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ORIGINAL ARTICLE

Production and maintenance planning for a failure-prone deteriorating manufacturing system: a hierarchical control approach

A. F. Kouedeu · J.-P. Kenné · P. Dejax · V. Songmene · V. Polotski

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Abstract The work presented in this paper examines the joint analysis of the optimal production and maintenance planning policies for a manufacturing system subject to random failures and repairs. When a machine fails, an imperfect corrective maintenance is undertaken. The objective of this study is to minimize a discounted overall cost consisting of preventive and corrective maintenance costs, inventory holding cost, and backlog cost. A two-level hierarchical decision-making approach is proposed, based on the determination of the mean time to failure (first level) and the statement of a joint optimization of production, preventive, and corrective maintenance policies (second level). Hence, the production, preventive, and corrective maintenance rates are determined in the second level, given the failure rates obtained from the first level. In the proposed model, the machine's failure rate depends on the number of imperfect repairs, and as a result, the control policies of the considered planning problems therefore depend on the number of failures. The structure of the optimal control policies and the usefulness of the proposed approach are illustrated through a numerical example and a sensitivity analysis.

Keywords Imperfect repairs · Flexible manufacturing systems · Production rate · Maintenance policies · Numerical methods

1 Introduction

The quality of a manufacturing system's design and the maintenance actions undertaken during its operation (production activities) are crucial factors determining its reliability. This paper models and illustrates the control problem of a stochastic manufacturing system. The stochastic nature of the system is due to the fact that the machine is subject to random breakdowns and repairs. The machine produces one part type; when one of the machine's components fails, an imperfect corrective maintenance action is undertaken. Here, the machine dynamics is assumed to be described by a finite-state semi-Markov chain. The decision variables are the production rate, the preventive maintenance rate, and the corrective maintenance rate, which influence the system's availability and the stock level. Many authors have contributed to the production planning and maintenance policies of manufacturing systems without considering the failure rates, depending on the number of imperfect repairs, and the simultaneous control of production, preventive, and corrective maintenance rates in the same model.

Based on the work of Rishel [1] on production planning for a system affected by jump disturbances, Boukas and Haurie in [2] combined production and preventive maintenance planning in cases where the machine's failure probability increases with its age, using the hedging point policy concept introduced by Kimemia and Gershwin [3]. For more details on this concept, we refer the reader to the age-dependent hedging point concept presented by Boukas [4] and Gharbi and Kenne [5]. Boukas and Haurie [2] determined production rate and maintenance rules which minimize the total expected cost of a two-machine system over infinite horizon. However, with the numerical scheme adopted in their work, it remains computationally difficult to realize optimal control of a large-scale manufacturing system. To cope with this difficulty, Kenne and Boukas in [6] formulated a hierarchical control problem based on production and preventive maintenance planning in manufacturing systems and obtained a limiting problem that was numerically more tractable. Gharbi and Kenne in [7] extended this approach to cover a large case of nonidentical

A. F. Kouedeu · J.-P. Kenné (✉) · P. Dejax · V. Songmene · V. Polotski
University of Quebec, ETS, Montreal, Quebec, Canada
e-mail: jean-pierre.kenne@etsmtl.ca

machine manufacturing systems. Zied et al. [8] investigated the result of a general class of stochastic production planning and maintenance scheduling problems via optimal procedure. The objective was to satisfy economically a random demand under some constraints like random failure rate and a subcontracting constraint. The manufacturing system considered was prone to random failures. Minimal repairs were adapted at every failure. So as to reduce the failure frequency, preventive maintenance actions were programmed according to the production rate. For more recent reviews of the production and maintenance planning, see Budai et al. in [9].

Many systems deteriorate with age, and are subject to stochastic random failures. This degradation may result in higher operating costs and less competitive products, thus making maintenance action highly essential (Yan et al. [10]). Conventional maintenance policies assume that the system is restored after repair or preventive maintenance activities, making it as good as new (see Boukas and Haurie [2], Kenne and Boukas [6] and Kenne et al. [11]) or as bad as an old machine (Nakagawa and Kowada [12]). The main limitation of these models is that they take into account only extreme maintenance actions (perfect or minimal), and do not consider the real efficiency of repairs, which can significantly improve the state of the system without returning it to an *as good as new* condition. Such a repair is called an *imperfect repair*. The level of such repair is known as the *intensity of repair*, as in Dehayem et al. [13]. The repair intensity could reflect the impact of the k repairs and be a function of the kth repair, as described in Love et al. [14], or be stochastic, as in Kijima [15]. This would depend on the quality of intervention performed and the skill level of the maintenance team as well as the number and nature of the components repaired (see Shin in [16]). However, little has been done in terms of developing a model taking into account the case where this factor is stochastic (Mohafid and Castanier in [17]). The repair intensity used to model the effectiveness of maintenance action undertaken is assumed to be known and constant in deterministic cases.

In Kijima [15], the author proposed that upon a failure, the repair undertaken could serve to reset the age of the machine only as far back as its age at the start of the last failure, called the virtual age. In the literature, this repair model is called Kijima's Type I imperfect repair model, and it has largely been used in cumulative damage models. The virtual age is equal to or less than the real age, as in [13]. Dehayem et al. in [13] extended Kijima's Type I imperfect repair model; they determined the production rate and the repair/replacement policy that minimizes the total expected cost when the system deteriorates with age, and is subject to damage failures. Jiwen and Lifeng in [18] modeled and analyzed various maintenance policies by incorporating the economic effects of maintenance actions, product deviation-related quality loss, and tool obsolescence cost. They provided a comparative analysis of various maintenance policies using the long-term average cost criterion and employed a quadratic loss function to characterize the cost resulting from the deviation of part dimension from its target value.

The main contribution of this work consists in its joint analysis of the optimal production and maintenance (preventive and corrective) planning problems for a manufacturing system under uncertainties and imperfect repairs, when the failure rate increases with the number of imperfect repairs. Following a preventive maintenance activity, the machine is as good as new. The proposed hierarchical approach involves developing a model in which, at the first level, the parameters of the stochastic machine failure process are derived for each number of imperfect repairs; at the second level, the optimal production, preventive, and corrective maintenance policies are determined for a system that deteriorates with the number of failures. The production and maintenance rates are obtained for the system by minimizing inventory, backlog, preventive, and corrective maintenance costs over an infinite planning horizon. The formulation, the approaches, and the numerical procedures used in this paper could possibly be applied to production planning in many industries, where resources can be subject to random failures and their production rates can also be controlled. The phenomenon has been experienced in machinery and mechanical assemblies, including at automobile, aircraft engine and machine tools, and paper manufacturing plants. Yin et al. in [19] obtained the optimal production policies of the paper manufacturing machine, for different machine capacity and demand processes.

The rest of this paper is organized as follows: Notations and assumptions are presented in Section 2. Section 3 presents the model of the problem under consideration. The optimality conditions described by the Hamilton-Jacobi-Bellman (HJB) equations and the numerical approach to solve the HJB equations obtained are presented in Section 4. In Section 5, a numerical example and results are presented; sensitivity analyses are presented to illustrate the usefulness of the proposed approach in Section 6, which also presents and discusses some extensions. Finally, the paper is concluded in Section 7.

2 Notations and assumptions

This section presents the notations and assumptions used throughout this article.

2.1 Notations

The model under consideration is based on the following notations:

k Number of imperfect repairs
$u(\cdot)$ Production rate (products/time unit)

$u_{\max}$	Maximal production rate (products/time unit)
x^+	Inventory (products)
x^-	Backlog (missing products)
d	Demand rate (products/time unit)
c^+	Inventory cost ($/product/time unit)
c^-	Backlog cost ($/missing product/time unit)
c_r	Corrective maintenance cost ($)
c_m	Preventive maintenance cost ($)
$\omega_r^{\min}$	Minimal corrective maintenance rate
$\omega_r^{\max}$	Maximal corrective maintenance rate
$\omega_m^{\min}$	Minimal preventive maintenance rate
$\omega_m^{\max}$	Maximal preventive maintenance rate
$\lambda_{\alpha\beta}$	Transition rate from state α to β
Q	Transition rate matrix
π	Vector of limiting probabilities
$g(\cdot)$	Instantaneous cost function
$J(\cdot)$	Total cost ($/time unit)
$\nu(\cdot)$	Value function
ρ	Discount rate

2.2 Assumptions

The following assumptions are made in this paper:

1. The failure rate increases with the number of imperfect repairs of the machine.
2. The lifetime of the machine decreases after each breakdown.
3. Corrective maintenance activities are imperfect.
4. Preventive maintenance activities are perfect.
5. Corrective and preventive maintenance activities are controlled (minimal and maximal rates).

Assumptions 1, 2, 3, and 5 are the major motivations of our approach. Other works often consider that the failure rate is constant and the corrective maintenance activities restore the machine as good as new state.

6. The customer demand is known and subject to a constant rate over time.

This assumption is common to deterministic demand models.

7. The maximal production rate of the machine is known.

This assumption is common in production planning.

8. The backlog cost depends on the shortage quantity and time (average value ($/product/unit of time)).
9. The holding cost depends on the mean inventory level (average value ($/product/unit of time)).

Assumptions 8 and 9 are common in inventory models.

3 Problem statement

The manufacturing system considered consists of a single machine which produces one part type. This machine is subject to random breakdowns and repairs. Its mode can be classified as operational, denoted by 1, under repair, denoted by 2, and under preventive maintenance, denoted by 3. Let $\xi(t)$ denote the mode of the machine with value in $B=\{1,2,3\}$. The dynamics of the machine is described by a continuous time semi-Markov process, with a transition rate from state α to state β denoted by $\lambda_{\alpha\beta}$ with $\alpha,\beta \in B$. The transition diagram, describing the dynamics of the machine considered is presented in Fig. 1.

The hierarchical approach proposed in this paper consists of two levels:

- Level 1: Determination of the mean time to failure. Therefore, the value of λ_{12}.
- Level 2: Joint determination of optimal production, preventive, and corrective maintenance, given the failure rate obtained at the first level.

At level 1, the failure rate at time t, over a finite planning horizon, is described by a Weibull distribution with two parameters μ and η. The Weibull law is often used in maintenance due to its flexibility to model survival times of systems and its ability to characterize their wear level through its shape parameter μ. The failure rate is given as follows:

$$\lambda_0(t) = \frac{\mu}{\eta}\left(\frac{t}{\eta}\right)^{\mu-1} \tag{1}$$

The failure rate between the kth and $(k+1)^{\text{th}}$ repair is given by the following:

$$\lambda_k(t) = \lambda_{k-1}.(t-\theta.(t_k-t_{k-1}))$$

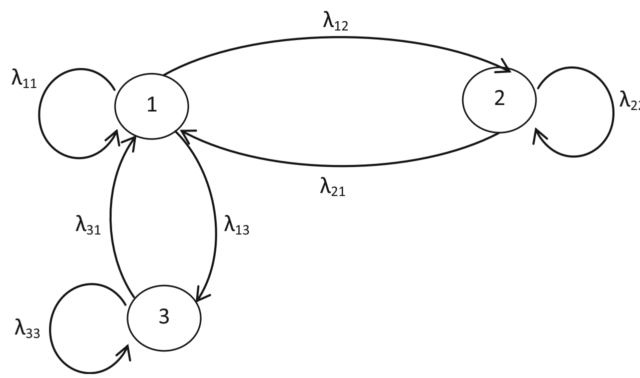


Fig. 1 Modes transition diagram of the considered system

When all the t_k times are known and considering the conditional distributions of successive interfailure times, this failure rate becomes

$$\lambda_k(t) = \lambda_0.(t-\theta.t_k) \tag{2}$$

where θ is the impact of the repair or the repair intensity and λ_0 is the initial intensity. The three values of θ are the following:

- $\theta=0$ for a minimal repair, which does not modify the state of the system (as bad as old); $\lambda_k(t)=\lambda_0.t$
- $\theta=1$ for a perfect repair, where the system is as good as new following a repair; $\lambda_k(t)=\lambda_0.(t-t_k)$
- $0<\theta<1$ for an imperfect repair.

In this paper, after system failures, imperfect repairs are performed to repair or replace the faulty component. Thus $0<\theta<1$.

The behavior of the failure rate at time t according to the random failures, over a finite planning horizon, is presented in Fig. 2 for $\mu=3$, $\eta=500$ and three values of θ (i.e., 0, 0.4, 0.7). The values of μ, η and θ can be estimated by using operating data and the maximum likelihood method, such as in Shin [16].

The reliability of the system before the first failure is $R_0(t) = e^{-\left(\frac{t}{\eta}\right)\mu}$, and after k imperfect repairs, it becomes $R_k(t) = e^{\left(-\left(\frac{t-\theta.t_k}{\eta}\right)^{\mu}\right)}$.

Let T, a positive random variable, which is interpreted as the operation times of repairable system (machine) with instantaneous repair upon failure. Following a repair in the date t_k, the system lifetime at time t is a random variable given by

$$R\left(t\middle|t_k\right) = P\left(T > (t+t_k)\middle|T > t_k\right) = \frac{R_k(t+t_k)}{R_k(t_k)} \tag{3}$$

The expected remaining lifetime (mean time to failure: MTTF=E_k) after time t_k, given that the system has survived after t_k is

$$E_k(T) = E\left((t-t_k)\middle|t > t_k\right) = \int_0^{\infty} P\left(T > (t+t_k)\middle|T > t_k\right)dt \tag{4}$$

Replacing $P(T>(t+t_k)|T>t_k)$ by $\frac{R_k(t+t_k)}{R_k(t_k)}$, $E_k(T)$ becomes

$$E_k(T) = \int_0^{\infty} \frac{R_k(t+t_k)}{R_k(t_k)}dt = \frac{1}{R_k(t_k)}\int_0^{\infty} R_k(t+t_k)dt$$
$$= \frac{1}{R_k(t_k)}\int_{t_k}^{\infty} R_k(t)dt$$

When lifetimes are distributed according to Weibull model $E_k(T) = e^{(\psi_k)\mu}\left(\eta\int_0^{\infty} e^{-t}t^{\frac{1}{\mu}}dt-\psi_k\right)$ where $\psi_k = \frac{(1-\theta)t_k}{\eta}$

Let $\Gamma(y) = \int_0^{\infty} e^{-t}t^{y-1}dt$, $E_k(T)$ becomes:

$$E_k(T) = e^{(\psi_k)\mu}\left(\eta\Gamma\left(1+\frac{1}{\mu}\right)-\psi_k\right) \tag{5}$$

For $\mu=3$, $\eta=500$, and $\theta=0.4$, the values of $E_k(T)$ are given in Table 1.

The system capacity is increased by controlling the transition rate from node 1 to 3 (preventive maintenance) and from node 2 to 1 (corrective maintenance). Hence, the transition rates matrix Q depends on ω_m and ω_r, defined as preventive and corrective maintenance rates, respectively. For the considered system, the mode of the machine $\xi(t)$ is described by the corresponding

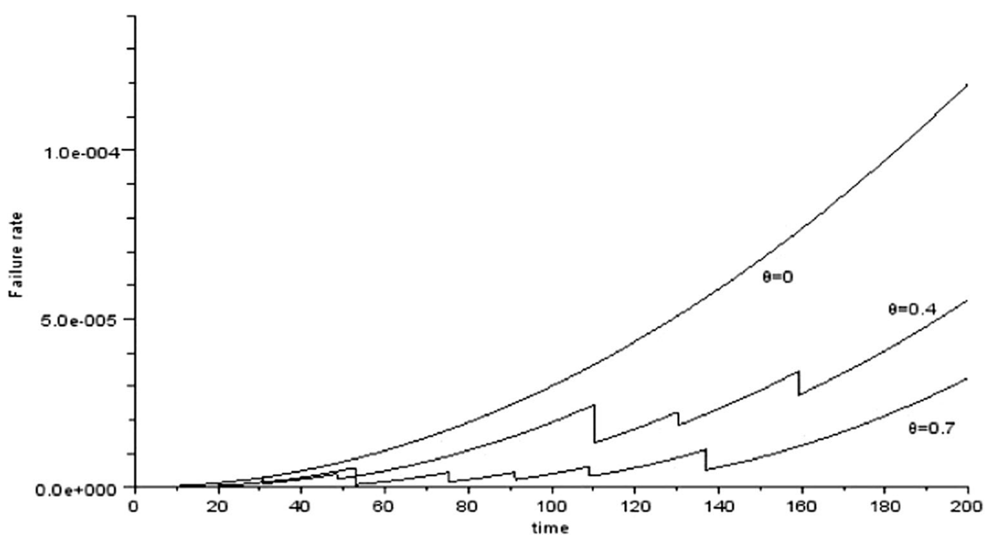


Fig. 2 Behavior of the failure rate for imperfect repairs

Table 1 Values of $E_k(T)$

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
t_k	0	642	689	874	957	1,020	1,256	1,270	1,307	1,385	1,388	1,394	1,454	1,559	1,562	1,577	1,582	1,623	1,648	1,662
$E_k(T)$	622	158	146	109	96.1	87.6	63.2	62.1	59.6	53.6	53.4	53.1	49.3	43.7	43.5	42.8	42.5	40.6	39.5	38.9

3×3 transition matrix $Q=[\lambda_{\alpha\beta}]$, where $\lambda_{\alpha\beta}$ verifies the following conditions:

$$\lambda_{\alpha\beta}(k,\omega_m,\omega_r)\geq 0 \quad (\alpha\neq\beta) \tag{6}$$

$$\lambda_{\alpha\alpha}(k,\omega_m,\omega_r) = -\sum_{\beta\neq\alpha}\lambda_{\alpha\beta} \tag{7}$$

A hybrid state comprising both a discrete and a continuous component, describes the system behavior. The discrete component consists of the discrete stochastic process $\xi(t)$ and the continuous component is the stock level defined later in this section. Let $u(x,k,\alpha,t)$ denote the production rate of the machine in mode α and at time t for a given stock level x and a given number of imperfect repairs k. The set of the feasible control policies $A(\alpha)$, including $u(\cdot)$, $\omega_m(\cdot)$ and $\omega_r(\cdot)$ is given by:

$$A(\alpha) = \left\{ \begin{array}{l} (u(\cdot),\ \omega_m(\cdot),\omega_r(\cdot)) \in \Re^3, 0\leq u(\cdot)\leq u_{\max}, \\ \omega_r^{\min} \leq\omega_r(\cdot)\leq\omega_r^{\max},\ \omega_m^{\min}\leq\omega_m(\cdot)\leq\omega_m^{\max} \end{array} \right\} \tag{8}$$

where $u(\cdot)$, $\omega_m(\cdot)$ and $\omega_r(\cdot)$ are known as control variables, and constitute the control policies of the problem under study. $u_{\max}$ is the maximal production rate, $\omega_m^{\min}$ and $\omega_m^{\max}$ are the minimal and maximal preventive maintenance rates, and $\omega_r^{\min}$ and $\omega_r^{\max}$ are the minimal and maximal corrective maintenance rates, respectively ($\xi(t)=\alpha$ in Eq. (8)).

The transition rates $\lambda_{\alpha\beta}(k,\omega_m,\omega_r)$ of the machine after the kth repair from mode $\xi(t)=\alpha$; $\alpha\in B$ to mode $\xi(t)=\beta$; $\beta\in B$ at instant t are defined by

$$\lambda_{12}(k,\omega_m,\omega_r) = \frac{1}{E_k(T)} \tag{9}$$

$$\lambda_{13}(k,\omega_m,\omega_r) = \omega_m(\cdot) \tag{10}$$

$$\lambda_{21}(k,\omega_m,\omega_r) = \omega_r(\cdot) \tag{11}$$

$$\lambda_{31}(k,\omega_m,\omega_r) \text{ is constant and} \neq 0 \tag{12}$$

The behavior of the failure rate $\lambda_{12}(k,\cdot)$ is shown in Fig. 3.

While the machine is submitted to preventive or corrective maintenance, the production has to be stopped. Then, the surplus (stock level) could be positive (inventory), or negative (backlog).

The stock level is given by the state equation:

$$\frac{dx(t)}{dt} = u(t)-d, \qquad x(0) = x_0 \tag{13}$$

where x_0 and d are given initial surplus and demand rate, respectively.

Let $g(\cdot)$ be the cost rate defined as follows:

$$g(\alpha,x,\cdot) = c^+x^+ + c^-x^- + c_m\omega_m\text{Ind}\{\alpha=1\} + c_r\omega_r\text{Ind}\{\alpha=2\} \tag{14}$$

with $\text{Ind}\{\Theta(\cdot)\} = \begin{cases} 1 & \text{if } \Theta(\cdot) \text{ is true} \\ 0 & \text{otherwise} \end{cases}$ for a given proposition $\Theta(\cdot)$. The constants c^+, c^-, c_m, and c_r are used to penalize inventory, backlog, preventive, and corrective maintenance, respectively, $x^+=\max(0,\ x)$

$x^- = \max(-x,\ 0)$.

The objective here is to control the production rate $u(\cdot)$, the preventive and the corrective maintenance rates $\omega_m(\cdot)$, and $\omega_r(\cdot)$, respectively, in order to minimize the expected discounted cost $J(\cdot)$ given by:

$$J(\alpha,x,k,u,\omega_m,\omega_r) = E\left\{ \int_0^\infty e^{-\rho t} g(\alpha,x,\cdot)dt | x(0) = x,\ \ \xi(0) = \alpha, k(t) = k \right\} \tag{15}$$

where ρ is the discounted rate. The value function of the problem described in this paper is defined as follows:

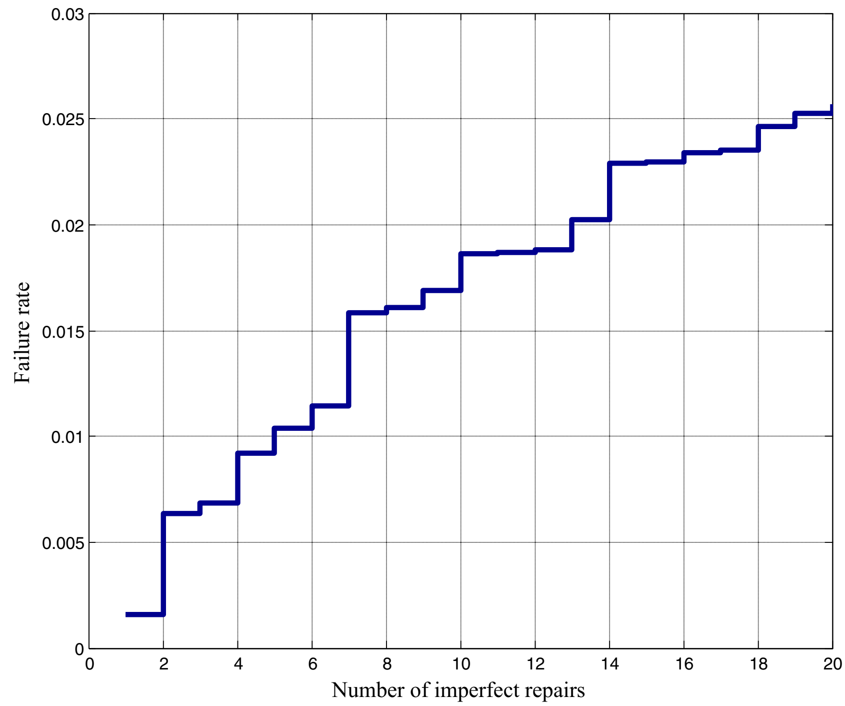
$$v(\alpha,\ x,k,\cdot) = \inf_{\left(u(\cdot),\omega_m(\cdot),\omega_r(\cdot)\right)\in A\left(\alpha\right)} J(\alpha,x,k,u,\omega_m,\omega_r) \quad \forall\alpha\in B \tag{16}$$

Section 4 presents the properties of the value function $v(\cdot)$ given by Eq. (16) and the numerical methods used to solve the proposed optimality conditions.

4 Optimality conditions and numerical methods

This section presents the optimality conditions satisfied by the value function presented in Eq. (16). The properties of the

Fig. 3 Failure rate of the machine



value function and the manner in which such equations are obtained can be found in Kenne and Nkeungoue [20]. They describe the optimal control policies (optimality conditions) for production, preventive, and corrective maintenance planning problems. Regarding the optimality principle, we can write the optimality conditions, given by Hamilton-Jacobi-Bellman (HJB) equations as follows:

$$\rho v(\alpha,k,x,\cdot)=\min_{(u,\omega_m,\omega_r)\in A(\alpha)}\left\{\begin{array}{l} g(\alpha,x,u,\omega_m,\omega_r)+\sum_{\beta\in B}\lambda_{\alpha\beta}v(\beta,k,x,\cdot)\\ +(u-d)\dfrac{\partial v(\alpha,k,x,\cdot)}{\partial x}\end{array}\right. \tag{17}$$

The optimal control policies over $A(\alpha)$ of the right hand side of Eq. (17) are $(u^*(\cdot),\omega_m^*(\cdot),\omega_r^*(\cdot))$. When the value function described by Eq. (16) is available, optimal control policies can be obtained as in Eq. (17). However, obtaining an analytical solution for Eq. (17) is almost impossible. The numerical solution of the HJB Eq. (17) used to be considered an insurmountable challenge, but Boukas and Haurie in [2] showed that implementing Kushner's method can solve such a problem in the context of production planning.

The numerical methods for solving the optimality conditions given by Eq. (17) are based on the Kushner approach, as in Kenne et al. [21], Hajji et al. [22], and references therein. We should recall that the primary premise of this approach consists in using an approximation scheme for the gradient of the value function $v(\alpha,k,x)$. Let h denote the length of the finite difference interval of the variable x. The value function $v(\alpha,k,x)$ is approximated by $v^h(\alpha,k,x)$, and $\frac{\partial v(\alpha,k,x)}{\partial x}$ is approximated using the following equation:

$$\frac{\partial v(x,\alpha)}{\partial x}=\begin{cases}\dfrac{1}{h}\left(v^h(\alpha,x+h,k)-v^h(\alpha,x,k)\right) & \text{if } (u-d)>0\\ \dfrac{1}{h}\left(v^h(\alpha,x,k)-v^h(\alpha,x-h,k)\right) & \text{otherwise.}\end{cases} \tag{18}$$

With approximations given by Eq. (18), and after a couple of straightforward manipulations, the HJB equations can be rewritten as follows:

$$v^h(\alpha,x,k)$$
$$=\min_{(u,\omega_m,\omega_r)\in A^h(\alpha)}\left\{\frac{g(\alpha,x,u,\omega_m,\omega_r)}{\Omega_h^\alpha(1+\rho/\Omega_h^\alpha)}+\frac{1}{(1+\rho/\Omega_h^\alpha)}\left(\begin{array}{l}p_x^\pm(\alpha)v^h(\alpha,x\pm h,k)\\ +\sum_{\beta\neq\alpha}p^\beta(\alpha)v^h(\alpha,x,k)\end{array}\right)\right\} \tag{19}$$

where $A^h(\alpha)$ is the numerical control grid. The other terms used in Eq. (19) are given as follows:

$$\Omega_h^\alpha=\left|\lambda_{\alpha\alpha}\right|+\frac{|u-d|}{h},\quad p_x^+(\alpha)=\begin{cases}\dfrac{u-d}{h\Omega_h^\alpha} & \text{if } (u-d)>0\\ 0 & \text{otherwise,}\end{cases}$$

$$p^\beta(\alpha)=\frac{\lambda_{\alpha\beta}}{\Omega_h^\alpha},\quad p_x^-(\alpha)=\begin{cases}\dfrac{d-u}{h\Omega_h^\alpha} & \text{if } (u-d)\leq 0\\ 0 & \text{otherwise.}\end{cases}$$

The system of Eq. (19) can be interpreted as the infinite horizon dynamic programming equation of a discrete-time, discrete-state decision process, as in Kenne and Nkeungoue [20]. The discrete event dynamic programming equations obtained can be solved using either policy improvement or successive approximation methods. In this paper, we use the value iteration procedure to approximate the value function given by Eq. (19). Kenne et al. in [21] and references therein provide details on such methods.

5 Numerical example and results

This section presents a numerical example for the manufacturing system presented in Section 3. A three-state semi-Markov process with the modes in $B=\{1,2,3\}$ describes the system capacity. The instantaneous cost is described by Eq. (14). The values of c^+, c^-, c_m, and c_r will be given later in this section. The transition rate matrix $Q(\cdot)$ is explicitly defined as follows:

$$Q(k,\omega_m,\omega_r) = \begin{pmatrix} -(\lambda_{12}(k)+\omega_m) & \lambda_{12}(k) & \omega_m \\ \omega_r & -\omega_r & 0 \\ \lambda_{31} & 0 & -\lambda_{31} \end{pmatrix}$$

where $\lambda_{12}(k)=1/E_k(T)$, with $E_k(T)$ defined in Eq. (5).

The following three equations are the discrete dynamic programming equations obtained from Eq. (19) for $\alpha=1,2,3$:

– Mode 1

$$\nu^h(1,x,k) = \min_{(u,\omega_m,\omega_r)\in A^h(1)} \left\{ \begin{array}{l} \dfrac{c^+x^+ + c^-x^- + c_m\omega_m}{\Omega_h^1(1+\rho/\Omega_h^1)} + \dfrac{1}{(1+\rho/\Omega_h^1)}\left(p_x^{\pm}(1,x\pm h,k)\right) \\ + \dfrac{1}{(1+\rho/\Omega_h^{\alpha})}\begin{pmatrix} p^2(1)\nu^h(2,x,k)+ \\ p^3(1)\nu^h(3,x,k) \end{pmatrix} \end{array} \right\} \tag{20}$$

– Mode 2

$$\nu^h(2,x,k) = \min_{\omega_r\in A^h(2)} \left\{ \begin{array}{l} \dfrac{c^+x^+ + c^-x^- + c_r\omega_r}{\Omega_h^2(1+\rho/\Omega_h^2)} + \\ \dfrac{1}{(1+\rho/\Omega_h^2)}\left(p_x^-(2)\nu^h(2,x-h,k) + p^1(2)\nu^h(1,x,k)\right) \end{array} \right\} \tag{21}$$

– Mode 3

$$\nu^h(3,x,k) = \min \left\{ \begin{array}{l} \dfrac{c^+x^+ + c^-x^-}{\Omega_h^3(1+\rho/\Omega_h^3)} + \\ \dfrac{1}{(1+\rho/\Omega_h^3)}\left(p_x^-(3)\nu^h(3,x-h,k) + p^1(3)\nu^h(1,x,k)\right) \end{array} \right\} \tag{22}$$

We use the computational domain D given by

$$D = \{(x,k): \quad -10\le x\le 50; \quad 0\le k\le 19\} \tag{23}$$

The condition to meet the customer demands, over an infinite horizon and reach a steady state is given by

$$\pi_1 * u_{\max} > d \tag{24}$$

where π_1 is the limiting probability at the operational mode of the machine. Note that the limiting probabilities of modes 1, 2, and 3 (i.e., π_1, π_2, and π_3), are computed as follows:

$$\pi\cdot Q(\cdot) = 0 \quad \text{and} \quad \sum_{i=1}^{3}\pi_i = 1 \tag{25}$$

where $\pi=(\pi_1, \pi_2, \pi_3)$ and $Q(\cdot)$ is the corresponding 3×3 transition rate matrix. Table 2 summarizes the parameters of the numerical example for which the feasibility condition given by Eq. (24) is satisfied. The policy improvement technique is used to solve the system of Eqs. (20)–(22). The results

Table 2 Parameters of numerical example

c_1^+	c_1^-	c_r	c_m	d	λ_{31}	$u_{\max}$	$\omega_m^{\min}$	$\omega_m^{\max}$	$\omega_r^{\min}$	$\omega_r^{\max}$	ρ
1	100	5,000	10	0.25	0.5	0.27	10^{-4}	0.5	0.02	0.1	0.01

obtained for the values in Table 2 are presented in Figs. 4, 5, and 6.

For illustrative purposes, the production rate for five failures of the machine, in its operational mode (i.e., mode 1), is presented in Fig. 4. This figure shows that the production rate is set to zero for comfortable stock levels. Then, there is no need to produce parts for comfortable stock levels.

The production rate is thus set to zero when there are more than 28 products in inventory. From the results obtained, the computational domain is divided into three regions where the optimal production control policy consists of one of the following rules:

1. Set the production rate of the machine to its maximal value when the current stock level is under the threshold value;
2. Set the production rate of the machine to the demand rate when the current stock level is equal to the threshold value;
3. Set the production rate of the machine to zero when the current stock level is larger than the threshold value.

The control policy obtained is an extension of the hedging point policy, given that the previous three rules respect the structure presented in reference [23] for production planning without the control of preventive and corrective maintenance activities. As shown within the numerical results and in Fig. 4, the optimal production rate can be expressed as follows:

$$u(x,k,1) = \begin{cases} u_{\max} & \text{if } x(\cdot) < X*(k) \\ d & \text{if } x(\cdot) = X*(k) \\ 0 & \text{otherwise,} \end{cases} \tag{26}$$

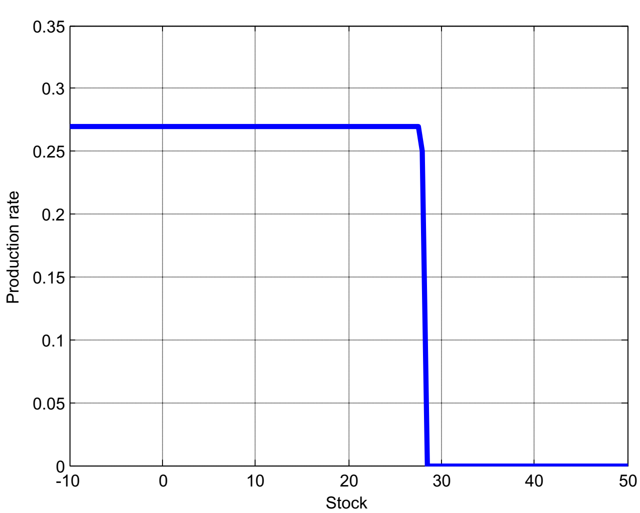


Fig. 4 Production rate of the machine at mode 1

where $X*(k)$ is the optimal threshold value for each value of the k number of imperfect repairs.

The preventive maintenance policy, plotted in Fig. 5, divides the computational domain into two regions where the preventive maintenance rate is set to its minimal and maximal values for uncomfortable stock levels (or for backlog situations) and for large stock levels, respectively. The optimal preventive maintenance policy, like the production policy, has a bang-bang structure, and is described as follows:

$$\omega_m(x,k,1) = \begin{cases} \omega_m^{\min} & \text{if } x(\cdot) < Y*(k) \\ \omega_m^{\max} & \text{otherwise,} \end{cases} \tag{27}$$

where $Y*(k)$ is the optimal stock level at which the preventive maintenance rate must be switched from $\omega_m^{\min}$ to $\omega_m^{\max}$.

Figure 6 presents the corrective maintenance policy. The computational domain is divided into two regions where the corrective maintenance rate is set to its maximal and minimal values for backlog situations and for large stock levels, respectively. The optimal corrective maintenance policy, like the production and the preventive maintenance policies, has a bang-bang structure, and is defined as follows:

$$\omega_r(x,k,2) = \begin{cases} \omega_r^{\max} & \text{if } x(\cdot) < Z*(k) \\ \omega_r^{\min} & \text{otherwise,} \end{cases} \tag{28}$$

where $Z*(k)$ is the optimal stock level at which the corrective maintenance rate must be switched from $\omega_r^{\max}$ to $\omega_r^{\min}$.

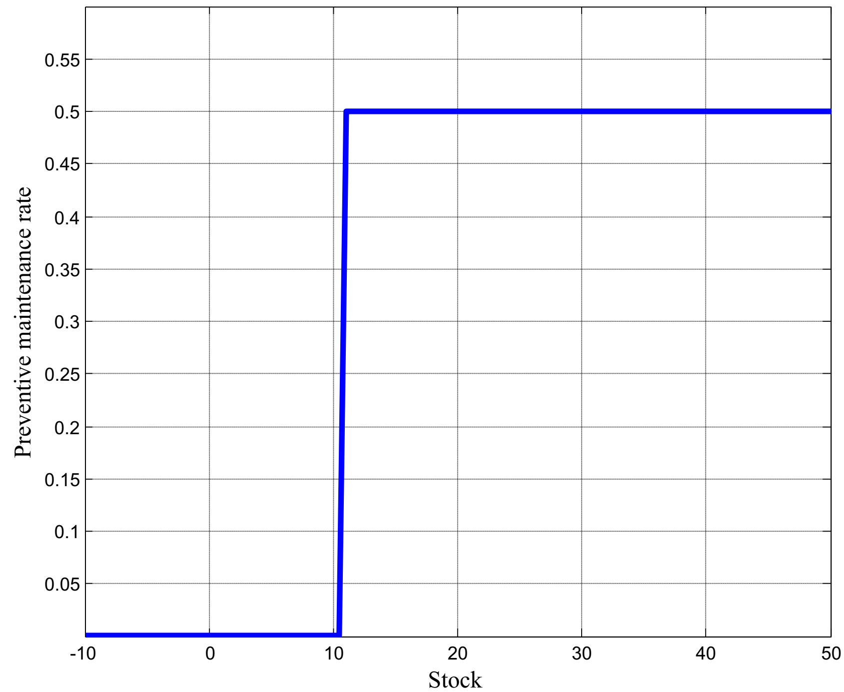
Using the control policies given by Eqs. (26), (27), and (28), the company will be able to minimize the total cost due to production, allowing it to eventually maximize its total profit.

In Section 6, we confirm such an observation through a sensitivity analysis, which can also validate and illustrate the usefulness of the model developed in this research.

6 Sensitivity analyses and extensions

This section analyses the sensitivity of the control policies according to the variation of the number of imperfect repairs and the costs parameters. Extensions of this paper are also discussed.

Fig. 5 Preventive maintenance rate of the machine at mode 1



6.1 Sensitivity analysis with respect to number of imperfect repairs

By combining a k-dependent failure rate, preventive and corrective maintenance actions with production activities, we obtained that the optimal threshold, and the other parameters of the control policy (X^*, Y^*, and Z^*) increase as the number of failures increases (see Table 3 and Figs. 7, 8, 9, and 10). Figure 8 illustrates the trend of threshold value versus number of imperfect repairs. The results of Figs. 7, 8, 9, and 10 show that when the number of imperfect repairs increases, the values of X^*, Y^*, and Z^* the increase as well. Then, we can then avoid backlogs when the machine is at modes 2 and 3. These results illustrate the contribution of the proposed model compared to one in which one value of the optimal threshold is used for production planning, without considering the fact

Fig. 6 Corrective maintenance rate of the machine at mode 2

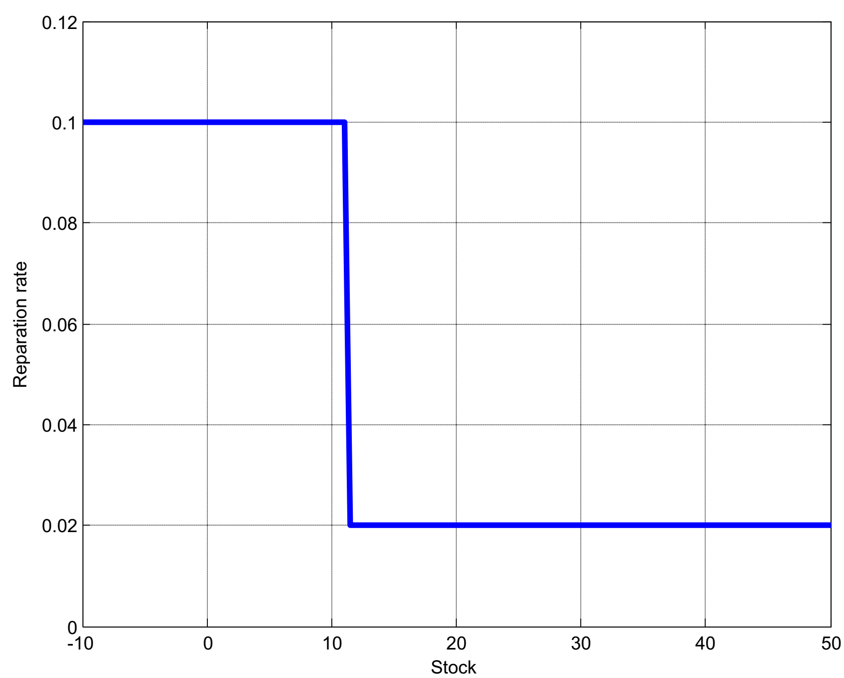


Table 3 Variation of the optimal threshold with the number of failures

c^+	c^-	c_m	c_r	k	X^*	Y^*	Z^*	Cost
1	100	10	5,000	2	20.50	9.09	10.00	16,926
1	100	10	5,000	5	28.00	10.31	10.62	19,179
1	100	10	5,000	10	39.00	13.12	13.12	23,436
1	100	10	5,000	20	46.50	14.69	14.69	26,882

that the failure rate depends on the number of imperfect repairs combined with control of the corrective and the preventive maintenance.

6.2 Sensitivity analysis with respect to costs parameters

Backlog, inventory, and preventive and corrective maintenance cost parameters are considered in the sensitivity analyses in order to gain insight into the proposed stochastic model. The numerical example presented previously was used to perform a couple of experiments, and the results shown in Table 4 illustrate four scenarios. The following variations are explored and compared to the basic case (highlighted lines).

6.2.1 Variation of the backlog cost

- Increasing c^-:X^*, Y^*, and Z^* increase. This must result in a tendency to increase the threshold value and the other parameters of the control policy in order to avoid further backlog costs. The overall cost increases as well.
- Decreasing c^-: The stock level decreases in order to avoid further inventory costs (second line of Table 4 : c^-=50).

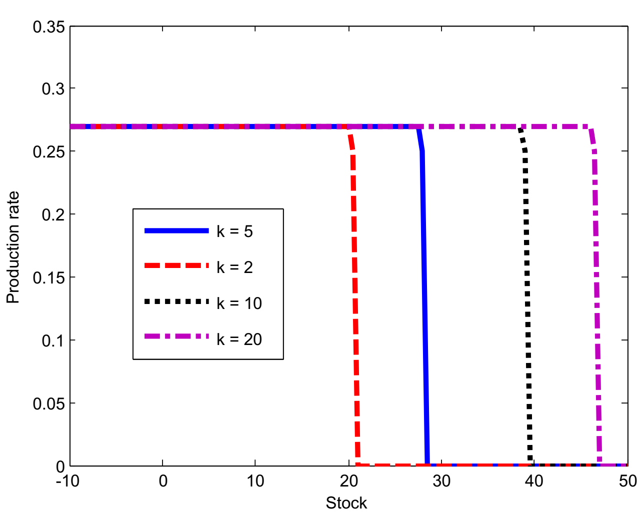


Fig. 7 Production rate versus number of imperfect repairs

6.2.2 Variation of the inventory cost

- Increasing c^+: The threshold value decreases and other parameters of the control policy move as predicted, from a practical view point, in order to avoid further inventory costs (second block of Table 4).

6.2.3 Variation of the preventive maintenance cost

- Increasing c_m: The threshold value decreases in order to avoid further inventory costs. The overall cost increases. For high values of preventive maintenance costs compared to the basic case, no preventive maintenance is required (the preventive maintenance rate is set to its minimal value). In these cases, the corrective maintenance parameter remains constant (third block of Table 4).

6.2.4 Variation of the corrective maintenance cost

- Increasing c_r: The corrective maintenance policy parameter decreases in order to avoid further repair cost. The threshold level increases in order to avoid further backlog costs with high levels of repair costs. The preventive maintenance parameter decreases and the overall cost increases (last block of Table 4).

The above sensitivity analyses validate the proposed approach and show that the control policy and parameters obtained from the results analyses are consistent.

6.3 Extensions

For given k−dependent failure rate parameters $X(k)$, $Y(k)$, and $Z(k)$, the control policy described by Eqs. (26) to (28) is completely known for the system proposed in this paper (one-machine and one-product). For a manufacturing system consisting of m machines producing n different part types, the production, preventive, and corrective maintenance policies could be defined by 3^{n+m} parameters or input factors because the control policy would depend on $k_1,\ldots,k_m$, $X_1^\alpha(k),\ldots,X_n^\alpha(k)$, $Y_1^\alpha(k),\ldots,Y_n^\alpha(k)$, and $Z_1^\alpha(k),\ldots,Z_n^\alpha(k)$ with $\alpha\in\{1,2,3\}$. In that case, the HJB equations (such as Eq. (17)) are impossible to solve for large values of m and n since the dimension of the numerical scheme to be implemented increases exponentially with the complexity of the system. The analytical models combined to simulation can be used to determine the effects of the factors considered on the

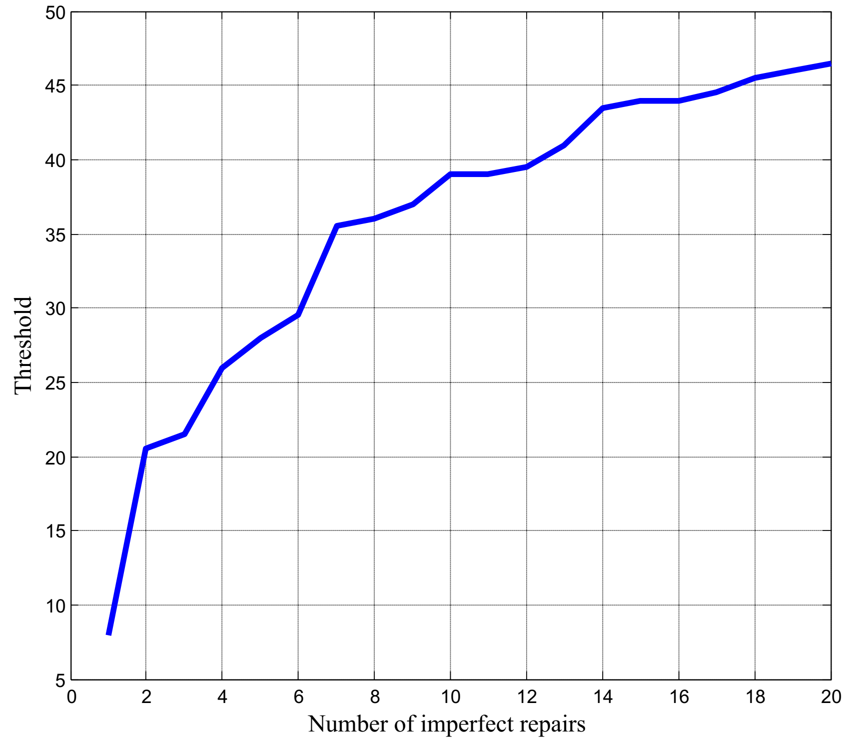


Fig. 8 Trend of threshold value versus number of imperfect repairs

incurred cost and to obtain a near-optimal control policy (see Gharbi and Kenne in [5], Boulet et al. in [24]).

For purposes of extension, the structure of a new approach to defining a near-optimal control policy in the context of a multiple-machine, multiple-product manufacturing system could consist of the following six sequential steps.

1. The control problem statement of the manufacturing system. Here, the objective is to find the production, preventive, and corrective maintenance control variables.
2. The structure of the HJB equations, the numerical methods, the policy improvement techniques, and the optimal control policies are obtained.

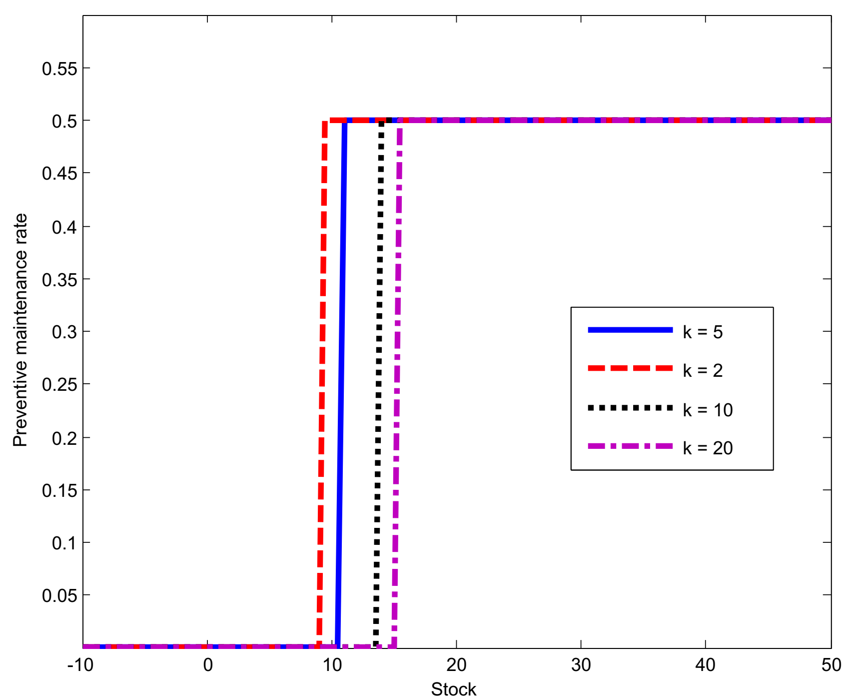


Fig. 9 Preventive maintenance rate versus number of imperfect repairs

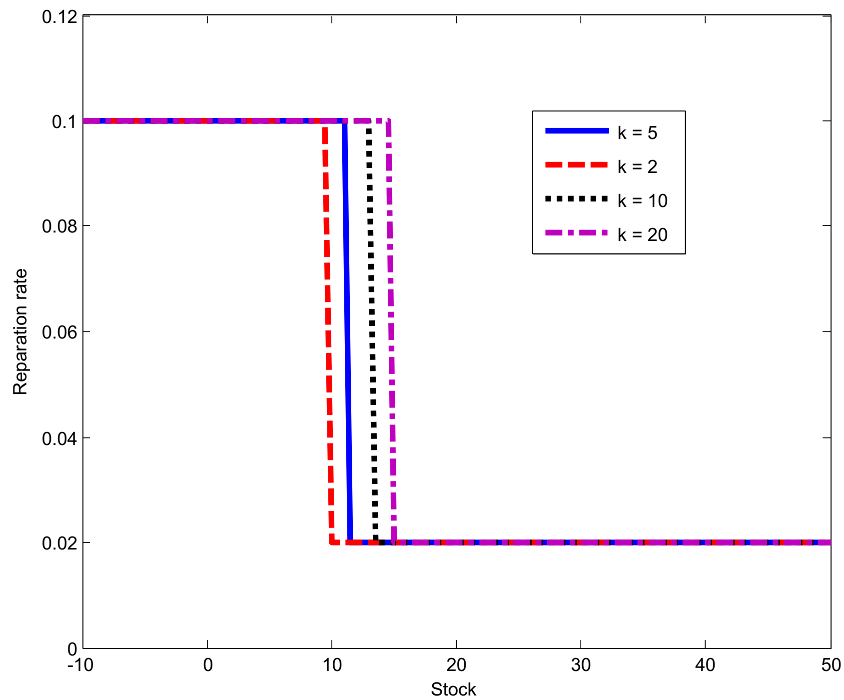


Fig. 10 Corrective maintenance rate versus number of imperfect repairs

3. The control production, preventive, and corrective maintenance factors for small size manufacturing systems (as in this paper) are determined.
4. The structure of the parameterized control policies is described and defined in simple cases. Then, the extension to more complex manufacturing systems is obtained.
5. The incurred cost is obtained from the simulation modeling according to the values of the control factors. The variations of the control factors, the effects of the main factors and their interactions on the cost are defined using the experimental design approach and an analysis of variance. Then, the response surface methodology is used to obtain the relationship between the cost and the significant main factors and interactions given in step 1. The optimization of the regression model obtained allows the determination of the best values of factors.
6. The near-optimal policies describing the production, preventive, and corrective maintenance parameters are approximated. Then, the robustness of the proposed approach is validated through a sensitivity analysis.

Table 4 Sensitivity analysis and policy parameters

c^+	c^-	c_m	c_r	k	X^*	Y^*	Z^*	*Cost*
1	100	10	5,000	5	28.00	10.31	10.62	19,179
1	50	10	5,000	5	25.50	5.00	5.00	18,829
1	200	10	5,000	5	30.50	13.13	13.75	19,571
1	300	10	5,000	5	32.50	14.38	15.00	19,821
1	100	10	5,000	5	28.00	10.31	10.62	19,179
5	100	10	5,000	5	12.50	6.88	10.00	55,213
10	100	10	5,000	5	8.50	5.00	8.75	99,221
20	100	10	5,000	5	5.50	3.13	8.13	18,6278
1	100	10	5,000	5	28.00	10.31	10.62	19,179
1	100	30	5,000	5	27.00	12.50	10.63	20,202
1	100	50	5,000	5	25.00	16.88	10.63	21,181
1	100	70	5,000	5	22.50	$\omega_m = \omega_m^{\min}$	10.63	21,376
1	100	10	5,000	5	28.00	10.31	10.62	19,179
1	100	10	10,000	5	34.50	8.75	8.75	27,772
1	100	10	15,000	5	39.00	7.50	7.50	36,145
1	100	10	20,000	5	42.00	6.25	6.25	44,398

7 Conclusion

This paper studied the impact of imperfect repairs, preventive and corrective maintenance scenarios for a single machine, and single product manufacturing system under uncertainties. We developed a stochastic optimization model of the problem considered, with three decision variables (production rate, preventive maintenance rate, and corrective maintenance rate) and one state variable (stock level). By controlling both production and maintenance rates, we obtained a near-optimal control policy for the system through the implementation of the policy improvement algorithm (numerical methods). We have shown that the number of parts to hold in inventory and preventive and corrective maintenance parameters, increase when the number of imperfect repairs increases.

We believe that this work represents a significant contribution to the literature on the production control of flexible two-level manufacturing systems, where, at the first level, the parameters of the machine failure stochastic process are derived for each number of imperfect repairs. At the second level, the optimal production, preventive, and corrective maintenance policies are determined for a system that deteriorates with the number of imperfect repairs. We illustrated and validated the proposed approach using a numerical example and sensitivity analyses yielding logical conclusions.

The proposed model is developed in the case of a constant demand rate, one-machine, and one-product manufacturing system. To cope with a real industrial environment case, we discussed the extensions of the proposed model to the case of manufacturing systems involving multiple products and multiple machines.

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