

A model of non-linear cumulative damage to tools at changing cutting speeds

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Abstract Damage to cutting tools is in most cases non-linear. In cutting at a constant speed, the traditional Taylor formula can be applied to determine tool life. However, it is frequently the case that the given tool is used at various cutting speeds until it becomes completely worn down, thereby rendering this formula unusable. In cutting at changing or alternating speeds, the generalised form of the Taylor formula provides a solution, which could be derived from the non-linear damage to the tool. Although damage to tools is mostly non-linear, it was proven that the linear model $\sum \Delta t_i / T_i = 1$ can serve well. By applying this formula, an equivalent cutting speed can be determined that can also be handled by means of the traditional Taylor formula, and in this way, tool life equations may even be determined under production conditions. Cutting experiments were conducted with an uncoated carbide tool and AISI1045 steel.

Keywords Cutting · Taylor formula · Changing speed

1 Introduction

We are familiar with a number of characteristics of cutting machinability, including surface quality of the workpiece and shape of chips. Tool life, however, undoubtedly receives the greatest attention. Progress has followed two distinct lines. On the one hand, researchers have tried to determine the empirical function of tool wear based on practical experiences and technological measurements. Zorev [1] used the

$$W \cong C_w t^u \quad (1)$$

power function for flank wear, where C_w and u are constant and exponent $u=0.5-1.0$. Although it is known that the curve describing flank wear measured as a function of cutting time bears an inflection, i.e. the wear rate decreases initially but then after a certain time starts to increase, Zorev's formula does not describe this second phase of the wear curve. It was probably Sipos [2] who first used an empirical function bearing such an inflection to describe flank wear.

$$VB(t) = te^{A+Bt+Ct^2} \quad (2)$$

where A , B and C are constants.

On the other hand, however, many researchers have sought to explore the physical processes of wear [3]. The complex approach to this issue, which may be considered up-to-date even by today's standards, probably originates from the research carried out by Shaw and Dirke [4] and Trigger and Chao [5]. Almost half a century later, Zhao et al. [6] enhanced the latter method by factoring in the thermal softening of the tool material. Takeyama and Murata [7] introduced a general equation for the description of the complex processes determining tool wear. Luo et al. [8] enhanced the method of Takeyama and Murata by using the formula of Child et al. [9] for the calculation of abrasive wear and the equation of Schmidt et al. [10] for diffusive wear. Importantly, the tool wear model of Attanasio et al. [11] also considered temperature for the diffusion coefficient, determining the relationship between the diffusion coefficient and temperature by means of experiments. El Wardany and Elbestawi regarded tool failure as a stochastic process [12] and defined tool life as a multiple injury. Singh and Rao modelled flank wear land with a non-linear autonomous differential equation which they could solve numerically with the MATLAB software programme [13]. Feng Ding and Zhengjia He also considered the vibration

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of the tool when they used a proportional hazards model for reliability analysis in monitoring the wear of the cutting tool [14]. Many researchers have studied the problems of measuring tool wear, with Siddhpura and Paurobally investing a great deal of effort in processing and formulating 45 years of publications [15].

As is widely known, tool life may be determined by one of these wear functions or by a direct empirical method.

This is most often calculated by means of the well-known Taylor formula [16]

$$vT^{-k} = C \quad (3)$$

where v is cutting speed, T is tool life, $-k$ and C are constants depending on the circumstances of cutting, which may include the characteristics of the workpiece and tool material, the tool geometrics and cutting parameters such as feed and depth of cut (Eq. 5.9 in [17]). A number of attempts have been made to develop the Taylor formula further, notable examples being the equations of Kronenberg [18] and König and Deperieux [19]. The Kundrák formula [20, 21] is valid for the whole technological range of the cutting speed. The same $T(v)$ functional relationship describing the whole v – T range is discussed in the paper of Punta and Hryniewicz [22]. Yier and Ukhidave described the probability of tool injury occurring during a Δt time by function $\lambda(v, t, \Delta t)$, with the integral by which the Taylor formula was obtained as a probability function [23]. Intensive cutting operations like the hard turning of hardened steels [24], or the turning of austenitic stainless steels are becoming more and more widespread. In these operations, for example, Fernández-Abia et al. [25] studied the effect of speed on wear, surface quality and cutting forces, with the specific force coefficient also being determined with regard to the latter [26], and these coefficients were examined in the case of PVD-coated tools [27]. Although important new findings have been revealed in these special topics, we can still conclude that the Taylor formula (3) continues to be valid for intensive cutting operations. The practical application of this formula, however, is problematic in all cases where the tool is used at alternating or changing cutting speeds. In such cases, both the specification and the interpretation of tool life are brought into question.

Wear of tools and pieces is the result of cumulative damage. The corresponding mathematical formula is described by Miner's rule [28, 29]. This linear model was originally used to describe fatigue processes, but its use was later extended to other types of damage, for instance, to the creep of metals observed at high temperature. The model in its original form can be described by the $\sum n_i/N_i=1$ formula, in which N_i is the number of load cycles at various levels of stress until deterioration, while n_i is the number of cycles actually occurring at

various stress levels. This linear rule was used by Pálmai [30] in cutting. He proved that formula

$$\sum_{i=1}^N \frac{\Delta t_i}{T_i} \approx 1 \quad (4)$$

can really be applied with proper approximation to the statistical average of the results of repeated experiments. Here, cutting is done at various v_i speed for Δt_i time, while T_i is tool life pertaining to a continuous v_i speed.

Jemielnik et al. [31] discussed experiments in which speed and feed were changed periodically. A significant scatter is known to be evidenced in the results of cutting measurements. Taking this into consideration, the calculations made based on the results also justify the validity of the approximation formula (4). Ojha and Dixit [32] regarded formula (4) as fact in determining economical tool life and conducted optimisation calculations on this basis. Lin [33] conducted tool life analysis of ceramic turning tools under the cumulative action of various cutting speeds. He also concluded that formula (4) is approximately valid and that $\sum > 1$ in cutting performed at periodically increasing speed and $\sum < 1$ when reversed. As the results were within the range $\sum = 1 \pm 2\%$, this trend may not be regarded as significant.

This all demonstrates the uncertainty in evaluating the validity of formula (4). On the other hand, as can be established from the known wear curves of tools and furthermore demonstrated by formulas (1) and (2), damage is typically a non-linear process. This raises the question of how formula (4) concerning the linear accumulation of damage may be reliably applied in cutting technology. The purpose of examinations summarised in this paper is to establish if the non-linear cumulative process of cutting tools can be described by the linear model with technically satisfactory precision.

2 Theoretical background

It is known that wear is usually characterised by a number of different parameters including wear of flank land or depth of the crater developing on the rake face. Figure 1 shows the interpretation of flank wear leaving aside the well-known fact that a distinction is usually drawn between average and maximum flank wear, notch wear and corner wear [34]. Typical wear is shown in Fig. 1.

The time when wear reaches the $W=W_{\text{crit}}$ value chosen as the tool life criterion is regarded as tool life $t=T$ pertaining to v speed. It is widely known that the v – T data pairs thus specified are situated along a straight line in an lg–lg scale system of

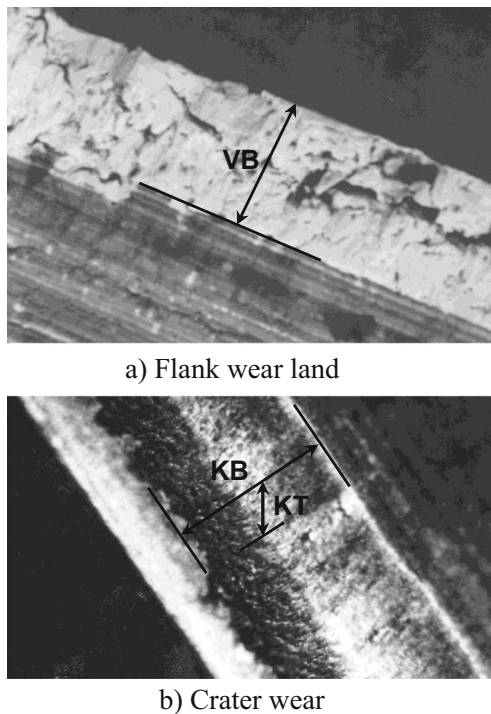


Fig. 1 Typical wear of the cutting tool (**a** flank wear land and **b** crater wear)

coordinates within a fairly broad range of cutting speeds (Fig. 2).

This is the case when the same v speed is used until the tool is worn down. The tool will, however, very often work at different speeds in successive operation. Even in standardised production, a number of operations are generally performed with a single edge, for instance, surfaces of varying diameters are turned at the same revolution of the workpiece. In such case (Fig. 3), formula (3) is rendered unusable in its original form and thus needs to be adapted for cutting at changing speeds. To this end, equation (4) may be used successfully.

The scope of validity of equation $\sum=1$ can be simply determined in the case of wear curves prepared at two different v_1 and v_2 cutting speeds. Wear can be flank wear (VB), corner wear (VC) and crater wear (KT), which are uniformly marked

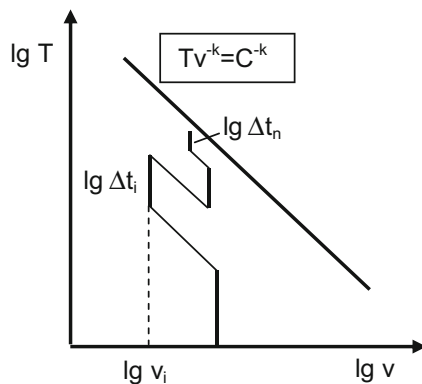


Fig. 2 Tool life at changing cutting speeds

here with W . According to Fig. 3, the process of wear in time is described by function

$$W_i = f_i(v_i, t), \quad i = 1, 2 \tag{5}$$

at two different speeds. Using the inverse of this function, based on the usual interpretation of tool life

$$T_{1,2} = f_{1,2}^{-1}(W_{cr}), \tag{6}$$

where W_{cr} is tool life criterion, and $t_i=f_1^{-1}(W_i)$ time pertains to any $W_i < W_{cr}$ wear.

Figure 3 shows a hypothetical operation in which first the tool cuts at v_1 speed for t_1 time; then, reaching $W=W_i$ wear values, the operation is continued at v_2 speed until wear reaches W_{cr} tool life criterion, the tool thus being worn according to W_1 wear curve for $t_1=f_1^{-1}(W_i)=\Delta t_1$ time, then wear continues according to W_2 wear curve for $T_2-f_2^{-1}(W_i)=\Delta t_{12}$ time. Wear is added up in cumulative damage. It may therefore be assumed that the tool having been worn as shown by one of the wear curves continues to be worn as shown by the other curve in such a way as if it had also been worn in the previous phase of the process according to this second curve. By substituting the time of the two phases to Eq. (4) we obtain equation.

$$\frac{f_1^{-1}(W_i)}{T_1} + \frac{T_2-f_2^{-1}(W_i)}{T_2} = 1 + \frac{f_1^{-1}(W_i)}{T_1} \cdot \frac{f_2^{-1}(W_i)}{T_2} = 1 \tag{7}$$

which is true if

$$\frac{f_1^{-1}(W_i)}{T_1} = \frac{f_2^{-1}(W_i)}{T_2} \tag{8}$$

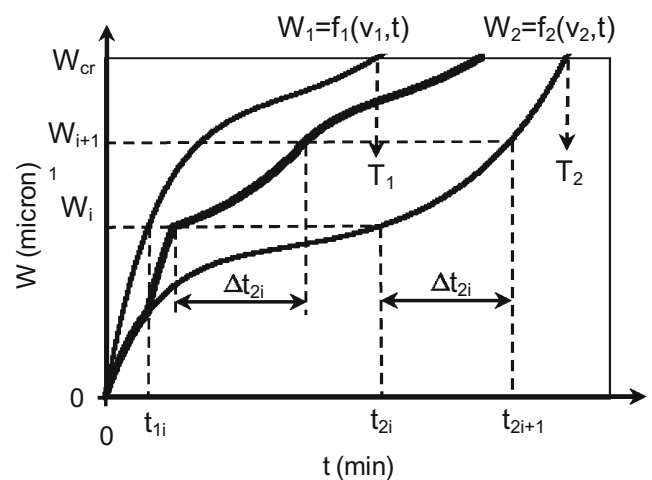


Fig. 3 Wear curves at constant [$W_1=f_1(v_1, t)$, $W_2=f_2(v_2, t)$] and changing cutting speeds

This is therefore the condition that should be met in order that the formula established earlier for linear damage accumulation can also be true for non-linear damage. It is important that there is no constraint in choosing W_i .

In Zorev's Eq. (1)

$$t = f^{-1}(W) = f^{-1}\left(\frac{W}{C_{w1,2}}\right)^{\frac{1}{u}} \tag{9}$$

consequently

$$\frac{f_i^{-1}(W_i)}{T_i} = \left(\frac{C_{wi}}{W_{kr}}\right)^{\frac{1}{u}} = \text{const} \tag{10}$$

As such, Zorev's power functions meet criterion (8), i.e. in their case always $\sum=1$. However, it may be stated with absolute certainty that this is not true either for wear function (2) or for processes described by non-linear wear differential equations [35], which therefore necessitates the experimental examination of non-linear damage.

Assuming for the present that Eq. (4) can be used with satisfactory precision, combining (3) and (4), we obtain

$$\sum_{i=1}^n v_i^{-k} \Delta t_i \cong C^{-k} \tag{11}$$

In flat turning or conical cutting

$$\int_{t=0}^{t=T} v(t)^{-k} dt \cong C^{-k} \tag{12}$$

(11) and (12) may be regarded as the generalisation of the Taylor formula, which is valid for cutting at sectionally or continuously changing speeds [36]. Combining (3) and (12), the tool life pertaining to any $v=\text{const.}$ speed is

$$T \cong \frac{C^{-k}}{v^{-k}} \cong \sum_{i=1}^{i=n} \Delta t_i \left(\frac{v_i}{v}\right)^{-k} \tag{13}$$

The use of this formula is simple if the $-k$ exponent is known, which, however, in practice is frequently not the case. Having the data pairs Δt_i and v_i at our disposal will be to no avail during the wear of the tool; formula (13), which is essentially a $T=f(v,-k)$ function of two variables, cannot be determined directly. Nevertheless, (13) does have an

important characteristic, namely, that it has a minimum value (Fig. 4), which can be calculated from equation

$$\frac{dT}{d(-k)} = \sum_{i=1}^{i=n} \Delta t_i \left(\frac{v_i}{v}\right)^{-k} \ln \frac{v_i}{v} = 0 \tag{14}$$

even though it is situated at a point that depends on the speed. This is useful insofar as exponent $-k$ practically has hardly any effect on tool life value T in the environment of the minimum value. Therefore, an estimated value of $-k$ will suffice, for example $-k \approx 4$ for P20 uncoated carbide tools, and using the solution of Eq. (14), an equivalent cutting speed $v=v_E$ can be calculated. The solution of Eq. (13) is

$$v_E = \exp C^k \sum_{i=1}^{i=n} \Delta t_i v_i^{-k} \ln v_i \tag{15}$$

by means of which, tool life T_E can also be calculated from (13), thereby enabling a value pair v_E-T_E that facilitates the determination of the Taylor formula (3) even in a manufacturing environment. Formulas (11)–(12) and (15), e.g. value pair v_E-T_E can be applied as accelerated cutting tests, where cutting speed is increased in the usual way until wear reaches tool life criteria.

3 The valorisation of the general Taylor formula

The verification of the correctness of Eq. (4) was made in the course of cutting first experiments that are based on an earlier paper [30] and were confirmed in publication [37]. The cutting speeds were 300 and 540 m/min, respectively, feed 0.11 mm/rev and the depth of cut 1 mm. Each operation was performed seven times. The hardness of the rolled steel of AISI1045 quality used for measurements was HV20 196, the chemical composition of which is shown in Table 1. The size

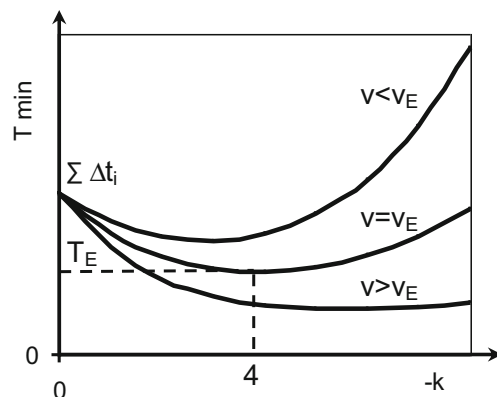


Fig. 4 Tool life calculated by means of formula (12) as a function of exponent $-k$

Table 1 Chemical composition of the experimental steel

C %	Mn %	Si %	P %	S %
0.46	0.70	0.27	0.027	0.021

of the workpiece was $\phi 120 \times 700$ mm with the geometrical data of the P20 uncoated carbide tools being included in Table 2.

Cutting was performed at a speed of 300 m/min on one edge and at 540 m/min on the other edge of each insert. Tool lives T_{300} and T_{540} , which are included in Table 3 and demonstrated by Fig. 5., were thus determined separately for each insert. The constants of the Taylor formula, which can be determined on the basis of seven measurements, are $-k = 3.6055$ and $C = 906.62$. A speed of 540 m/min was initially set for cutting the third edge of the insert, with the process being interrupted after various cutting times had elapsed. Cutting was subsequently continued at a speed of 300 m/min until $VB_{crit} = 0.3$ mm was reached, thereby data pairs Δt_{300} and Δt_{540} could be obtained with the results summarised in Table 3. It can be established that the values of $\sum \Delta t_i / T_i$ are in the region of 1, their average being 0.963 in spite of the significant scatter (Fig. 6).

Using the results obtained for cutting speed $v = \text{const}$, the constants of the Taylor formula (3) could be calculated for each insert; these may in turn be used together with Δt data to calculate the tool life relating to the third edge for any speed. Table 3 contains the T_{1ch} and T_{2ch} values determined for $v = 300$ and 540 m/min speeds. The correspondence of these values can be considered satisfactory, $R^2 = 0.9769$. Equally, satisfactory correspondence is demonstrated by the values of the C constant of the Taylor formula which can be calculated from the data relating to continuous and sectional cutting.

Experiment 1 did not provide an answer to the question of what effect a technological order in cutting not of $v = 560 \rightarrow 300$ m/min but rather $v = 300 \rightarrow 560$ m/min would produce. For this reason, two technological orders were applied in the second experiment performed with the fourth edge of the inserts in two cycles, namely: $v = 300 \rightarrow 560 \rightarrow 300 \rightarrow 560$ m/min for inserts 1, 4, 5 and 6, and $v = 560 \rightarrow 300 \rightarrow 560 \rightarrow 300$ m/min for inserts 2, 3 and 7. Cutting times are summarised in Table 4. The other technological parameters remained the same as those applied for the first experiment.

On the basis of the findings of the second experiment, it can be concluded that (a) the sum of cumulative damage in cutting performed in two cycles is on average $\sum = 0.986$, i.e. there is less deviation from the $\sum = 1$ theoretical value of the linear model. The scatter of data is smaller than in the one-cycle first

Table 2 Geometry of the tool

α_o	γ_o	λ_s	κ_r	ε_r	r_ε
8°	6°	0°	70°	90°	0.8 mm

experiment shown in Fig. 6 and (b) the sum of the cumulative damage depends on the direction of the speed change. In the “up” operation, the average is $\sum = 0.918 < 1$, while in the “down” operation, it is $\sum = 1.076$. This difference can be regarded as significant.

Table 4 also includes the $C_{ch,i}$ Taylor constant calculated from formula (13), where the current $-k$ value was considered for all the inserts. The $C_{ch,i}$ values calculated from the two experiments are shown in Fig. 7 as a function of Taylor constants determined at $v = \text{const}$ speeds. The relationship is obviously close and it is even more apparent in two-cycle cutting. With the help of $C_{ch,i}$ constant, $T_{ch,i}$ tool life pertaining to the conditions of 300–560 m/min changing speeds could then be calculated similarly to the first experiment. The results of the two experiments are shown in Fig. 8 as a function of tool lives obtained at $v = \text{const}$.

4 Discussion

Knowing the constants of the Taylor formula (3), how much cutting work can be done at arbitrarily changing speeds until we reach the W_{crit} wear can easily be calculated. A typical example is, during the manufacture of such workpieces, when cutting with the same edge used at speeds $v_1, v_2 \dots$ for a period of time $\Delta t_1, \Delta t_2 \dots$ the N number of workpieces that can be produced until the edge is worn down needs to be specified. Based on Eq. (11)

$$N \cong \frac{C^{-k}}{\sum_{i=1}^{i=n} \Delta t_i v_i^{-k}} \tag{16}$$

However, when constant C and exponent $-k$ are unknown and only value pairs $v_E - T_E$ specified at changing speeds can be used, it must be considered that the equivalent speed in a particular sectional cutting depends on exponent $-k$ in accordance with Eq. (15). Figure 9 illustrates this relationship in the case of three inserts.

The margin of error calculation which the application of this approximation method requires can be determined as follows based on Fig. 10. If $-k$ is known, v_E equivalent speed can be specified, which in the case of an approximated exponent $-k_a$, is v_E' . Calculating with the latter, the exact value of the tool life would be T_E pertaining to exponent $-k$, with the approximate calculation resulting in tool life T_E' due to exponent $-k_a$. Deviation ΔT_E is demonstrated in Fig. 10 which summarises the results of the following model calculation.

The object of the examination is a component subject to three cutting operations with the same time Δt . The operations are performed at cutting speeds $v_1 = qv, v_2 = v$ and $v_3 = v/q$.

Table 3 Results and evaluations of cutting experiments

Name	Number of inserts							Note
	1	2	3	4	5	6	7	
T_{1i} (min)	46.8	51.0	43.4	66.8	70.9	54.6	49.8	$v_1=300$ m/min
T_{2i} (min)	6.5	5.6	7.3	5.0	5.3	3.9	7.0	$v_2=540$ m/min
$-k_i$	3.1620	3.5393	2.8560	4.1532	4.1553	4.2282	3.1436	$-k_{common}=3.6055$
$C_{const,i}$	1,012.1	911.14	1,123.2	825.06	836.54	772.64	1,040.0	$C_{common}=906.62$
Δt_{1i} (min)	22.0	22.1	16.5	32.7	13.4	11.5	22.1	$v_1=300$ m/min
Δt_{2i} (min)	3.1	3.1	6.5	1.4	3.7	2.1	4.8	$v_2=540$ m/min
$\sum \Delta t_i / T_i$	0.95	0.99	1.26	0.77	0.89	0.75	1.13	Avg. 0.963
$C_{ch,i}$	994.80	907.75	1,218.4	774.62	812.77	721.61	1,081.0	
T_{1chi}	44.3	50.3	54.7	51.4	62.9	40.9	56.2	
T_{2chi}	6.2	5.5	9.2	3.9	4.7	2.9	7.9	
v_E (m/min)	405.9	420.6	460.4	372.0	484.2	464.1	432.9	
T_E (min)	17.0	15.2	16.1	21.0	8.6	6.5	17.8	
$v_{-k=3}$	399.5	399.2	465.6	339.7	442.5	415.9	427.1	
$T_{-k=3}$	17.9	17.9	15.6	28.8	11.7	9.4	18.5	
$v_{-k=4}$	439.4	439.5	497.5	367.0	479.2	455.6	465.9	
$T_{-k=4}$	13.0	13.0	12.6	22.2	9.0	7.0	13.8	

Here, $q^2 = v_{max}/v_{min}$ characterises the range of speed change. In this special case, the solution of Eq. (14) can be written in the form

$$v_E = v \exp \left[\frac{q^{-k} - q^k}{q^{-k} + 1 + q^k} \ln q \right] \tag{17}$$

and the tool life calculated exactly from (13). This value may be compared to a value that was calculated by the approximation method, for example, in the case of P20 uncoated carbide with exponent $-k \approx 4$. The relative error made in the approximate calculation is characterised by quotient $100 \Delta T_E' / T_E'$, shown in Fig. 11 as a function of exponent $-k$ and q . The error resulting from applying the approximate method at a

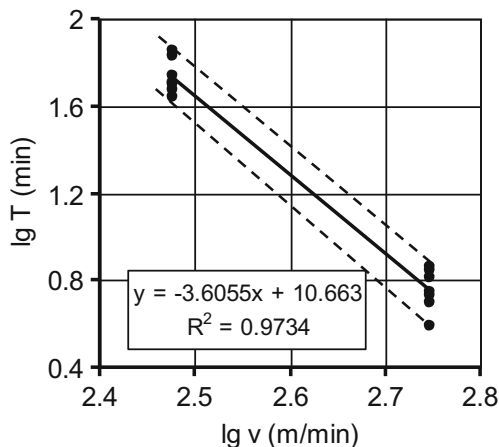


Fig. 5 Tool lives measured in cutting performed at constant speed

speed change of, for example, $v=100-140$ m/min stays below 1 %.

These error calculations are based on the $\sum = 1$ linear model of wear accumulation. The calculation error may be increased by the fact that cumulative damage is non-linear; therefore, formula (4) can in most cases only render an approximate result. The correctness of approximation can also depend on technological parameters. To verify this, using the wear curves

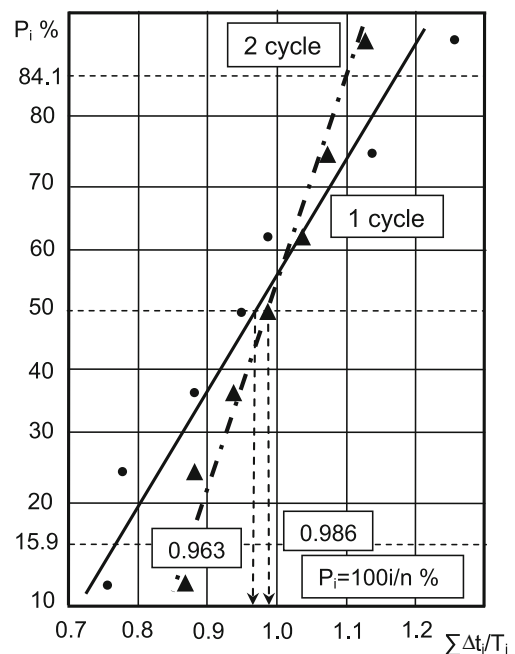


Fig. 6 Scatter of sum $\Delta t_i / T_i$

Table 4 The results of the second cutting experiments and their evaluation

Name	Number of the inserts							Note
	1	2	3	4	5	6	7	
Δt_{11i} (min)	12	↑ 12	↑ 12	12	12	12	↑ 12	1× cycle
Δt_{22i} (min)	2	↑ 2	↑ 2	2	2	2	↑ 2	
Δt_{21i} (min)	5	↑ 4.6	↑ 12.8	5	5	2	↑ 12.8	2× cycles
Δt_{22i} (min)	↓ 2	↑ 2	↑ 2	↓ 1.1	↓ 1.7	↓ 0.4	↑ 2	
$\sum \Delta t_i / T_i$	0.98	1.04	1.12	0.88	0.94	0.87	1.07	Avg. 0.986
$C_{ch,i}$	1005.2	921.2	1168.5	798.8	823.7	748.0	1062.4	
T_{1chi}	45.8	53.0	48.6	58.4	66.5	47.6	53.3	
T_{2chi}	6.4	5.8	8.2	4.4	5.0	3.4	7.5	
v_E (m/min)	438.8	455.0	402.3	461.3	471.5	460.4	413.7	
T_E (min)	13.8	12.2	21.0	9.8	10.2	7.8	19.4	
$C_{ch,i}^4$	853.7	852.4	878.0	815.6	841.6	768.9	878.0	
v_E m/min	470.5	471.8	447.9	455.5	466.0	451.7	447.9	-k=4
T_E min	10.8	10.7	14.8	10.3	10.6	8.4	14.8	

Long arrow: $v=300 \rightarrow 560 \rightarrow 300 \rightarrow 560$ m/min, short arrow: $v=560 \rightarrow 300$ m/min

determined by Pálmai [32] at $v=const$ speed, the turning of an axle was assumed at a speed of $v_1=125$ m/min for t_1 time, then at a speed of $v_2=200$ m/min for t_2 time. The axle used was so long that the tool was completely worn down during the performance of the two operations. The quotient of the two operating times obviously influences the value of \sum cumulative damage. The results of calculations made using wear curves published in [32] are summarised in Fig. 12. It can also be concluded here that the order of the two speeds influences the result, as $\sum < 1$ if $v=125 \rightarrow 200$ m/min, while $\sum > 1$ if it is reversed. The curves have an extreme value where the deviation from the $\sum=1$ linear model is the largest. Maximum deviation also depends on the relationship of the two speeds: as demonstrated by Fig. 13, where sum max values are displayed as a function of $v/125$ ratio calculated from various v speeds, deviation can even approach 5 %.

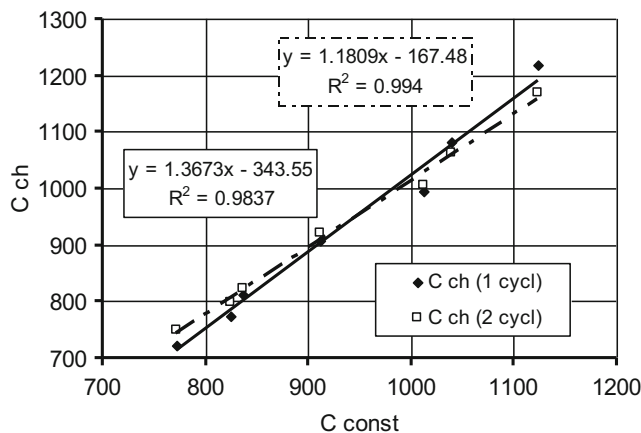


Fig. 7 Relationship between C Taylor constants calculated at constant and changing speeds

However, the fact that the “up” and “down” operating modes change in the case of serially repeated operations needs to be taken into consideration in evaluating this, so sometimes the upper, sometimes the lower curve of Fig. 12 shall apply. The two-cycle experiment showed that the error of the linear model decreases in the case of repeated operations. Continuing the model calculation shown in Figs. 12 and 13, this trend can clearly be shown. According to Fig. 14, error within the linear model decreases rapidly during the serial turning of shorter and shorter axles.

In summary, it can be concluded that the cumulative damage to cutting tools is a non-linear process. However, the linear model can be applied for repeated operations. Thus, the formula concerning v_E equivalent speed and the related T_E tool life can be applied reliably in practice.

Table 3 not only contains the equivalent v_E-T_E data pairs pertaining to sectional cutting that can be calculated with the

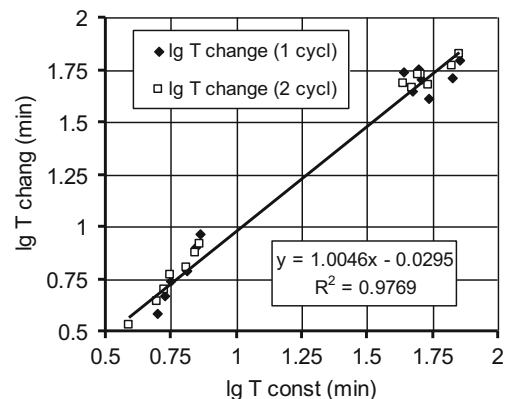


Fig. 8 Relationship between tool lives measured at constant and changing speeds ($v=300, 560$ m/min)

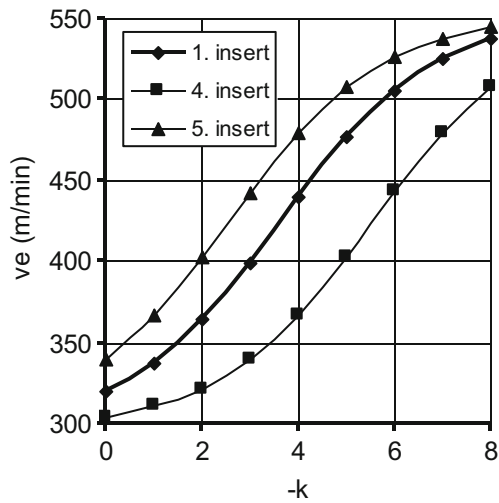


Fig. 9 Relationship between $-k$ and v_E for inserts 1, 4 and 5 (Table 3)

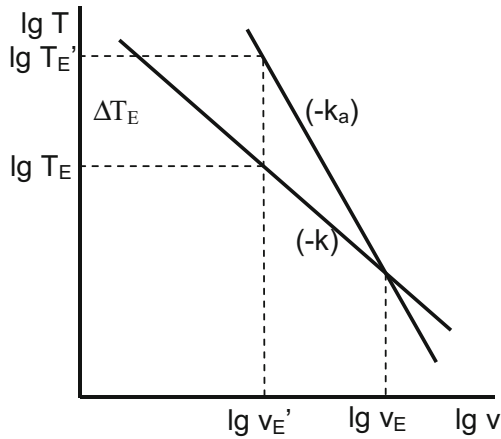


Fig. 10 Error of equivalent tool life at estimated exponent $-k$

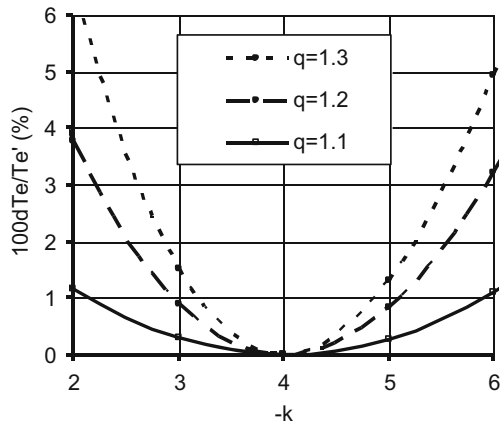


Fig. 11 Dependence of the tool life determined from formula (12) from exponent $-k$

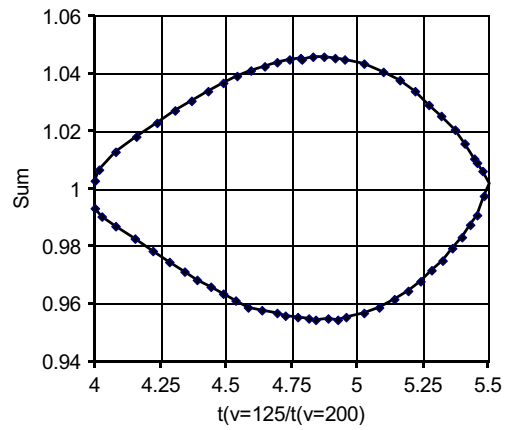


Fig. 12 Impact of interim operational times on the value of cumulative damage

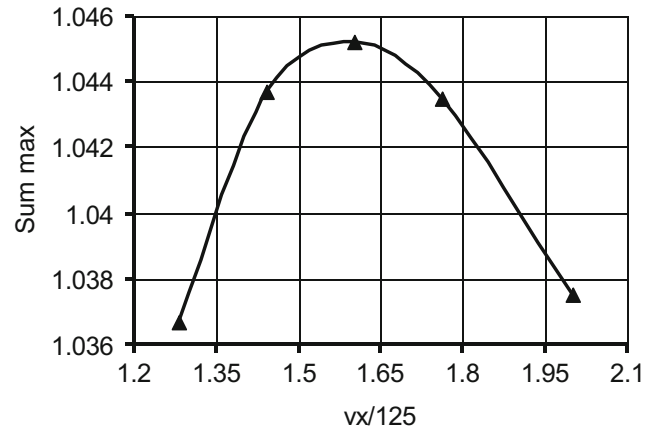


Fig. 13 The impact of $v/125$ speed ratio on the value of sum max cumulative wear (see Fig. 12)

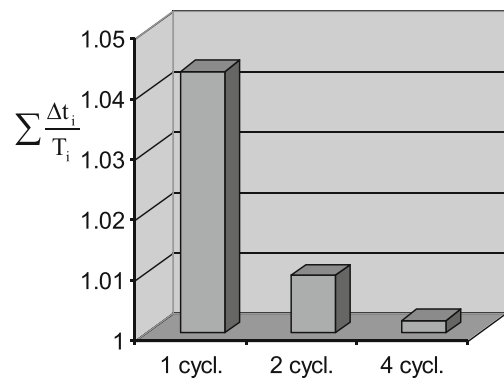


Fig. 14 The impact of the number of $v_1 \rightarrow v_2$ cycles on the value of maximum cumulative damage

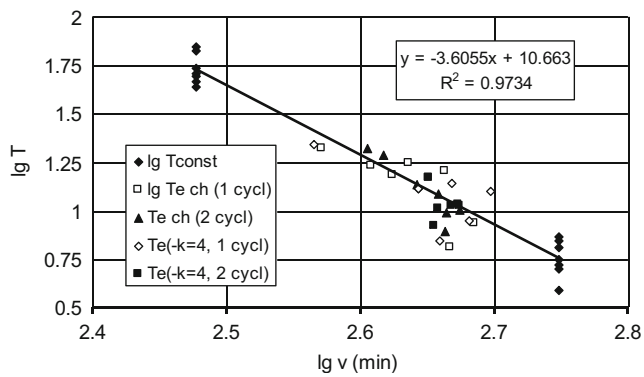


Fig. 15 Correspondence of equivalent tool lives calculated with various $-k$ exponents to the Taylor function

$-k$ exponents known for each insert but also those that could be calculated with $-k=4$ values. These are shown in Fig. 15 together with the data obtained during continuous cutting. As can be seen, the scatter band of v_E-T_E data determined by sectional cutting falls within the scatter band of the Taylor function as determined for the most part by the traditional method. It is especially true for two-cycle experimental results, which also show that the error in the calculation of equivalent v_E , T_E technological values decrease in multiple-cycle cutting and series production.

4.1 Summary

The most important aspect of the evaluation of machinability is tool life. This is usually described by means of the well-known Taylor formula which originally specifies tool life as a function of cutting speed. That cutting is performed with the same tool at various successive speeds, thereby rendering the Taylor formula unusable, is, however, a common problem. In the case of sectionally changing cutting speeds, it can be verified both theoretically and through experiments that equation $\sum \Delta t_i / T_i \cong 1$, which was originally formulated for linear damage accumulation, is valid. Although damage to the cutting tool is in most cases non-linear, the experiments and calculations showed that the model of linear damage can be used. The generalised form of the Taylor formula, which can also be applied for cutting at sectionally changing speeds, could be derived from this equation. If cutting is performed at various speeds, an equivalent speed and a corresponding tool life, which can also be handled by means of the traditional Taylor formula, may be specified. The correctness of the new Taylor formula was verified by means of cutting experiments. The calculation of equivalent speed enables the standard Taylor formula to be used under production manufacturing conditions even in cutting at changing speeds; moreover, it can also be determined by factory measurements. The model can also be used for accelerated cutting tests.

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