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ORIGINAL ARTICLE

Integrated cell formation and layout problem considering multi-row machine arrangement and continuous cell layout with aisle distance

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Abstract Cellular manufacturing is an important application of group technology and the cell formation process is one of the important steps in designing cellular manufacturing system. In recent years, researchers have noticed potential benefits when the layout problem is considered within the cell formation process. Nevertheless, there are not sufficient studies about consideration of real-life features in the cell formation and layout design process. In this paper, a new approach is presented to integrate the cell formation and its layout design. The proposed approach has three important design features not found in other papers. These design features are multi-row intra-cell layout (layout of machines within the cells), continuous inter-cell layout (layout of rectangular shape cells on the planar area), and aisle distance. The objective of the proposed approach is to form machine cells, find the layout of machine cells, and obtain the arrangement of machines within the cells such that the total material handling cost is minimized. In order to have a more accurate layout design, the material handling cost is calculated in terms of the actual position of machines within the cell. Due to the computational complexity of the proposed problem, a heuristic method is proposed to solve medium- and large-scale problems in a reasonable computational time. Three lower bounds are developed for the proposed integrated problem in which the tightest of them is chosen for evaluating the solution of the heuristic method. Finally, numerical examples adopted from the literature are solved to verify the performance of the proposed heuristic method and illustrate the advantages of the proposed integrated approach. The results indicated that the heuristic method is both effective and efficient in solving real-sized problems. The results also demonstrated that the proposed layout approach gives better layout design in comparison with the existing approaches.

Keywords Cellular manufacturing system · Cellular layout · Cell formation · Heuristic method · Lower bound · Linearization method

1 Introduction

Group technology (GT) is a manufacturing philosophy that identifies and explores the similarities of product design and manufacturing process. Cellular manufacturing system (CMS) is an application of GT. It can be used to enhance both flexibility and efficiency in today's small-to-medium lot production environment. According to Wemmerlöv and Hyer [1], the design of a CMS includes (1) cell formation (CF)—grouping parts with similar design features or processing requirements into part families and grouping machines into machine cells on the basis of the operations required by the part families; (2) group layout—laying out machines within each cell (intra-cell layout) and cells with respect to one another (inter-cell layout); (3) group scheduling—scheduling parts and part families for production; and (4) resource allocation—assigning tools, human, and material resources. The CMS has more advantages such as decreased setup times, reduced work-in-process inventories, improved product quality, shorter lead times, reduced tool requirements, improved productivity, better quality and production control, increment in flexibility, and decreased material handling cost [2, 3].

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The CF process is one of the important steps in designing CMS and generally aims to minimize the total intracellular and intercellular part movements. The issue of CF has widely been investigated in the literature and researchers have noticed potential benefits when the layout problem is considered within the CF process. Heragu and Kakuturi [4] attempted to integrate the machine grouping problem with layout problem. The machine cells are first formed by a heuristic algorithm, and then, a hybrid simulated annealing algorithm is employed to construct a near-optimal inter- and intra-cell layout. Aktürk and Turkcan [5] proposed a solution methodology to simultaneously solve the part-family and machine cell formation problem by considering the intra-cell layout problem. A hedonistic approach was used to maximize the profit of not only the overall system but also individual cells. Lee and Chiang [6] addressed the joint problem of CF and its layout assignment to minimize the inter-cell material flow cost. It was assumed that cell locations are approximately equally spaced and machine cells are located along a bidirectional linear layout. They proposed a new graphic approach based on the multi-terminal cut tree network model to form machine cells. A partition procedure was developed to separate the cut tree into a number of sub-graphs (cells) and assigns the location sequence of each cell by comparing the cut capacities. Also, Chiang and Lee [7] employed a simulated annealing approach augmented with dynamic programming algorithm for solving the same problem presented by Lee and Chiang [6]. In the proposed approach, the configuration of a solution is comprised of a string of integer values, where each value is associated with each machine. Then, a dynamic programming algorithm is applied to partition each string into several segments (cells) such that the total inter-cell flow cost is minimized. Yin et al. [8] incorporated operation sequences, production volumes, and alternative process routings of parts into a nonlinear mathematical model and aimed to minimize a weighted sum of both inter-cell and intra-cell movements in which the weights are based on the actual unit costs of inter-cell/intra-cell movements. A heuristic methodology was also developed for solving such a nonlinear problem. Hicks [9] developed a genetic algorithm (GA) design tool that can be used for the layout design of cellular and noncellular manufacturing facilities. The tool can solve layout problems directly or indirectly by optimizing the results obtained from a CF algorithm. Chan et al. [10] proposed a two-stage GA base solution approach for solving the CF problem as well as the cell layout problem. The first stage is to identify machine cells and part families. Also, the second stage is to arrange the layout sequence of machine cells (linear inter-cell layout) in such a way that the total inter-cell material handling cost is minimized. In the proposed approach, the inter-cell layout is considered as a quadratic assignment problem (QAP). Tavakkoli-Moghaddam et al. [11] presented a new mathematical model with stochastic demands to minimize the total costs of inter- and intra-cell movements in both machine and cell layout problems simultaneously. They assumed that the CF is known in advance and formulated a biquadratic assignment problem to obtain the inter- and intra-cell layouts. Wu et al. [12] developed a GA approach for solving an integrated CF and group layout problem in CMS considering operation sequences, work load, machine capacity, demand, batch size, and layout type. Most recently, Jolai et al. [13] presented a modified version of proposed model by Wu et al. [12] considering different parameters such as forward and backward transportation, different batch sizes for parts, different cell sizes, operation sequences, and the number of exceptional elements (an exceptional element is a part which needs to be processed in more than one cell). They developed an electromagnetism-like algorithm with a heuristic local search to minimize the total material handling cost and the number of exceptional elements. Yalaoui et al. [14] solved a combined GT problem with a facility layout problem by using a three-stage method. In the first stage, a GA is used to create part families and machine cells based on a volume data matrix. In the second stage, considering the solution of the first stage, an ant colony optimization mixed with a guided local search is employed to assign machines to fixed locations (this assignment problem is represented by the QAP). Finally, in the third stage, a loop on cells is carried out using a minimum and maximum number of cells to choose the appropriate number of cells. Kia et al. [15] presented a mathematical model for the layout design of dynamic CMSs under an uncertain environment. Single-row layout was considered in the arrangement of machines within the cells and prespecified locations were used for the cell layout. This model incorporates several design features including operation sequence, operation time, alternative process routing, duplicate machines, machine capacity, production volume of parts, and cell reconfiguration. They used optimization software for solving the proposed problem. Chang et al. [16] formulated a two-stage mathematical programming model to integrate the CF, cell layout, and intracellular machine sequence with the consideration of alternative process routings, operation sequences, and production volume. The aim of stage I is to simultaneously solve the CF and cell layout problems, whereas the primary function of stage II is to determine the machine layout in each cell on the basis of the CF determined in stage I. In this study, the linear single- and double-row layouts were considered as two alternatives for the cell layout. A tabu search algorithm was employed to solve the proposed problem. Mahdavi et al. [17] presented an integrated mathematical model considering CF and cell layout simultaneously. In this research, machines are located in a linear form and machine cells are allocated to a set of predetermined positions in the plant. The proposed model was solved by an optimization software in order to minimize the cost of intra-cell moves (regarding forward and backtracking movements), the cost value of inter-cell traveled distance which is acquired via product of travel cost between two cells and number of travels between them, and the number of exceptional elements (regarding the production volume of each part).

Most of the approaches to the CF problem are concerned with creating machine cells with the minimal number of inter-cell movements (see [18–24]). However, a CF with the minimal number of inter- and intra-cell movements is not always consistent with the one with the minimal inter- and intra-cell material handling costs, due to the lack of layout data in the design process. From the other side, those approaches that consider the inter- and intra-cell material handling costs in the CMS design usually have some weakness. These weakness are consideration of predetermined locations in both inter- and intra-cell layouts, disregarding the aisle distance and the actual dimensions of the cells in the inter-cell layout, calculation of the material handling cost in terms of the center-to-center distances between the cells (rather than the actual position of machines), and consideration of the single-row layout in both inter- and intra-cell layouts (see [4–17]). These weaknesses result in inappropriate layout design and inefficient material handling system. To fill these gaps, the present paper addresses a new integrated approach to the CF and its inter- and intra-cell layouts. The proposed approach has three important design features not found in other papers. These design features are multi-row intra-cell layout (layout of machines within the cells), continuous inter-cell layout (layout of rectangular shape cells on the planar area), and aisle distance. The objective of the proposed approach is to form machine cells, find the layout of machine cells, and obtain the arrangement of machines within the cells such that the total material handling cost is minimized. In order to have a more accurate layout design, the material handling cost is calculated in terms of the actual position of machines within the cell. Due to the computational complexity of the proposed problem, a heuristic method is proposed to solve medium- and large-scale problems in a reasonable computational time. Three lower bounds are developed for the proposed integrated problem in which the tightest of them is chosen for evaluating the solution of the heuristic method. Finally, numerical examples adopted from the literature are solved to verify the performance of the proposed heuristic method and illustrate the advantages of the proposed integrated approach.

2 Problem formulation

In this paper, the CF and inter- and intra-cell layouts are simultaneously determined by an integrated approach which is called as integrated cell formation and layout problem (ICFLP). It is assumed that cells are rectangular in shape. The inter-cell layout (i.e., the layout of unequal-sized cells on the plant area) is represented by the continuous layout problem (two-dimensional layout problem) regarding the aisles and the orientation of cells. The aisle distance is specified to provide the necessary distance between each pair of cells. Also, the intra-cell layout (the multi-row arrangement of equal-sized machines within the cells) is formulated by the QAP. To do so, each cell is divided into a rectangular grid (candidate locations) where each grid cell can be allocated to a machine. Both the inter- and intra-cell material handling costs are calculated in terms of the actual position of machines within the cells and considering part demands, operation sequences, and unit material handling costs per unit distance. The proposed approach for the inter- and intra-cell layouts are illustrated in Fig. 1.

The following notation is used throughout this paper.

Sets

i	Index for parts (i=1,…,P)
k,k'	Indexes for machines (k,k'=1,…,M)
l,l'	Indexes for cells (l,l'=1,…,C)
m	Index for columns (m=1,…,Co_l)
n	Index for rows (n=1,…,Ro_l)

Parameters

P	Number of parts
C	Maximum number of cells allowed
M	Number of machines

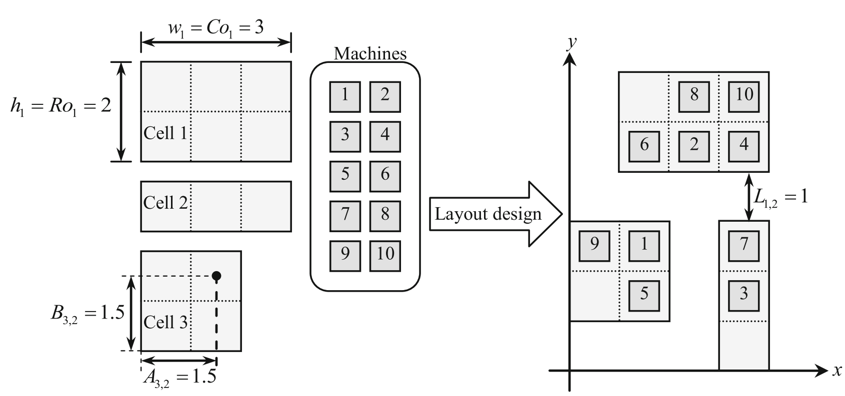


Fig. 1 Schematic illustration on the proposed approach for the inter- and intra-cell layouts

d_i	Demand of part i
$c^A_{i,k,k'}$	Unit intra-cell material handling cost for transporting part i from machine k to machine k' per unit distance
$c^E_{i,k,k'}$	Unit inter-cell material handling cost for transporting part i from machine k to machine k' per unit distance
$f_{i,k,k'}$	Number of times that an operation at machine k immediately follows an operation at machine k' for part i
w_l	Width of cell l
h_l	Height of cell l
Ro_l	Number of rows in cell l
Co_l	Number of columns in cell l
$L_{l,l'}$	Aisle distance between cells l and l'
$A_{l,m}$	Length of the center of column m of cell l in the x-axis with respect to the lower left corner of cell l
$B_{l,n}$	Length of the center of row n of cell l in the y-axis with respect to the lower left corner of cell l

Decision variables

(x_l,y_l)	Horizontal and vertical coordinates of the centroid of cell l, respectively
u_l	=1 if cell l is located vertically; 0 otherwise
$z_{k,l}$	=1 if machine k is assigned to cell l; 0 otherwise
$z^x_{k,l,m}$	=1 if machine k is assigned to column m of cell l; 0 otherwise
$z^y_{k,l,n}$	=1 if machine k is assigned to row n of cell l; 0 otherwise
$d^x_{k,k',l,l'}$	Horizontal distance between machine k in cell l and machine k' in cell l'
$d^y_{k,k',l,l'}$	Vertical distance between machine k in cell l and machine k' in cell l'
$d_{k,k',l}$	Rectilinear distance between machines k and k' in cell l

Proposed model According to the description given above, the ICFLP can be formulated as the following mixed-integer nonlinear programming model.

$$\text{ICFLP}:\min\sum_{l}\sum_{k'>k}\sum_{i} d_i c^A_{i,k,k'} f_{i,k,k'} z_{k,l} z_{k',l} d_{k,k',l} + \sum_{l\neq l'}\sum_{k'>k}\sum_{i} d_i c^E_{i,k,k'} f_{i,k,k'} z_{k,l} z_{k',l'}\left(d^x_{k,k',l,l'}+d^y_{k,k',l,l'}\right). \tag{1.1}$$

Subject to

$$d^x_{k,k',l,l'}=\left|x_l-\frac{w_l+(h_l-w_l)u_l}{2}+\sum_{m=1}^{Co_l}A_{l,m}z^x_{k,l,m}+u_l\left(\sum_{n=1}^{Ro_l}B_{l,n}z^y_{k,l,n}-\sum_{m=1}^{Co_l}A_{l,m}z^x_{k,l,m}\right)-x_{l'} + \frac{w_{l'}+(h_{l'}-w_{l'})u_{l'}}{2}-\sum_{m=1}^{Co_{l'}}A_{l',m}z^x_{k',l',m}-u_{l'}\left(\sum_{n=1}^{Ro_{l'}}B_{l',n}z^y_{k',l',n}-\sum_{m=1}^{Co_{l'}}A_{l',m}z^x_{k',l',m}\right)\right|,\forall k'>k,l\neq l', \tag{1.2}$$

$$d^y_{k,k',l,l'}=\left|y_l-\frac{h_l+(w_l-h_l)u_l}{2}+\sum_{n=1}^{Ro_l}B_{l,n}z^y_{k,l,n}+u_l\left(\sum_{m=1}^{Co_l}A_{l,m}z^x_{k,l,m}-\sum_{n=1}^{Ro_l}B_{l,n}z^y_{k,l,n}\right)-y_{l'} + \frac{h_{l'}+(w_{l'}-h_{l'})u_{l'}}{2}-\sum_{n=1}^{Ro_{l'}}B_{l',n}z^y_{k',l',n}-u_{l'}\left(\sum_{m=1}^{Co_{l'}}A_{l',m}z^x_{k',l',m}-\sum_{n=1}^{Ro_{l'}}B_{l',n}z^y_{k',l',n}\right)\right|,\forall k'>k,l\neq l', \tag{1.3}$$

$$d_{k,k',l}=\left|\sum_{m=1}^{Co_l}A_{l,m}\left(z^x_{k,l,m}-z^x_{k',l,m}\right)\right|+\left|\sum_{n=1}^{Ro_l}B_{l,n}\left(z^y_{k,l,n}-z^y_{k',l,n}\right)\right|,\forall k'>k,l, \tag{1.4}$$

$$\sum_{l}\sum_{m=1}^{Co_l}z^x_{k,l,m}=1,\forall k, \tag{1.5}$$

$$\sum_{l}\sum_{n=1}^{Ro_l}z^y_{k,l,n}=1,\forall k, \tag{1.6}$$

$$\sum_{l} z_{k,l} = 1, \forall k, \tag{1.7}$$

$$z_{k,l} = \sum_{m=1}^{Co_l} \sum_{n=1}^{Ro_l} z^x_{k,l,m} z^y_{k,l,n}, \forall k, l, \tag{1.8}$$

$$\sum_{k} z^x_{k,l,m} z^y_{k,l,n} \leq 1, \forall l, m, n, \tag{1.9}$$

$$\left\{ \begin{array}{c} |x_l - x_{l'}| \geq \dfrac{w_l + (h_l - w_l)u_l + w_{l'} + (h_{l'} - w_{l'})u_{l'}}{2} + L_{l,l'} \\ \text{(or)} \\ |y_l - y_{l'}| \geq \dfrac{h_l + (w_l - h_l)u_l + h_{l'} + (w_{l'} - h_{l'})u_{l'}}{2} + L_{l,l'}, \forall l' > l, \end{array} \right\} \tag{1.10}$$

$$x_l, y_l, z_{k,l}, d^x_{k,k',l,l'}, d^y_{k,k',l,l'}, d_{k,k',l} \geq 0, \forall k' > k, l \neq l', \tag{1.11}$$

$$u_l, z^x_{k,l,m}, z^y_{k,l,n} \in \{0, 1\}, \forall k, l, m, n. \tag{1.12}$$

Objective function (1.1) minimizes the total material handling cost, where the first term is the intra-cell material handling cost, associated with the movement of parts between machines within each cell, and the second term is the inter-cell material handling cost due to the exceptional elements. Constraints (1.2) and (1.3) calculate the vertical and horizontal distances between two machines in two distinct cells, respectively. Constraint (1.4) measures the rectilinear distance between the machines belonged to a same cell. Constraints (1.5) and (1.6) ensure that each machine is assigned to one column and one row, respectively. Constraint (1.7) ensures that each machine is assigned to one cell. Constraint (1.8) computes the cell which machine k is allocated. Constraint (1.9) represents that each candidate location in each cell can only be occupied by one machine. Constraint (1.10) ensures that cells do not overlap; in this constraint, the 'or' operator between the first and second terms implies that at least one of these terms must be satisfied. Finally, set of constraints (1.11) and (1.12) is the logical binary and nonnegativity requirements on the decision variables. Note that in the above model, constraints (1.7) and (1.8) satisfy the binary requirement on decision variable $z_{k,l}$. So, decision variable $z_{k,l}$ is considered as a positive variable.

The proposed model for the ICFLP is a mixed-integer nonlinear programming problem with absolute operators in constraints (1.2)–(1.4) and (1.10) as well as quadratic terms in objective function (1.1) and constraints (1.2), (1.3), (1.8), and (1.9). The presence of nonlinear terms in the model makes it difficult to solve the problem to optimality even for small instances. Thus, the linearization techniques should be applied to convert the model into a mixed-integer linear programming problem. To do so, we can use the generic linearization methods given in Appendix 1. In this way, the product terms within constraints (1.2) and (1.3) as well as the absolute operators in constraints (1.2)–(1.4) are first linearized by methods (4.1) and (4.2), respectively. Then, the product terms in objective function (1.1) is linearized according to method (4.3). Next, the product term in constraint (1.8) is linearized by (4.4). Finally, the absolute operators as well as the 'or' operator in constraint (1.10) are linearized by method (4.5). The linearized model will be used to evaluate the quality of the lower bounds in the next section.

3 Lower bounds for the ICFLP

Since both the CF and layout problems are NP-hard [25–28], the ICFLP becomes NP-hard; it means that time for obtaining an optimal solution becomes unlikely large as the problem size grows. In this section, three lower bounds, namely, L1, L2, and L3, are presented for the ICFLP. The quality of the lower bounds is evaluated by the optimal solution of the ICFLP through solving several small and small-to-medium-scale instances from the literature. The tights lower bound will be used to evaluate the solutions of the proposed heuristic method in Sect. 6.

3.1 Lower bound L1

The mathematical model of the first lower bound considered for the ICFLP is as follows:

$$\text{L1}: \min \sum_{k'>k} F^A_{k,k'} \left(\sum_{l} z_{k,l} z_{k',l} \right) + \sum_{k'>k} F^E_{k,k'} \left(1 - \sum_{l} z_{k,l} z_{k',l} \right) \left(1 + L^{\min} \right). \tag{2.1}$$

Subject to

$$\sum_{k} z_{k,l} \leq Ro_l \times Co_l, \forall l, \tag{2.2}$$

$$\sum_{l} z_{k,l} = 1, \forall k, \tag{2.3}$$

$$z_{k,l} \in \{0, 1\}, \forall k, l. \tag{2.4}$$

Where parameters $F^A_{k,k'}$, $F^E_{k,k'}$, and $L^{\min}$ are calculated as follows:

$$F^A_{k,k'} = \sum_{i} c^A_i d_i f_{i,k,k'}, \forall k' > k, \tag{2.5}$$

$$F^{E}_{k,k'} = \sum_{i} c_i^E d_i f_{i,k,k'}, \forall k' > k, \tag{2.6}$$

$$L^{\min} = \min_{l>l'} \left\{ L_{l,l'} \right\}. \tag{2.7}$$

In order to linearize the above model, objective function (2.1) is rearranged as follows:

$$\text{L1}: \min\left(1+L^{\min}\right)\sum_{k'>k} F^{E}_{k,k'} - \sum_{l}\sum_{k} z_{k,l} \left(\sum_{k'>k} \left(F^{E}_{k,k'}\left(1+L^{\min}\right) - F^{A}_{k,k'} \right) z_{k',l} \right). \tag{2.8}$$

Now, model L1 can be linearized according to method (4.6) give in Appendix 1.

Proposition.1 *The optimal objective function value of model L1 is a lower bound for the ICFLP.*

Proof Assuming that the dimensions of all candidate locations are 1×1, the distances between the machines in a same cell will always be greater than or equal to 1 unit. Therefore, the term $z_{k,l} z_{k',l} d_{k,k',l}$ in objective function (1.1) can be replaced by $z_{k,l} z_{k',l}$. Also, it is clear that the distances between the machines in distinct cells are always greater than or equal to $1+L^{\min}$. So, the term $\sum_{l'\neq l} z_{k,l} z_{k',l'} \left(d^{x}_{k,k',l,l'} + d^{y}_{k,k',l,l'} \right)$ in objective function (1.1) can be estimated by $\left(1+L^{\min}\right) \left(1 - \sum_{l} z_{k,l} z_{k',l} \right)$. From the other side, constraints (1.5)–(1.9) can be relaxed into constraints (2.2)–(2.4) and the remaining constraints can be ignored. Therefore, it is concluded that the optimal objective function value of model L1 is better than or equal to that of the ICFLP. Consequently, the optimal objective function value of model L1 is a lower bound for the ICFLP. □

3.2 Lower bound L2

This model is a modified version of the QAP proposed in [29] and called as the grid representation quadratic assignment problem (GRQAP). The GRQAP has less number of binary variables in comparison with the traditional QAP. The GRQAP can solve small- and small-to-medium-scale problems almost two times faster than the QAP and also with less physical memory. According to [29], two sets of binary variables z^{x}_{km} and z^{y}_{kn} are introduced for the assignment of facilities (machines) to the candidate locations (columns and rows, respectively). The formulation of the GRQAP is as follows:

$$\text{L2}: \min \sum_{k'>k} F^{A}_{k,k'} \left(\left| \sum_{m} A_m \left(z^{x}_{k,m} - z^{x}_{k',m} \right) \right| + \left| \sum_{n} B_n \left(z^{y}_{k,n} - z^{y}_{k',n} \right) \right| \right). \tag{2.9}$$

Subject to

$$\sum_{k} z^{x}_{k,m} z^{y}_{k,n} \leq 1, \forall m, n, \tag{2.10}$$

$$\sum_{m} z^{x}_{k,m} = 1, \forall k, \tag{2.11}$$

$$\sum_{n} z^{y}_{k,n} = 1, \forall k, \tag{2.12}$$

$$z^{x}_{k,m}, z^{y}_{k,n} \in \{0,1\}, \forall k, m, n. \tag{2.13}$$

Where parameter $F^{A}_{k,k'}$ is computed by Eq. (2.5).

Objective function (2.9) minimizes the total intra-cell material handling cost. Note that, in the above model, the restriction on the cell capacities is not taken into account. It means that all machines are assigned to one cell. Constraint (2.10) ensures that each candidate location can be occupied by one machine. Constraints (2.11) and (2.12), respectively, represent that each machine is assigned to one column and one row.

Model L2 can be linearized according to the generic linearization methods given in Appendix 1. In this way, the absolute operators in objective function (2.9) and the product term in constraint (2.10) are linearized by applying methods (4.2) and (4.4), respectively.

Proposition.2 *The optimal objective function value of model L2 is a lower bound for the ICFLP.*

Proof Enforcement of facilities to get arranged within prespecified layout shapes may increase the total material handling cost [30]. So, regardless of the cell structures (i.e., constraints (1.5)–(1.10)), each machine can be placed anywhere on the planar area and, as a result, the total material handling cost may decrease. From the other side, as it was assumed that the dimensions of machines are equal in size (i.e., 1×1), the problem can be formulated by the QAP [31]. Finally, since model L2 is a modified version of the QAP, it is concluded that the optimal objective function value of model L2 is a lower bound for the ICFLP. □

3.3 Lower bound L3

The previous lower bounds do not consider the capacity of the cells as well as the aisle distance between the cells. In order to overcome these shortcomings, lower bound L3 is presented which considers these parameters. The formulation of lower bound L3 is as follows:

$$\begin{aligned}\mathrm{L3}:\min\sum_{k'>k} F^{A}_{k,k'}\left(\sum_{l} z_{k,l}z_{k',l}\right)\left(\left|\sum_{m} A_m\left(z^{x}_{k,m}-z^{x}_{k',m}\right)\right|+\left|\sum_{n} B_n\left(z^{y}_{k,n}-z^{y}_{k',n}\right)\right|\right)\\ +\sum_{k'>k} F^{E}_{k,k'}\left(1-\sum_{l} z_{k,l}z_{k',l}\right)\left(\left|\sum_{m} A_m\left(z^{x}_{k,m}-z^{x}_{k',m}\right)\right|+\left|\sum_{n} B_n\left(z^{y}_{k,n}-z^{y}_{k',n}\right)\right|+L^{\min}\right).\end{aligned} \quad (2.14)$$

Subject to (2.2)–(2.4) and (2.10)–(2.13).

Where parameters $F^{A}_{k,k'}$, $F^{E}_{k,k'}$, and $L^{\min}$ are calculated by Eqs. (2.5)–(2.7).

Model L3 can be linearized according to the generic linearization methods given in Appendix 1. First, the product terms and the absolute operators are linearized by using methods (4.4) and (4.2), respectively. Then, the problem is linearized by applying method (4.7).

Proposition.3 *The optimal objective function value of model L3 is a lower bound for the ICFLP.*

Proof The objective function and the constraints of model L3 are a combination of those in models L1 and L2. As models L1 and L2 are lower bounds for the ICFLP, it can be concluded that the optimal objective function value of model L3 is also a lower bound for the ICFLP. □

4 Evaluating the lower bounds

In this section, the quality of lower bounds L1, L2, and L3 is tested on small- and small-to-medium-scale problems adopted from the literature. Each problem is solved with different configurations (i.e., the number of cells, the dimensions of cells, and aisle distance between the cells). It should be noted that all the problems are solved by the GAMS/CPLEX solver on a personal computer with 2.4 GHz CPU, 2 GB memory, and Windows 7 operating system. The characteristic of the problems as well as computational results are given in Table 1. In this table, column '$C\times(Col\times Rol)$' indicates the physical configuration of the cells. For instance, the term '$2\times(2\times1)$, (3×1)' indicates that we have three cells. The first two cells are composed of two columns and one row and the third cell is composed of three columns and one row. Also, the relative gap between the optimal objective function value of each lower bound and that of the ICFLP is shown in Fig. 2.

As can be seen in Table 1, all the problems were solved optimally except for problems 13 and 19 which were not solved optimally by the ICFLP within a reasonable computational time (in this paper 10,000 s). Nevertheless, as the objective function value of lower bound L3 is equal to that of the ICFLP, we can conclude that the objective function value of the *ICFLP* for problems 13 and 19 is optimal. The average gap between the ICFLP and lower bounds L1, L2, and L3 are 15.55, 16.38, and 4.46 %, respectively. According to this measure, we can conclude that lower bound L3 is the tightest bound for the ICFLP with a minimum gap of 0 % and a maximum gap of 14.6 %. As a result, this lower bound will be used in Sect. 6.2 to evaluate the solution of the heuristic method on medium- and large-scale problems.

5 Proposed heuristic method

As mentioned earlier, the ICFLP is NP-hard. In this section, a heuristic method composed of four sub-models, namely, M1, M2, M3, and M4, is suggested to effectively solve medium- and large-scale problems. In fact, the proposed heuristic method decomposes the ICFLP into four smaller sub-problems which are easier to solve than the ICFLP. After creating an initial solution, the iteration between these sub-problems is considered to gradually improve the solution. Figure 3 shows the flowchart of the proposed heuristic method.

In the following sub-sections, a detailed description of the proposed heuristic method is given.

5.1 Stage 1 (forming machine cells)

In this stage, the machine cells are formed according to the following model:

$$\mathrm{M1}:\max\sum_{k'>k}\sum_{l} F^{A}_{k,k'} z_{k,l} z_{k',l}. \quad (3.1)$$

Table 1 Comparison results for evaluating lower bounds

Problem					ICFLP		Lower bound L1		Lower bound L2		Lower bound L3	
No.	Source	Size ($M\times P$)	Aisle distance	Cell sizes $C\times(Co_l\times Ro_l)$	OFV	Time (s)	OFV	Time (s)	OFV	Time (s)	OFV	Time (s)
1.1	[13]	6×9	0.5	2×(3×1)	39.25	4.75	30	0.04	26	0.75	35.5	6.48
1.2	[13]	6×9	0	2×(3×1)	32.5	5.00	24	0.06	26	0.75	31	8.71
1.3	[13]	6×9	0.5	3×(2×1)	46.25	152	35	0.05	26	0.75	39.5	6.51
1.4	[13]	6×9	0	3×(2×1)	35	174	26	0.07	26	0.75	34.5	8.17
2.1	[5]	6×20	0.5	2×(3×1)	34,678.44	4.96	27,187.15	0.11	27,533.94	0.64	32,314.72	8.38
2.2	[5]	6×20	0	2×(3×1)	30,294.53	5.86	22,803.24	0.06	27,533.94	0.64	30,294.53	8.53
2.3	[5]	6×20	0.5	3×(2×1)	43,624.035	162	33,192.86	0.06	27,533.94	0.64	37,421.135	6.08
2.4	[5]	6×20	0	3×(2×1)	32,604	172	24,516.14	0.07	27,533.94	0.64	32,135.42	8.57
3.1	[11]	7×8	0.5	1×(3×1), 1×(4×1)	2800	15.05	2600	0.04	2800	0.63	2800	2.92
3.2	[11]	7×8	0.5	1×(3×1), 1×(2×2)	2800	7.65	2600	0.04	2800	0.63	2800	2.92
3.3	[11]	7×8	0.5	2×(2×1), 1×(3×1)	2800	91.25	2600	0.04	2800	0.63	2800	9.92
3.4	[11]	7×8	0.5	4×(2×1)	3300	11,836	3100	0.16	2800	0.63	3200	7.73
3.5	[11]	7×8	0	4×(2×1)	2800	>10,000*	2600	0.05	2800	0.63	2800	34.38
4.1	[32]	8×7	0.5	2×(4×1)	16	18.37	13	0.07	14	1.01	14	6.49
4.2	[32]	8×7	0.5	2×(2×2)	14	28.56	13	0.07	14	1.01	14	6.49
4.3	[32]	8×7	0.5	2×(2×1), 1×(2×2)	14	244	13	0.04	14	1.01	14	8.42
4.4	[32]	8×7	0.5	2×(3×1), 1×(2×1)	21.5	1462	18	0.06	14	1.01	18.75	37.48
4.5	[32]	8×7	0.5	4×(2×1)	19.5	8281	18	0.06	14	1.01	18.75	43.71
4.6	[32]	8×7	0	4×(2×1)	16.5	>10,000*	15	0.07	14	1.01	16.5	68.21

M no. of machines, *P* no. of parts, *C* no. of cells, Co_l no. of columns in the cells, Ro_l no. of rows in the cells *OFV* objective function value

*In this case the solve procedure was interrupted after 10,000 seconds

Subject to (2.2)–(2.4).

Where parameter $F^{A}_{k,k'}$ is computed by Eq. (2.5).

Objective function (3.1) maximizes the total intra-cell material movements cost (this leads to the minimization of the total inter-cell movement cost). Also, the remaining constraints are the same as explained before. Note that the resulting solution of this stage (i.e., $\widetilde{z}_{k,l}$) becomes the input of the next stage.

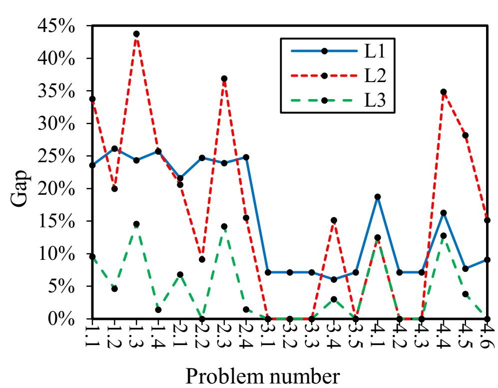


Fig. 2 Relative gap between the optimal objective function value of the lower bounds and that of the ICFLP (for the small-scale problems)

The product term in objective function (3.1) can be linearized by applying method (4.4) given in Appendix 1.

5.2 Stage 2 (creating a primary cell layout)

In this stage, according to the output of model M1, a primary cell layout is created. As the cell layout is still unknown, the centroid of each cell is used as the measuring point to calculate the distances between the machines. The following model is applied to obtain an initial cell layout.

$$\mathrm{M2}:\min\sum_{l'>l}\left(\sum_{i}\sum_{k'\neq k} d_i c^{E}_{i,k,k'} f_{i,k,k'}\widetilde{z}_{k,l}\widetilde{z}_{k',l'}\right)\left(|x_l-x_{l'}|+|y_l-y_{l'}|\right). \quad (3.2)$$

Subject to (1.10)–(1.12).where, $\widetilde{z}_{k,l}$ is the optimal value of variable $z_{k,l}$ obtained by model M1.

Objective function (3.2) minimizes the total inter-cell material handling cost which is estimated by considering the

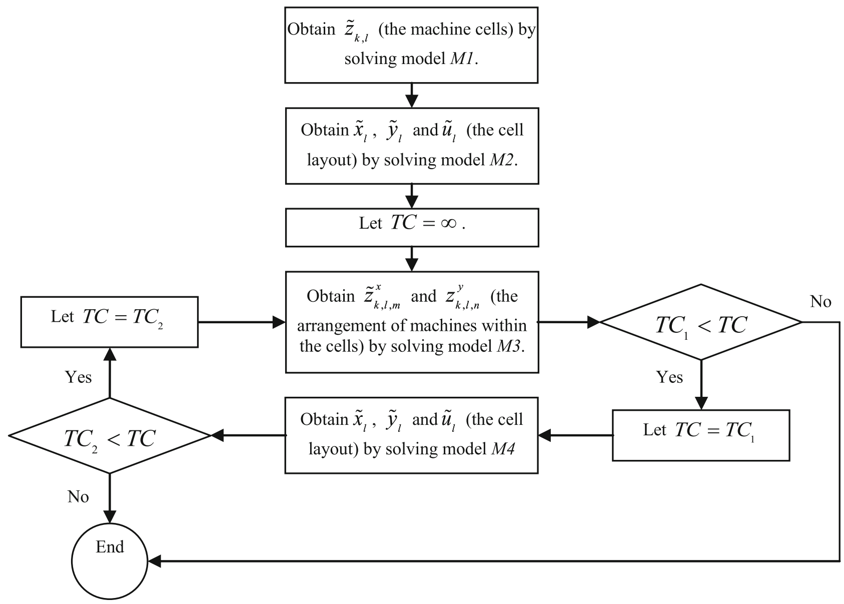


Fig. 3 Follow chart of the proposed heuristic method

center-to-center distances between the cells. The remaining parameters and constraints are the same explained before.

The nonlinear terms in model M2 are linearized according to the generic linearization methods given in Appendix 1. First, the absolute operators in objective function (3.2) are linearized by using method (4.2). Then, method (4.3) is applied to resolve the nonlinearity of the product terms.

5.3 Stage 3 (arranging/rearranging machines within the cells)

In this stage, by considering the assignment of machines to the cells and the cell layout, a mathematical model is presented to determine the arrangement of machines within the cells such that the total martial handling cost is minimized. The proposed model is as follows:

$$\begin{aligned}\text{M3} : \min TC_1 = &\sum_{k>k'}\sum_{l} F^A_{k,k',l}\tilde{z}_{k,l}\tilde{z}_{k',l}d_{k,k',l}\\ &+\sum_{k>k'}\sum_{l\neq l'} F^E_{k,k',l,l'}\tilde{z}_{k,l}\tilde{z}_{k',l'}\left(d^x_{k,k',l,l'}+d^y_{k,k',l,l'}\right).\end{aligned} \tag{3.3}$$

Subject to (1.9), (1.11), and (1.12).

$$\begin{aligned}d^x_{k,k',l,l'} = \Bigg|&\tilde{x}_l - \frac{w_l+(h_l-w_l)\tilde{u}_l}{2} + \sum_{m=1}^{Co_l} A_{l,m}z^x_{k,l,m} + \tilde{u}_l\left(\sum_{n=1}^{Ro_l} B_{l,n}z^y_{k,l,n} - \sum_{m=1}^{Co_l} A_{l,m}z^x_{k,l,m}\right) - \tilde{x}_{l'}\\ &+\frac{w_{l'}+\left(h_{l'}-w_{l'}\right)\tilde{u}_{l'}}{2} - \sum_{m=1}^{Co_{l'}} A_{l',m}z^x_{k',l',m} - \tilde{u}_{l'}\left(\sum_{n=1}^{Ro_{l'}} B_{l',n}z^y_{k',l',n} - \sum_{m=1}^{Co_{l'}} A_{l',m}z^x_{k',l',m}\right)\Bigg|, \forall \tilde{z}_{k,l} = \tilde{z}_{k',l'} = 1, k' > k, l\neq l',\end{aligned} \tag{3.4}$$

$$\begin{aligned}d^y_{k,k',l,l'} = \Bigg|&\tilde{y}_l - \frac{h_l+(w_l-h_l)\tilde{u}_l}{2} + \sum_{n=1}^{Ro_l} B_{l,n}z^y_{k,l,n} + \tilde{u}_l\left(\sum_{m=1}^{Co_l} A_{l,m}z^x_{k,l,m} - \sum_{n=1}^{Ro_l} B_{l,n}z^y_{k,l,n}\right) - \tilde{y}_{l'} + \frac{h_{l'}+\left(w_{l'}-h_{l'}\right)\tilde{u}_{l'}}{2}\\ &-\sum_{n=1}^{Ro_{l'}} B_{l',n}z^y_{k',l',n} - \tilde{u}_{l'}\left(\sum_{m=1}^{Co_{l'}} A_{l',m}z^x_{k',l',m} - \sum_{n=1}^{Ro_{l'}} B_{l',n}z^y_{k',l',n}\right)\Bigg|, \forall \tilde{z}_{k,l} = \tilde{z}_{k',l'} = 1, k' > k, l\neq l'.\end{aligned} \tag{3.5}$$

$$d_{k,k',l} = \left|\sum_{m=1}^{Co_l} A_{l,m}\left(z^x_{k,l,m} - z^x_{k',l,m}\right)\right| + \left|\sum_{n=1}^{Ro_l} B_{l,n}\left(z^y_{k,l,n} - z^y_{k',l,n}\right)\right|, \forall z_{k,l} = z_{k',l} = 1, k' > k, l, \tag{3.6}$$

$$\sum_{m=1}^{Co_l} z^x_{k,l,m} = \widetilde{z}_{k,l}, \forall k, l, \tag{3.7}$$

$$\sum_{n=1}^{Ro_l} z^y_{k,l,n} = \widetilde{z}_{k,l}, \forall k, l, \tag{3.8}$$

Where $\widetilde{z}_{k,l}$ is the optimal value of decision variable $z_{k,l}$ obtained by model M1. Also, $\widetilde{x}_l$, $_l$, and $\tilde{u}_l$ are the optimal value of decision variables $\widetilde{x}_l$, $\widetilde{x}_l$, and $\widetilde{x}_l$ derived from model M2 or model M4 which is presented in the next subsection.

Objective function (3.3) minimizes the total material handling cost (the first term is the total intra-cell material handling cost and the second one is the total inter-cell material handling cost). Constraints (3.4) and (3.5), respectively, calculate the vertical and the horizontal distances between the machines in distinct cells. Also, constraint (3.6) calculates the rectilinear distance between the machines in a same cell. Constraints (3.7) and (3.8), respectively, ensure that each machine is assigned to one column and one row in its corresponding cell. The remaining constraints are the same explained before.

According to the generic linearization methods given in Appendix 1, the nonlinear terms in model M3 are linearized as follows. First, the absolute operators in constraints (3.4)–(3.6) are linearized by method 4.2. Then, method (4.3) is applied to resolve the nonlinearity of the product terms in objective function (3.3).

5.4 Stage 4 (re-designing cell layout)

In this stage, the cell layout is re-designed by assuming that the intra-cell layout is fixed. As the layout of machines within the cells is assumed to be fixed, the total intra-cell material handling cost becomes constant. The major difference between the model which is presented in this subsection (i.e., model M4) and model M2 is that in model M2, the material handling cost is calculated in terms of the center-to-center distances between the cells. However, in model M4, the actual position of machines is taken into account; this results in more accurate cell layout. To formulate such a problem, we need to consider eight possible positions for the placement of machine cells on the planar area. These possible positions for a machine cell with five machines are illustrated in Fig. 4.

The proposed model for re-designing cell layout is as follows:

$$\text{M4}: \min TC_2 = \sum_{k'>k}\sum_{l} f^A_{k,k'}\widetilde{z}_{k,l}\widetilde{z}_{k',l}\left(\left|\widetilde{A}_k - \widetilde{A}_{k'}\right| + \left|\widetilde{B}_k - \widetilde{B}_{k'}\right|\right) + \sum_{k'>k}\sum_{l\neq l'} f^E_{k,k'}\widetilde{z}_{k,l}\widetilde{z}_{k',l'}\left(d^x_{k,k',l,l'} + d^y_{k,k',l,l'}\right) \tag{3.9}$$

Subject to (1.10)–(1.12).

$$d^x_{k,k',l,l'} = x_l - \frac{w_l + (h_l - w_l)u_l}{2} + \tilde{A}_k + u_l u'_l\left(h_l - w_l + 2\tilde{A}_k - 2\tilde{B}_k\right) + u'_l\left(w_l - 2\tilde{A}_k\right) + u_l\left(\tilde{B}_k - \tilde{A}_k\right)\Big| - x_{l'} + \frac{w_{l'} + (h_{l'} - w_{l'})u_{l'}}{2} - \tilde{A}_{k'} u_{l'} u_{l'}{}'\left(h_{l'} - w_{l'} + 2\tilde{A}_{k'} - 2\tilde{B}_{k'}\right) u'_{l'}\left(w_{l'} - 2\tilde{A}_{k'}\right) + u_{l'}\left(\tilde{B}_{k'} - \tilde{A}_{k'}\right)\Big|, \forall \tilde{z}_{k,l} = \tilde{z}_{k',l'} = 1, k' > k, l \neq l', \tag{3.10}$$

$$d^y_{k,k',l,l'} = y_l - \frac{h_l + (w_l - h_l)u_l}{2} + \widetilde{B}_k + u_l u''_l\left(w_l - h_l - 2\widetilde{A}_k + 2\widetilde{B}_k\right) + u''_l\left(h_l - 2\widetilde{B}_k\right) + u_l\left(\widetilde{A}_k - \widetilde{B}_k\right) - y_{l'} + \frac{h_{l'} + (w_{l'} - h_{l'})u_{l'}}{2} - \widetilde{B}_{k'} - u_{l'}u''_{l'}\left(w_{l'} - h_{l'} - 2\widetilde{A}_{k'} + 2\widetilde{B}_{k'}\right) - u''_{l'}\left(h_{l'} - 2\widetilde{B}_{k'}\right) - u_{l'}\left(\widetilde{A}_{k'} - \widetilde{B}_{k'}\right)|, \forall z_{k,l} = z_{k',l'} = 1, k' > k, l \neq l', \tag{3.11}$$

$$u'_l, u''_l \in \{0, 1\}, \forall l. \tag{3.12}$$

Where $\widetilde{A}_k = \sum_l \sum_{m=1}^{Co_l} A_{l,m}\widetilde{z}^x_{k,l,m}$ and $\widetilde{B}_k = \sum_l \sum_{n=1}^{Ro_l} B_{l,n}\widetilde{z}^y_{k,l,n}$.

In the above model, $\widetilde{z}^x_{k,l,m}$ and $\widetilde{z}^y_{k,l,n}$ are the optimal values of decision variables $z^x_{k,l,m}$ and $z^y_{k,l,n}$ obtained by model M3, and $\widetilde{z}_{k,l}$ is the optimal value of decision variable $z_{k,l}$ obtained from model M1. Also, decision variables u'_l and u''_l besides decision variable u_l are used to specify the direction of machine cells and represent one of eight possible positions for the placement of a machine cell. Table 2 shows the possible coordinates of a hypothetical machine within a cell considering different values of these variables.

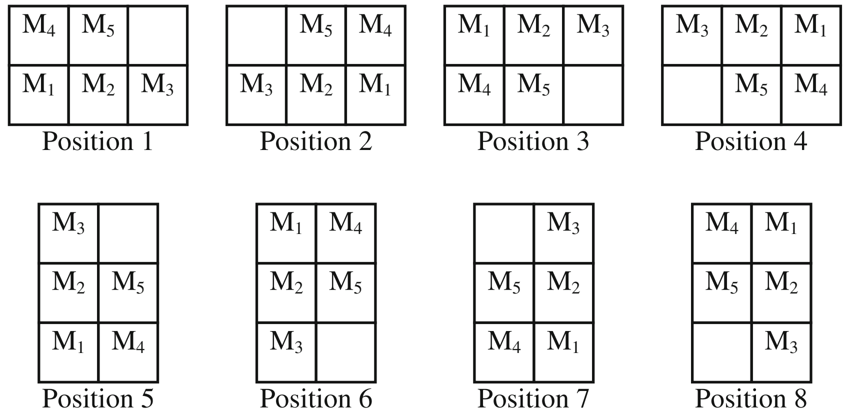


Fig. 4 Possible positions for placement of a hypothetical machine cell with five machines

Objective function (3.9) minimizes the total material handling cost. As mentioned before, the arrangement of machines within the cells are assumed to be fixed. Therefore, the first term of objective function (3.9), i.e., the total intra-cell material handling cost, becomes constant. Constraints (3.10) and (3.11), respectively, calculate the horizontal and vertical distances between the machines in distinct cells. Constraint (3.12) is the logical binary requirement on decision variables. The remaining constraints are the same explained before.

According to the generic linearization methods given in Appendix 1, model M4 can be linearized as follows. First, the product term and the absolute operators in the first and the second part of objective function (3.9) are linearized by using methods (4.4) and (4.2), respectively. Then, the product term in the second part of objective function (3.9) is linearized by applying method (4.3).

6 Computational results

In this section, the performance of the proposed heuristic method is evaluated by solving several problems from the literature. The heuristic method was formulated in the GAMS modeling software and the CPLEX solver was chosen for solving the problems. The proposed problems are classified into two sets and each set is solved separately. Also, in the rest of this section, the proposed integrated approach is compared to a similar approach from the literature. It should be noted that all experiments in this section are performed on the same computer used in Sect. 4.

6.1 Small- and small-to-medium-scale problems

The first set of problems includes the same small- and small-to-medium-scale problems which were solved in Sect. 4. These problems are solved by the heuristic method and the results are compared with the optimal objective function value of the ICFLP. The computational results are given in Table 3. In this table, column 'No. iter.' indicates the number of submodels solved in the heuristic method; also, column 'Error (%)' shows the relative difference between the objective function value of the ICFLP and that of the heuristic method. This measure is calculated as follows: Error=(OFV$^{\text{Heuristic}}$– OFV$^{\text{ICFLP}}$)/OFV$^{\text{ICFLP}}$×100.

The results show that except for problems 1.1, 1.2, 2.3, and 4.4, the remaining problems were solved optimally by the heuristic method. The average error for those problems which

Table 2 possible coordinates of a machine within a cell in terms of various combinations of decision variables u_l, u_l', and u_l''

Position	u_l	u_l'	u_l''	x-axis	y-axis
1	0	0	0	$x_l - \frac{w_l}{2} + \widetilde{A}_k$	$y_l - \frac{h_l}{2} + \widetilde{B}_k$
2	0	0	1	$x_l - \frac{w_l}{2} + \widetilde{A}_k$	$y_l + \frac{h_l}{2} - \widetilde{B}_k$
3	0	1	0	$x_l + \frac{w_l}{2} - \widetilde{A}_k$	$y_l - \frac{h_l}{2} + \widetilde{B}_k$
4	0	1	1	$x_l + \frac{w_l}{2} - \widetilde{A}_k$	$y_l + \frac{h_l}{2} - \widetilde{B}_k$
5	1	0	0	$x_l - \frac{h_l}{2} + \widetilde{B}_k$	$y_l - \frac{w_l}{2} + \widetilde{A}_k$
6	1	0	1	$x_l - \frac{h_l}{2} + \widetilde{B}_k$	$y_l + \frac{w_l}{2} - \widetilde{A}_k$
7	1	1	0	$x_l + \frac{h_l}{2} - \widetilde{B}_k$	$y_l - \frac{w_l}{2} + \widetilde{A}_k$
8	1	1	1	$x_l + \frac{h_l}{2} - \widetilde{B}_k$	$y_l + \frac{w_l}{2} - \widetilde{A}_k$

Table 3 Computation results of the proposed heuristic method on small- and small-to-medium-scale problems

Problem no.	ICFLP	Heuristic			Error (%)
	OFV	OFV	No. iter.	Time (s)	
1.1	39.25	40	4	0.292	1.9
1.2	32.5	34	4	0.314	4.6
1.3	46.25	46.25	5	0.722	0.0
1.4	35	35	5	1.711	0.0
2.1	34,678.44	34,678.44	4	0.286	0.0
2.2	30,294.53	30,294.53	4	0.323	0.0
2.3	43,624.035	44,286.99	5	0.779	1.5
2.4	32,604	32,604	5	0.822	0.0
3.1	2800	2800	4	0.288	0.0
3.2	2800	2800	4	0.282	0.0
3.3	2800	2800	4	0.284	0.0
3.4	3300	3300	4	0.659	0.0
3.5	2800	2800	4	0.511	0.0
4.1	16	16	4	0.251	0.0
4.2	14	14	4	0.274	0.0
4.3	14	14	4	0.253	0.0
4.4	21.5	22.5	4	0.302	4.7
4.5	19.5	19.5	4	0.285	0.0
4.6	16.5	16.5	4	0.270	0.0

were not solved optimally is about 3.18 %. Also, our computations show that the proposed heuristic method is able to solve each problem in less than 0.7 s. The average computation time for these problems is almost 0.47 s. These imply the good performance of the proposed heuristic method in solving small- and small-to-medium-scale problems in terms of both solution quality and computation time.

6.2 Medium- and large-scale problems

The second set of problems includes 10 medium- and large-scale problems adopted from the literature. The characteristics of these problems are presented in Table 4. Note that for these problems, some necessary information was not available in the source paper (e.g., part demands, material handling costs, etc.). Hence, we added the required data to each problem. Each problem is investigated with different configurations (including the number of cells, the physical shape of cells, and the aisle distance between the cells). This group of problems is solved by the heuristic method and the solutions are compared with the results derived from lower bound L3. As lower bound L3 is on the basis of the QAP, some of these problems may not be solved optimally within a reasonable computational time (because the QAP is NP-hard). As a result, depending on the scale of each problem, the solver is interrupted after a specified time. The computational results are given in Table 5. In this table, column '$C\times(Co_l\times Rol)$' indicates the physical configuration of the cells, column 'Time limit' shows the maximum allowable time that can be spent in heuristic method for solving each sub-model, column 'No.

Table 4 Characteristic of medium- and large-scale problems

Problem no.	Scale ($M\times P$)	Source	Supplementary information			Descriptions
			$d_i,\forall i$	$c^A_{i,k,k'},\forall i,k,k'$	$c^E_{i,k,k'},\forall i,k,k'$	
5	Medium (8×20)	[32]	1	1	2	–
6	Medium (11×7)	[24]	1	1	1.5	Processes of parts are done according to the machine indexes in increasing order
7	Medium (10×20)	[33]	–	1	1.5	Processes of parts are done by the first routing
8	Medium (12×10)	[11]	–	–	10	Mean demand of parts is considered
9	Medium (12×18)	[34]	–	1	1.5	–
10	Large (15×30)	[35]	–	1	1.5	Processes of parts are done according to the machine indexes in increasing order
11	Large (16×43)	[36]	1	1	1.5	Processes of parts are done according to the machine indexes in increasing order
12	Large (17×20)	[24]	1	1	1.5	Processes of parts are done according to the machine indexes in increasing order
13	Large (20×20)	[37]	1	1	2	–
14	Large (24×40)	[38]	–	1	1.5	–

Table 5 Computation results of the proposed heuristic method on medium- and large-scale problems

Problem			Lower bound L3		Heuristic method				Gap (%)
No.	Aisle distance	Cell sizes $C\times(Co_l\times Ro_l)$	OFV	Time (s)	OFV	Time limit (s)	No. iter.	Time (s)	
5.1	0.25	2×(2×2)	81	123.785	91	–	4	0.368	11.0
5.2	0.25	2×(2×1), 1×(2×2)	88.5	351.419	99	–	6	2.202	10.6
5.3	0.25	4×(2×1)	102.5	278.94	114.5	–	5	15.065	10.5
6.1	0.25	2×(3×2)	16.75	7168.957	16.75	–	4	1.215	0.0
6.2	0.25	3×(2×2)	16.75	7200[a]	16.75	–	4	0.380	0.0
6.3	0.5	3×(2×2)	17.5	4127.069	17.5	–	4	0.369	0.0
6.4	0.25	4×(3×1)	18.5	7200[a]	18.5	–	4	0.457	0.0
6.5	0.5	4×(3×1)	20	7200[a]	20	–	4	0.668	0.0
7.1	0.5	2×(3×2)	27,488.25	7200[a]	29,393.2	–	4	1.039	6.5
7.2	0.5	2×(5×1)	27,588.5	7200[a]	32,203.25	–	4	0.492	14.3
7.3	0.5	3×(2×2)	29,596	7200[a]	34,807.75	–	5	1.709	15.0
7.4	0.5	3×(4×1)	29,596	7200[a]	34,324.75	–	4	0.772	13.8
7.5	0.5	4×(3×1)	30,948.25	7200[a]	36,133.75	–	5	13.759	14.4
7.6	0.5	5×(2×1)	34,039.5	7200[a]	40,279.5	10	5	20.338	15.5
8.1	0.5	3×(2×2)	10,650	7200[a]	10,720	–	4	0.994	0.7
8.2	0.5	4×(3×1)	11,590	7200[a]	12,800	–	4	0.598	9.5
8.3	1	4×(3×1)	12,410	7200[a]	13,910	–	4	0.593	10.8
8.4	0	6×(2×1)	10,550	7200[a]	10,550	5	4	12.758	0.0
8.5	0.5	6×(2×1)	12,380	7200[a]	13,105	5	4	12.793	5.5
8.6	1	6×(2×1)	14,110	7200[a]	15,660	5	4	12.729	9.9
9.1	0.5	3×(2×2)	10,197.5	7200[a]	11,343.75	–	4	1.107	10.1
9.2	0.5	4×(3×1)	11,330	7200[a]	14,018.75	–	6	32.849	19.2
9.3	0.5	4×(2×1), 1×(2×2)	11,575	7200[a]	13,505.00	5	5	10.548	14.3
9.4	0.5	2×(3×1), 3×(2×1)	12,410	7200[a]	13,448.75	5	6	15.449	7.7
9.5	0.5	6×(2×1)	12,735	7200[a]	14,810.00	10	5	29.698	14.0
10.1	0.25	3×(3×2)	20,061.500	7200[a]	20,237.75	20	5	40.353	0.9
10.2	0.25	3×(5×1)	19,770.875	7200[a]	20,305.25	–	5	1.192	2.6
10.3	0.25	4×(4×1)	23,368.875	7200[a]	24,173.875	–	5	1.515	3.3
10.4	0.25	5×(3×1)	24,560.875	7200[a]	25,636	10	5	14.779	4.2
11.1	0.5	3×(3×2)	149.75	7200[a]	166.50	–	4	7.606	10.1
11.2	0.5	4×(2×2)	161.75	7200[a]	181.25	–	5	18.497	10.8
11.3	0.5	3×(2×2), 2×(2×1)	161.5	7200[a]	181.25	10	5	12.822	10.9
11.4	0.5	6×(3×1)	164.5	7200[a]	189.25	20	5	40.798	13.1
12.1	0.25	3×(3×2)	63.25	10,000[a]	64.50	–	4	18.548	1.9
12.2	0.25	3×(2×2), 1×(3×2)	66.25	10,000[a]	66.25	–	5	3.525	0.0
12.3	0.25	3×(2×2), 2×(3×1)	68.75	10,000[a]	73.25	–	5	2.050	6.1
12.4	0.25	6×(3×1)	70.25	10,000[a]	78.125	10	5	21.102	10.1
13.1	0.25	3×(4×2)	114.5	18,000[a]	118.5	20	4	40.634	3.4
13.2	0.25	4×(3×2)	117	18,000[a]	127	20	8	89.963	7.9
13.3	0.25	5×(2×2)	149	18,000[a]	167	10	5	23.081	10.8
13.4	0.25	5×(4×1)	149	18,000[a]	161	10	6	27.989	7.5
13.5	0.5	7×(3×1)	143	18,000[a]	172	200	8	921.401	16.9
14.1	0.25	3×(4×2)	63,582.5	18,000[a]	64,490	100	7	300.919	1.4
14.2	0.25	4×(3×2)	72,537.5	18,000[a]	77,127.5	50	4	51.734	6.0
14.3	0.25	5×(5×1)	73,906	18,000[a]	79,912.5	–	4	34.535	7.5
14.4	0.25	6×(2×2)	78,665	18,000[a]	79,140	–	5	21.151	0.6
14.5	0.25	8×(3×1)	89,650	18,000[a]	96,235	100	7	401.659	6.8

[a] In this case, the solver was interrupted after a specified time

iter.' indicates the number of sub-models solved in the heuristic method, and column 'Gap (%)' shows the relative gap between the objective function value of model L3 and that of the heuristic method. This measure is defined as follows: Gap=($OFV^{Heuristic}-OFV^{L3}$)/$OFV^{L3}\times 100$.

From Table 5, it can be observed that the heuristic method is able to find good solution in a reasonable amount of computation time for the proposed medium- and large-scale problems. For this set of problems, the average gap is equal to 7.6 %. Particularly, in problems 6.1 and 6.3, the relative gap is equal to 0. From the other side, for these two problems, the lower bound model was solved optimally. Therefore, we can conclude that the solutions of the heuristic method for problems 6.1 and 6.3 are optimal. The results also show that the number of cells, the aisle distance between the cells, and the layout type within the cells are important factors in designing manufacturing cells. For instance, in problem 13, when double-row layout with four cells is considered (i.e., problem 13.3), the material handling cost is obtained 127 units. However, when the same problem is solved for linear layout with five cells (i.e., problem 13.4) the material handling cost is increased to 161 units by 26.77 %. Therefore, careful attention should be paid to these parameters in order to achieve the best configuration of cells.

6.3 Comparison of the proposed approach with a similar study

To show the advantages of the proposed layout approach (continuous cell layout and multi-row machine arrangement), this approach is compared with one of the similar approaches in the literature. To do so, six test instances adopted from Chang et al. [16] are solved by the proposed heuristic method and the solutions are compared with their solutions. The data set of these instances can be obtained from the following link: http://sites.google.com/site/chinjuchang/data/Production_data.pdf. In order to compare the solutions under comparable setting, both the inter- and intra-cell material handling costs are assumed to be 1 unit (i.e., $c^{E}_{i,k,k'}=c^{A}_{i,k,k'}=1, \forall i,k,k'$). Also, the aisle distance between the cells is assumed to be 1 unit. Chang et al. [16] solved these problems by using a tabu search algorithm. They considered linear double-row layout for the placement of machine cells (discrete cell layout) and linear layout for the arrangement of machines within the cells. Table 6 shows a comparison between the layout approach presented in this study and the ones proposed in [16]. In this table, column 'Imp. (%)' indicates the improvement percent in the total material handling cost. This measure is calculated as follows: Imp.=(TC_C-TC_H)/$TC_C\times 100$. Where TC_C is the total material handling cost calculated for the solutions presented in [16] and TC_H is the total material handling cost obtained by the proposed heuristic method. Also, the solutions of the heuristic method as well as the solutions reported in [16] are illustrated in Appendix 2.

From Table 6, we can see that for all the problems, our solutions are considerably better than those found by Chang et al. [16]. For problems 15, 17, and 19, the CF result (the assignment of the machines to the cells) found by the heuristic method is identical with that found by Chang et al. [16] (see Appendix 2). For all the problems, the total inter-cell material handling cost, TC^E, found by the heuristic method, is far better than that found by Chang et al. [16]. This demonstrates the potential benefits that could be derived from considering continuous cell layout rather than discrete cell layout. Moreover, for all the problems, the total intra-cell material handling cost, TC^A, found by the heuristic method, is better than or the same as that calculated for the solutions reported in [16]. This also implies the advantage of the proposed multi-row machine arrangement over the single-row machine arrangement. In spite of the fact that the heuristic method costs more computation time compared to the tabu search algorithm

Table 6 Comparison results between the proposed approach and Chang et al. [16] approach

Problem		Chang et al. [16] approach (discrete cell layout and linear machine arrangement)				Proposed approach (continuous cell layout and multi-row machine arrangement)						Imp. (%)
No.	Size ($M\times P$)	TC^A	TC^E	TC_C	Time (s)	Cell sizes $C\times(Co_l\times Ro_l)$	TC^A	TC^E	TC_H	Res. time (s)	Time (s)	
15	8×20	38	37	78	0.31	2×(4×1)	38	31	69	–	0.40	11.54
16	12×19	72	156	228	0.65	3×(2×2)	67	112	179	–	0.75	21.49
17	18×35	15,400	26,085	41,485	1.74	3×(3×2)	13,675	16,705	30,380	10	21.36	26.77
18	20×20	57	91	148	1.14	5×(5×1)	56	53	109	10	62.42	26.35
19	20×51	298	493	791	2.07	5×(5×1)	293	277	570	10	50.59	27.94
20	25×40	79	147	226	2	7×(2×2)	71	113	184	100	257.95	18.58

TC^A total intra-cell material handling cost, TC^E total inter-cell material handling cost, TC_C and $TC_H=TC^A+TC^E$ total material handling cost proposed in [16], but it still can solve these problems in reasonably less computation time (less than 258 s).

7 Conclusions and directions for further research

In this paper, we presented a new integrated approach, namely, ICFLP, to the CF and its inter- and intra-cell layout design. The ICFLP has three design features that were not studied in previous research. These design features include multi-row intra-cell layout (for the arrangement of machines within the cells), continuous cell layout, and aisle distance. Also, in the proposed problem, the material handling cost is calculated in terms of the actual position of machines within the cells. This measure results in the precise evaluation of the material handling cost and consequently leads to a more accurate layout design. As the ICFLP is NP-hard, a heuristic method was developed to solve medium- and large-scale problems in a reasonable computational time. Also, three lower bounds were proposed for the ICFLP. The performance of the proposed heuristic method was evaluated in two sections. In the first section, a set of small-scale test instances was solved by the heuristic method and the results were compared to the optimal solution of the ICFLP. The results indicated that the heuristic method is able to solve most of small-scale problems to optimality. Also, for those cases which were not solved optimally, the average error is about 3.18 %. In the second section, a set of medium- and large-scale test instances were solved by the proposed heuristic method and the results were evaluated by the tightest lower bound. Our computations indicated that the proposed heuristic method can solve these set of problems in a short amount of time and with an average gap of 7.6 %. In another part of this research, the proposed layout approach (continuous inter-cell and multi-row intra-cell layout) was compared to a conventional layout approach in which the inter-cell layout is assumed to be discrete and the intra-cell layout is assumed to be single-row. The comparisons revealed that the continuous inter-cell layout is far better than discrete inter-cell layout and it can lead to a considerable improvement in the total material handling cost. Moreover, it was demonstrated that in some cases, the multi-row intra-cell layout is better than the single-row intra-cell layout and it can reduce the total material handling cost.

Finally, the following issues are suggested for further research:

- The consideration of other design factors in the proposed problem, such as alternative process routings, capacity of machines, actual dimensions of machines in the intra-cell layout, pick-up and drop-off points in the layout design, etc.
- The extension of the proposed problem to include machine duplication and part subcontracting
- The integration of the proposed problem with other important issues in CMS, particularly group scheduling and production planning
- The implementation of meta-heuristic algorithms, especially Simulated Annealing and GA, for solving problems of larger scale for this integrated CMS problem

Appendix 1. Generic linearization methods adopted from [22, 28–31, 39, 40]

$$\begin{array}{l}\min f(x,y).\\ S.t.:\\ g(x,y)+yh(x)\le(\ge)a,\\ x\ge 0,\\ y\in\{0,1\}.\end{array} \equiv \begin{array}{l}\min f(x,y).\\ S.t.:\\ g(x,y)+z\le(\ge)a,\\ -My\le z\le My,\\ h(x)-M(1-y)\le z\le h(x)+M(1-y),\\ x\ge 0,\\ z:\text{free},\\ y\in\{0,1\}.\end{array} \tag{4.1}$$

Where a is a constant and M is a large enough number. $f(x,y)$ and $g(x,y)$ are linear functions in term of variables x and y and $h(x)$ is a linear function in terms of decision variable x. Also, z is an auxiliary positive variable used to resolve the nonlinearity of the product term $yh(x)$.

$$\min|g(x)|. \quad \equiv \quad \begin{array}{l}\min x^{+}+x^{-}.\\ S.t.:\\ x^{+}-x^{-}=g(x),\\ x^{+},x^{-}\ge 0.\end{array} \tag{4.2}$$

Where $g(x)$ is a linear function of variable x. Also, x^+ and x^- are auxiliary variables use to resolve the nonlinearity of the absolute term $|g(x)|$.

$$\begin{array}{l}\min xyf(z).\\ S.t.:\\ x,y\in\{0,1\},\\ z\ge 0.\end{array} \quad \equiv \quad \begin{array}{l}\min w.\\ S.t.:\\ w\ge f(x)-M(2-x-y),\\ x,y\in\{0,1\},\\ w,z\ge 0.\end{array} \tag{4.3}$$

Where $f(z)$ is a positive linear function in terms of continuous variable z. Also, w is an auxiliary positive variable used to resolve the nonlinearity of the product term $xyf(z)$.

$$
\begin{array}{l}
\min(\max) f(x,y) + cxy. \\
S.t.: \\
g(x,y) + axy \le b, \\
x, y \in \{0,1\}.
\end{array}
\equiv
\begin{array}{l}
\min(\max) f(x,y) + cz. \\
S.t.: \\
g(x,y) + az \le b, \\
z \le x, \\
z \le y, \\
z \ge x + y - 1, \\
z \ge 0, \\
x, y \in \{0,1\}.
\end{array}
\tag{4.4}
$$

Where a, b, and c are constants; $f(x,y)$ and $g(x,y)$ are linear functions in term of variables x and y. Also, z is an auxiliary positive variable used to resolve the nonlinearity of the product term xy.

$$
\begin{array}{l}
\min f\left(x, x', y, y'\right). \\
S.t.: \\
\left|x - x'\right| \ge a, \\
\quad \text{or} \\
\left|y - y'\right| \ge b, \\
x, x', y, y' \ge 0.
\end{array}
\equiv
\begin{array}{l}
\min f\left(x, x', y, y'\right). \\
S.t.: \\
x - x' + M(p + r) \ge a, \\
x' - x + M(1 - p + r) \ge a, \\
y - y' + M(1 + q - r) \ge b, \\
y' - y + M(2 - q - r) \ge b, \\
p, q, r \in \{0,1\}, \\
x, x', y, y' \ge 0.
\end{array}
\tag{4.5}
$$

Where a and b are constants. p and q are the binary auxiliary variables used to resolve the nonlinearity of the absolute operators. Also, r is an auxiliary binary variable used to represent the 'or' operator.

$$
\begin{array}{l}
\min c - x \sum_i a_i y_i. \\
S.t.: \\
x, y_i \in \{0,1\}, \forall i.
\end{array}
\equiv
\begin{array}{l}
\min c - z. \\
S.t.: \\
z \le \sum_i a_i y_i, \\
z \le Mx, \\
x, y_i \in \{0,1\}, \forall i, \\
z \ge 0.
\end{array}
\tag{4.6}
$$

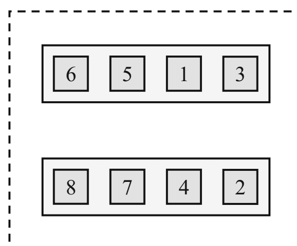
Where c and a_i are positive constants and M is a large enough number. Also, z is an auxiliary positive variable used to resolve the nonlinearity of the product term $x \sum_i a_i y_i$.

$$
\begin{array}{l}
\min axy + b(1-x)(y+c). \\
S.t.: \\
x \in \{0,1\}, \\
y \ge 0.
\end{array}
\equiv
\begin{array}{l}
\min aw_1 + bw_2. \\
S.t.: \\
w_1 \ge y - M(1-x), \\
w_2 \ge y + c - Mx, \\
x \in \{0,1\}, \\
y, w_1, w_2 \ge 0.
\end{array}
\tag{4.7}
$$

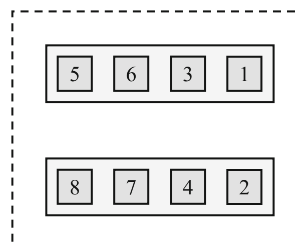
Where a, b, and c are positive constants and M is a large enough number. Also, w_1 and w_2 are auxiliary variables used to resolve the nonlinearity of the product terms xy and $(1-x)(y+c)$, respectively.

Appendix 2. Solutions of the problems solved in Sect. 6.3

Problem 15

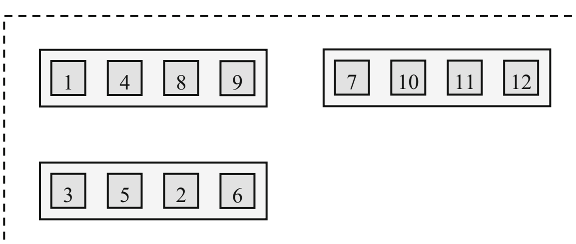


Chang et al. [16] approach

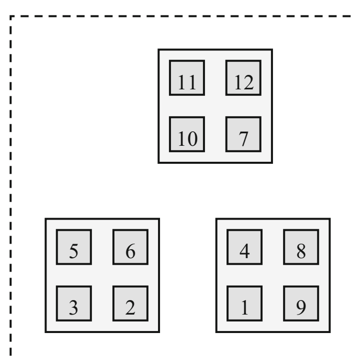


Proposed approach

Problem 16

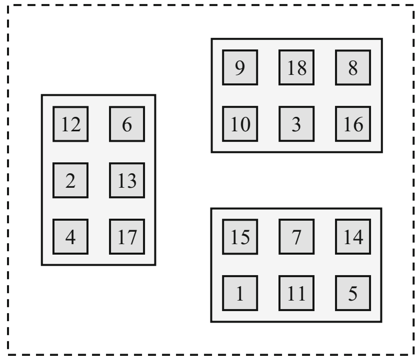


Chang et al. [16] approach

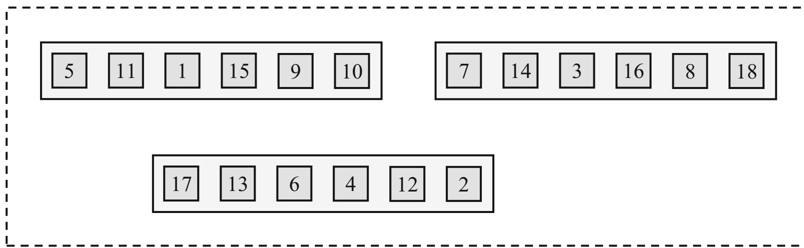


Proposed approach

Problem 17

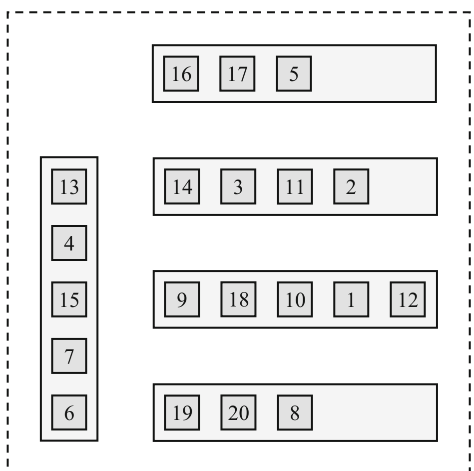


Proposed approach

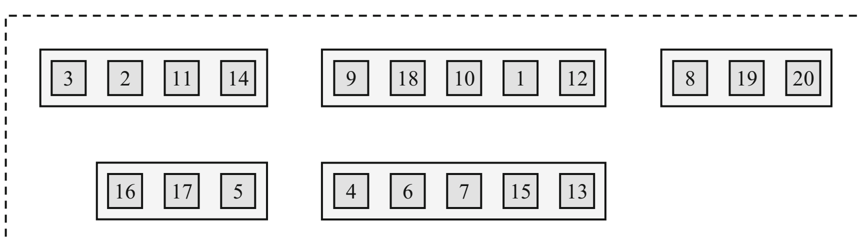


Chang et al. [16]

Problem 18

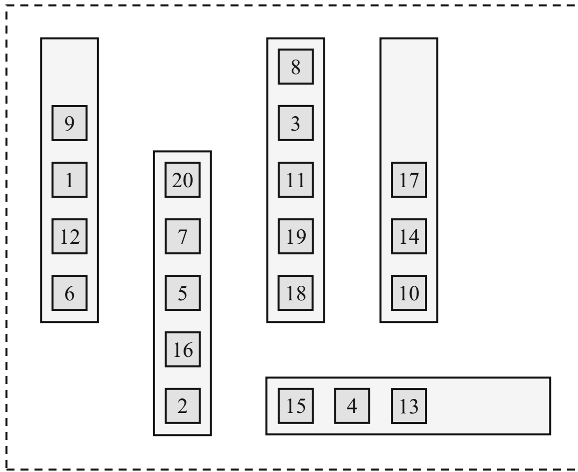


Proposed approach

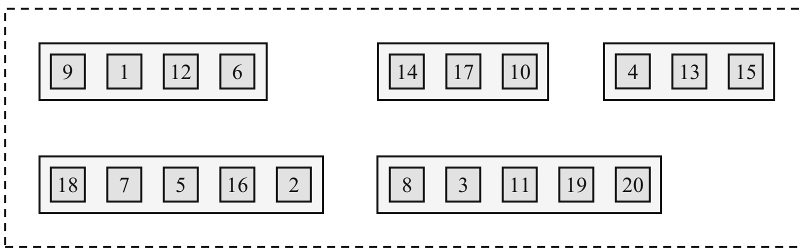


Chang et al. [16] approach

Problem 19

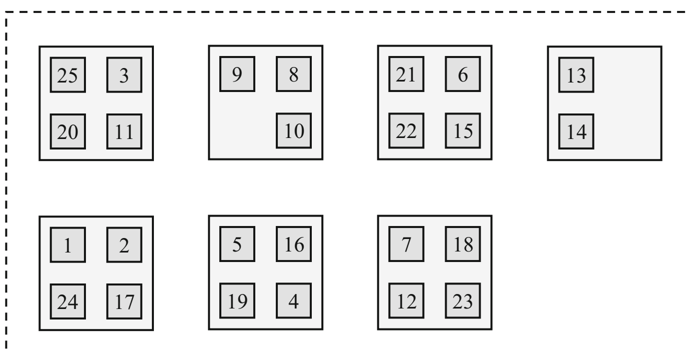


Proposed approach

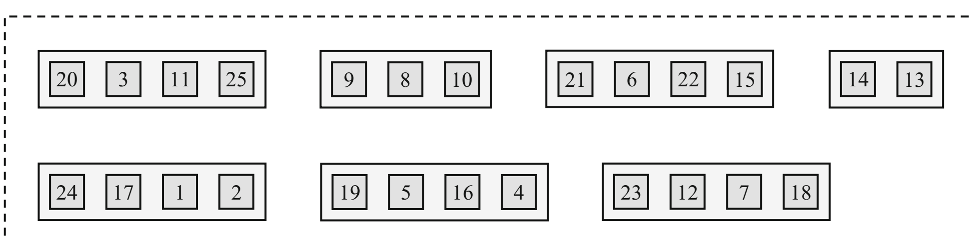


Chang et al. [16] approach

Problem 20



Proposed approach



Chang et al. [16] approach

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