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Empirical models for cutting forces in finish dry hard turning of hardened tool steel at different hardness levels

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Abstract In this research, the exponential and quadratic polynomial empirical models for three-component cutting forces by employing five factors, such as the cutting speed, depth of cut, feed, workpiece hardness, and nose radius, were developed by utilizing the orthogonal regression methodology (ORM) and response surface methodology (RSM). On the other hand, an attempt has been made to experimentally investigate the effects of those factors on three-component cutting forces in finish dry hard turning (FDHT) of tool steel AISI D2 with the PCBN tool. In this investigation, based on five-factor three-level orthogonal experiments, threecomponent cutting forces were measured, and then, analysis of variance (ANOVA) was performed to estimate the significance of developed models and analyze the main and interaction effects of the factors. The experimental results indicated that the RSM quadratic polynomial empirical model (RSMQPEM) is much more accurate and credible than the ORM exponential empirical model (ORMEM) in predicting the three-component cutting forces. It was also found that the cutting speed and feed are the two dominant factors affecting the main cutting forces F_Z ; the feed is the one dominant factor affecting radial cutting force $F_{\rm Y}$ and the feed cutting force $F_{\rm X}$. Additionally, the optimum cutting parameters for the hardened materials with 51, 55, 60, and 64 HRC was found.

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Keywords Hardened tool steel \cdot FDHT \cdot Three-component cutting forces \cdot ORM \cdot RSM \cdot ANOVA \cdot RSMQPEM \cdot ORME \cdot M

Nomenclature

- v Cutting speed (m/min)
- $a_{\rm p}$ Depth of cut (mm)
- f Feed (mm/rev)
- *H* Workpiece hardness (HRC)
- r_{ε} Nose radius (mm)
- *b* Chamfer width
- β Chamfer angle (°)
- $F_{\rm X}$ Feed cutting force (N)
- $F_{\rm Y}$ Radial cutting force (N)
- F_Z Main cutting force (N)
- $\kappa_{\rm r}$ Major cutting-edge angle (°)
- $\kappa_{\rm r}$ End-cutting edge angle (°)
- *a* Rake angle (°)
- γ_0 Clearance angle (°)
- γ_0 Side clearance angle (°)
- $\lambda_{\rm s}$ Inclination angle (°)

1 Introduction

The hard turning is defined as the single-point turning process of materials with hardness greater than 45 Rockwell Hardness measured on the C scale (HRC) under appropriate cutting tools [1]. It has gained more attention owing to its substantial advantages, such as reducing time of finish machining, declining cost of manufacturing, and offsetting the environment concerns compared to grinding [2–4].

Therefore, numerous investigations have been carried out to study the tool life, surface integrity, and the cutting forces in turning operations. One of those studies is to focus on the empirical models about the cutting forces. Tang et al. [5] have

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investigated the effect of the effects of cutting speed, depth of cut, feed, workpiece hardness (51, 55, 58, 62, and 65 ± 1 HRC), tool flank wear, and nose radius on cutting forces and not established the empirical models for cutting forces. More et al. [6] have analyzed the effect of cutting speed and feed on cutting forces using an analysis of variance (ANOVA) technique. Sharma et al. [7] utilized a neural network to construct a model in finish hard turning. Then, the model obtained was tested with the experimental data. Gaitonde et al. [1] analyze the effects of depth of cut and machining time on machining force using second-order mathematical models in turning of AISI D2 tool steel. Sieben [8] have established the empirical model for cutting forces based on such parameters as cutting speed, feed, and depth of cut. Based on Taguchi's method, the empirical model for cutting forces in the hard milling of AISI H13 steel has been developed by Ding et al. [9]. Gopalsamy [10] have studied the performance characteristics of machining process parameters, such as cutting speed, feed, depth of cut, and width of cut by utilizing an orthogonal array and ANOVA and then obtained optimum process parameters by gray relational analysis. Fnides et al. [11] have established statistical models of the cutting forces in hard turning of AISI H11 hot work tool steel and analyzed the effect of the main cutting variables, such as cutting speed, feed, and depth of cut on cutting force components. Besides, Aouici et al. [12] have experimentally investigated the effects of cutting speed, feed rate, workpiece hardness, and depth of cut on cutting force in hard turning of AISI H11 steel (40, 45, and 50 HRC); then, mathematical model for cutting force components was developed by using the response surface methodology (RSM).

It is revealed from the literature reviewed above that the models proposed by scholars were mainly focused on the influence of cutting speed, depth of cut, and feed. In fact, there are a large number of parameters which affect the cutting forces. These include cutting tool variables, workpiece material variables, cutting conditions, etc. Therefore, the influence of these process parameters on the models and analyses of the effect are limited. Here, a study aims to develop models for cutting forces and investigate the effect of process parameters, such as cutting speed, depth of cut, feed, hardness of workpiece, and nose radius, using the orthogonal regression methodology (ORM) and RSM approach. Then, ANOVA is performed to estimate the significance of developed models and analyze the main and interaction effects of factors.

2 Experimental details

2.1 Workpiece

In this study, the bar of tool steel AISI D2 (Cr12MoV, China) was used. The chemical composition is presented in Table 1. The bars of 48-mm diameter and 300-mm length were used.

 Table 1
 Chemical composition of the tool steel AISI D2 (wt%)

С	Cr	Мо	Mn	Si	Р	S	V
1.55	11.25	0.45	0.35	0.35	0.025	0.025	0.20

2.2 Heat treatment

In order to effectively utilize the finish dry hard turning (FDHT) process in the manufacturing industries, the hardened tool steel at different hardness levels was considered for the study with a polycrystalline cubic boron nitride (PCBN) insert. The results showed that the tool steel AISI D2 could get fine-needle martensite and high-diffusion and uniform distribution fine-grain carbide if using the quenching temperatures of 1,000–1,040 °C[13]. According to the methods of heat treatment in the literature [14], the specimens were put into an electrical resistance furnace at 1,000–1,040 °C, then quenched in oil, and finally tempered at various low temperatures. The obtained hardened specimens were in different hardness levels of 51 ± 1 , 58 ± 1 , and 64 ± 1 HRC.

2.3 PCBN cutting tool

2.3.1 Choice of the contents of CBN

At present, in the tool and die industries, the PCBN tool has been extensively utilized to dry hard machine-hardened steel, refractory steel, and high-temperature alloy steel, specially for difficult-cut materials with the hardness of 55– 65 HRC [15, 16].

In this paper, according to this literature [17], the PCBN cutting tools (type: GE2100, America) which had an approximate chemical composition of 50% CBN by volume and a CBN grain size of 2 μ m were selected to turn the hardened tool steels at different hardness levels (51–64 HRC). In hard cutting, the carbide provides shock resistance; the CBN provides very high wear resistance and cutting-edge strength. Therefore, the composite PCBN inserts (type: SCGN150404) made in Beijing World Company were utilized to finish dry hard turn, as shown in Fig. 1.



(a) Construction of a PCBN insert

(b) PCBN insert

Fig. 1 Design schematic of a composite PCBN insert. **a** Construction of a PCBN insert. **b** PCBN insert

2.3.2 Geometry parameter of the PCBN tool

The inserts were clamped in a piezoelectric three-component turning dynamometer (type: YDC-III89A) tool holder. Except for the nose radius, all the composite PCBN inserts used for the experiments had the same tool geometry. The effective geometry parameters of PCBN cutting tool are presented in Table 2.

2.4 Experimental setups and procedures

2.4.1 Test system of cutting force

FDHT tests were conducted at room temperature of about 22 °C and relative humidity of about 40 %. A schematic of an FDHT setup and force measurement system is presented in Fig. 2. The piezoelectric three-component turning dynamometer was mounted on a CNC lathe in which speed can vary from 0 to 2,200 rpm and there is a maximum power of 9.5 kW, as shown in Fig. 3.

The test system of cutting forces was made up of a computer installed with Windows XP 2003 Professional and GDFMS dynamic measurement software for cutting forces, multifunctional data acquisition card (type: PCI-9118DG), multichannel charge amplifier (type: YE5850), and piezoelectric quartz crystal three-component force dynamometer (type: YDC-III89A).

2.4.2 Cutting force measurements

Three-component cutting force's signals obtained from the dynamometer were transferred to a computer by means of the multifunctional data acquisition card and then were evaluated by utilizing the GDFMS dynamic measurement software. In order to acquire exact cutting force, three-component forces were acquired during steady-state phase in FDHT. Each test was repeated for three times, and the mean cutting force was used for further analysis.

2.5 Experimental design

Turning is a complex process because many parameters, such as cutting speed, depth of cut, feed, tool geometry, workpiece material condition, turning environment, etc. are involved. In this study, five parameters, including cutting speed (v), depth of cut (a_p), feed (f), workpiece hardness (H), and nose radius

 Table 2
 The effective geometry parameters of the PCBN cutting tool

$\kappa_{\rm r}$ (°)	$\kappa_{\mathrm{r}}^{\mathrm{'}}(^{\mathrm{o}})$	α (°)	γ_0 (°)	$\gamma'0~(^\circ)$	$\lambda_{\rm s}(^{\rm o})$	r_{ε} (mm)	β (°)	<i>b</i> (mm)
65	25	-5	5	5	-3	0.8, 1.2, 1.6	-15	0.1

 (r_{ε}) , were taken into consideration for conducting the experiments. Three levels were defined for each cutting variable as given in Table 3. Orthogonal design of experiments was utilized to design the experimentation. Therefore, the L27 (3)⁵ (three levels-five factors) orthogonal array for this experimentation led to a total of 27 tests.

RSM is a type of modeling to develop the relationship between various factors with the response [18, 19]. It is an effective technique to design the experiments and analyze problems by applying ANOVA and regression analysis. The established empirical models for three-component cutting forces describe the interaction of various parameters with respect to response factors.

3 Analysis and discussions

The experimental layout and experimental results of threecomponent cutting forces (F_X , F_Y , and F_Z) are presented in Table 4.

3.1 Modeling process

3.1.1 Method of the ORM exponential empirical model (ORMEM)

The functions of representing the three-component cutting forces can be expressed as [20]

$$F_{\rm X} = C_{\rm X} v^{l_{\rm X}} a_{\rm p}^{m_{\rm X}} f^{n_{\rm X}} H^{p_{\rm X}} r_{\varepsilon}^{q_{\rm X}} \tag{1}$$

$$F_{\rm Y} = C_{\rm Y} \nu^{l_{\rm Y}} a_{\rm p}^{m_{\rm Y}} f^{n_{\rm Y}} H^{p_{\rm Y}} r_{\varepsilon}^{q_{\rm Y}}$$

$$\tag{2}$$

$$F_Z = C_Z v^{l_Z} a_p^{m_Z} f^{n_Z} H^{p_Z} r_{\varepsilon}^{q_Z}$$
(3)

where C_X , C_Y , and C_Z are respectively the correction coefficient of F_X , F_Y and F_Z . l_X , m_X , n_X , p_X , and q_X ; l_Y , m_Y , n_Y p_Y and q_Y ; and l_Z , m_Z , n_Z , p_Z , and q_Z are exponents of corresponding parameters (v, ap, f, H, and $r\varepsilon$) in the ORMEM F_X , F_Y , and F_Z .

In order to make this paper concise, here, we only introduce the modeling process of the main cutting force F_Z .

Equation (3) may be transformed into the following linear model equation [21]:

$$\ln F_Z = \ln C_Z + l_Z \ln \nu + m_z \ln a_p + n_z \ln f + p_Z \ln H + q_Z \ln r_{\varepsilon}$$
(4)

Suppose

$$y = \ln F_z, b_0 = \ln C_z, x_1 = \ln v, x_2 = \ln a_p, x_3 = \ln f, x_4 = \ln H, x_5$$
$$= \ln r_{\varepsilon}; b_1 = l_Z, b_2 = m_Z, b_3 = n_Z, b_4 = p_Z, b_5 = q_Z$$





Thus, Eq. (4) becomes

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5$$
(5)

If there are m influential factors and n tests were conducted, then a multiple linear regression model can be generally described as [22, 23]

$$y = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_m x_{im} + \varepsilon_i, i$$
(6)

where β_0 , β_1 , and β_m are the predictable variables and x_1 , x_2 , and x_m are strictly controlled the natural elements. ε_1 , ε_2 , and ε_n are random variables which are independent mutually and obey the same normal distribution $N(0, \sigma^2)$.

If defined,

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \ \mathbf{x} = \begin{pmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1m} \\ 1 & x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n1} & x_{n3} & \cdots & x_{nm} \end{pmatrix} \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_m \end{pmatrix} \boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

A multiple linear regression model is represented in matrix form, which is generally described as

$$y = x\beta + \varepsilon \tag{7}$$

Fig. 3 FDHT

In linear regression analysis, Eq. (6) can be expressed as

$$\widehat{y} = b_0 + \sum_{j=1}^m b_j x_j \widehat{y} j \tag{8}$$

The least-squares estimate of regression coefficient β is generally described as [22]

$$b = \left(x^{\mathsf{T}}x\right)^{-1} \left(x^{\mathsf{T}}y\right) \tag{9}$$

3.1.2 Method of RSM quadratic polynomial empirical model (RSMQPEM)

In a general case, a second-order polynomial empirical model used to represent the response surface for k factors is given by [24]

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_i \sum_{j \neq i}^k \beta_{ij} x_i x_j + \varepsilon \quad (10)$$



imental error of the nth observations.

described as

Table 3 Assignment of the levels to the variables

Level	v (m/min)	$a_{\rm p} ({\rm mm})$	f(mm/rev)	$H(\mathrm{HRC})$	$r_{\varepsilon} (\mathrm{mm})$
1	75	0'.10	0.10	51±1	0.8
2	150	0.20	0.20	58±1	1.2
3	226	0.30	0.30	64±1	1.6

In the above equation, the β_0 , β_i , β_{ii} , and β_{ij} are the

In the case of engineering problems, higher order

regression coefficients and the residual; ε measures the exper-

interactions, such as interactions of three or more fac-

tors, are not practically of significance. Thus, three and

four factor interactions are ignored in the empirical model. The simplified model is a polynomial which is

$$y = \beta_0 + \beta_1(A) + \beta_2(B) + \beta_3(C) + \beta_4(D) + \beta_5(E) + \beta_6(AB) + \beta_7(AC)\beta_8(AD) + \beta_9(AE) + \beta_{10}(BC) + \beta_{11}(BD) + \beta_{12}(BE) + \beta_{13}(CD) + \beta_{14}(CE) + \beta_{15}(DE)$$
(11)

where β_0 is a constant; β_0 , β_1 , and β_{15} are coefficients based on the main as well as interaction effects.

3.1.3 Test of regression coefficient

The significance of the regression equation does not imply that each of the independent variables has a significant effect. Apparently, if the effect of a certain independent variable x_j on y is not significant, its coefficient β_j may equal to zero in this regression model. Therefore, whether a tested variable is significant or not is equivalent to hypothesis of testing shown in Eq. (12):

$$H_0: \beta_i = 0 \tag{12}$$

Table 4 Experimental results for three-component cutting forces

Cr12MoV	Factors		Experiment	Experimental results				
Tests	A v (m/min)	$B_{a_{p}(mm)}$	C f (mm/rev)	D H (HRC)	$E_{r_{\varepsilon}}$ (mm)	$F_{\rm X}$ (N)	$F_{\rm Y}$ (N)	$F_{\rm Z}$ (N)
1	75 (1)	0.10(1)	0.10(1)	51 (1)	0.80 (1)	29.95	45.95	66.79
2	75 (1)	0.10(1)	0.10(1)	51 (1)	1.20 (2)	25.44	46.73	68.20
3	75 (1)	0.10(1)	0.10(1)	51 (1)	1.60 (3)	26.14	50.26	71.58
4	75 (1)	0.20 (2)	0.20 (2)	58 (2)	0.80(1)	37.38	78.89	91.68
5	75 (1)	0.20 (2)	0.20 (2)	58 (2)	1.20 (2)	39.83	85.75	98.83
6	75 (1)	0.20 (2)	0.20 (2)	58 (2)	1.60 (3)	35.10	87.80	118.84
7	75 (1)	0.30 (3)	0.30 (3)	64 (3)	0.80(1)	42.19	87.76	95.59
8	75 (1)	0.30 (3)	0.30 (3)	64 (3)	1.20 (2)	43.06	91.89	100.38
9	75 (1)	0.30 (3)	0.30 (3)	64 (3)	1.60 (3)	36.36	100.09	115.59
10	150 (2)	0.10(1)	0.20 (2)	64 (3)	0.80(1)	37.20	109.23	93.13
11	150 (2)	0.10(1)	0.20 (2)	64(3)	1.20 (2)	36.00	122.46	101.55
12	150 (2)	0.10(1)	0.20 (2)	64 (3)	1.60 (3)	35.40	134.34	103.76
13	150 (2)	0.20 (2)	0.30 (3)	51 (1)	0.80(1)	39.07	80.98	103.12
14	150 (2)	0.20 (2)	0.30 (3)	51 (1)	1.20 (2)	29.07	87.01	107.07
15	150 (2)	0.20 (2)	0.30 (3)	51 (1)	1.60 (3)	24.05	94.08	120.78
16	150 (2)	0.30 (3)	0.10(1)	58 (2)	0.80(1)	48.11	145.45	119.19
17	150 (2)	0.30 (3)	0.10(1)	58 (2)	1.20 (2)	43.46	150.73	126.71
18	150 (2)	0.30 (3)	0.10(1)	58 (2)	1.60 (3)	48.99	177.75	137.18
19	226 (3)	0.10(1)	0.30 (3)	58 (2)	0.80(1)	27.02	80.42	75.71
20	226 (3)	0.10(1)	0.30 (3)	58 (2)	1.20 (2)	27.82	81.10	82.45
21	226(3)	0.10(1)	0.30 (3)	58 (2)	1.60 (3)	25.43	88.12	87.01
22	226 (3)	0.20(2)	0.10(1)	64 (3)	0.80(1)	40.93	144.54	136.74
23	226 (3)	0.20 (2)	0.10(1)	64 (3)	1.20 (2)	49.52	155.69	140.68
24	226 (3)	0.20 (2)	0.10(1)	64 (3)	1.60 (3)	36.66	176.28	142.78
25	226 (3)	0.30 (3)	0.20 (2)	51 (1)	0.80(1)	34.45	94.21	105.08
26	226 (3)	0.30 (3)	0.20 (2)	51 (1)	1.20 (2)	30.05	107.53	122.50
27	226 (3)	0.30 (3)	0.20 (2)	51 (1)	1.60 (3)	33.47	121.64	149.61

b and $\hat{\sigma}^2 = S_{\text{res}}/(n-m-1)$ is mutual independence, and, $b \sim N(\beta \sigma^2 (x^T x)^{-1}), E(b_j) = \beta_j, D(b_j) = c_{jj}\sigma^2$, where c_{jj} is a *j*th element along a diagonal line in matrix $(x^T x)^{-1}$.

Hence,

$$(b_j - \beta_j) / \sqrt{c_{jj}\sigma^2} N(0, 1)$$

Under the condition of Eq. (12), utilizing $t = \frac{b_i}{\sqrt{c_{jj}S_{res}/(n-m-1)}}$ to test β_j is zero or not. Namely, tested independent variable x_i is significant or not.

3.1.4 Test of regression equation

Suppose

 $H_0:\beta_1=\beta_2=\ldots=\beta_m=0$

Thus, the statistics F calculated according to regression and residual variance is compared and estimated after selecting confidence level and finding out the critical value of F in Table 5.

If the statistics $F > F_{\alpha}(m, n-m-1)$, the assumption prerequisite is not supported, and namely, the regression equation is significant in confidence level 100(1- α)%. Contrarily, the regression equation is significant.

3.2 Empirical model of three-component cutting forces

In this paper, the applicable conditions of the ORMEM and RSMQPEM are as follows:

- 1. Material of the workpiece is Cr12MoV.
- Material of the tool is the PCBN which had an approximate chemical composition of 50% CBN.
- 3. Geometrical parameters are in Table 2.

4. Input variables should be v=75-226 m/min, f=0.10-0.30 mm/rev, $a_p=0.10-0.30$ mm, $H=51-64\pm 1$ HRC, $r_{\varepsilon}=0.80-1.60$ mm, and $VB \le 0.15$ mm.

3.2.1 Empirical model of the main cutting force F_Z

The statistical software SPSS [25] was utilized to establish the regression model and calculate constants and regression coefficients of these models. After removing the variables with negligible regression coefficients, the ORMEM and RSMQPEM established of the main cutting force are described in Eqs. (13) and (14):

$$F_Z = 4.8067 v^{0.213} a_p^{0.346} f^{-0.074} H^{0.608} r_{\varepsilon}^{0.230}$$
(13)

$$F_{Z} = -62.17 + 0.674v + 59.999a_{p} + 923.956f + 0.221H + 41.934r_{\varepsilon} + 1.06va_{p} - 4.373vf + 0.028vr_{\varepsilon} -1724.17a_{p}f + 116.25a_{p}r_{\varepsilon} + 41.958fr_{\varepsilon} - 0.962Hr_{\varepsilon}$$
(14)

The significance of the regression model was tested using the ANOVA method. Table 5 shows the ANOVA for the ORMEM and RSMQPEM of the three-component cutting forces. It also presents the sum of squares (SS), degrees of freedom (DF), mean squares (MS), calculated value of F_{cal} , and the critical value of $F_{0.05}$, in addition to the correlation coefficient R^2 (called R-squared).

The regression model is evaluated by an F test. If the calculated value of F_{cal} is greater than the critical value of $F_{0.05}$ (95 % confidence level), the null hypothesis is rejected, which also implies that the model is significant. In addition, it indicates a good correlation between experimental and predicted values when R^2 comes closer to the value of unity.

It can be seen in Table 5 that the values of F_{cal} are more than $F_{0.05}$ for the two models, which implies that both of them are all significant. Moreover, the R^2 for the RSMQPEM is found to be greater than that for the ORMEM; namely, the

Cutting forces	ORM							RSM					
	Model	SS	DF	MS	F _{cal}	F _{0.05}	R^2	SS	DF	MS	F _{cal}	F _{0.05}	R^2
F_Z	Regression	1.150	5	0.230	23.960	2.68	0.851	13,975.711	12	1,164.643	51.859	2.53	0.978
	Residual	0.202	21	0.010				314.408	14	22.458			
	Total	1.352	26					14,290.118	26				
F_Y	Regression	3.044	5	0.609	33.702	2.68	0.889	32,400.482	12	2,700.040	23.417	2.53	0.953
	Residual	0.379	21	0.018				1,614.236	14	115.303			
	Total	3.423	26					34,014.718	26				
F_X	Regression	0.918	5	0.184	16.023	2.68	0.792	1,230.713	12	102.559	7.150	2.53	0.860
	Residual	0.241	21	0.011				200.813	14	14.344			
	Total	1.159	26					1,431.525	26				

 Table 5
 ANOVA for empirical model of three-component cutting forces

fitting degree for the RSMQPEM is greater than that for the ORMEM, which shows that the RSMQPEM can explain the variation to the extent of 0.978, while the ORMEM can explain the variation to the extent of only 0.851.

Now, ANOVA is utilized to determine and analyze the effect of parameters. According to the *t* distribution [21], $t_{0.01/2}$ is 2.831; $t_{0.05/2}$ is 2.080. The parameters are very significant if $|t| > t_{0.01/2}$. The parameters are significant if $t_{0.01/2} > |t| > t_{0.05/2}$. And, the parameters are not significant if $|t| < t_{0.05/2}$.

Table 6 ANOVA for the main and interaction effects of parameters on three-component cutting forces (F_Z , F_Y , and F_X)

Model	Factors	t	Test	Р	Significance
F_Z	V	4.355	$ t_v > t_{0.01/2}$	0.001	VS
	ap	0.818	$ t_{\rm ap} \le t_{0.05/2}$	0.427	NS
	f	7.938	$ t_{\rm f} > t_{0.01/2}$	0.000	VS
	Н	0.285	$ t_{\rm H} \le t_{0.05/2}$	0.780	NS
	r_{ε}	1.284	$ t_{r\varepsilon} \le t_{0.05/2}$	0.220	NS
	vap	2.222	$t_{0.01/2} > t_{vap} > t_{0.05/2}$	0.043	S
	vf	-9.408	$ t_{\rm vf} > t_{0.01/2}$	0.000	VS
	$V r_{\varepsilon}$	0.607	$ t_{\rm vr\epsilon} < t_{0.05/2}$	0.553	NS
	$a_{\rm p}f$	-6.817	$ t_{\rm H} > t_{0.01/2}$	0.000	VS
	$a_{\rm p}r_{\varepsilon}$	3.399	$ t_{\rm apr\varepsilon} > t_{0.01/2}$	0.004	VS
	fr_{ε}	1.227	$ t_{\rm fre} < t_{0.05/2}$	0.240	NS
	Hr_{ε}	-1.830	$ t_{\rm Hr\varepsilon} < t_{0.05/2}$	0.089	S
F_Y	v	1.189	$ t_{\rm v} < t_{0.05/2}$	0.254	NS
	$a_{\rm p}$	2.047	$ t_{\rm ap} < t_{0.05/2}$	0.060	NS
	f	4.348	$ t_{\rm f} > t_{0.01/2}$	0.001	VS
	Η	0.592	$ t_{\rm H} < t_{0.05/2}$	0.563	NS
	r _e	-0.520	$ t_{r\varepsilon} \le t_{0.05/2}$	0.611	NS
	vap	1.295	$ t_{\rm vap} \le t_{0.05/2}$	0.216	NS
	vf	-3.598	$ t_{\rm vf} > t_{0.01/2}$	0.003	VS
	νrε	1.108	$ t_{\rm vre} < t_{0.05/2}$	0.287	NS
	$a_{\rm p}f$	-5.269	$ t_{\rm H} > t_{0.01/2}$	0.000	VS
	$a_{\rm p}r_{\varepsilon}$	0.939	$ t_{apre} < t_{0.05/2}$	0.363	NS
	fr_{ε}	-0.947	$ t_{\rm fre} < t_{0.05/2}$	0.360	NS
	Hr_{ε}	0.643	$ t_{\rm Hr\varepsilon} < t_{0.05/2}$	0.531	NS
F_X	v	0.477	$ t_{\rm v} < t_{0.05/2}$	0.640	NS
	$a_{\rm p}$	1.957	$ t_{\rm ap} < t_{0.05/2}$	0.071	NS
	f	2.142	$t_{0.01/2} > t_{\rm f} > t_{0.05/2}$	0.050	S
	Н	0.106	$ t_{\rm H} < t_{0.05/2}$	0.917	NS
	r_{ε}	-0.649	$ t_{r\varepsilon} < t_{0.052}$	0.527	NS
	vap	0.105	$ t_{\rm vap} < t_{0.05/2}$	0.918	NS
	vf	-1.664	$ t_{\rm vf} < t_{0.05/2}$	0.118	NS
	vr _e	0.389	$ t_{\rm vre} < t_{0.05/2}$	0.703	NS
	$a_{\rm p}f$	-2.409	$t_{0.01/2} > t_{\rm H} > t_{0.05/2}$	0.030	S
	$a_{\rm p}r_{\varepsilon}$	0.097	$ t_{apr\varepsilon} < t_{0.05/2}$	0.924	NS
	frε	-1.162	$ t_{\rm fr\epsilon} < t_{0.05/2}$	0.265	NS
	Hr_{ε}	0.653	$ t_{\rm Hr\epsilon} < t_{0.05/2}$	0.525	NS





(a) Normal probability plot of residuals



Fig. 4 Residual plots for F_z . a Normal probability plot of residuals. b Scatter diagram

The ANOVA for the main and interaction effects of parameters on the three-component cutting forces is shown in Table 6. It is clear from the results in Table 6 that the cutting speed and feed are the two dominant factors determining the main cutting force; however, the effect of depth of cut, workpiece hardness, and nose radius on the main cutting force is less. Moreover, interaction effects of the cutting speed and depth of cut and the workpiece hardness and nose radius are significant followed by interaction effects of the cutting speed and feed,



Fig. 5 Comparison between the ORMEM and RSMQPEM in the predicted main cutting forces



(a) Normal probability plot of residuals





(a) Normal probability plot of residuals



(b) Scatter diagram

Fig. 6 Residual plots for $F_{\rm Y}$

depth of cut and feed, and depth of cut and radius, but the interaction effects of the cutting speed and nose radius and feed and nose radius are less.

The normal probability plot of residuals and scatter diagram for the main cutting force F_z in Fig. 4a, b can be utilized to further estimate the RSMQPEM. The predictions will be exact if the points are plotted on a straight line. Figure 4a



Fig. 7 Comparison between the ORMEM and RSMQPEM in the predicted radial cutting forces

Fig. 8 Residual plots of RSM for F_X . a Normal probability plot of residuals. b Scatter diagram

shows that the residuals lie reasonably close to a straight line, which implies that the distribution of the errors is normal. Figure 4b indicates that the points present a random state and are all located in $\pm 2\sigma$. All of them show that the RSMQPEM is very good and there is no reason to doubt its correctness.

In order to understand the capability of the two models, the 18 experimental results were conducted by randomly selecting



Fig. 9 Comparison between the ORMEM and RSMQPEM in the predicted feed cutting forces

Fig. 10 Response surface plot for F_Z . a f=0.10 mm/rev, $H=58\pm 1$, $r_{\varepsilon}=$ 0.8 mm. **b** $a_p=0.15$ mm, $r_{\varepsilon}=0.8$ mm, $H=58\pm1$. **c** $a_p=0.15$ mm, $r_{\varepsilon}=0.15$ mm, 0.8 mm, f=0.10 mm/rev. **d** $a_p=0.15$ mm, $H=58\pm1$, f=0.10 mm/rev

the input variables under the applicable conditions for the ORMEM and RSMQPEM. The absolute error can be determined with Eq. (15).

$$\Delta = \left| \frac{F_{expt} - F_{pred}}{F_{expt}} \right| * 100 \tag{15}$$

where

 Δ (%) is the absolute error,

 F_{exp} is the experimental value and

 F_{pred} is the simulated value

The experimental results indicate that the absolute error of the ORMEM is 11.12 %, while that of the RSMQPEM is only 4.45 %, as shown in Fig. 5. As can be seen from the figure, it is obvious that the predicted values of the RSMOPEM closely match with the variation of the main cutting forces.

3.2.2 Empirical model of the radial cutting force F_{Y}

Likewise, the established ORMEM and RSMOPEM of the radial cutting force are described in Eqs. (16) and (17):

$$F_Y = 0.0038067 v^{0.436} a_p^{0.366} f^{-0.157} H^{2.060} r_{\varepsilon}^{0.225}$$
(16)

$$F_{Y} = -149.85 + 0.417v + 340.337a_{p} + 1146.687f + 1.04H - 38.483r_{\varepsilon} + 1.4va_{p} - 3.79vf + 0.114vr_{\varepsilon} -3019.599a_{p}f + 72.792a_{p}r_{\varepsilon} - 73.375fr_{\varepsilon} + 0.765Hr_{\varepsilon}$$
(17)

The significance of the regression model $F_{\rm Y}$ was also tested using the ANOVA method. It is clear in Table 5 that the values of F_{cal} are higher than $F_{0.05}$ for two models, which implies that both of them are all significant. Moreover, as shown in Table 5, the R^2 for the RSMQPEM is higher than that for the ORMEM, which implies the RSMQPEM is more significant than the ORMEM. The ANOVA for the main and interaction effects of cutting parameters on the radial cutting force $F_{\rm V}$ is presented in Table 6. It is also clear in Table 6 that only the feed factor is very significant in determining the radial cutting force. Moreover, interaction effects of the cutting speed and feed and depth and cut and feed are very significant, but the interaction effects of the other parameters are less.

Likewise, in order to further estimate the RSMQPEM F_{y} , the normal probability plot of residuals and scatter diagram are also presented in Figs 6a, b, respectively. It is clear from the Fig. 6a, b that the residuals lie reasonably close to a straight line, and the points present a random state and are all located in $\pm 2\sigma$, which is similar to the model F_Z . All of them show that the statistical model is very good.



0.5 (d) $a_{\rm p} = 0.15 \text{ mm}, H = 58 \pm 1, f = 0.10 \text{ mm/r}$

50

Nose radius m(mm)

Cutting speed v(m/min)

Fig. 11 Response surface plot for $F_{\rm Y}$. **a** f=0.10 mm/rev, $H=58\pm1$, $r_{\varepsilon}=$ **b**.8 mm. **b** $a_{\rm p}=0.15$ mm, $r_{\varepsilon}=0.8$ mm, $H=58\pm1$. **c** $a_{\rm p}=0.15$ mm, $r_{\varepsilon}=$ 0.8 mm, f=0.10 mm/rev. **d** $a_{\rm p}=0.15$ mm, f=0.10 mm/rev, $H=58\pm1$

As can be seen from Fig. 7, it is evident that the predicted results of the RSMQPEM do show much more accuracy by comparing the model output with the directly measured radial cutting forces. The experimental results indicate that the absolute error of the ORMEM is 13.01 %, while that of the RSMQPEM is only 4.75 %.

3.2.3 Empirical model of the feed cutting force F_X

Likewise, the developed ORMEM and RSMQPEM of the feed cutting force are described in Eqs. (18) and (19):

$$F_X = 0.31474 v^{-0.019} a_p^{-0.262} f^{-0.143} H^{1.241} r_c^{-0.167}$$
(18)

$$F_X = 2.795 + 0.059v + 114.782a_p + 199.28f +0.066H - 16.926r_{\varepsilon} + 0.04va_p - 0.618vf +0.014vr_{\varepsilon} - 486.909a_pf + 2.646a_pr_{\varepsilon} -31.75fr_{\varepsilon} + 0.274Hr_{\varepsilon}$$
(19)

Table 5 shows that the calculated value of F_{cal} is larger than the critical value of $F_{0.05}$, and its R^2 is higher than the other. Thus, the reliability for the RSMQPEM is higher than that for the ORMEM. The ANOVA for the main and interaction effects of parameters on the feed cutting force is in Table 6. It can also be seen in Table 6 that only feed is the dominant factor determining the feed cutting force. Moreover, interaction effects of only the depth and cut and feed are significant.

The normal probability plot of residuals and scatter diagram for the RSMQPEM F_X are presented in Fig. 8a, b. It is obvious that distribution characteristics of the figure are similar to the model F_Z and F_Y ; hence, the RSMQPEM for F_Y is very good.

Figure 9 represents a comparison between the experimental results and predicted values of the ORMEM and RSMQPEM for the feed cutting forces. The experimental results show that the absolute error of the ORMEM is 8.15 %, while that of the RSMQPEM is only 4.49 %. From the analysis of this figure, it can be asserted that the developed RSMQPEM gives closer correlation with experimental results.

3.3 Influence of process parameters on three-component cutting forces

The influences of process parameters on three-component cutting forces were analyzed above by ANOVA. Now, the complex RSMQPEM is visualized via 3-day plots. In each plot, two parameters are varied, and the others are to remain constant. The effects of parameters on three-component



(d) $a_p = 0.15 \text{ mm}, f = 0.10 \text{ mm/r}, H = 58 \pm 1$

Fig. 12 Response surface plot for F_X . **a** f=0.10 mm/rev, $H=58\pm1$, $r_{\varepsilon}=$ **b**.8 mm. **b** $a_p=0.15$ mm, $r_{\varepsilon}=0.8$ mm, $H=58\pm1$. **c** $a_p=0.15$ mm, $r_{\varepsilon}=$ 0.8 mm, f=0.10 mm/rev. **d** $a_p=0.15$ mm, f=0.10 mm/rev, $H=58\pm1$

cutting forces are simulated by utilizing an available software package (Matlab).

3.3.1 Influence of process parameters on the main cutting force

Figure 10a–d presents interaction effects between the cutting speed and the other factors on the main cutting force in the case of FDHT of hardened steel tool (58 ± 1 HRC). It can be seen from the 3-day plots (Fig. 10a–d) that the main cutting force increases with the increase in the cutting speed, depth of cut, the workpiece hardness, and the nose radius, while it decreases with increments of the feed. It is also clear from this figure that the interaction effects between the cutting speed and feed on the main cutting force are the most significant.

3.3.2 Influence of process parameters on the radial cutting force

Figure 11a–d presents interaction effects between the cutting speed and the other factors on the radial cutting force. It can be seen from the 3-day plots that the radial cutting force sharply decreases and then gradually decreases with the increments of the feed, while as the cutting speed, depth of cut, workpiece hardness, and nose radius increase, the radial cutting force gradually increases. It is also clear from these figures that the interaction effects between the cutting speed and the feed on the main cutting force are the most significant.

3.3.3 Influence of process parameters on the feed cutting force

Figure 12a–d shows that the influence of the feed, depth of cut, workpiece hardness, and nose radius on the feed cutting force is very significant except that of the cutting speed. It can be observed from the 3-day plots that the feed cutting force slowly increases with the increase in the cutting speed, while it nonlinearly gradually increases and then sharply increases with the increments of depth of cut. However, it gradually decreases after a sudden decline with increments of the feed and nose radius, while it linearly increases with the rise of the workpiece hardness.

3.4 Optimization of cutting parameters

Optimization of cutting parameters is of great significance in not only increasing the machining efficiency but also improving the surface qualities and tool life. In this paper, an effort has been made to optimize the cutting parameters attaining the



(d) $a_{\rm p}=0.15$ mm, f=0.10 mm/r, $H=58\pm1$

 Table 7 Optimized cutting parameters and the corresponding results of the three-component cutting forces

	v (m/min)	$a_{\rm p}$ (mm)	f(mm/rev)	$H(\mathrm{HRC})$	$r_{\varepsilon} (\mathrm{mm})$	n (N)
$F_{\rm Z}$	75	0.10	0.10	51	0.8	64.6
	75	0.10	0.10	55	0.8	62.4
	75	0.10	0.10	60	0.8	59.7
	75	0.10	0.10	64	1.6	56.1
$F_{\rm Y}$	75	0.10	0.10	51	0.8	42.2
	75	0.10	0.10	55	0.8	48.8
	75	0.10	0.10	60	0.8	57.1
	75	0.10	0.10	64	0.8	63.7
$F_{\rm X}$	75	0.10	0.10	51	0.8	25.1
	75	0.10	0.10	55	0.8	27.1
	75	0.10	0.10	60	1.6	29.6
	75	0.10	0.10	64	1.6	31.7

lowest cutting forces. Here, the Eqs. (11), (17) and (19) are utilized to optimize the cutting parameters within the ranges given in this paper by an available software package (Matlab). The final results are summarized in Table 7.

4 Conclusions

In this paper, experimentations utilizing hardened tool steel AISI D2 at different hardness levels were conducted by using a tool insert PCBN. The influence of five factors (cutting speed, feed, depth of cut, workpiece hardness, and nose radius) on the three-component cutting forces in a FDHT process has been comprehensively analyzed by using ANOVA. And, the ORMEM and RSMQPEM for the three-component cutting forces by employing the five factors were developed by utilizing the ORM and RSM. The conclusions can be drawn from the above analysis:

- 1. ANOVA tests for two empirical models of threecomponent cutting forces show that the RSMQPEM is more significant than the ORMEM.
- 2. The experimental results show that the predicted values of RSMQPEM for three-component cutting forces are much more close to the experimental values than those of ORMEM. Therefore, it can be concluded that the developed RSMQPEM is more credible compared to the ORMEM in the considered parameter ranges. This model would be helpful in selecting the tool geometry and cutting conditions in FDHT of hardened tool steel AISI D2.
- ANOVA tests for the main and interaction effects of parameters on three-component cutting forces show that the cutting speed and feed are of great influence on the main cutting force. And, all of the second interaction

effects are very significant for the main cutting force except interactions of feed and nose radius and cutting speed and nose radius.

- 4. Among the main effect factors considered, the feed has more influence on the radial cutting force than the other factors. In addition, interaction effects of the cutting speed and feed and depth of cut and feed are very significant, but the interaction effects of the other parameters are less.
- 5. The influence of only feed is significant for the feed cutting force among five main effect factors. Moreover, the interaction effect of only the depth and cut and feed is significant.

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