \neq i \tag{21}$$

6 Numerical analysis

In this section, we demonstrate the developed models through a numerical example. From the earlier literature survey, we generate the parameter-values for a supply chain system having five manufacturers as given in Table 1.

These parameter-values are very much appropriate for our model as well as suitable in practice. As for example, $a_1 = 0.52 > b_2 + b_3 + b_4 + b_5 (= 0.20)$, $\alpha_1 = 0.22 > \beta_2 + \beta_3 + \beta_4 + \beta_5 (= 0.12)$. In practice, all supply chain players have an interest to cooperate if $p_i > w_i > c_{mi}$.

Table 2 contains the optimal results for different pricing strategies. It shows that as the number of manufacturer in the system increases, the system profit as well as the retail prices increase. It implies that if there are more

Table 2 Optimal results under different pricing strategies

n	Centralized	Collusion	Cournot
2	$x_1 = 0.9801$; $x_2 = 0.9672$; $p_1 = 70.1901$; $p_2 = 67.0110$; $\Pi = 2984.91$	$x_1 = 0.6525$; $x_2 = 0.6917$; $p_1 = 88.6235$; $p_2 = 85.4562$; $\Pi_m = 1321.62$; $\Pi_r = 1330.21$; $\Pi = 2651.83$	$x_1 = 0.7450$; $x_2 = 0.7842$; $p_1 = 88.6422$; $p_2 = 85.4747$; $\Pi_m = 1320.85$; $\Pi_r = 1331.54$; $\Pi = 2652.39$
3	$x_1 = 0.9441$; $x_2 = 0.9295$; $x_3 = 0.9605$; $p_1 = 76.8949$; $p_2 = 73.3798$; $p_3 = 70.2167$; $\Pi = 4913.78$	$x_1 = 0.56$; $x_2 = 0.5992$; $x_3 = 0.6650$; $p_1 = 95.3202$; $p_2 = 91.8173$; $p_3 = 88.6597$; $\Pi_m = 2079.41$; $\Pi_r = 2370.28$; $\Pi = 4449.69$	$x_1 = 0.745$; $x_2 = 0.7842$; $x_3 = 0.85$; $p_1 = 95.3555$; $p_2 = 91.8524$; $p_3 = 88.696$; $\Pi_m = 2077.01$; $\Pi_r = 2374.3$; $\Pi = 4451.31$
4	$x_1 = 0.8981$; $x_2 = 0.8815$; $x_3 = 0.9167$; $x_4 = 0.9203$; $p_1 = 84.9034$; $p_2 = 80.9871$; $p_3 = 77.462$; $p_4 = 75.2864$; $\Pi = 7285.13$	$x_1 = 0.4675$; $x_2 = 0.5067$; $x_3 = 0.5725$; $x_4 = 0.6058$; $p_1 = 103.324$; $p_2 = 99.4204$; $p_3 = 95.8993$; $p_4 = 93.7296$; $\Pi_m = 2911.1$; $\Pi_r = 3806.06$; $\Pi = 6717.16$	$x_1 = 0.745$; $x_2 = 0.7842$; $x_3 = 0.85$; $x_1 = 0.8833$; $p_1 = 103.374$; $p_2 = 99.4696$; $p_3 = 95.9505$; $p_4 = 93.7814$; $\Pi_m = 2906.18$; $\Pi_r = 3814.01$; $\Pi = 6720.18$
5	$x_1 = 0.8387$; $x_2 = 0.8197$; $x_3 = 0.8601$; $x_4 = 0.8645$; $x_5 = 0.9244$; $p_1 = 94.643$; $p_2 = 90.2387$; $p_3 = 86.2732$; $p_4 = 83.8262$; $p_5 = 81.5447$; $\Pi = 10,245.4$	$x_1 = 0.375$; $x_2 = 0.4142$; $x_3 = 0.48$; $x_4 = 0.5133$; $x_5 = 0.5758$; $p_1 = 113.064$; $p_2 = 108.672$; $p_3 = 104.709$; $p_4 = 102.267$; $p_5 = 99.983$; $\Pi_m = 3816.73$; $\Pi_r = 5784.01$; $\Pi = 9600.74$	$x_1 = 0.745$; $x_2 = 0.7842$; $x_3 = 0.85$; $x_4 = 0.8833$; $x_5 = 0.9458$; $p_1 = 113.124$; $p_2 = 108.732$; $p_3 = 104.772$; $p_4 = 102.331$; $p_5 = 100.051$; $\Pi_m = 3808.24$; $\Pi_r = 5797.04$; $\Pi = 9605.28$

Table 3 Optimal results under unique pricing strategy

Policy	x_1	x_2	x_3	p	Π_m	Π_r	Π
Centralized	0.7089	0.7574	0.8418	61.1673	–	–	3816.84
Collusion	0.5600	0.5992	0.6650	79.6418	1801.91	1449.75	3251.66
Cournot	0.7450	0.7842	0.8500	79.6709	1799.51	1452.60	3252.11

Table 4 A comparison of profits under two pricing strategies

Strategy	Centralized	Collusion			Cournot		
	Π	Π_m	Π_r	Π	Π_m	Π_r	Π
Different pricing	4913.78	2079.41	2370.28	4449.69	2077.01	2374.3	4451.31
Unique pricing	3816.84	1801.91	1449.75	3251.66	1799.51	1452.60	3252.11
% profit decrease	22.32	13.34	38.84	26.92	13.36	38.82	26.94

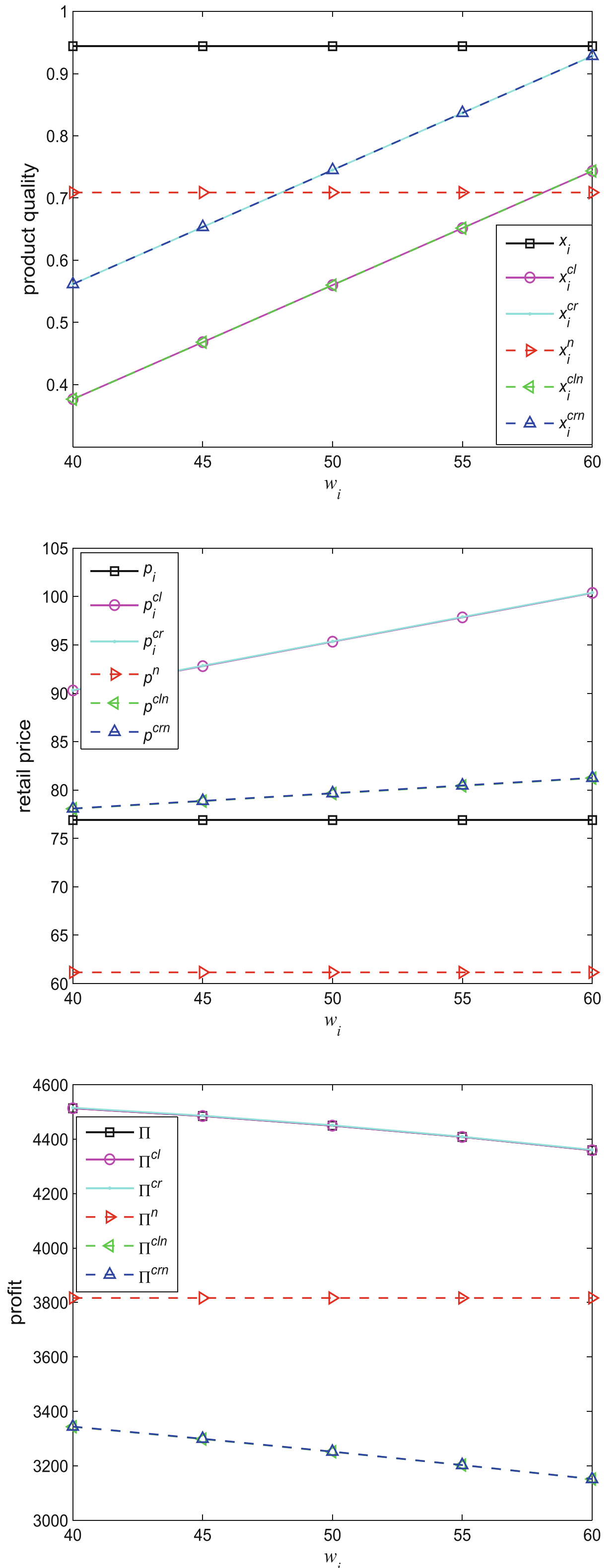
manufacturers in the system then the system's profit tends to the profit of the centralized system but the product quality becomes lower and selling prices become higher. At the same time, the manufacturers' product qualities decrease quite significantly in the Collusion solution. If the manufacturers are cooperative among themselves, the consumers suffer the most. So, from the consumers point of view, it is unacceptable that there are so many manufacturers in the system. For this reason, Collusion is regarded as illegal in many countries (USA, Canada, and Germany) but implicit collusion in the form of price leadership and tacit understandings still takes place. Further, from Table 2, we also see that the profits of Collusion and Cournot models are not much different. The reason is that the impact of quality on demand is low and consequently, its impact on retailer's profit is minimal.

Table 3 shows the optimal results of the system with three manufacturers under unique pricing strategy. It is observed that the retail price under unique pricing strategy is lower than those under different pricing strategy. The product quality is lower in the centralized policy under unique pricing strategy than that under different pricing strategy. So, from consumer's point of view, the centralized and Collusion policies in the unique pricing strategy are unacceptable but Cournot policy is very much acceptable.

Table 4 shows that profits of all players as well as the whole system decrease in the unique pricing strategy. Moreover, the retailer is the maximum looser. The decrease in profit of the whole system in each of the two-retailer Stackelberg games is greater than that of the centralized system. In practice, a dominant retailer sets different prices for different qualities of product. That means, he shares information of product's quality with consumers. But if he declares the same price for all qualities of the product then he actually conceals the products' qualities. The consumers are obviously not reliable to him. As a result, the retailer's demand and profit decrease and the same happens in the whole system's profit.

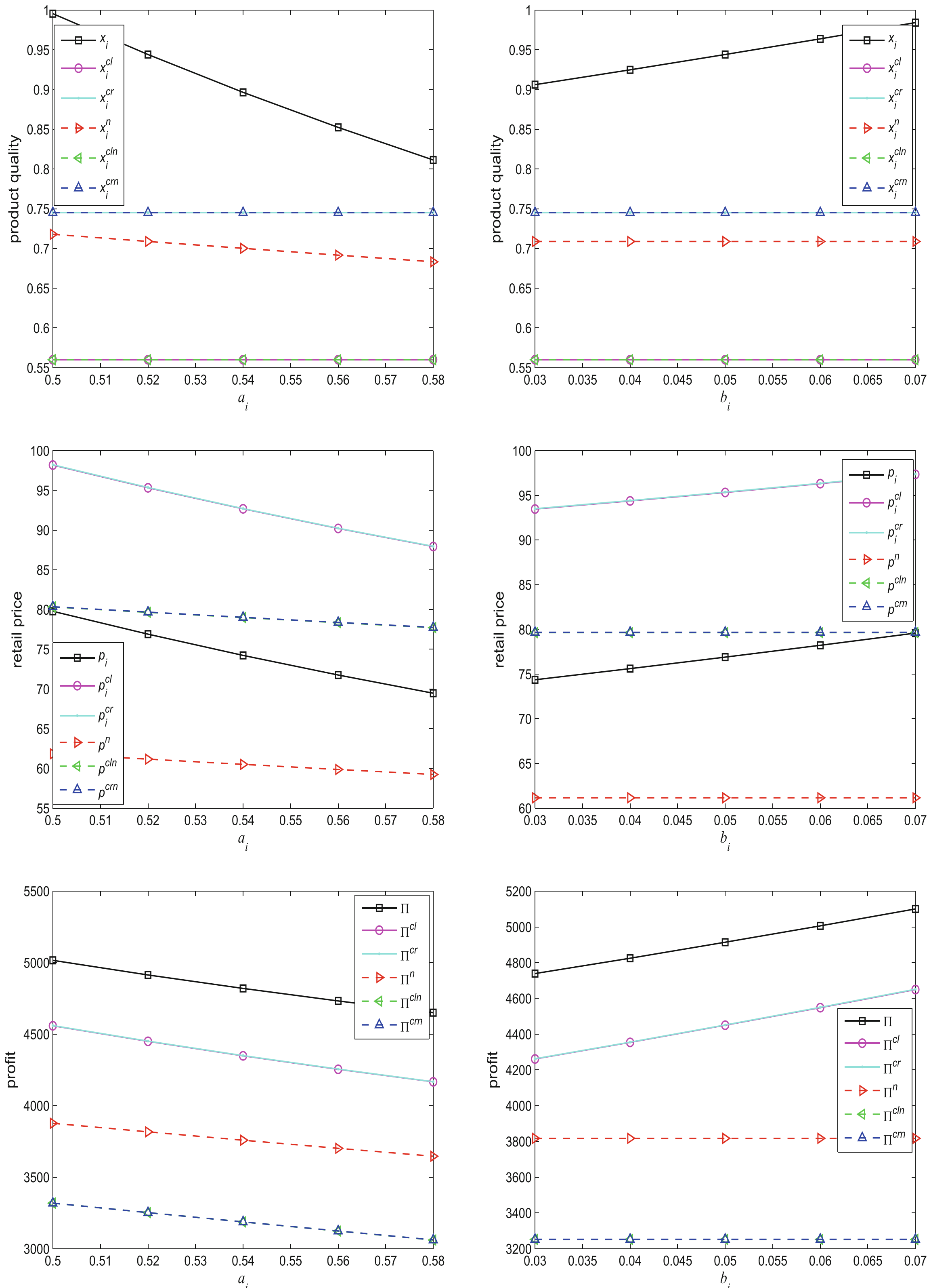
7 Sensitivity analysis

In this section, we present the sensitivity analysis of the model with respect to its key parameters. To explore the impact of the game theoretic strategy on the supply chain decisions, we consider the model with three manufactures. Six optimal decisions under centralized, Cournot and Collusion for unique pricing strategy and different pricing strategy are derived and plotted in a single figure for comparison among qualities, retail prices, and profits with five different parameters (see Figs. 1a–e). We carry out the analysis by changing the value of one parameter at a time while



(a) Sensitivity with respect to parameter w_i

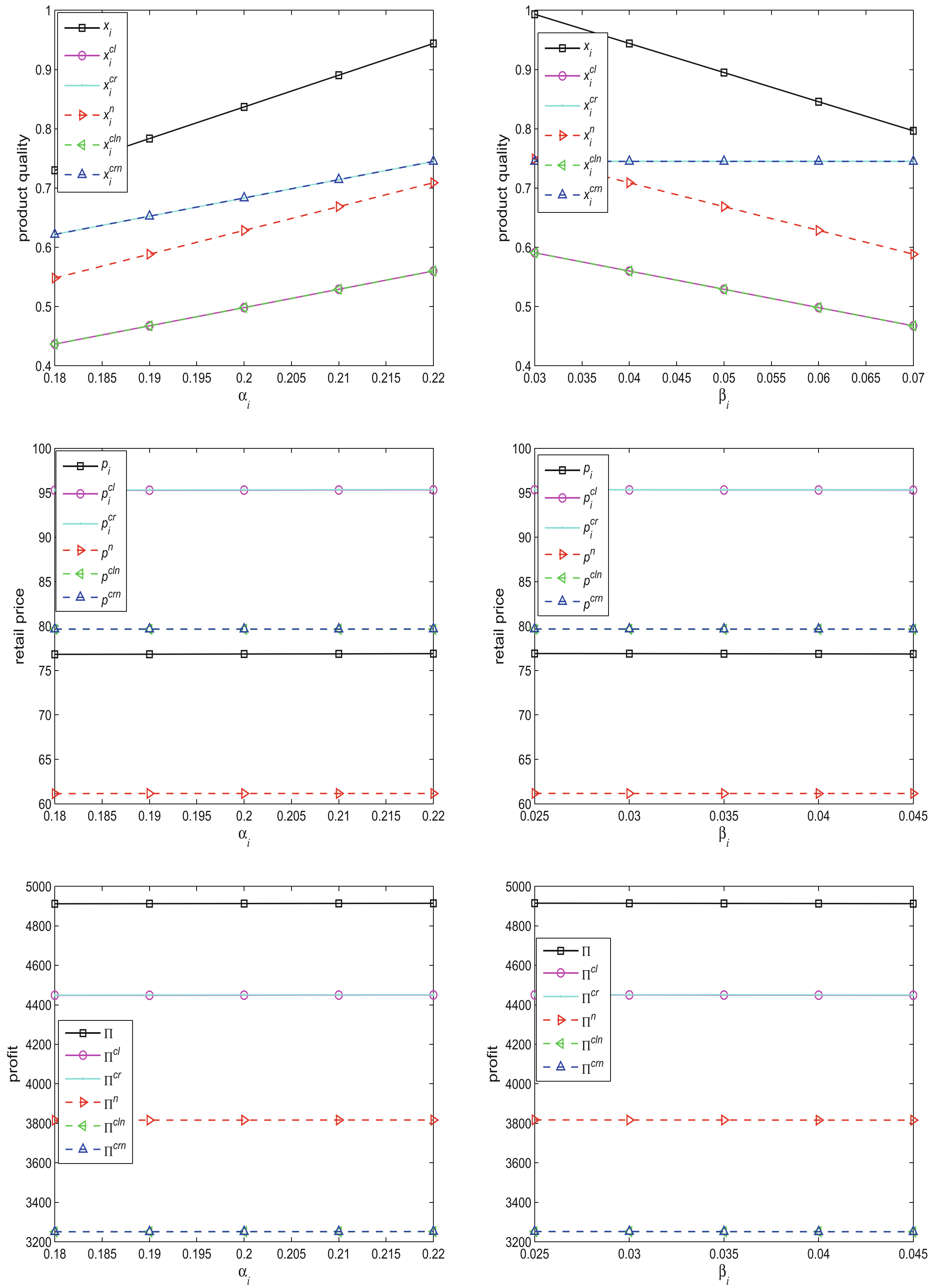
Fig. 1 **a** Sensitivity with respect to parameter w_i. **b** Sensitivity with respect to parameter a_i. **c** Sensitivity with respect to parameter b_i. **d** Sensitivity with respect to parameter α_i. **e** Sensitivity with respect to parameter β_i



(b) Sensitivity with respect to parameter a_i

(c) Sensitivity with respect to parameter b_i

Fig. 1 (continued)



(d) Sensitivity with respect to parameter α_i

(e) Sensitivity with respect to parameter β_i

Fig. 1 (continued)

keeping all other parameter-values unchanged. From these figures, we can draw the following conclusions:

(1) For changes in wholesale price, quality level remains the same in the centralized system (see Fig. 1a). This is obvious from Eq. 4, because optimum quality level (x_i) is not dependent upon wholesale price (w_i). The optimum service levels in Cournot and Collusion game models increase and interestingly those levels nearly coincide with that of the unique or different pricing strategy. It indicates that the manufacturer's wholesale price in all the game models is independent of pricing strategy adopted.

On the contrary, when the supplier's wholesale price changes, the optimum retail price remains the same for both the Cournot and Collusion games, whether the unique pricing strategy or different pricing strategy is adopted. We observe that the rate of change of optimal price is more when different pricing strategy is adopted. So, the optimal retail price for different pricing strategy is more sensitive than unique pricing strategy. The profit of the supply chain decreases when the wholesale price increases in both the game models. If suppliers charge higher wholesale price then the profit of the supply chain decreases. From manager's point of view, the supplier should not offer lower wholesale price in supplier's Stackelberg game in order to improve the quality of the product.

(2) The parameters b_i, and a_i (see Figs. 1c and b) are sensitive according to their characteristics. As b_i increases, the profit of the supply chain increases in different pricing strategy. So, depending on the decision made by the retailer, the supply chain profit increases gradually or remains the same. For retailer Stackelberg game, the retailer tries to adopt different pricing strategy and increase the value of b_i in order to gain more profit. On the other hand, a_i has the same level of sensitivity to the quality and supply chain profit but in opposite sense. It is also noticed that the supply chain profit as well as retail price gradually decrease for increase in a_i. So, in retailer-Stackelberg game, the retailer should try to adopt demand for lower value of a_i.

(3) The parameters α_i and β_i are directly related to the product quality of the manufacturer but in opposite sense (see Figs. 1d and e). The product quality increases and decreases with the increase in α_i and β_i, respectively. These parameters almost have no impact on the retail price as well as on the whole system profit.

8 Conclusions and future research directions

The product quality and pricing strategy of a product in a two-echelon supply chain are the main issues considered in this paper. As multiple manufacturers are involved in the supply chain, the quality of the product is decided by the respective manufacturer. The retailer sells different variations of the product with different prices to the consumers in the marketplace. We have considered centralized policy and retailer-Stackelberg game strategy when manufacturers pursue Cournot or Collusion strategy to react to the product quality. We have also considered a special case where the retailer chooses the unique pricing strategy for all variations of the product. From the numerical analysis, we have found that the retailer chooses lower retail price for the unique pricing strategy. The product qualities chosen by the manufactures are the same in the retailer-Stackelberg game while those are different in the centralized policy under both unique and different pricing strategies. In the unique pricing strategy, the profits of the retailer, manufacturers as well as the whole system are lower than those in the different pricing strategy. From the consumers point of view, it is more acceptable if retailer chooses the unique retail pricing strategy in the retailer-Stackelberg game. However, in the centralized case, consumers are in doubt which pricing policy is more desirable to them because the product qualities and also the retail price are lower in the unique pricing strategy than those in different pricing strategy. Consumers who are conscious about the quality of the product, not retail price, obviously want the retailer to choose different pricing strategy. On the other hand, the consumers who are conscious about the price of the product and not so much for the quality, want the retailer to choose the unique pricing strategy.

In this paper, we have developed the proposed model under deterministic demand. An obvious extension would be to consider the stochastic demand. Consideration of manufacturer's uncertain yield would also be interesting. A dual channel at the retailer place or e-market selling may be considered for future research. The multi-retailer situation with the same or different pricing strategy may also be considered as an extension of the present work.

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