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ORIGINAL ARTICLE

Study on the orthomogonalization for hybrid motion/force control and its application in aspheric surface polishing

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Abstract In the process of curved surface polishing and buffing, the tools have to be compressed on the workpiece and moved to wear the surface, so the normal force and tangential feed movements should be provided and controlled synchronously. It is a typical kind of compliant control, which would most likely be done under the hybrid motion/force control policy. Although its related theory is already almost perfect, this method could seldom be used in the process of curved surface polishing and buffing, if ever, for it is always not easy to orthogonalize force control space from movement control space. In this paper, a force control subsystem is developed and fixed on a computer numerical control (CNC) lathe to control the normal force independently, and the normal force control space is orthogonalized from the feed movement control space by geometry defining for aspheric surface polishing. Furthermore, the orthogonalization in the domain of time is taken to make the force control not to interpolate with the displacement control of the feed movements. Experiments in controlling and polishing the normal force show that the hybrid motion/force control policy could be used in the process of aspheric surface polishing and buffing by keeping the normal force control and feed movement control independently in both domains of space and time.

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Nomenclatures

K	Elasticity coefficient of the spring, N/m
μ	Damping coefficient of the damper
F_n	Projected force in the normal direction of the cutting point, N
F_t	Projected force in the tangential direction of the cutting point, N
T_n	Output torque of the torque-servo device, $N\cdot m$
$p(x,y)$	Position of the cutting point on the machined surface, m
$p_1(x_1,y_1)$	Position of the rotational center of the rotatable shaft, m
L_1	Length of the axes of the polishing tool, m
L_2	Length of the shaft of the polishing tool, m
α	Angle between the tangential line of the machined surface at the cutting point of $p(x,y)$ and the reverse direction of x-axis, rad
β	Angle between L_2 and the reverse direction of x-axis, rad
$Vol(x,y,z)$	Volume of total material removal, m^3
$h(x,y,z)$	Depth of the material removal at $p(x,y,z)$, m
k	Material removal coefficient determined by the cutting conditions
Δt	Machining time between two adjacent cutter locations, s
R_x	Rotational radius of $p(x,y,z)$, m
n	Rotational speed of the spindle of the CNC lathe, rad/s
v_f	Resultant speed of the feed movement on the cutting surface, m/s
v_{fx}	Projected speed of v_f in the direction of the rotational speed of the spindle, m/s

v_{f1}	Resultant speed of the feed movements in CNC codes, *m*/*s*
R_a	Value of surface roughness, μm

1 Introduction

When fixed or free abrasives are compressed to move along the surface of a workpiece, they would wear and cut the material away from the surface. That is, the contact force between the abrasives and the workpiece has to be kept or controlled all the time. So, most of researchers believe that the normal force, relative speed, and machining time determine the quantity or volume of material removal in the process of polishing [1, 2]. Until now, the most acceptable theory on polishing is developed by Preston [3] who believed that, when the hardware of a polishing system have been constructed and the abrasive is chosen, the material removal is simply proportional to the product of the normal force, the relative speed, and the machining time. For curved surfaces, the normal direction at one point would differ from that of another point as the direction of grads changes everywhere, and so does the direction of the normal force. The direction of the relative speed should be kept in a tangential plane at every cutting point on the workpiece surface. Therefore, in the process of curved surface polishing, a computer numerical control (CNC) system should drive a polishing tool to move along the surface at the calculated speed of feed movements in the machining time and put the expected force on the workpiece in the normal direction at the cutting point. That is a typical kind of compliant control.

When space orthogonalization between force control and displacement control happens in the same direction, you could just control either the contact force or the displacement in a single direction, never both. Consequently, the hybrid motion/force control policy is regarded as the most reasonable method of compliant control [4–8], under which the whole control space is divided into two orthotropic spaces: one as the force control space and the other as the motion control space. In this way, the famous Jacobian matrix could be used to deal with space coupling of the hybrid motion/force control. However, according to Li et al. [8], force control results from motion control in a constrained space. He divides the actual force control procedure into two basic movement states and their transitions. (a) For movement in a free space, when a manipulator controls the displacements and forces at the same time in a free space, the emergence of any unexpected forces will mean a serious error condition. (b) For movement in a constrained space, when a manipulator controls the forces in the absence of detecting displacements in a constrained environment, the emergence of any unexpected displacements will indicate a serious error condition as well. (c) When transitions occur between the two state spaces, from the non-contact state to the contact state and from the contact state to the non-contact state, both control mode and monitoring mode need to be changed. When the contact force reaches the desired value in the right direction, the control mode switches from the motion control mode to the force control one to maintain the contact force at an expected value. When the interaction force becomes zero, the control mode switches from the force control mode to the motion control one. Any actual force control strategy should include the non-contact state, contact state, as well as transitional states from the former to the latter and from the latter to the former, which demands that any description should establish the two states and the switch model.

For the hybrid motion/force control policy in the process of curved surface compliant polishing, much more research fellows [9–13] propose that the compliant control of the polishing tool could be regarded as the compromise of the complete displacement control and complete force control, which is different from both the complete motion control and complete force control. In the complete motion control mode, the displacement control of 6 degrees of freedom in the Cartesian space should be provided for the polishing tool which would move freely in any direction. Similarly, in the complete force control mode, the force control of 6 degrees of freedom should be provided for the polishing tool, which could get the required force in any direction.

Since the displacement control and force control should be kept in orthogonality exactly, and just be taken in its own space, why should the variables of displacement control should be interpolated with the variables of force control? Is it possible to take the displacement control and force control by dual CNC systems? Under the hybrid motion/force control policy, this paper studies a new way to get force controlled just in its space and displacement controlled in the other space in the process of aspheric surface polishing, and by variable optimization, the force control is made completely independent from the motion control.

2 Control space orthogonalization

Figure 1 shows a typical force control system of the elastic-damping mechanism, in which K denotes the elasticity coefficient of the spring, μ is the damp coefficient of the damper, and m is the mass of the moved object. Suppose that x'' denotes an accelerated speed and x' denotes a line speed, then its dynamic function could be described as

$$m\cdot x'' = K\cdot(x - x_0) + \mu\cdot x' \quad (1)$$

where x is the position of the moved object, and x_0 is the original length of the spring.

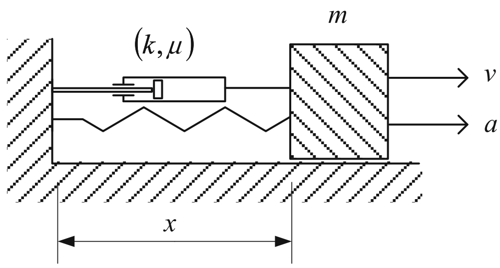

Fig. 1 Force control system of elastic-damping mechanism

In the process of curved surface polishing, the polishing tool could be regarded as the above moved object which is driven by the feed movements. At any cutter location (CL), the accelerated and line speeds of the feed movements in any direction should reach 0 at the same time when the feed movement is controlled by a CNC servo system. Suppose that F denotes the outside force on the moved object, then Eq. 1 at any cutter location could be rewritten as

$$F = K\cdot(x - x_0) \tag{2}$$

Obviously, the relationship between F and x is a linear correlation, and only one of them could be set as the controlled variable. It means that you could control either the contact force or the displacement in a single direction, never both.

Generally, the force between a cutter and its workpiece could be described as the resultant one of $\vec{F}_n$ and $\vec{F}_t$. Force test sensors would usually be set in the directions of x,y and z, so

$$\vec{F}_n \neq \vec{F}_x + \vec{F}_y + \vec{F}_z \tag{3}$$

If the CNC system could always be programmed to work in the best and easy case, the cutter could be moved to its expected cutter location without any trends of movement in any direction. In that case, $\vec{F}_t = 0$ and $\vec{F}_n = \vec{F}_x + \vec{F}_y + \vec{F}_z$. So, it seems reasonable to make it by just getting every component force be controlled in its own direction, but when a cutter or a polishing tool cuts some material from the surface of the workpiece, $\vec{F}_t \neq 0$ and $\vec{F}_n \neq \vec{F}_x + \vec{F}_y + \vec{F}_z$. It is absolutely not a good idea to get the normal force and displacements controlled in three directions by the same feed movement control in the same space of (x,y,z). In the processing of free-form surface polishing, the normal force should still be controlled in the normal direction and the displacements should be done in the orthogonal space, so the controlled vector in the process of curved surface polishing could be described as

$$\begin{cases} X = [x_u, x_v, x_w]^T \\ F_n = x_w \end{cases} \tag{4}$$

where (u,v,w) are directions of the cutter location in the curved surface, w denotes the normal direction, and F_n is the contact force in the normal direction of the point. (u,v,w) are not the directions of (x,y,z) of the Cartesian coordinate system of CNC machine tools, so the controlled vector of Eq. 4 could not be used to control CNC machine tools.

3 Aspheric surface polishing system

Aspheric optics are always of extremely high shape precision and low surface roughness, which are widely used in the military field, astronautics field, and some high-tech fields. The ultra-precision machining needs to be precisely grinded and polished after cutting using NC machine tools [14–16]. Lots of great research works on measurement, control, and machining have been taken for aspheric surface cutting, grinding, and polishing [17–20]. In the process of aspheric surface polishing, the contact force in the normal direction of the cutting point should be controlled on-line along with the feed movement control. The key issue on aspheric surface polishing is how to deal with the coupling relationship of the movement control and force control. Based on the orthogonality relation of force control space and movement control space, this paper develops a new 3-axis CNC compliant polishing method or system by controlling the movement and force in its own space separately [21, 22]. In this way, two computers work in parallel: the one that is used to fulfill the polishing of movement control of 2-axis interpolation and the other to fulfill the polishing of force control [23, 24].

On an ordinary CNC lathe of 2-axis interpolation, this paper develops a compliant polishing system for aspheric surface finishing under the hybrid motion/force control policy, in which a torque-servo device is set to control the normal force, as shown in Fig. 2. The compliant control system includes three subsystems. The first subsystem is the cutting motion control subsystem, which is composed of parts 1 and 2. The cutting motion is driven by the rotational motion of the spindle of the CNC lathe. The second one is the feed movement control subsystem, made up of parts 4, 5, and 10. The polishing tool holder could rotate around parts 9, 5 which provides the feed movements in two orthogonal directions and 10 which detects the pose of the polishing tool. By moving part 5 and rotating part 4, we could get the normal force just in the normal direction. The third one is the normal force control subsystem, composed of parts 6, 7, 8, and 9. As the stepper motor, device 6, transmits the required rotational displacement to part 7, the output torque of part 7 would be transmitted to part 9 through part 8. A torque sensor installed on part 7 would detect the output torque. When the output torque changes from the required value, the input current would be adjusted to drive the stepper motor to change its output displacement automatically to make the normal force at its expected value.

The feed movements of CNC lathes are in a plane. Conventionally, this plane is defined as the plane of xoz in the

Cartesian coordinate system of the CNC lathe. As shown in Fig. 2, for the sake of simplicity, the plane of *xoz* is defined as *xoy*, in which x is the right direction of the rotational speed of the spindle of the CNC lathe, and y is the right direction of the other feed movement. When the polishing tool is driven to move along the surface in the feed movement control space, the plane of *xoy*, the contact force at the cutting point, could be kept just in normal direction in the same plane. In this way, the controlled vector gets three variables, not just x and y as usual. Accordingly, in the process of aspheric surface polishing, the controlled vector could be simplified as

$$\begin{cases} X = [x_u, x_v, x_w]^T \\ F_n = x_w \end{cases} \tag{5}$$

where x_x and x_y are the displacements of the feed movements of CNC lathes.

4 Hybrid motion/force control policy

Kinematics of the aspheric surface compliant polishing system is shown in Fig. 3, in which $p(x,y)$ is the cutting point on the machined surface, and $p_1(x_1,y_1)$ is the rotational center of the rotatable shaft, part 9 in Fig. 2. As shown in Fig. 3, taking L_1 as the axis of polishing tool and L_2 as the shaft of polishing tool, L_1 is set vertical to L_2. Suppose that α denotes the angle between the tangential line of the machined surface at the cutting point of $p(x,y)$ and the reverse direction of x-axis, β denotes the angle between L_2 and the reverse direction of x-axis, and T_n denotes the output torque of parts 6 and 7 of Fig. 2, then the geometry relationship between the cutting point $p(x,y)$ of the machined surface and the rotational center $p_1(x_1,y_1)$ is determined by β

$$\begin{cases} x_1 = x + L_1\sin\beta + L_2\cos\beta \\ y_1 = y + L_1\cos\beta - L_2\sin\beta \end{cases} \tag{6}$$

Especially, when $\alpha=\beta$,

$$F_n = T_n/L_2 \tag{7}$$

5 Further orthogonalization in the domain of time

If one space is orthogonal to the other, variables in one space would not change, while those in the other would change. In the above aspheric surface compliant polishing system, the position of the rotational center, $p_1(x_1,y_1)$, has been changed by the CNC servo system, the polishing tool could not reach the original cutting point $p(x,y)$, and F_n is not in the normal direction. It means that the normal force control would be disturbed by the feed movement control. Fortunately, orthogonalization in the domain of time is totally different from that of space, in which variables do not need to interpolate with each other. When $\alpha=\beta$, you could change the normal force freely. Based on Preston's well-known polishing theory [3], the volume of total material removal of the workpiece is in direct proportion to the product of the contact force in the normal direction, the machining time, and the relative speed between the polishing tools and the workpiece at the point, and for the point of $p(x,y,z)$, it could be described in an equation as follows:

$$Vol(x,y,z) = \int_0^{\Delta t} k\cdot F_n(x,y,z,t)\cdot v(x,y,z,t)dt \tag{8}$$

where $Vol(x,y,z)$ is the volume of total material removal at the point of (x,y,z), $F_n(x,y,z,t)$ is the contact force in normal direction at the point of (x,y,z), $v(x,y,z,t)$ is the relative speed at the point of (x,y,z), k is the material removal coefficient determined by cutting conditions, and t is the real machining time.

While $\alpha=\beta$ and $F_n=T_n/L_2$, there are still some other ways to control the polishing process. If the material on the surface should be removed more or less somewhere, the normal force could be increased or decreased, and we could also meet it by increasing or decreasing the relative speed or the machining time. So, it should be a feasible way to keep the normal force at a constant value in the process of curved surface compliant polishing. When $F_n(x,y,z,t)$=constant, Eq. 8 could be rewritten as

$$Vol(x,y,z) = k\cdot F_n\cdot\int_0^{\Delta t} v(x,y,z,t)dt \tag{9}$$

When the aspheric part is rotating with the spindle of the CNC lathe, the cutting point of $p(x,y,z)$ becomes all points of $x=x$. Suppose that the depth of material removal at $p(x,y,z)$ is $h(x,y,z)$, the rotational radius of $p(x,y,z)$ is R_x, the rotational speed the spindle of the CNC lathe is n, and the speed of the feed movement is v_f. According to the existed CNC policy, the speed of the feed movement would be 0 at every cutter location. The distance of the curve between the two adjacent cutter locations is usually very small in the process of curved surface machining, and the value of the relative speed would be kept at the same value. When the polishing tool is moved from a certain cutter location to the next adjacent one, the machining time between the two adjacent cutter locations is Δt and the distance is Δx. When n is

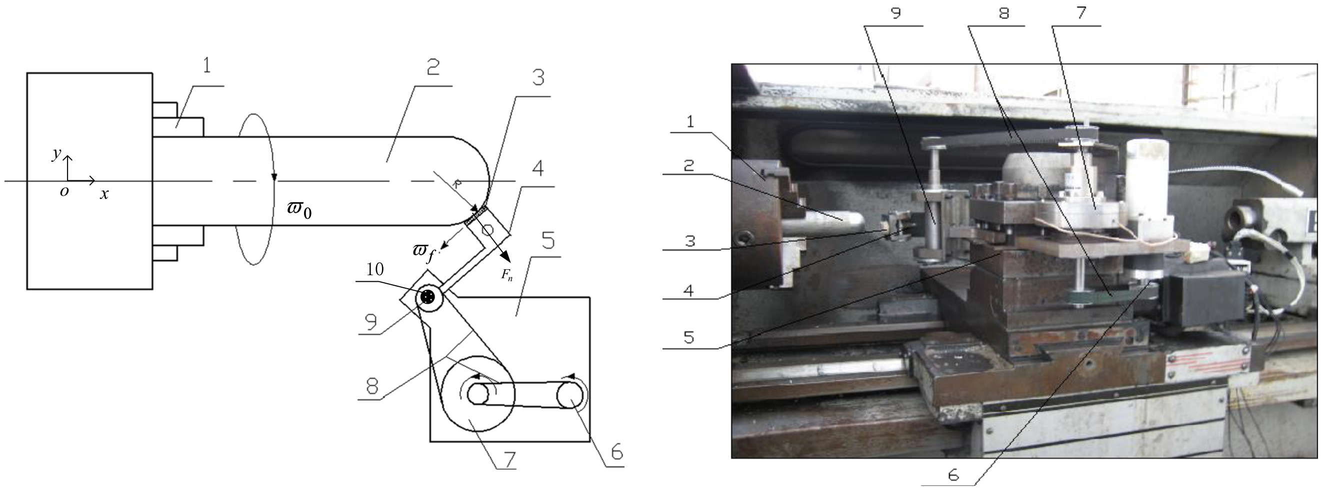

Fig. 2 Compliant polishing system for aspheric surfaces. *1* Spindle of the lathe, *2* workpiece, *3* polishing tool, *4* polishing tool holder, *5* Cutter carrier of the CNC lathe, *6* stepper motor, *7* elastic-damping element, *8* belt, *9* rotating shaft, *10* angle sensor

constant in a short period of Δt and with $F_n(x,y,z,t)$=constant, then

$$\begin{cases} \int_0^{\Delta t} v(x,y,z,t)dt = (2\pi R_x \cdot n)\cdot \Delta t \\ Vol(x,y,z) = 2\pi R_x \cdot \Delta x \cdot h(x,y,z) \end{cases} \tag{10}$$

From Eq. 9,

$$2\pi R_x \cdot \Delta x \cdot h(x,y,z) = k \cdot F_n \cdot (2\pi R_x \cdot n)\cdot \Delta t \tag{11}$$

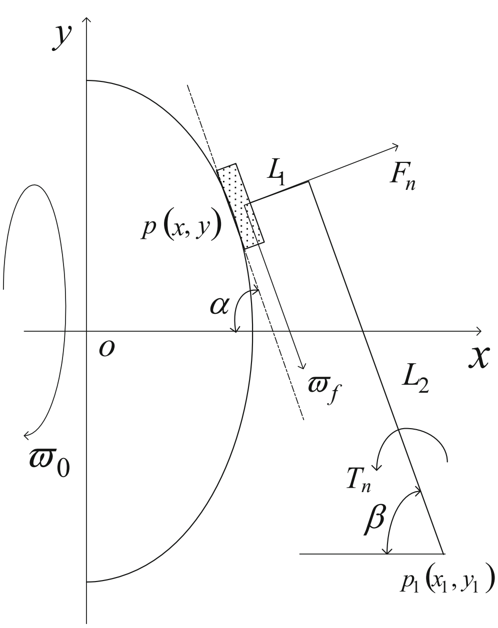

Fig. 3 Kinematics of the aspheric surface compliant polishing system

So,

$$h(x,y,z) = k \cdot F_n \cdot n \cdot \frac{\Delta t}{\Delta x} \tag{12}$$

The speed of feed movement which is v_f could be defined as $v_f = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$. Define $v_{fx} = \frac{dx}{dt}$ and $v_{fy} = \frac{dy}{dt}$ for a CNC lathe in Fig. 3. In this way,

$$h(x,y,z) = k \cdot F_n \cdot \frac{n}{v_{fx}} \tag{13}$$

More intuitively, when L_2,

$$h(x,y,z) \propto \frac{n}{v_{fx}} \tag{14}$$

It means that, by adjusting the rotational speed of the spindle of the CNC lathe and the speed of feed movement in the direction of the rotational speed of the spindle on-line, the volume of material removal could reach its expected value even if $F_n(x,y,z,t)$=constant. In this way, the force control space is completely independent from the feed movement control space.

6 CNC programming

CNC programming for CNC machine tools not only has to calculate the cutter location data and create the CNC codes but also has to decide the manufacturing variables.

6.1 Cutter location data and CNC codes

According to the traditional CNC programming, the CNC codes could be created from the contour of a workpiece with a kind of CAM software, such as Mastercam. While machined on a lathe, an aspheric part would be fixed on the spindle of the machine tool and rotates along with the spindle. The intersecting curve of the surface of the aspheric part and the plane of xoy in Fig. 3 would be a curve in the plane of xoy. The curve of A, the blue curve in Fig. 4, denotes this intersecting curve in the first quadrant, and it could be described as $f(x,y)=0$. When the curve of A is driven to rotate around its principal axis, it would become the surface of the aspheric part. It seems that the curve of A should be used to create the CNC codes for CNC lathes. As a matter of fact, the cutter location data come from the path of the cutter carrier of CNC lathes, not the curve of A. Suppose that the curve of B denotes the path of the cutter carrier, the path of $p_1(x_1,y_1)$. Under $\alpha=\beta$, the path of $p_1(x_1,y_1)$ could be computed from Eq. 6 and shown as the curve of B, the red curve in Fig. 4. The path of the cutter carrier of the CNC lathe is the real path of feed movements. So, the curve of B should be used to create the CNC codes.

Generally, the cutter location in x-axis of the feed movements for a lathe should always be in the negative direction. When the cutter location data in the x-axis are positive numbers, the cutter could not reach the locations. In the curve of B, some cutter location data are positive numbers, from which CAM software could not create the CNC code for CNC lathes. In this paper, we do it in the milling mode by setting the cutter radius equals to 0. In this way, we could get all the cutter location data for (x_1,y_1). When the feed movements move from a (x_1,y_1) to another, the angle of β is measured on-line, and with the output torque T_n on the polishing tool, then

$$\begin{cases} x = x_1-(L_1\sin\beta + L_2\cos\beta) \\ y = y_1-(L_1\cos\beta-L_2\sin\beta) \end{cases} \tag{15}$$

So does F_n and T_n,

$$\begin{cases} F_n = \dfrac{T_n\cdot\cos(\alpha-\beta)}{L_2} \\ \alpha = \arctan\dfrac{dy}{dx} \end{cases} \tag{16}$$

6.2 Manufacturing variables

Based on Eq. 14, the volume of material removal could reach its expected value by adjusting the rotational speed of the spindle of the CNC lathe and the speed of feed movement in the direction of the rotational speed of the spindle on-line, where v_{fx} is the projected one in the x-axis of v_f. In Fig. 5, v_f is the resultant feed speed on the workpiece. When the CNC servo system drives the polishing tool to move along the surface of a workpiece, the feed movements defined in CNC codes are not v_f, but the resultant speed of the cutter carrier, the speed of $p_1(x_1,y_1)$. Suppose that the speed of feed movements in CNC codes is v_{f1} and define $v_{fx_1} = \frac{dx_1}{dt}$ and $v_{fy_1} = \frac{dy_1}{dt}$, then

$$v_{f1} = \sqrt{{v_{fx_1}}^2 + {v_{fy_1}}^2} \tag{17}$$

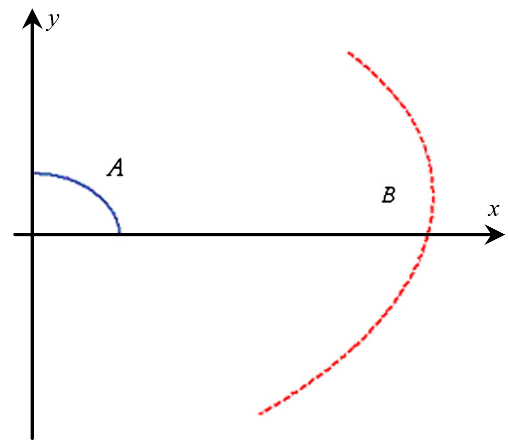

Fig. 4 Programming curve and the real contour

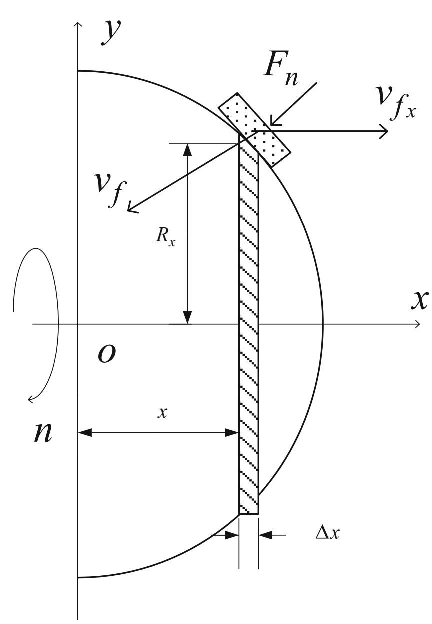

Fig. 5 Material removing in the aspheric surface compliant polishing

Table 2 Polishing procedures

	Step 1				Step 2
	Metallographic sandpaper				Polishing paste
Size of abrasive	#400	#1000	#2000	#3000	W0.5
Polishing times	3	3	3	3	10

As defined in Eq. 10, Δt is the machining time between the two adjacent cutter locations. In this time period, the plashing tool is moved from (x,y) to a new point that defines the new location (x_t,y_t) which could be calculated from Eq. 15. Suppose that the contour of the workpiece of aspheric surface is y=F(x), then

$$\begin{cases} x_t = x + v_{fx}\cdot\Delta t \\ y_t = F(x_t) \end{cases} \tag{18}$$

When v_{fx} is set from Eq. 14, (x,y) and (x_t,y_t) are computed from Eq. 15, and Δt could be solved as

$$\Delta t = \frac{x_t - x}{v_{fx}}. \tag{19}$$

Define α as the angle between the tangential line at (x,y) and the reserve direction of x-axis and α_t as the angle between the tangential line at (x_t,y_t) and the reserve direction of x-axis, then

$$\begin{cases} \alpha = \arctan\left.\frac{dF(x)}{dx}\right|_x \\ \alpha_t = \arctan\left.\frac{dF(x)}{dx}\right|_{x_t} \end{cases} \tag{20}$$

Define ϖ_t as the rotational speed of the polishing tool holder, part 4 in Fig. 2, i.e., the line L_2 in Fig. 3. For Δt is a very small period of time, so ϖ_t could be solved as

$$\varpi_t = \frac{d\alpha}{dt} = \frac{\alpha_t - \alpha}{\Delta t} \tag{21}$$

Table 1 Parameters of experiments

Rotational speed of spindle	600 rad/min
Feed speed	0.06 mm/rad
Surface roughness before polished	0.8 μm
Material of abrasive	Diamond
Material of workpiece	Steel #45

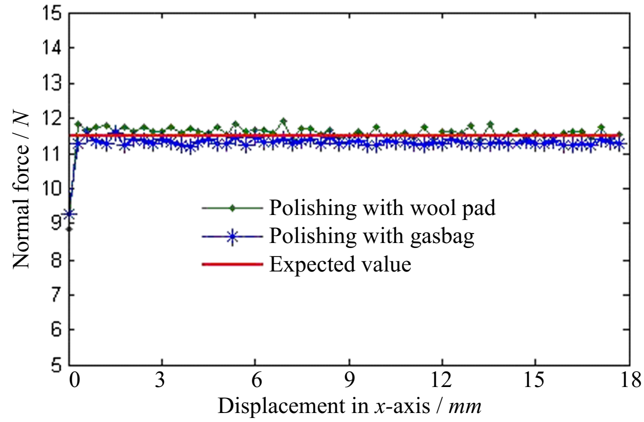

Fig. 6 Normal force measured with wool pad and gasbag

In addition,

$$\tan\alpha = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \tag{22}$$

So,

$$\frac{dy}{dt} = \frac{dx}{dt}\cdot\tan\alpha = v_{fx}\cdot(\tan\alpha) \tag{23}$$

Based on Eq. 5,

$$\begin{cases} \frac{dx_1}{dt} = \frac{d(x + L_1\sin\beta + L_2\cos\beta)}{dt} \\ \frac{dy_1}{dt} = \frac{d(y + L_1\cos\beta - L_2\sin\beta)}{dt} \end{cases} \tag{24}$$

Always keep β=α, then

$$\begin{cases} \frac{dx_1}{dt} = \frac{dx}{dt} + (L_1\cos\alpha - L_2\sin\alpha)\frac{d\alpha}{dt} \\ \frac{dy_1}{dt} = \frac{dy}{dt} - (L_1\sin\alpha + L_2\cos\alpha)\frac{d\alpha}{dt} \end{cases} \tag{25}$$

Table 3 Report of surface roughness test

Workpiece no.	#1
Report no.	2011-07-09
Cutoff wavelength	0.25 mm
Scanning length	5×cutoff
Date/time	24 March 2013
R_a (μm)	0.030
R_z ISO (μm)	0.204
R_y (μm)	0.339

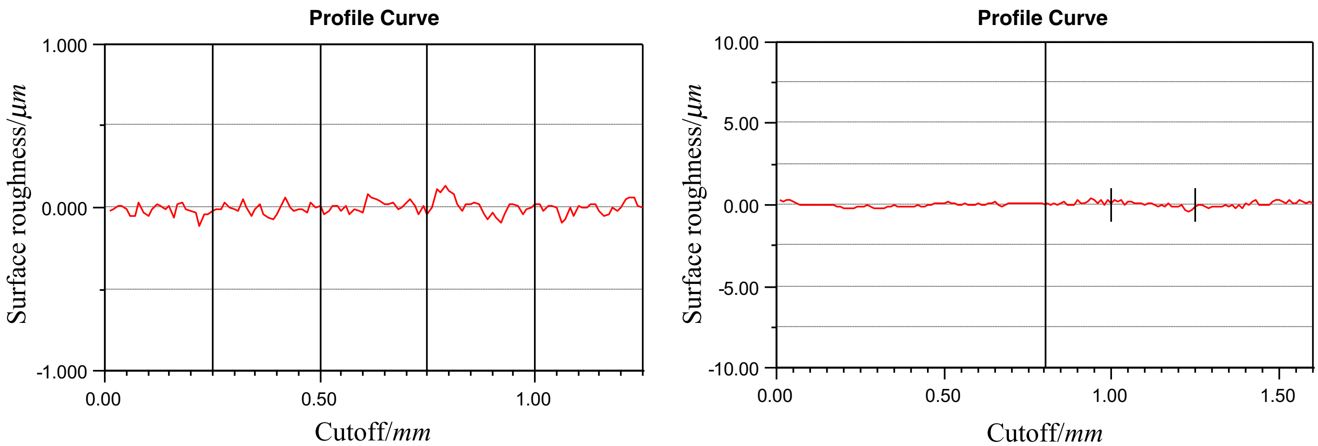

Fig. 7 Curves of the profile of aspheric parts scanned. **a** With the new method. **b** By the traditional way

Associating with Eqs. 20 and 22,

$$\begin{cases} \dfrac{dx_1}{dt} = v_{fx} + (L_1\cos\alpha - L_2\sin\alpha)\cdot\varpi_t \\ \dfrac{dy_1}{dt} = v_{fx}\cdot\tan\alpha - (L_1\sin\alpha + L_2\cos\alpha)\cdot\varpi_t \end{cases} \quad (26)$$

where $\alpha = \arctan\frac{dF(x)}{dx}\Big|_x$

From Eq. 17, the speed of feed movements in CNC codes v_{f1} could be calculated as

$$\begin{aligned} {v_{f1}}^2 = & \left[v_{fx} + (L_1\cos\alpha - L_2\sin\alpha)\cdot\varpi_t\right]^2 \\ & + \left[v_{fx}\cdot\tan\alpha - (L_1\sin\alpha + L_2\cos\alpha)\cdot\varpi_t\right]^2 \end{aligned} \quad (27)$$

When the cutter location data have been calculated from the curve of B, the real polishing point at the curve of A could be calculated using Eq. 15, and when v_{fx} is set from the expected depth of the material removal, α and ϖ_t could be measured and computed on-line, so the speed of the feed movements in CNC codes v_{f1} could be solved using Eq. 27.

Table 4 Report of surface roughness test

Workpiece no.	#2
Report no.	2010-05-04
Cutoff wavelength	0.8 mm
Scanning length	2×cutoff
Date/time	15 August 2013
R_a (μm)	0.110
R_z ISO (μm)	0.688
R_y (μm)	0.797

7 Experiments

To verify the feasibility and real efficiency of the compliant polishing system, machining experiments are taken according to the steps in Table 2 with the parameters as shown in Table 1. The surface of the aspheric part in the axial section plane is described in Eq. 28. In this polishing system, an aspheric part would be controlled to rotate by the rotational speed control for the spindle of the machine tool. The feed movements include two line motions and a rotational motion. The two line motions are taken by the motions of the cutter holder of the CNC lathe, and the rotational motion is driven by the torque-servo device automatically. When the two line motions are controlled to meet the path of (x_t,y_t), the polishing tool would polish the workpiece at the right position in its right pose. If the workpieces are polished more times and by much more finer abrasive rather than that of Table 2, lower surface roughness would be created definitely.

$$\frac{x^2}{30^2} + \frac{y^2}{75^2} = 1 \quad (28)$$

By orthogonalization in the control space and in the domain of time, the process of polishing the normal force control could be taken under an independent control system rather than the CNC servo system. It could also be dealt with as an ordinary manufacturing variable under the only CNC servo system of a machine tool. An ordinary manufacturing variable need not to be dealt with by an interpolator of the CNC servo system, but by a PLC controller. In this paper, the normal force control is taken with an additional force control system which is completely independent to the existed CNC servo system. When the aspheric part of steel #45 is polished using polishing tools of a wool pad and a gasbag and with polishing paste, the normal force is measured and shown in Fig. 6. By setting the output torque as a calculated constant value, the normal force could be controlled at the expected value. When the value of the expected value of the force is set at 11.5*N*, the measured value of the normal force by polishing with a wool pad fluctuates from about 11.5*N* to 12.0*N*, and it fluctuates from about 11.0*N* to 11.5*N* by polishing with a gasbag. Since the error of the normal force control is less than 5 %, the force control system does its job pretty well.

Before and after being polished, the same workpiece has been scanned, and the surface roughness is decreased from R_a=0.80 μm in Table 1 to R_a=0.030 μm in Table 3, and the section of the profile of the polished aspheric part is shown in Fig. 7a. By contrast, another steel #45 workpiece of Eq. 28 is polished according to the steps in Table 2 with the parameters as shown in Table 1 by F_n interpolating with (x_t, y_t), the surface roughness is decreased from R_a=0.80 μm to R_a=0.110 μm in Table 4, and the section of the profile is shown in Fig. 7b. Obviously, the new method in this paper is a better way than the traditional one.

8 Conclusions

In the process of curved surface polishing and buffing, the cutters should be driven to move along the surface of workpieces by keeping the contact force at an expected value. Until now, the process of curved surface polishing and buffing is most likely to be solved under the hybrid motion/force control policy, which is regarded as the most reasonable method for compliant control. The control vector of the compliant control in the process of curved surface polishing could be defined as $[x_u, x_v, x_w]^T$ and $F_n = x_w$; then the controlled force is constrained in the normal direction at the cutting points, and the feed movements are constrained in the tangential plane of the surface. Especially, the control vector of the compliant control in the process of aspheric part polishing could be defined as $[x_x, x_y, x_w]^T$ and $F_n = x_w$, and the three variables are in the same plane of *XOY*. Under the geometry constraints, the control space could be orthogonalized as two complete independent control spaces, and the existed CNC servo systems could make the control vector be controlled by all the variables interpolating on-line.

For an existed CNC servo system fixed in a machine tool, adding a new variable to interpolate with others is absolutely not a good idea. This paper avoids the tough task by further orthogonalizing force control from movement control in the domain of time to make it possible to keep the value of the normal force at a constant one, and the normal force control could be taken under an independent control system rather than the fixed CNC servo system. By setting the output torque as a calculated constant value, the normal force could be controlled at the expected value. The machining speed is provided by a CNC lathe with the rotational speed being controlled by the spindle of the lathe. The feed movements could be provided by the movements of the cutter holder of the CNC lathe and by the torque-servo device automatically. When the two line motions are controlled to meet the path of the cutter holder, the polishing tool would polish the workpiece at the right position in its right pose. Experiments in controlling and polishing force show that it could make the hybrid motion/force control policy be used in the process of aspheric surface polishing and buffing by providing normal force control and feed movement control independently.

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