Int J Adv Manuf Technol (2014) 75:1115–1130
DOI 10.1007/s00170-014-6142-7

ORIGINAL ARTICLE

Multiple criteria group decision-making for supplier selection based on COPRAS method with interval type-2 fuzzy sets

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Received: 19 February 2014 / Accepted: 3 July 2014 / Published online: 13 August 2014

Abstract Supplier selection is one of the most critical activities of purchasing management in a supply chain because of the key role of supplier's performance in achieving the objectives of a supply chain. Supplier selection problem requires a trade-off between multiple criteria exhibiting vagueness and imprecision with the involvement of a group of experts. This paper presents a multiple criteria group decision-making approach for supplier selection problem in the context of interval type-2 fuzzy sets. A new method for ranking interval type-2 fuzzy numbers, based on the centroid of fuzzy sets, is proposed and compared with some methods. The proposed ranking method is used for extending complex proportional assessment (COPRAS) method for group decision-making with interval type-2 fuzzy numbers. The developed method uses a stepwise procedure for ranking and evaluating the alternatives, in terms of significance and utility degree, and selects the best solution considering both the positive-ideal and the negative-ideal solutions. To demonstrate the applicability of the proposed approach in supplier selection problems, an illustrative example is presented and the results are analyzed.

Keywords COPRAS method · Interval type-2 fuzzy sets · Multiple criteria group decision-making · Supplier selection

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1 Introduction

In the past decades, supply chain management (SCM) and supplier selection problem have greatly been considered by researchers in the business management literature and practice [1–8]. Supply chain management (SCM) encompasses the management of participants' transactions in a supply chain for maximization of total supply chain profitability. SCM focuses on minimization of overall costs of the supply chain from side to side and maximization of the revenue earned from the customer in association with organization partners. Companies within a supply chain can reach the stability in competitive advantages through expanding much closer connections with all corporations. They can meaningfully decrease time and costs based on the suitable management of the supply chain as well [9]. In most situations, SCM appears from several companies that have made their own supply chain. They have to work with more capable partners to reach a competitive chain. Manufacturers must prefer more cooperative ones among a variety of available suppliers which are competent to develop long-period relationships [10]. Hence, one of the most essential activities of firms is supplier selection. Selecting suitable suppliers can significantly increase the organization competitiveness by decreasing the purchasing costs [11]. We can classify the supplier selection problems in the multiple criteria problems category which include both qualitative and quantitative performance indicators. It is necessary to make a trade-off between these tangible and intangible factors, in order to select the desired suppliers [12]. Consequently, a purchasing manager must scrutinize the trade-off among these criteria. Multiple criteria decision-making (MCDM) techniques support the decision makers (DMs) in appraising an assortment of alternatives. Depending on the purchasing conditions, criteria have varying importance and the DMs have to weigh them [13].

In practice, for supplier selection problems, most of the input information is not known precisely. In these cases, the theory of fuzzy sets is one of the best tools for handling uncertainty [14]. In order to deal with the fuzziness in the real world, the values of criteria are usually represented by type-1 fuzzy sets proposed by Zadeh [15]. Several methods have been proposed for dealing with type-1 multiple criteria decision-making problems [16–18]. Tolga and Kahraman [19] proposed a modified fuzzy TOPSIS method in the case of uncertain information which took the form of linguistic terms, and the developed method was utilized for solving energy planning problem. Li and Yang [20] presented a linear programming method in the context of MCDM. Ma et al. [21] proposed a decision support system based on a model to increase the level of overall contentment in the multiple criteria group decision-making. Mohammad et al. [22] developed a new method to handle the problem of parametric form of fuzzy numbers and applied it to a case study of diversion of water. Yeh and Chang [23] introduced a hierarchical weighting method to appraise weights, and further developed an algorithm for grouping MCDM to incorporate criteria weights involving decision makers' subjective judgments.

The membership value of type-1 fuzzy set is a crisp number in [0, 1]. However, we are generally faced with the situations where it is hard to find out the exact membership function for a fuzzy set. Utilizing type-1 fuzzy sets is not suitable in these cases. To handle this issue, Zadeh [24] proposed type-2 fuzzy as an extension of type-1 fuzzy sets. Type-2 fuzzy sets are three dimensional and their membership function is represented by a fuzzy set on the interval [0,1] and delineated by both primary and secondary membership to provide more degrees of freedom and flexibility. Hence, type-2 fuzzy sets are more accurate in the modeling of uncertainty in comparison with type-1 fuzzy sets. Nevertheless, we are confronted with heavy computations when type-2 fuzzy sets are employed to solve problems [25]. To resolve the difficulties in establishing and handling type-2 fuzzy sets, interval type-2 fuzzy sets are used by researchers with some representations such as vertical-slice and wavy-slice representations [26]. The concept of interval type-2 fuzzy sets is defined by an interval-valued membership function. Interval type-2 fuzzy sets contain membership values that are crisp intervals and are extremely advantageous for theoretical and computational studies of the higher order fuzzy sets because of their relative simplicity [27]. Some basic definitions of interval type-2 fuzzy sets were proposed by Mendel et al. [25]. Mitchell [28] and Zeng and Li [29] suggested methods to estimate the similarity between interval type-2 fuzzy sets. Wu and Mendel [30] proposed a new method to reduce the limitations in methods of Mitchell [28] and Zeng and Li [29] named vector similarity method (VSM) to transform interval type-2 fuzzy sets into words more effectively.

The other aspect of interval type-2 fuzzy sets is their application in MCDM problems. Developing methods for MCDM problems within the context of interval type-2 fuzzy sets has been considered by researchers. Chen and Lee [31] developed an interval type-2 fuzzy technique for order preference by similarity to an ideal solution (TOPSIS) to deal with group decision-making problems based on interval type-2 fuzzy sets. Lu et al. [32] proposed an interval-valued fuzzy linear programming method based on infinite α-cuts and applied this method to the problem of water resource management. Vahdani et al. [33] developed an elimination and choice-translating reality (ELECTRE) method with interval weights and data to handle MCDM problems. Vahdani and Hadipour [34] developed an ELECTRE method based on interval type-2 fuzzy sets to solve a problem of selecting a maintenance strategy. Vahdani et al. [35] proposed a method based on VIKOR and interval-valued fuzzy numbers to solve MCDM problems. Chen [36] introduced a multiple criteria group decision-making method with generalized interval-valued fuzzy numbers under incomplete weight information. Chen [37] presented a new linear assignment method to produce an optimal preference ranking of the alternatives in conformity with a set of criterion-wise rankings and a set of criterion importance within the context of interval type-2 trapezoidal fuzzy numbers. Wang et al. [38] proposed multiple-criteria group decision-making methods based on the ranking values and the arithmetic operations of interval type-2 fuzzy sets. Chen et al. [39] presented a method to deal with multiple criteria group decision-making problems based on ranking interval type-2 fuzzy sets. Chen [40] developed a method for determining the objective importance of criteria and handling multiple criteria group decision-making problems within the context of interval type-2 fuzzy sets. Hu et al. [41] introduced a new approach based on possibility degree to solve multiple criteria decision-making (MCDM) problems in which the criteria values take the form of interval type-2 fuzzy number. Chen and Lee [42] presented a new method to handle fuzzy multiple criteria group decision-making problems based on the ranking values and the arithmetic operations of interval type-2 fuzzy sets. Zhang and Zhang [43] developed a novel approach to multiple criteria group decision-making under interval type-2 fuzzy environment. Celik et al. [44] proposed an interval type-2 fuzzy MCDM method based on TOPSIS and GRA, to evaluate and improve customer satisfaction in Istanbul public transportation. Chen and Wang [45] presented a new method for fuzzy multiple criteria decision-making based on interval type-2 fuzzy sets, and a new fuzzy ranking method based on the α-cuts of interval type-2 fuzzy sets. Razavi Hajiagha et al. [46] suggested an extended form of the complex proportional assessment (COPRAS) method for group decision-making with interval-valued intuitionistic fuzzy sets.

Although a considerable amount of research work has already been conducted by the past researchers on supplier selection using different MCDM methods, there is still a need to employ a simple and systematic mathematical approach for handling ambiguity and fuzziness in supplier selection problems. In this paper, the COPRAS method [47], a new MCDM method which has been developed to solve multiple criteria decision-making (MCDM) problems with conflicting and non-commensurable (different units) criteria, is extended to propose a new method for dealing with fuzzy multiple criteria group decision-making problems within the context of interval type-2 fuzzy sets. We present a new method for ranking interval type-2 fuzzy numbers, based on the centroid [48] of fuzzy sets, and compare it with some other methods. We also use an example to describe the application of the proposed method in supplier selection problems. The proposed method provides us with a useful way to handle fuzzy multiple criteria group decision-making problems in a more flexible and smarter manner and the basis for developing supplier selection models.

The rest of this paper is organized as follows: in Section 2, we briefly review the concepts of interval type-2 fuzzy sets. In Section 3, the arithmetic operations between trapezoidal interval type-2 fuzzy sets are presented. In Section 4, we describe a method for ranking interval type-2 fuzzy sets. In Section 5, we present a new method for handling fuzzy multiple criteria group decision-making problems based on the COPRAS method and the proposed ranking method. In Section 6, we use an example to illustrate the application of the proposed method in supplier selection problems. The conclusions are discussed in Section 7.

2 Concepts of interval type-2 fuzzy sets

The theory of type-1 fuzzy sets was proposed by Zadeh in 1965 [15]. The membership value in a type-1 fuzzy set is represented by a real value in the range of 0 to 1. Let $\widetilde{A}$ be a type-1 trapezoidal fuzzy set, where $\widetilde{A} = (a_1, a_2, a_3, a_4; H_1(\widetilde{A}), H_2(\widetilde{A}))$ and $0 \leq H_1(\widetilde{A}) \leq 1$ and $0 \leq H_2(\widetilde{A}) \leq 1$, as shown in Fig. 1. If $a_2 = a_3$, then the type-1 fuzzy set $\widetilde{A}$ becomes a triangular type-1 fuzzy set.

Type-2 fuzzy sets, described by primary and secondary membership values, are the extension of type-1 fuzzy sets.

Definition 2.1 [25] A type-2 fuzzy set $\tilde{\tilde{A}}$ can be represented by a type-2 membership function, expressed as follows:

$$\tilde{\tilde{A}} = \left\{ \left((x,u),\ \mu_{\tilde{\tilde{A}}}(x,u) \right) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1],\ 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1 \right\}, \quad (1)$$

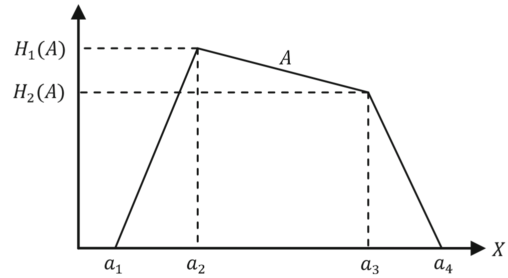

Fig. 1 A type-1 trapezoidal fuzzy set

where X denotes the domain of $\tilde{\tilde{A}}$ and $\mu_{\tilde{\tilde{A}}}$ denotes the membership function (secondary membership function) of $\tilde{\tilde{A}}$ and J_X is an interval between zero and one and denotes the primary membership function. The type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_X} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u), \quad (2)$$

where $J_x \subseteq [0,1]$ and $\iint$ denotes the union over all admissible x and u.

Definition 2.2 [25] For a type-2 fuzzy set $\tilde{\tilde{A}}$, if all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is called interval type-2 fuzzy set. An interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be considered as a special case of type-2 fuzzy set, described as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_X} 1/(x,u), \quad (3)$$

where $J_x \subseteq [0,1]$.

Definition 2.3 [25] Footprint of uncertainty (FOU) is an uncertain bounded region for the primary membership function, which is the union of all primary memberships. FOU is described by upper membership function (UMF) and lower membership function (LMF). UMF and LMF are type-1 fuzzy sets.

Definition 2.4 [42] An interval type-2 fuzzy number is called trapezoidal interval type-2 fuzzy number if and only if the UMF and the LMF are both trapezoidal fuzzy numbers. Let $\tilde{\tilde{A}}$ be a trapezoidal interval type-2 fuzzy set. $\tilde{\tilde{A}}$ can be expressed as follows:

$$\tilde{\tilde{A}}\ (\tilde{A}^U, \tilde{A}^L) = \Big(\big(a_1^u,\ a_2^u,\ a_3^u,\ a_4^u;\ H_1(\tilde{A}^U),\ H_2(\tilde{A}^U) \big), \big(a_1^L,\ a_2^L,\ a_3^L,\ a_4^L;\ H_1(\tilde{A}^L),\ H_2(\tilde{A}^L) \big) \Big), \quad (4)$$

where $\widetilde{A}^U$ and $\widetilde{A}^L$ denote the UMF and LMF of $\widetilde{\widetilde{A}}$, respectively, and $H_j(\widetilde{A}^U)$ and $H_j(\widetilde{A}^L)$ ($H_j(\widetilde{A}^U) \in [0,1]$, $H_j(\widetilde{A}^L) \in [0,1]$ and j=1,2) denote the membership values of the corresponding elements a^L_{j+1} and a^U_{j+1}, respectively, as shown in Fig. 2.

In this study, we concentrate on utilizing interval type-2 fuzzy sets for dealing with fuzzy multiple criteria group decision-making problems.

3 The arithmetic operations of trapezoidal interval type-2 fuzzy sets

Suppose that $\widetilde{\widetilde{A}}_1$ and $\widetilde{\widetilde{A}}_2$ are two trapezoidal interval type-2 fuzzy numbers:

$$\widetilde{\widetilde{A}}_1 = \left(\widetilde{A}_1^U, \widetilde{A}_1^L\right) = \left(\left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1\left(\widetilde{A}_1^U\right), H_2\left(\widetilde{A}_1^U\right)\right), \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1\left(\widetilde{A}_1^L\right), H_2\left(\widetilde{A}_1^L\right)\right)\right), \tag{5}$$

$$\widetilde{\widetilde{A}}_2 = \left(\widetilde{A}_2^U, \widetilde{A}_2^L\right) = \left(\left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1\left(\widetilde{A}_2^U\right), H_2\left(\widetilde{A}_2^U\right)\right), \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1\left(\widetilde{A}_2^L\right), H_2\left(\widetilde{A}_2^L\right)\right)\right), \tag{6}$$

Then arithmetic operations are represented in the following definitions.

Definition 3.1 [42] The addition operation is defined as follows:

$$\widetilde{\widetilde{A}}_1 \oplus \widetilde{\widetilde{A}}_2 = \left(\widetilde{A}_1^U, \widetilde{A}_1^L\right) \oplus \left(\widetilde{A}_2^U, \widetilde{A}_2^L\right) = \left(\left(a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \min\left(H_1\left(\widetilde{A}_1^U\right), H_1\left(\widetilde{A}_2^U\right)\right), \min\left(H_2\left(\widetilde{A}_1^U\right), H_2\left(\widetilde{A}_2^U\right)\right)\right), \left(a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \min\left(H_1\left(\widetilde{A}_1^L\right), H_1\left(\widetilde{A}_2^L\right)\right), \min\left(H_2\left(\widetilde{A}_1^L\right), H_2\left(\widetilde{A}_2^L\right)\right)\right)\right), \tag{7}$$

Definition 3.2 [42] The subtraction operation is defined as follows:

$$\widetilde{\widetilde{A}}_1 \ominus \widetilde{\widetilde{A}}_2 = \left(\widetilde{A}_1^U, \widetilde{A}_1^L\right) \ominus \left(\widetilde{A}_2^U, \widetilde{A}_2^L\right) = \left(\left(a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U; \min\left(H_1\left(\widetilde{A}_1^U\right), H_1\left(\widetilde{A}_2^U\right)\right), \min\left(H_2\left(\widetilde{A}_1^U\right), H_2\left(\widetilde{A}_2^U\right)\right)\right), \left(a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L; \min\left(H_1\left(\widetilde{A}_1^L\right), H_1\left(\widetilde{A}_2^L\right)\right), \min\left(H_2\left(\widetilde{A}_1^L\right), H_2\left(\widetilde{A}_2^L\right)\right)\right)\right), \tag{8}$$

Definition 3.3 The multiplication operation is defined as follows:

$$\widetilde{\widetilde{A}}_1 \otimes \widetilde{\widetilde{A}}_2 = \left(\widetilde{A}_1^U, \widetilde{A}_1^L\right) \otimes \left(\widetilde{A}_2^U, \widetilde{A}_2^L\right) = \left(\left(X_1^U, X_2^U, X_3^U, X_4^U; \min\left(H_1\left(\widetilde{A}_1^U\right), H_1\left(\widetilde{A}_2^U\right)\right), \min\left(H_2\left(\widetilde{A}_1^U\right), H_2\left(\widetilde{A}_2^U\right)\right)\right), \left(X_1^L, X_2^L, X_3^L, X_4^L; \min\left(H_1\left(\widetilde{A}_1^L\right), H_1\left(\widetilde{A}_2^L\right)\right), \min\left(H_2\left(\widetilde{A}_1^L\right), H_2\left(\widetilde{A}_2^L\right)\right)\right)\right), \tag{9}$$

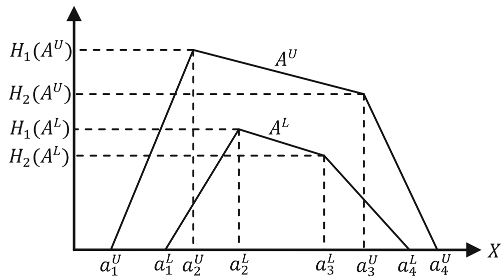

Fig. 2 A trapezoidal interval type-2 fuzzy number

where

$$X_i^T = \begin{cases} \min\left(a_{1i}^T a_{2i}^T, a_{1i}^T a_{2(5-i)}^T, a_{1(5-i)}^T a_{2i}^T, a_{1(5-i)}^T a_{2(5-i)}^T\right) & if\ i = 1,2 \\ \max\left(a_{1i}^T a_{2i}^T, a_{1i}^T a_{2(5-i)}^T, a_{1(5-i)}^T a_{2i}^T, a_{1(5-i)}^T a_{2(5-i)}^T\right) & if\ i = 3,4 \end{cases} \tag{10}$$

and $T \in \{U, L\}$.

Definition 3.4 The division operation is defined as follows:

$$\tilde{\tilde{A}}_1 \oslash \tilde{\tilde{A}}_2 = \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \oslash \left(\tilde{A}_2^U, \tilde{A}_2^L\right) = \Big(\Big(Y_1^U, Y_2^U, Y_3^U, Y_4^U; \min\Big(H_1\Big(\tilde{A}_1^U\Big), H_1\Big(\tilde{A}_2^U\Big)\Big), \min\Big(H_2\Big(\tilde{A}_1^U\Big), H_2\Big(\tilde{A}_2^U\Big)\Big)\Big), \Big(Y_1^L, Y_2^L, Y_3^L, Y_4^L; \min\Big(H_1\Big(\tilde{A}_1^L\Big), H_1\Big(\tilde{A}_2^L\Big)\Big), \min\Big(H_2\Big(\tilde{A}_1^L\Big), H_2\Big(\tilde{A}_2^L\Big)\Big)\Big)\Big), \tag{11}$$

where

$$Y_i^T = \begin{cases} \min\left(a_{1i}^T / a_{2i}^T, a_{1i}^T / a_{2(5-i)}^T, a_{1(5-i)}^T / a_{2i}^T, a_{1(5-i)}^T / a_{2(5-i)}^T\right) & if\ i = 1,2 \\ \max\left(a_{1i}^T / a_{2i}^T, a_{1i}^T / a_{2(5-i)}^T, a_{1(5-i)}^T / a_{2i}^T, a_{1(5-i)}^T / a_{2(5-i)}^T\right) & if\ i = 3,4 \end{cases}, \tag{12}$$

$a_{2j}^T \neq 0, j = 1,2,3,4$ and $T \in \{U, L\}$.

The reciprocal influence of different trapezoidal interval type-2 fuzzy number is considered in multiplication and division definitions. Therefore, there is no limitation in multiplication or division of two trapezoidal interval type-2 fuzzy numbers with negative elements.

Definition 3.5 [37] Multiplication by an ordinary number is defined as follows:

$$k.\tilde{\tilde{A}}_1 = \begin{cases} \Big(\Big(k.a_{11}^U, k.a_{12}^U, k.a_{13}^U, k.a_{14}^U; H_1\Big(\tilde{A}_1^U\Big), H_2\Big(\tilde{A}_1^U\Big)\Big), \Big(k.a_{11}^L, k.a_{12}^L, k.a_{13}^L, k.a_{14}^L; H_1\Big(\tilde{A}_1^L\Big), H_2\Big(\tilde{A}_1^L\Big)\Big)\Big) & if\ k \geq 0 \\ \Big(\Big(k.a_{14}^U, k.a_{13}^U, k.a_{12}^U, k.a_{11}^U; H_1\Big(\tilde{A}_1^U\Big), H_2\Big(\tilde{A}_1^U\Big)\Big), \Big(k.a_{14}^L, k.a_{13}^L, k.a_{12}^L, k.a_{11}^L; H_1\Big(\tilde{A}_1^L\Big), H_2\Big(\tilde{A}_1^L\Big)\Big)\Big) & if\ k \leq 0 \end{cases}, \tag{13}$$

Definition 3.6 [37] Division by an ordinary number (l is a nonzero number) is defined as follows:

$$\tilde{\tilde{A}}_1 / l = \begin{cases} \Big(\Big(a_{11}^U/l, a_{12}^U/l, a_{13}^U/l, a_{14}^U/l; H_1\Big(\tilde{A}_1^U\Big), H_2\Big(\tilde{A}_1^U\Big)\Big), \Big(a_{11}^L/l, a_{12}^L/l, a_{13}^L/l, a_{14}^L/l; H_1\Big(\tilde{A}_1^L\Big), H_2\Big(\tilde{A}_1^L\Big)\Big)\Big) & if\ l > 0 \\ \Big(\Big(a_{14}^U/l, a_{13}^U/l, a_{12}^U/l, a_{11}^U/l; H_1\Big(\tilde{A}_1^U\Big), H_2\Big(\tilde{A}_1^U\Big)\Big), \Big(a_{14}^L/l, a_{13}^L/l, a_{12}^L/l, a_{11}^L/l; H_1\Big(\tilde{A}_1^L\Big), H_2\Big(\tilde{A}_1^L\Big)\Big)\Big) & if\ l < 0 \end{cases}, \tag{14}$$

4 Ranking values of trapezoidal interval type-2 fuzzy sets

A new method for computing the ranking value of trapezoidal interval type-2 fuzzy sets is presented in this section. The centroid of a trapezoidal fuzzy set (type-1 fuzzy set) that was presented by Wang et al. [49] is used for calculating possibility degree of a trapezoidal interval type-2 fuzzy set. Then the possibility degree is used for determining its ranking value.

Definition 4.1 [49] Let $\widetilde{A}_i$ be a type-1 trapezoidal fuzzy set:

$$\widetilde{A}_i = \left(a_{i1}, a_{i2}, a_{i3}, a_{i4}; H\left(\widetilde{A}_i\right)\right)$$

In order to determine the centroid point (C_{ix}, C_{iy}) of a type-1 trapezoidal fuzzy number $(\widetilde{A}_i)$, the following formulas are used:

$$C_{ix} = \frac{1}{3}\left(a_{i1} + a_{i2} + a_{i3} + a_{i4} - \frac{a_{i3}a_{i4} - a_{i1}a_{i2}}{(a_{i3} + a_{i4}) - (a_{i1} + a_{i2})}\right), \quad (15)$$

$$C_{iy} = \frac{H\left(\widetilde{A}_i\right)}{3}\left(1 + \frac{a_{i3} - a_{i2}}{(a_{i3} + a_{i4}) - (a_{i1} + a_{i2})}\right) \quad (16)$$

Definition 4.2 Suppose that $\widetilde{\widetilde{A}}_i$ is a trapezoidal interval type-2 fuzzy set:

$$\widetilde{\widetilde{A}}_i = \left(\widetilde{A}_i^U, \widetilde{A}_i^L\right) = \left(\left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\widetilde{A}_i^U\right), H_2\left(\widetilde{A}_i^U\right)\right)\right.$$
$$\left.\left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1\left(\widetilde{A}_i^L\right), H_2\left(\widetilde{A}_i^L\right)\right)\right)$$

The previous definition is extended for a trapezoidal interval type-2 fuzzy set $\left(\widetilde{\widetilde{A}}_i\right)$ as follows:

$$C_{ix}^T = \frac{1}{3}\left(a_{i1}^T + a_{i2}^T + a_{i3}^T + a_{i4}^T - \frac{a_{i3}^T a_{i4}^T - a_{i1}^T a_{i2}^T}{\left(a_{i3}^T + a_{i4}^T\right) - \left(a_{i1}^T + a_{i2}^T\right)}\right), \ T\in\{U, L\}, \quad (17)$$

$$C_{iy}^T = \frac{\omega^T}{3}\left(1 + \frac{a_{i3}^T - a_{i2}^T}{\left(a_{i3}^T + a_{i4}^T\right) - \left(a_{i1}^T + a_{i2}^T\right)}\right), \ T\in\{U, L\}, \quad (18)$$

where

$$\omega^T = \frac{H_1\left(\widetilde{A}_i^T\right) + H_2\left(\widetilde{A}_i^T\right)}{2}, \ T\in\{U, L\}, \quad (19)$$

Definition 4.3 Suppose that $\widetilde{\widetilde{A}}_s$ and $\widetilde{\widetilde{A}}_t$ are two trapezoidal interval type-2 fuzzy sets:

$$\widetilde{\widetilde{A}}_s = \left(\widetilde{A}_s^U, \widetilde{A}_s^L\right) = \left(\left(a_{s1}^U, a_{s2}^U, a_{s3}^U, a_{s4}^U; H_1\left(\widetilde{A}_s^U\right), H_2\left(\widetilde{A}_s^U\right)\right), \left(a_{s1}^L, a_{s2}^L, a_{s3}^L, a_{s4}^L; H_1\left(\widetilde{A}_s^L\right), H_2\left(\widetilde{A}_s^L\right)\right)\right)$$
$$\widetilde{\widetilde{A}}_t = \left(\widetilde{A}_t^U, \widetilde{A}_t^L\right) = \left(\left(a_{t1}^U, a_{t2}^U, a_{t3}^U, a_{t4}^U; H_1\left(\widetilde{A}_t^U\right), H_2\left(\widetilde{A}_t^U\right)\right), \left(a_{t1}^L, a_{t2}^L, a_{t3}^L, a_{t4}^L; H_1\left(\widetilde{A}_t^L\right), H_2\left(\widetilde{A}_t^L\right)\right)\right)$$

Let,

$$E_x = \frac{\left(C_{tx}^U + C_{tx}^L\right) - \left(C_{sx}^U + C_{sx}^L\right)}{\left|C_{sx}^U + C_{sx}^L\right| + \left|C_{tx}^U + C_{tx}^L\right|}, \quad (20)$$

$$E_x' = \max\left(\left(1 - \max\left(E_x, 0\right)\right), 0\right), \quad (21)$$

$$E_y = \frac{\left(C_{ty}^U + C_{ty}^L\right) - \left(C_{sy}^U + C_{sy}^L\right)}{C_{sy}^U + C_{sy}^L + C_{ty}^U + C_{ty}^L}, \quad (22)$$

$$E_y' = \max\left(\left(1 - \max\left(E_y, 0\right)\right), 0\right), \quad (23)$$

$$M = \alpha E_x' + (1 - \alpha) E_y', \quad (24)$$

Where E_x' and E_y' are defined as the strength of $\widetilde{\widetilde{A}}_s$ over $\widetilde{\widetilde{A}}_t$ with respect to the domain and the membership function of them, respectively. We use α as a trade-off factor between E_x and E_y. Then, the possibility degree of $\widetilde{\widetilde{A}}_s$ over $\widetilde{\widetilde{A}}_t$ is defined as:

$$p\left(\widetilde{\widetilde{A}}_s \geq \widetilde{\widetilde{A}}_t\right) = \min(M, 1), \quad (25)$$

If n trapezoidal interval type-2 fuzzy sets need to be compared, the possibility degree matrix (P) can be obtained, shown as follows:

$$P=\begin{bmatrix} p\left(\tilde{\tilde{A}}_1\geq\tilde{\tilde{A}}_1\right) & p\left(\tilde{\tilde{A}}_1\geq\tilde{\tilde{A}}_2\right) & \cdots & p\left(\tilde{\tilde{A}}_1\geq\tilde{\tilde{A}}_n\right)\\ p\left(\tilde{\tilde{A}}_2\geq\tilde{\tilde{A}}_1\right) & p\left(\tilde{\tilde{A}}_2\geq\tilde{\tilde{A}}_2\right) & \cdots & p\left(\tilde{\tilde{A}}_2\geq\tilde{\tilde{A}}_n\right)\\ \vdots & \vdots & \ddots & \vdots\\ p\left(\tilde{\tilde{A}}_n\geq\tilde{\tilde{A}}_1\right) & p\left(\tilde{\tilde{A}}_n\geq\tilde{\tilde{A}}_2\right) & \cdots & p\left(\tilde{\tilde{A}}_n\geq\tilde{\tilde{A}}_n\right) \end{bmatrix}, \tag{26}$$

in other words

$$p_{ij}=p\left(\tilde{\tilde{A}}_i\geq\tilde{\tilde{A}}_j\right). \tag{27}$$

Definition 4.4 Suppose that $\tilde{\tilde{A}}_i\,(i=1,2,\ldots,n)$ is a collection of n trapezoidal interval type-2 fuzzy sets. Based on previous definition, all possibility degrees (p_{ij}) can be obtained by comparing every two trapezoidal interval type-2 fuzzy sets. Then, the following formula is used for calculating the ranking values of alternatives [42]:

$$\mathrm{Rank}\left(\tilde{\tilde{A}}_i\right)=\frac{1}{n(n-1)}\left(\sum\nolimits_{j=1}^{n}p\left(\tilde{\tilde{A}}_i\geq\tilde{\tilde{A}}_j\right)+\frac{n}{2}-1\right), \tag{28}$$

where $1\leq i\leq n$.

Thirteen fuzzy sets that were provided by Bortolan and Degani [49] is used for computing and comparing the ranking values. These fuzzy sets are shown in Table 1. Three values of α (α=0.5, 0.8,1) are chosen for this computation. The presented method is compared with some existing methods that proposed by Lee and Li [50], Baas and Kwakernaak [51], Chang et al. [52] and, Chen and Lee [42]. The calculated results are shown in Table 2.

According to Table 2, some drawbacks of the existing methods [42, 50–52] can be observed. Drawing a comparison between these existing methods and the proposed one can be described as follows:

1. According to Set 1 in Table 2, the methods of Chen and Lee [42], Lee and Li [50] (in Uniform mode) and the proposed method get the same ranking order.
2. According to Set 5, Set 6 and Set 8, as shown in Table 2, the same results can be obtained based on Chen and Lee's method [42], Lee and Li's method [50], the method of Chang et al. [52], and the proposed method (in α=0.8 and α=1 for Set 5), while Baas and Kwakernaak's method [51] cannot individualize the fuzzy sets in ranking order.
3. According to Set 10 in Table 2, the ranking order of proposed method in α=0.8 *and* 1 is consistent with the methods proposed by Chen and Lee [42], Baas and Kwakernaak [51] and Lee and Li [50], whereas in α=0.5 gets a different ranking result. The result from the method of Chang et al. [52] in α=0.5 and β=0.5 is inconsistent with other approaches as well.
4. According to Set 11 which has been represented in Table 2, the same outcomes can be obtained based on methods proposed by Chen and Lee [42], Chang et al. [52], Baas and Kwakernaak [51], Lee and Li [50], and the method that is presented in this study.
5. According to Set 12 in Table 2, it can be seen that Lee and Li's method [50], Baas and Kwakernaak's method [51], and the proposed method in α=1 cannot distinguish

Table 1 Thirteen sets of fuzzy sets given by Bortolan and Degani [49]

Sets of fuzzy sets		Trapezoidal interval type-2 fuzzy sets
Set 1	$\tilde{\tilde{A}}_1$	((0.35,0.4,0.4,1;1,1), (0.35,0.4,0.4,1;1,1))
	$\tilde{\tilde{A}}_2$	((0.15,0.7,0.7,0.8;1,1), (0.15,0.7,0.7,0.8;1,1))
Set 2	$\tilde{\tilde{A}}_1$	((0,0.1,0.5,1;1,1), (0,0.1,0.5,1;1,1))
	$\tilde{\tilde{A}}_2$	((0.5,0.6,0.6,0.7;1,1), (0.5,0.6,0.6,0.7;1,1))
Set 3	$\tilde{\tilde{A}}_1$	((0,0.1,0.5,1;1,1), (0,0.1,0.5,1;1,1))
	$\tilde{\tilde{A}}_2$	((0.6,0.7,0.7,0.8;1,1), (0.6,0.7,0.7,0.8;1,1))
Set 4	$\tilde{\tilde{A}}_1$	((0.4,0.9,0.9,1;1,1), (0.4,0.9,0.9,1;1,1))
	$\tilde{\tilde{A}}_2$	((0.4,0.7,0.7,1;1,1), (0.4,0.7,0.7,1;1,1))
	$\tilde{\tilde{A}}_3$	((0.4,0.5,0.5,1;1,1), (0.4,0.5,0.5,1;1,1))
Set 5	$\tilde{\tilde{A}}_1$	((0.5,0.7,0.7,0.9;1,1), (0.5,0.7,0.7,0.9;1,1))
	$\tilde{\tilde{A}}_2$	((0.3,0.7,0.7,0.9;1,1), (0.3,0.7,0.7,0.9;1,1))
	$A^{\approx}_3$	((0.3,0.4,0.7,0.9;1,1), (0.3,0.4,0.7,0.9;1,1))
Set 6	$\tilde{\tilde{A}}_1$	((0.3,0.5,0.8,0.9;1,1), (0.3,0.5,0.8,0.9;1,1))
	$\tilde{\tilde{A}}_2$	((0.3,0.5,0.5,0.9;1,1), (0.3,0.5,0.5,0.9;1,1))
	$\tilde{\tilde{A}}_3$	((0.3,0.5,0.5,0.7;1,1), (0.3,0.5,0.5,0.7;1,1))
Set 7	$\tilde{\tilde{A}}_1$	((0.2,0.5,0.5,0.8;1,1), (0.2,0.5,0.5,0.8;1,1))
	$\tilde{\tilde{A}}_2$	((0.4,0.5,0.5,0.6;1,1), (0.4,0.5,0.5,0.6;1,1))
Set 8	$\tilde{\tilde{A}}_1$	((0,0.4,0.6,0.8;1,1), (0,0.4,0.6,0.8;1,1))
	$\tilde{\tilde{A}}_2$	((0.2,0.5,0.5,0.9;1,1), (0.2,0.5,0.5,0.9;1,1))
	$\tilde{\tilde{A}}_3$	((0.2,0.6,0.7,0.8;1,1), (0.2,0.6,0.7,0.8;1,1))
Set 9	$\tilde{\tilde{A}}_1$	((0,0.2,0.2,0.4;1,1), (0,0.2,0.2,0.4;1,1))
	$\tilde{\tilde{A}}_2$	((0.6,0.8,0.8,1;0.8,0.8), (0.6,0.8,0.8,1;0.8,0.8))
Set 10	$\tilde{\tilde{A}}_1$	((0.4,0.6,0.6,0.8;1,1), (0.4,0.6,0.6,0.8;1,1))
	$\tilde{\tilde{A}}_2$	((0.8,0.9,0.9,1;0.2,0.2), (0.8,0.9,0.9,1;0.2,0.2))
Set 11	$\tilde{\tilde{A}}_1$	((0,0.2,0.2,0.4;0.2,0.2), (0,0.2,0.2,0.4;0.2,0.2))
	$\tilde{\tilde{A}}_2$	((0.6,0.8,0.8,1;1,1), (0.6,0.8,0.8,1;1,1))
Set 12	$\tilde{\tilde{A}}_1$	((0.2,0.6,0.6,1;1,1), (0.2,0.6,0.6,1;1,1))
	$\tilde{\tilde{A}}_2$	((0.2,0.6,0.6,1;0.2,0.2), (0.2,0.6,0.6,1;0.2,0.2))
Set 13	$\tilde{\tilde{A}}_1$	((0.6,1,1,1,;1,1), (0.6,1,1,1,;1,1))
	$\tilde{\tilde{A}}_2$	((0.8,1,1,1,;0.2,0.2), (0.8,1,1,1,;0.2,0.2))

Table 2 A comparison of the ranking results for different methods

Sets of fuzzy sets		Lee and Li's method [50]		Baas and Kwakernaak's method [51]	The method of Chang et al. [52]		Chen and Lee's method [42]	The proposed method		
		Uniform	Proportional		α=0.1, β=0.9	α=0.5, β=0.5		α=0.5	α=0.8	α=1
Set 1	$\tilde{\tilde{A}}_1$	0.58	0.54	0.84	0.417	0.519	0.52	1.000	1.000	1.000
	$\tilde{\tilde{A}}_2$	0.55	0.59	1	0.462	0.544	0.48	0.993	0.988	0.985
Set 2	$\tilde{\tilde{A}}_1$	0.41	0.38	0.82	0.158	0.45	0.4	0.954	0.927	0.908
	$\tilde{\tilde{A}}_2$	0.6	0.60	1	0.554	0.55	0.6	0.969	0.988	1
Set 3	$\tilde{\tilde{A}}_1$	0.41	0.38	0.66	0.158	0.45	0.36	0.936	0.897	0.872
	$\tilde{\tilde{A}}_2$	0.70	0.70	1	0.644	0.6	0.64	0.969	0.988	1
Set 4	$\tilde{\tilde{A}}_1$	0.77	0.80	1	0.878	0.65	0.39	0.583	0.583	0.583
	$\tilde{\tilde{A}}_2$	0.70	0.70	0.74	0.788	0.6	0.33	0.580	0.577	0.576
	$\tilde{\tilde{A}}_3$	0.63	0.60	0.6	0.698	0.55	0.28	0.571	0.564	0.559
Set 5	$\tilde{\tilde{A}}_1$	0.70	0.70	1	0.752	0.6	0.4	0.571	0.579	0.583
	$\tilde{\tilde{A}}_2$	0.63	0.65	1	0.743	0.575	0.32	0.567	0.572	0.575
	$\tilde{\tilde{A}}_3$	0.58	0.57	1	0.73	0.538	0.28	0.572	0.564	0.560
Set 6	$\tilde{\tilde{A}}_1$	0.62	0.63	1	0.775	0.563	0.39	0.583	0.583	0.583
	$\tilde{\tilde{A}}_2$	0.57	0.55	1	0.653	0.525	0.34	0.568	0.572	0.576
	$\tilde{\tilde{A}}_3$	0.50	0.50	1	0.572	0.5	0.27	0.557	0.556	0.555
Set 7	$\tilde{\tilde{A}}_1$	0.50	0.50	1	0.608	0.5	0.5	1	1	1
	$\tilde{\tilde{A}}_2$	0.50	0.50	1	0.536	0.5	0.5	1	1	1
Set 8	$\tilde{\tilde{A}}_1$	0.44	0.46	1	0.635	0.475	0.28	0.566	0.555	0.548
	$\tilde{\tilde{A}}_2$	0.53	0.53	0.88	0.649	0.513	0.35	0.568	0.575	0.580
	$\tilde{\tilde{A}}_3$	0.56	0.58	1	0.694	0.538	0.37	0.581	0.583	0.583
Set 9	$\tilde{\tilde{A}}_1$	0.20	0.20	0	0.158	0.35	0.28	0.85	0.76	0.7
	$\tilde{\tilde{A}}_2$	0.80	0.80	0.8	0.688	0.6	0.72	0.972	0.989	1
Set 10	$\tilde{\tilde{A}}_1$	0.60	0.60	0	0.518	0.55	0.49	0.95	0.92	0.9
	$\tilde{\tilde{A}}_2$	0.90	0.90	0.2	0.784	0.5	0.51	0.833	0.933	1
Set 11	$\tilde{\tilde{A}}_1$	0.20	0.20	0	0.118	0.15	0.25	0.683	0.693	0.7
	$\tilde{\tilde{A}}_2$	0.80	0.80	0.2	0.698	0.65	0.75	1	1	1
Set 12	$\tilde{\tilde{A}}_1$	0.60	0.60	0.2	0.446	0.55	0.63	1	1	1
	$\tilde{\tilde{A}}_2$	0.60	0.60	0.2	0.406	0.35	0.37	0.833	0.933	1
Set 13	$\tilde{\tilde{A}}_1$	0.87	0.90	0.2	0.932	0.7	0.63	0.991	0.985	0.981
	$\tilde{\tilde{A}}_2$	0.95	0.95	0.2	0.901	0.525	0.37	0.833	0.933	1

between fuzzy sets, whereas the same results can be obtained from Chen and Lee's method [42], the method of Chang et al. [52] and the proposed method in α=0.5 and α=0.8.

6. According to Set 13, as shown in Table 2, the result of the proposed method in α=0.5 and α=0.8 is consistent with Chen and Lee's method [42] and the method of Chang et al. [52], while in α=1 proposed method gets result that consistent with Lee and Li's method [50]. Baas and Kwakernaak's method [51] cannot distinguish between fuzzy sets of this set.

With overall view of Table 2, we can say that the proposed method in α=0.8 gets the results that are consistent with other methods. Thus, α=0.8 is chosen as a good value for α in this study.

The simplicity of the proposed method is the major advantage of it. The computational steps of the proposed method are less than those of other methods, while it gets the same results in most cases.

5 A new method fuzzy multiple criteria group decision-making based on COPRAS method

The COPRAS method is an MCDM technique that was introduced by Zavadskas et al. [48]. This method determines a solution with respect to the positive-ideal solution and the negative-ideal solution and therefore can be considered as a compromising MCDM method. Originally, the COPRAS method has been developed for decision-making under deterministic conditions. Since uncertainty is an inevitable feature of decision-making, an extended form of the COPRAS method is proposed in this study that can be used for group decision-making problems in uncertain conditions where such uncertainties are taken into account by means of interval type-2 fuzzy sets.

In this section, a new method for dealing with multiple criteria group decision-making problems is proposed. This method is based on the presented arithmetic operations between interval type-2 fuzzy sets, the proposed fuzzy ranking method and the COPRAS method. Let L be a set of alternatives, $L=\{l_1,l_2,\ldots l_n\}$, and let R be a set of criteria, $R=\{r_1,r_2,\ldots r_m,\}$. Assume that there are k decision makers $D_1,D_2,\ldots,$ andD_k. The proposed method is now shown as follows:

Step 1: Construct the decision matrix M_p of the pth decision maker, shown as follows:

$$M_p=\left[\tilde{\tilde{X}}_{ij}^p\right]_{n\times m}=\begin{bmatrix}\tilde{\tilde{X}}_{11}^p & \tilde{\tilde{X}}_{12}^p & \cdots & \tilde{\tilde{X}}_{1m}^p\\ \tilde{\tilde{X}}_{21}^p & \tilde{\tilde{X}}_{22}^p & \cdots & \tilde{\tilde{X}}_{2m}^p\\ \vdots & \vdots & \vdots & \vdots\\ \tilde{\tilde{X}}_{n1}^p & \tilde{\tilde{X}}_{n2}^p & \cdots & \tilde{\tilde{X}}_{nm}^p\end{bmatrix}, \quad (29)$$

where $\tilde{\tilde{X}}_{ij}^p$ denotes the performance value of alternative l_i on criterion r_j assigned by the pth decision maker, $1\le i\le n$, $1\le j\le m$, $1\le p\le k$.

Step 2: Construct the average decision matrix $\overline{Y}$, shown as follows:

$$\tilde{\tilde{X}}_{ij}=\left(\left(\tilde{\tilde{X}}_{ij}^1\oplus\tilde{\tilde{X}}_{ij}^2\oplus\ldots\oplus\tilde{\tilde{X}}_{ij}^k\right)/k\right), \quad (30)$$

$$\overline{Y}=\left[\tilde{\tilde{X}}_{ij}\right]_{n\times m}, \quad (31)$$

where $\tilde{\tilde{X}}_{ij}$ denotes the average performance value of alternative l_i on criterion r_{j}, $1\le i\le n$, $1\le j\le m$.

Step 3: Construct the weighting matrix W_p of the criteria of the pth decision maker, shown as follows:

$$W_p=\left[\tilde{\tilde{w}}_j^p\right]_{m\times 1}=\begin{bmatrix}\tilde{\tilde{w}}_1^p\\ \tilde{\tilde{w}}_2^p\\ \vdots\\ \tilde{\tilde{w}}_m^p\end{bmatrix}, \quad (32)$$

where $\tilde{\tilde{w}}_j^p$ denotes the weight of criterion r_j assigned by the pth decision maker, $1\le j\le m$, $1\le p\le k$.

Step 4: Construct the average weighting matrix $\overline{W}$, shown as follows:

$$\tilde{\tilde{w}}_j=\left(\left(\tilde{\tilde{w}}_j^1\oplus\tilde{\tilde{w}}_j^2\oplus\ldots\oplus\tilde{\tilde{w}}_j^k\right)/k\right), \quad (33)$$

$$\overline{W}=\left[\tilde{\tilde{w}}_j\right]_{m\times 1}. \quad (34)$$

Step 5: Normalize the average decision matrix $\overline{Y}$ and construct the normalized matrix N using the following equations:

$$\tilde{\tilde{v}}_j=\left(\tilde{\tilde{X}}_{1j}\oplus\tilde{\tilde{X}}_{2j}\oplus\ldots\oplus\tilde{\tilde{X}}_{nj}\right), \quad (35)$$

$$\tilde{\tilde{n}}_{ij}=\tilde{\tilde{X}}_{ij}\oslash\tilde{\tilde{v}}_j, \quad (36)$$

$$N=\left[\tilde{\tilde{n}}_{ij}\right]_{nxm}, \quad (37)$$

where $1\le i\le n$, $1\le j\le m$.

Step 6: Determine the weighted normalized decision matrix E, shown as follows:

$$\tilde{\tilde{e}}_{ij}=\tilde{\tilde{n}}_{ij}\otimes\tilde{\tilde{w}}_j, \quad (38)$$

$$E=\left[\tilde{\tilde{e}}_{ij}\right]_{n\times m}. \quad (39)$$

Step 7: Calculate the sum of weighted normalized values for both the beneficial criteria $\tilde{\tilde{S}}_{+i}$ and non-beneficial criteria $\tilde{\tilde{S}}_{-i}$, shown as follows:

$$\tilde{\tilde{S}}_{+i}=\left(\tilde{\tilde{e}}_{+i1}\oplus\tilde{\tilde{e}}_{+i2}\oplus\ldots\oplus\tilde{\tilde{e}}_{+im}\right), \quad (40)$$

$$\tilde{\tilde{S}}_{-i}=\left(\tilde{\tilde{e}}_{-i1}\oplus\tilde{\tilde{e}}_{-i2}\oplus\ldots\oplus\tilde{\tilde{e}}_{-im}\right), \quad (41)$$

where $\tilde{\tilde{e}}_{+ij}$ and $\tilde{\tilde{e}}_{-ij}$ are the weighted normalized values for the beneficial and non-beneficial criteria, respectively. The greater value of $\tilde{\tilde{S}}_{+i}$, the better is the alternative, and the lower the value of $\tilde{\tilde{S}}_{-i}$, the better is the alternative. In other words, the $\tilde{\tilde{S}}_{+i}$ and

$\tilde{\tilde{S}}_{-i}$ Values describe the degree of goals achieved by each alternative.

Step 8: Determine the ranking values for both $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ ($1\leq i\leq n$) with the equations presented in Section 4.

Step 9: Determine the relative significances Q_i of the alternatives, shown as follows:

$$Q_i = \mathrm{Rank}\left(\tilde{\tilde{S}}_{+i}\right) + \frac{\mathrm{Rank}_{\min}\left(\tilde{\tilde{S}}_{-i}\right)\sum_{i=1}^{n}\left(\mathrm{Rank}\left(\tilde{\tilde{S}}_{-i}\right)\right)}{\mathrm{Rank}\left(\tilde{\tilde{S}}_{-i}\right)\sum_{i=1}^{n}\left(\mathrm{Rank}_{\min}\left(\tilde{\tilde{S}}_{-i}\right)\Big/\mathrm{Rank}\left(\tilde{\tilde{S}}_{-i}\right)\right)}, \quad (42)$$

where $\mathrm{Rank}_{\min}\left(\tilde{\tilde{S}}_{-i}\right)$ denotes the minimum value of $\mathrm{Rank}\left(\tilde{\tilde{S}}_{-i}\right)$. The relative significance value describes the degree of satisfaction attained by an alternative. The above formula can be written as follows:

$$Q_i = \mathrm{Rank}\left(\tilde{\tilde{S}}_{+i}\right) + \frac{\sum_{i=1}^{n}\left(\mathrm{Rank}\left(\tilde{\tilde{S}}_{-i}\right)\right)}{\mathrm{Rank}\left(\tilde{\tilde{S}}_{-i}\right)\sum_{i=1}^{n}\left(1\Big/\mathrm{Rank}\left(\tilde{\tilde{S}}_{-i}\right)\right)}. \quad (43)$$

Step 10: Calculate the quantitative utility U_i. The degree of an alternative's utility is determined by comparing the relative significances of all alternatives with the most efficient one, shown as follows:

$$U_i = \left(\frac{Q_i}{Q_{\max}}\right) \times 100, \quad (44)$$

where $Q_{\max}$ is the maximum relative significance value. The larger the value of U_i, the more preference of the alternative l_i, where $1\leq i\leq n$.

Table 3 Linguistic terms and their corresponding interval type-2 fuzzy sets

Linguistic terms	Interval type-2 fuzzy sets
Very low (VL)	((0,0,0,0.1;1,1), (0,0,0,0.05;0.9,0.9))
Low (L)	((0,0.1,0.15,0.3;1,1), (0.05,0.1,0.15,0.2;0.9,0.9))
Medium low (ML)	((0.1,0.3,0.35,0.5;1,1), (0.2,0.3,0.35,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.55,0.7;1,1), (0.4,0.5,0.55,0.6;0.9,0.9))
Medium high (MH)	((0.5,0.7,0.75,0.9;1,1), (0.6,0.7,0.75,0.8;0.9,0.9))
High (H)	((0.7,0.85,0.9,1;1,1), (0.8,0.85,0.9,0.95;0.9,0.9))
Very high (VH)	((0.9,1,1,1;1,1), (0.95,1,1,1;0.9,0.9))

Table 4 Weights of the criteria evaluated by the decision makers

Criteria	Decision makers		
	D_1	D_2	D_3
Responsiveness	VH	H	VH
Cost	MH	M	M
Defect rate	H	H	MH
Delivery reliability	VH	MH	VH
Flexibility	ML	L	M

Table 5 Evaluating values of alternatives of the decision makers with respect to different criteria

Criteria	Suppliers	Decision makers		
		D_1	D_2	D_3
Responsiveness	l_1	VL	L	ML
	l_2	H	MH	VH
	l_3	H	MH	MH
	l_4	M	M	ML
	l_5	H	H	MH
	l_6	VH	VH	H
Cost	l_1	VH	H	H
	l_2	ML	ML	VL
	l_3	M	M	ML
	l_4	ML	ML	ML
	l_5	VL	L	L
	l_6	VL	VL	L
Defect rate	l_1	M	ML	M
	l_2	L	L	VL
	l_3	ML	L	L
	l_4	VL	VL	L
	l_5	M	ML	M
	l_6	L	VL	VL
Delivery reliability	l_1	L	ML	MH
	l_2	H	H	VH
	l_3	MH	M	M
	l_4	H	MH	H
	l_5	M	ML	MH
	l_6	VH	VH	MH
Flexibility	l_1	VL	L	ML
	l_2	MH	MH	H
	l_3	M	M	ML
	l_4	M	MH	M
	l_5	H	MH	M
	l_6	VH	MH	MH

Table 6 The average decision matrix $(\overline{Y})$

	$\widetilde{X}_{ij}^{U}$						$\widetilde{X}_{ij}^{L}$					
	x_{1ij}^{U}	x_{2ij}^{U}	x_{3ij}^{U}	x_{4ij}^{U}	$H_1(\widetilde{X}_{ij}^{U})$	$H_2(\widetilde{X}_{ij}^{U})$	x_{1ij}^{L}	x_{2ij}^{L}	x_{3ij}^{L}	x_{4ij}^{L}	$H_1(\widetilde{X}_{ij}^{L})$	$H_2(\widetilde{X}_{ij}^{L})$
$\tilde{\tilde{X}}_{11}$	0.03	0.13	0.17	0.30	1.00	1.00	0.08	0.13	0.17	0.22	0.90	0.90
$\tilde{\tilde{X}}_{12}$	0.70	0.85	0.88	0.97	1.00	1.00	0.78	0.85	0.88	0.92	0.90	0.90
$\tilde{\tilde{X}}_{13}$	0.57	0.75	0.80	0.93	1.00	1.00	0.67	0.75	0.80	0.85	0.90	0.90
$\tilde{\tilde{X}}_{14}$	0.23	0.43	0.48	0.63	1.00	1.00	0.33	0.43	0.48	0.53	0.90	0.90
$\tilde{\tilde{X}}_{15}$	0.63	0.80	0.85	0.97	1.00	1.00	0.73	0.80	0.85	0.90	0.90	0.90
$\tilde{\tilde{X}}_{16}$	0.83	0.95	0.97	1.00	1.00	1.00	0.90	0.95	0.97	0.98	0.90	0.90
$\tilde{\tilde{X}}_{21}$	0.77	0.90	0.93	1.00	1.00	1.00	0.85	0.90	0.93	0.97	0.90	0.90
$\tilde{\tilde{X}}_{22}$	0.07	0.20	0.23	0.37	1.00	1.00	0.13	0.20	0.23	0.28	0.90	0.90
$\tilde{\tilde{X}}_{23}$	0.23	0.43	0.48	0.63	1.00	1.00	0.33	0.43	0.48	0.53	0.90	0.90
$\tilde{\tilde{X}}_{24}$	0.10	0.30	0.35	0.50	1.00	1.00	0.20	0.30	0.35	0.40	0.90	0.90
$\tilde{\tilde{X}}_{25}$	0.00	0.07	0.10	0.23	1.00	1.00	0.03	0.07	0.10	0.15	0.90	0.90
$\tilde{\tilde{X}}_{26}$	0.00	0.03	0.05	0.17	1.00	1.00	0.02	0.03	0.05	0.10	0.90	0.90
$\tilde{\tilde{X}}_{31}$	0.23	0.43	0.48	0.63	1.00	1.00	0.33	0.43	0.48	0.53	0.90	0.90
$\tilde{\tilde{X}}_{32}$	0.00	0.07	0.10	0.23	1.00	1.00	0.03	0.07	0.10	0.15	0.90	0.90
$\tilde{\tilde{X}}_{33}$	0.03	0.17	0.22	0.37	1.00	1.00	0.10	0.17	0.22	0.27	0.90	0.90
$\tilde{\tilde{X}}_{34}$	0.00	0.03	0.05	0.17	1.00	1.00	0.02	0.03	0.05	0.10	0.90	0.90
$\tilde{\tilde{X}}_{35}$	0.23	0.43	0.48	0.63	1.00	1.00	0.33	0.43	0.48	0.53	0.90	0.90
$\tilde{\tilde{X}}_{36}$	0.00	0.03	0.05	0.17	1.00	1.00	0.02	0.03	0.05	0.10	0.90	0.90
$\tilde{\tilde{X}}_{41}$	0.20	0.37	0.42	0.57	1.00	1.00	0.28	0.37	0.42	0.47	0.90	0.90
$\tilde{\tilde{X}}_{42}$	0.77	0.90	0.93	1.00	1.00	1.00	0.85	0.90	0.93	0.97	0.90	0.90
$\tilde{\tilde{X}}_{43}$	0.37	0.57	0.62	0.77	1.00	1.00	0.47	0.57	0.62	0.67	0.90	0.90
$\tilde{\tilde{X}}_{44}$	0.63	0.80	0.85	0.97	1.00	1.00	0.73	0.80	0.85	0.90	0.90	0.90
$\tilde{\tilde{X}}_{45}$	0.30	0.50	0.55	0.70	1.00	1.00	0.40	0.50	0.55	0.60	0.90	0.90
$\tilde{\tilde{X}}_{46}$	0.77	0.90	0.92	0.97	1.00	1.00	0.83	0.90	0.92	0.93	0.90	0.90
$\tilde{\tilde{X}}_{51}$	0.03	0.13	0.17	0.30	1.00	1.00	0.08	0.13	0.17	0.22	0.90	0.90
$\tilde{\tilde{X}}_{52}$	0.57	0.75	0.80	0.93	1.00	1.00	0.67	0.75	0.80	0.85	0.90	0.90
$\tilde{\tilde{X}}_{53}$	0.23	0.43	0.48	0.63	1.00	1.00	0.33	0.43	0.48	0.53	0.90	0.90
$\tilde{\tilde{X}}_{54}$	0.37	0.57	0.62	0.77	1.00	1.00	0.47	0.57	0.62	0.67	0.90	0.90
$\tilde{\tilde{X}}_{55}$	0.50	0.68	0.73	0.87	1.00	1.00	0.60	0.68	0.73	0.78	0.90	0.90
$\tilde{\tilde{X}}_{56}$	0.63	0.80	0.83	0.93	1.00	1.00	0.72	0.80	0.83	0.87	0.90	0.90

Table 7 The average weighting matrix $(\overline{W})$

	$\widetilde{w}_{ij}^{U}$						$\widetilde{w}_{ij}^{L}$					
	w_{1ij}^{U}	w_{2ij}^{U}	w_{3ij}^{U}	w_{4ij}^{U}	$H_1(\widetilde{w}_{ij}^{U})$	$H_2(\widetilde{w}_{ij}^{U})$	w_{1ij}^{L}	w_{2ij}^{L}	w_{3ij}^{L}	w_{4ij}^{L}	$H_1(\widetilde{w}_{ij}^{L})$	$H_2(\widetilde{w}_{ij}^{L})$
$\tilde{\tilde{w}}_1$	0.83	0.95	0.97	1.00	1.00	1.00	0.90	0.95	0.97	0.98	0.90	0.90
$\tilde{\tilde{w}}_2$	0.37	0.57	0.62	0.77	1.00	1.00	0.47	0.57	0.62	0.67	0.90	0.90
$\tilde{\tilde{w}}_3$	0.63	0.80	0.85	0.97	1.00	1.00	0.73	0.80	0.85	0.90	0.90	0.90
$\tilde{\tilde{w}}_4$	0.77	0.90	0.92	0.97	1.00	1.00	0.83	0.90	0.92	0.93	0.90	0.90
$\tilde{\tilde{w}}_5$	0.13	0.30	0.35	0.50	1.00	1.00	0.22	0.30	0.35	0.40	0.90	0.90

6 Illustrative example

A high-technology manufacturing company desires to select a suitable material supplier to purchase the key components of new products. After initial screening, six candidates ($l_1, l_2, l_3, l_4, l_5,$ and l_6) remain for further assessment. A committee of three decision makers, $D_1, D_2,$ and D_3, has been formed to select the most appropriate supplier.

Five criteria are considered:

1. Responsiveness
2. Cost
3. Defect rate
4. Delivery reliability
5. Flexibility

It should be noted that, "Responsiveness", "Delivery reliability" and "Flexibility" are the beneficial criteria and, "Cost" and "Defect rate" are the non-beneficial criteria. Decision makers use the linguistic terms shown in Table 3 to assess the importance of the criteria and evaluate the ratings of candidates with respect to each criterion. The importance weights of the criteria determined by these decision makers are shown in Table 4 and the ratings of the six candidates given by the decision makers under the various criteria are shown in Table 5.

Table 8 The normalized matrix (N)

	$\tilde{n}_{ij}^U$						$\tilde{n}_{ij}^L$					
	n_{1ij}^U	n_{2ij}^U	n_{3ij}^U	n_{4ij}^U	$H_1(\tilde{n}_{ij}^U)$	$H_2(\tilde{n}_{ij}^U)$	n_{1ij}^L	n_{2ij}^L	n_{3ij}^L	n_{4ij}^L	$H_1(\tilde{n}_{ij}^L)$	$H_2(\tilde{n}_{ij}^L)$
$\tilde{n}_{11}$	0.01	0.03	0.04	0.10	1.00	1.00	0.02	0.03	0.04	0.06	0.90	0.90
$\tilde{n}_{12}$	0.15	0.20	0.23	0.32	1.00	1.00	0.18	0.20	0.23	0.26	0.90	0.90
$\tilde{n}_{13}$	0.12	0.18	0.20	0.31	1.00	1.00	0.15	0.18	0.20	0.24	0.90	0.90
$\tilde{n}_{14}$	0.05	0.10	0.12	0.21	1.00	1.00	0.08	0.10	0.12	0.15	0.90	0.90
$\tilde{n}_{15}$	0.13	0.19	0.22	0.32	1.00	1.00	0.17	0.19	0.22	0.26	0.90	0.90
$\tilde{n}_{16}$	0.17	0.23	0.25	0.33	1.00	1.00	0.20	0.23	0.25	0.28	0.90	0.90
$\tilde{n}_{21}$	0.26	0.42	0.48	0.86	1.00	1.00	0.35	0.42	0.48	0.62	0.90	0.90
$\tilde{n}_{22}$	0.02	0.09	0.12	0.31	1.00	1.00	0.05	0.09	0.12	0.18	0.90	0.90
$\tilde{n}_{23}$	0.08	0.20	0.25	0.54	1.00	1.00	0.14	0.20	0.25	0.34	0.90	0.90
$\tilde{n}_{24}$	0.03	0.14	0.18	0.43	1.00	1.00	0.08	0.14	0.18	0.26	0.90	0.90
$\tilde{n}_{25}$	0.00	0.03	0.05	0.20	1.00	1.00	0.01	0.03	0.05	0.10	0.90	0.90
$\tilde{n}_{26}$	0.00	0.02	0.03	0.14	1.00	1.00	0.01	0.02	0.03	0.06	0.90	0.90
$\tilde{n}_{31}$	0.11	0.31	0.41	1.27	1.00	1.00	0.20	0.31	0.41	0.64	0.90	0.90
$\tilde{n}_{32}$	0.00	0.05	0.09	0.47	1.00	1.00	0.02	0.05	0.09	0.18	0.90	0.90
$\tilde{n}_{33}$	0.02	0.12	0.19	0.73	1.00	1.00	0.06	0.12	0.19	0.32	0.90	0.90
$\tilde{n}_{34}$	0.00	0.02	0.04	0.33	1.00	1.00	0.01	0.02	0.04	0.12	0.90	0.90
$\tilde{n}_{35}$	0.11	0.31	0.41	1.27	1.00	1.00	0.20	0.31	0.41	0.64	0.90	0.90
$\tilde{n}_{36}$	0.00	0.02	0.04	0.33	1.00	1.00	0.01	0.02	0.04	0.12	0.90	0.90
$\tilde{n}_{41}$	0.04	0.09	0.10	0.19	1.00	1.00	0.06	0.09	0.10	0.13	0.90	0.90
$\tilde{n}_{42}$	0.15	0.21	0.23	0.33	1.00	1.00	0.19	0.21	0.23	0.27	0.90	0.90
$\tilde{n}_{43}$	0.07	0.13	0.15	0.25	1.00	1.00	0.10	0.13	0.15	0.19	0.90	0.90
$\tilde{n}_{44}$	0.13	0.19	0.21	0.32	1.00	1.00	0.16	0.19	0.21	0.25	0.90	0.90
$\tilde{n}_{45}$	0.06	0.12	0.14	0.23	1.00	1.00	0.09	0.12	0.14	0.17	0.90	0.90
$\tilde{n}_{46}$	0.15	0.21	0.23	0.32	1.00	1.00	0.18	0.21	0.23	0.26	0.90	0.90
$\tilde{n}_{51}$	0.01	0.04	0.05	0.13	1.00	1.00	0.02	0.04	0.05	0.08	0.90	0.90
$\tilde{n}_{52}$	0.13	0.21	0.24	0.40	1.00	1.00	0.17	0.21	0.24	0.30	0.90	0.90
$\tilde{n}_{53}$	0.05	0.12	0.14	0.27	1.00	1.00	0.09	0.12	0.14	0.19	0.90	0.90
$\tilde{n}_{54}$	0.08	0.16	0.18	0.33	1.00	1.00	0.12	0.16	0.18	0.23	0.90	0.90
$\tilde{n}_{55}$	0.11	0.19	0.22	0.37	1.00	1.00	0.15	0.19	0.22	0.27	0.90	0.90
$\tilde{n}_{56}$	0.14	0.22	0.25	0.40	1.00	1.00	0.18	0.22	0.25	0.30	0.90	0.90

Table 9 The weighted normalized decision matrix (*E*)

	$\tilde{e}_{ij}^U$						$\tilde{e}_{ij}^L$					
	e_{1ij}^U	e_{2ij}^U	e_{3ij}^U	e_{4ij}^U	$H_1(\tilde{e}_{ij}^U)$	$H_2(\tilde{e}_{ij}^U)$	e_{1ij}^L	e_{2ij}^L	e_{3ij}^L	e_{4ij}^L	$H_1(\tilde{e}_{ij}^L)$	$H_2(\tilde{e}_{ij}^L)$
$\tilde{\tilde{e}}_{11}$	0.01	0.03	0.04	0.10	1.00	1.00	0.02	0.03	0.04	0.06	0.90	0.90
$\tilde{\tilde{e}}_{12}$	0.12	0.19	0.22	0.32	1.00	1.00	0.16	0.19	0.22	0.26	0.90	0.90
$\tilde{\tilde{e}}_{13}$	0.12	0.19	0.22	0.32	1.00	1.00	0.16	0.19	0.22	0.26	0.90	0.90
$\tilde{\tilde{e}}_{14}$	0.04	0.10	0.12	0.21	1.00	1.00	0.07	0.10	0.12	0.15	0.90	0.90
$\tilde{\tilde{e}}_{15}$	0.11	0.18	0.21	0.32	1.00	1.00	0.15	0.18	0.21	0.25	0.90	0.90
$\tilde{\tilde{e}}_{16}$	0.14	0.22	0.24	0.33	1.00	1.00	0.18	0.22	0.24	0.28	0.90	0.90
$\tilde{\tilde{e}}_{21}$	0.10	0.24	0.30	0.66	1.00	1.00	0.16	0.24	0.30	0.41	0.90	0.90
$\tilde{\tilde{e}}_{22}$	0.01	0.05	0.07	0.24	1.00	1.00	0.03	0.05	0.07	0.12	0.90	0.90
$\tilde{\tilde{e}}_{23}$	0.03	0.11	0.15	0.42	1.00	1.00	0.06	0.11	0.15	0.23	0.90	0.90
$\tilde{\tilde{e}}_{24}$	0.01	0.08	0.11	0.33	1.00	1.00	0.04	0.08	0.11	0.17	0.90	0.90
$\tilde{\tilde{e}}_{25}$	0.00	0.02	0.03	0.15	1.00	1.00	0.01	0.02	0.03	0.06	0.90	0.90
$\tilde{\tilde{e}}_{26}$	0.00	0.01	0.02	0.11	1.00	1.00	0.00	0.01	0.02	0.04	0.90	0.90
$\tilde{\tilde{e}}_{31}$	0.07	0.25	0.35	1.22	1.00	1.00	0.15	0.25	0.35	0.58	0.90	0.90
$\tilde{\tilde{e}}_{32}$	0.00	0.04	0.07	0.45	1.00	1.00	0.01	0.04	0.07	0.16	0.90	0.90
$\tilde{\tilde{e}}_{33}$	0.01	0.10	0.16	0.71	1.00	1.00	0.04	0.10	0.16	0.29	0.90	0.90
$\tilde{\tilde{e}}_{34}$	0.00	0.02	0.04	0.32	1.00	1.00	0.01	0.02	0.04	0.11	0.90	0.90
$\tilde{\tilde{e}}_{35}$	0.07	0.25	0.35	1.22	1.00	1.00	0.15	0.25	0.35	0.58	0.90	0.90
$\tilde{\tilde{e}}_{36}$	0.00	0.02	0.04	0.32	1.00	1.00	0.01	0.02	0.04	0.11	0.90	0.90
$\tilde{\tilde{e}}_{41}$	0.03	0.08	0.09	0.18	1.00	1.00	0.05	0.08	0.09	0.12	0.90	0.90
$\tilde{\tilde{e}}_{42}$	0.12	0.19	0.21	0.32	1.00	1.00	0.16	0.19	0.21	0.25	0.90	0.90
$\tilde{\tilde{e}}_{43}$	0.06	0.12	0.14	0.24	1.00	1.00	0.09	0.12	0.14	0.17	0.90	0.90
$\tilde{\tilde{e}}_{44}$	0.10	0.17	0.19	0.31	1.00	1.00	0.13	0.17	0.19	0.24	0.90	0.90
$\tilde{\tilde{e}}_{45}$	0.05	0.11	0.12	0.22	1.00	1.00	0.07	0.11	0.12	0.16	0.90	0.90
$\tilde{\tilde{e}}_{46}$	0.12	0.19	0.21	0.31	1.00	1.00	0.15	0.19	0.21	0.24	0.90	0.90
$\tilde{\tilde{e}}_{51}$	0.00	0.01	0.02	0.06	1.00	1.00	0.00	0.01	0.02	0.03	0.90	0.90
$\tilde{\tilde{e}}_{52}$	0.02	0.06	0.08	0.20	1.00	1.00	0.04	0.06	0.08	0.12	0.90	0.90
$\tilde{\tilde{e}}_{53}$	0.01	0.04	0.05	0.14	1.00	1.00	0.02	0.04	0.05	0.07	0.90	0.90
$\tilde{\tilde{e}}_{54}$	0.01	0.05	0.06	0.16	1.00	1.00	0.03	0.05	0.06	0.09	0.90	0.90
$\tilde{\tilde{e}}_{55}$	0.02	0.06	0.08	0.19	1.00	1.00	0.03	0.06	0.08	0.11	0.90	0.90
$\tilde{\tilde{e}}_{56}$	0.02	0.07	0.09	0.20	1.00	1.00	0.04	0.07	0.09	0.12	0.90	0.90

Table 10 The calculated values for $\tilde{\tilde{S}}_{+i}$

	$\tilde{s}_{+i}^U$						$\tilde{s}_{+i}^L$					
	s_{+i1}^U	s_{+i2}^U	s_{+i3}^U	s_{+i4}^U	$H_1(\tilde{s}_{+i}^U)$	$H_2(\tilde{s}_{+i}^U)$	s_{+i1}^L	s_{+i2}^L	s_{+i3}^L	s_{+i4}^L	$H_1(\tilde{s}_{+i}^L)$	$H_2(\tilde{s}_{+i}^L)$
$\tilde{\tilde{S}}_{+1}$	0.04	0.12	0.15	0.34	1.00	1.00	0.07	0.12	0.15	0.21	0.90	0.90
$\tilde{\tilde{S}}_{+2}$	0.26	0.45	0.51	0.84	1.00	1.00	0.35	0.45	0.51	0.63	0.90	0.90
$\tilde{\tilde{S}}_{+3}$	0.19	0.35	0.41	0.70	1.00	1.00	0.26	0.35	0.41	0.51	0.90	0.90
$\tilde{\tilde{S}}_{+4}$	0.15	0.31	0.38	0.68	1.00	1.00	0.23	0.31	0.38	0.48	0.90	0.90
$\tilde{\tilde{S}}_{+5}$	0.17	0.34	0.41	0.73	1.00	1.00	0.26	0.34	0.41	0.52	0.90	0.90
$\tilde{\tilde{S}}_{+6}$	0.28	0.47	0.53	0.84	1.00	1.00	0.38	0.47	0.53	0.64	0.90	0.90

In this section, the proposed method is applied to solve this problem. The computational procedure is summarized as follows:

Step 1: Based on Table 5 and Eq. 29, the decision matrices $M_1, M_2,$ and M_3 of the alternatives (suppliers) $l_1, l_2, l_3, l_4, l_5,$ and l_6 are constructed respectively:

$$M_1 = \begin{bmatrix} \text{VL} & \text{H} & \text{H} & \text{M} & \text{H} & \text{VH} \\ \text{VH} & \text{ML} & \text{M} & \text{ML} & \text{VL} & \text{VL} \\ \text{M} & \text{L} & \text{ML} & \text{VL} & \text{M} & \text{L} \\ \text{L} & \text{H} & \text{MH} & \text{H} & \text{M} & \text{VH} \\ \text{VL} & \text{MH} & \text{M} & \text{M} & \text{H} & \text{VH} \end{bmatrix},$$

$$M_2 = \begin{bmatrix} \text{L} & \text{MH} & \text{MH} & \text{M} & \text{H} & \text{VH} \\ \text{H} & \text{ML} & \text{M} & \text{ML} & \text{L} & \text{VL} \\ \text{ML} & \text{L} & \text{L} & \text{VL} & \text{ML} & \text{VL} \\ \text{ML} & \text{H} & \text{M} & \text{MH} & \text{ML} & \text{VH} \\ \text{L} & \text{MH} & \text{M} & \text{MH} & \text{MH} & \text{MH} \end{bmatrix},$$

$$M_3 = \begin{bmatrix} \text{ML} & \text{VH} & \text{MH} & \text{ML} & \text{MH} & \text{H} \\ \text{H} & \text{VL} & \text{ML} & \text{ML} & \text{L} & \text{L} \\ \text{M} & \text{VL} & \text{L} & \text{L} & \text{M} & \text{VL} \\ \text{MH} & \text{VH} & \text{M} & \text{H} & \text{MH} & \text{MH} \\ \text{ML} & \text{H} & \text{ML} & \text{M} & \text{M} & \text{MH} \end{bmatrix}.$$

Step 2: Based on the results of Step 1, Eqs. 30 and 31, we can get the average decision matrix $\overline{Y}$, shown as follows:

$$\overline{Y} = \begin{bmatrix} \tilde{\tilde{X}}_{11} & \tilde{\tilde{X}}_{12} & \tilde{\tilde{X}}_{13} & \tilde{\tilde{X}}_{14} & \tilde{\tilde{X}}_{15} & \tilde{\tilde{X}}_{16} \\ \tilde{\tilde{X}}_{21} & \tilde{\tilde{X}}_{22} & \tilde{\tilde{X}}_{23} & \tilde{\tilde{X}}_{24} & \tilde{\tilde{X}}_{25} & \tilde{\tilde{X}}_{26} \\ \tilde{\tilde{X}}_{31} & \tilde{\tilde{X}}_{32} & \tilde{\tilde{X}}_{33} & \tilde{\tilde{X}}_{34} & \tilde{\tilde{X}}_{35} & \tilde{\tilde{X}}_{36} \\ \tilde{\tilde{X}}_{41} & \tilde{\tilde{X}}_{42} & \tilde{\tilde{X}}_{43} & \tilde{\tilde{X}}_{44} & \tilde{\tilde{X}}_{45} & \tilde{\tilde{X}}_{46} \\ \tilde{\tilde{X}}_{51} & \tilde{\tilde{X}}_{52} & \tilde{\tilde{X}}_{53} & \tilde{\tilde{X}}_{54} & \tilde{\tilde{X}}_{55} & \tilde{\tilde{X}}_{56} \end{bmatrix}$$

Table 6 presents the results.

Step 3: Based on Table 4 and Eq. 32, we can get the weighting matrices $W_1, W_2,$ and W_3, respectively, where

$$W_1 = \begin{bmatrix} \text{VH} \\ \text{MH} \\ \text{H} \\ \text{VH} \\ \text{ML} \end{bmatrix}, \quad W_2 = \begin{bmatrix} \text{H} \\ \text{M} \\ \text{H} \\ \text{MH} \\ \text{L} \end{bmatrix}, \quad W_3 = \begin{bmatrix} \text{VH} \\ \text{M} \\ \text{MH} \\ \text{VH} \\ \text{M} \end{bmatrix}$$

Step 4: Based on Step 3, Eqs. 33 and 34, we can get the average weighting matrix $\overline{W}$, where

$$W = \begin{bmatrix} \tilde{\tilde{w}}_1 \\ \tilde{\tilde{w}}_2 \\ \tilde{\tilde{w}}_3 \\ \tilde{\tilde{w}}_4 \\ \tilde{\tilde{w}}_5 \end{bmatrix},$$

The results are shown in Table 7.

Step 5: Based on Table 6, Eqs. 35, 36, and 37, normalized matrix N is constructed, where

$$N = \begin{bmatrix} \tilde{\tilde{n}}_{11} & \tilde{\tilde{n}}_{12} & \tilde{\tilde{n}}_{13} & \tilde{\tilde{n}}_{14} & \tilde{\tilde{n}}_{15} & \tilde{\tilde{n}}_{16} \\ \tilde{\tilde{n}}_{21} & \tilde{\tilde{n}}_{22} & \tilde{\tilde{n}}_{23} & \tilde{\tilde{n}}_{24} & \tilde{\tilde{n}}_{25} & \tilde{\tilde{n}}_{26} \\ \tilde{\tilde{n}}_{31} & \tilde{\tilde{n}}_{32} & \tilde{\tilde{n}}_{33} & \tilde{\tilde{n}}_{34} & \tilde{\tilde{n}}_{35} & \tilde{\tilde{n}}_{36} \\ \tilde{\tilde{n}}_{41} & \tilde{\tilde{n}}_{42} & \tilde{\tilde{n}}_{43} & \tilde{\tilde{n}}_{44} & \tilde{\tilde{n}}_{45} & \tilde{\tilde{n}}_{46} \\ \tilde{\tilde{n}}_{51} & \tilde{\tilde{n}}_{52} & \tilde{\tilde{n}}_{53} & \tilde{\tilde{n}}_{54} & \tilde{\tilde{n}}_{55} & \tilde{\tilde{n}}_{56} \end{bmatrix},$$

and the results are shown in Table 8.

Step 6: Based on Tables 7 and 8, Eqs. 38 and 39, we can determine the weighted normalized decision matrix E, where

Table 11 The calculated values for $\tilde{\tilde{S}}_{-i}$

	$\tilde{s}^U_{-i}$						$\tilde{s}^L_{-i}$					
	s^U_{-i1}	s^U_{-i2}	s^U_{-i3}	s^U_{-i4}	$H_1(\tilde{s}^U_{-i})$	$H_2(\tilde{s}^U_{-i})$	s^L_{-i1}	s^L_{-i2}	s^L_{-i3}	s^L_{-i4}	$H_1(\tilde{s}^L_{-i})$	$H_2(\tilde{s}^L_{-i})$
$\tilde{\tilde{S}}_{-1}$	0.16	0.49	0.65	1.88	1.00	1.00	0.31	0.49	0.65	0.99	0.90	0.90
$\tilde{\tilde{S}}_{-2}$	0.01	0.09	0.15	0.69	1.00	1.00	0.04	0.09	0.15	0.28	0.90	0.90
$\tilde{\tilde{S}}_{-3}$	0.04	0.21	0.31	1.13	1.00	1.00	0.11	0.21	0.31	0.51	0.90	0.90
$\tilde{\tilde{S}}_{-4}$	0.01	0.10	0.15	0.65	1.00	1.00	0.05	0.10	0.15	0.28	0.90	0.90
$\tilde{\tilde{S}}_{-5}$	0.07	0.27	0.38	1.38	1.00	1.00	0.15	0.27	0.38	0.64	0.90	0.90
$\tilde{\tilde{S}}_{-6}$	0.00	0.03	0.05	0.43	1.00	1.00	0.01	0.03	0.05	0.15	0.90	0.90

Table 12 The ranking values of $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$

i	$Rank\left(\tilde{\tilde{S}}_{+i}\right)$	$Rank\left(\tilde{\tilde{S}}_{-i}\right)$
1	0.2029	0.2666
2	0.2660	0.2332
3	0.2600	0.2551
4	0.2557	0.2325
5	0.2602	0.2608
6	0.2665	0.1987

$$E = \begin{bmatrix} \tilde{\tilde{e}}_{11} & \tilde{\tilde{e}}_{12} & \tilde{\tilde{e}}_{13} & \tilde{\tilde{e}}_{14} & \tilde{\tilde{e}}_{15} & \tilde{\tilde{e}}_{16} \\ \tilde{\tilde{e}}_{21} & \tilde{\tilde{e}}_{22} & \tilde{\tilde{e}}_{23} & \tilde{\tilde{e}}_{24} & \tilde{\tilde{e}}_{25} & \tilde{\tilde{e}}_{26} \\ \tilde{\tilde{e}}_{31} & \tilde{\tilde{e}}_{32} & \tilde{\tilde{e}}_{33} & \tilde{\tilde{e}}_{34} & \tilde{\tilde{e}}_{35} & \tilde{\tilde{e}}_{36} \\ \tilde{\tilde{e}}_{41} & \tilde{\tilde{e}}_{42} & \tilde{\tilde{e}}_{43} & \tilde{\tilde{e}}_{44} & \tilde{\tilde{e}}_{45} & \tilde{\tilde{e}}_{46} \\ \tilde{\tilde{e}}_{51} & \tilde{\tilde{e}}_{52} & \tilde{\tilde{e}}_{53} & \tilde{\tilde{e}}_{54} & \tilde{\tilde{e}}_{55} & \tilde{\tilde{e}}_{56} \end{bmatrix},$$

and Table 9 presents the results.

Step 7: Based on Table 9, Eqs. 40 and 41, we can calculate $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$. The results are shown in Tables 10 and 11.

Step 8: Based on Tables 10 and 11 and the method that presented in section 4, ranking values of $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ are calculated and shown in Table 12.

Step 9 and 10: Based on Table 12, Eqs. 43 and 44, we can determine the relative significances Q_i and the quantitative utility U_i of the alternatives, as shown in Table 13.

With respect to Table 13 the optimal ranking of six alternatives (suppliers) is $l_6 \succ l_2 \succ l_4 \succ l_3 \succ l_5 \succ l_1$. Thus, l_6 is the best choice.

7 Conclusion

In this paper, a new method for ranking interval type-2 fuzzy numbers, based on the centroid of fuzzy sets has been proposed and compared with some existing methods. Although the proposed method is simpler than others, the comparison results validate the performance of it. We have presented a new method for fuzzy multiple criteria group decision-making based on the proposed ranking method and COPRAS method within the context of interval type-2 fuzzy sets. We have also illustrated the application of the proposed method in supplier selection problems through a numerical example. The proposed method provides us with a useful way to handle fuzzy multiple criteria group decision-making problems based on interval type-2 fuzzy sets.

Table 13 The relative significances and the quantitative utility of the alternatives

i	Q_i	U_i
1	0.4189	75.3
2	0.5129	92.2
3	0.4857	87.31
4	0.5034	90.49
5	0.4810	86.46
6	0.5563	100

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