Int J Adv Manuf Technol (2014) 71:1873–1882
DOI 10.1007/s00170-014-5635-8

ORIGINAL ARTICLE

Adaptive feedrate interpolation with multiconstraints for five-axis parametric toolpath

Jianfeng Zhou · Yuwen Sun · Dongming Guo

Received: 4 July 2013 / Accepted: 13 January 2014 / Published online: 25 January 2014

Abstract A good adaptive feedrate will be helpful for improving machining accuracy and efficiency, as well as avoiding the excess of the machine's physical capabilities and feed fluctuations during machining. Therefore, it is highly desirable to consider the constraints of geometric error, cutting performance, and drive constraints in the feedrate scheduling of the parametric curve interpolator for five-axis computer numerical control machining. In this paper, a novel multiconstraints feedrate scheduling method is proposed for the parametric curve interpolator in five-axis machining. In the method, the feed optimization model is first built with the constraints of geometric error, the maximum feedrate and acceleration of cutter tip, and the maximum feedrate and acceleration of five-drive axes. Then, the relations between each constraint and the cutter tip feedrate are derived by means of near arc length parameterization. After that, a linear programming algorithm is applied to obtain the optimal feed profile on the sampling positions of the given tool path. Finally, illustrated examples are given to validate the feasibility and applicability of the proposed feedrate scheduling method. The comparison results show that the proposed method has an ability of the simultaneous guarantees of geometric accuracy, cutting performance, and drive characters of machine tools.

Keywords Feedrate scheduling · Interpolator · NURBS · Five-axis machining

J. Zhou (✉) · Y. Sun · D. Guo
Key Laboratory for Precision and Non-Traditional Machining Technology of the Ministry of Education, Dalian University of Technology, Dalian 116024, China
e-mail: zhoujfdut@163.com

1 Introduction

With the development of modern CAD design system, parametric surface is widely used in the expression of sculptured parts such as impellers, propellers, biomedical parts, and molds. Accordingly, during numerical control (NC), machining parametric curve interpolator has aroused more and more attention. On the other hand, conventional computer numerical control (CNC) machine tools often provide only linear and circular interpolators, so the tool path needs to be approximated with tiny lines or arc segments according to the predefined chord error. Actually, the linear approximated path is not smooth enough and thus it often leads to feed fluctuations. Moreover, relatively large code data, which will drastically increase the data transform load of the CNC controller, will be generated in curved surface machining.

In order to overcome the disadvantages of the linear interpolators, various kinds of parametric interpolators have been developed respecting different application backgrounds for three-axis machining [1–7]. For example, uniform interpolator on B-spline curve [8], constant feedrate interpolator [9], PH curved-based interpolation [10], universal real-time non-uniform rational basis spline (NURBS) interpolator [11], adaptive NURBS interpolator with confined chord error or/and acceleration [12–16], curve evolution based jerk-limited feedrate interpolation [17], feedrate interpolation with constant material removal rate [18, 19], feedrate scheduling with drive constraints [20–24], and NURBS interpolator with minimal feedrate fluctuations or other special purposes [25–34]. Generally, most of the existing parametric interpolators in three-axis machining are capable of making the geometric error, the drive constraints of machine, and the kinematic characters of cutter under control.

However, as is well known, due to the two additional rotation axes in five-axis machining, the relations between the pivot space and Cartesian space usually is nonlinear.

When machining along a curved path at high speed, it is necessary to prevent excess feed fluctuation of the cutter tip in order to maintain even cutting load, and at the same time preventing each drive axis beyond its saturation limit. Hence, developing an adaptive parametric interpolator algorithm available for five-axis machining is becoming increasingly important to improve the machining efficiency and ensure machining quality, especially under the condition of high-speed machining.

Recently, some researchers have focused their efforts to the issues of the control of geometric error and kinematic performances in five-axis toolpath planning and feedrate scheduling. Feng [35] presented a method to access the geometry-based errors for interpolated tool paths in five-axis surface machining. Lavernhe et al. [36] investigated the kinematical performance prediction in five-axis machining for process optimization. Li et al. [37] presented a NURBS pre-interpolator based on a series of linear/circular segments. In [38], a NURBS machining strategy is proposed for five-axis machining and the nonlinear error could be adaptively controlled within a user-defined tolerance. Sencer et al. [39] established a feed optimization model on cubic B-spline curve, respecting velocity, acceleration, and jerk limits for each axis with an objective function aiming to minimize the machining time. Generally, the calculation related to the nonlinear optimization is time-consuming, and an improvement work was done in [40–42] to reduce the computational complexity. Sun [43] proposed an adaptive feedrate scheduling method for dual NURBS curve interpolator, respecting geometric and kinematic constraints, which paid an attention to develop a universal parametric interpolator regardless of what type of configuration the machine has.

As above mentioned, most of the existing feedrate scheduling methods mainly consider the drive constraints or geometric error constraints. However, the simultaneous constraints of drive axes, geometric error, and cutting performance are seldom considered in the existing works despite that the cutting performance of the cutter also has an influence on the quality of the machined surface. In this paper, as shown in Fig. 1a suitable multiconstraint feedrate scheduling method is proposed for five-axis machining to ensure the satisfaction of the drive constraints and the cutting performance of cutter tip simultaneously. In the method, chord error is also confined within a specified tolerance. The implementation of this proposed method is mainly based on the following two premises. (1) Cutter tip path curve and axis displacement curve can be described with the same parameter which is the normalized arc length parameter in cutter tip path curve. By taking the first and second derivatives with respect to time for the cutter tip path curve and the coordinate curve of each axis, the analytical relation between the cutter tip and axis displacements can be built. (2) Sampled cutter tip points are enough to appropriate the analytical solution of real continuous movement of the cutter tip. Hence, if the given tool path curve is sampled with a suitable number of sample points, it is reasonable to substitute the original nonlinear analytic expressions with discrete

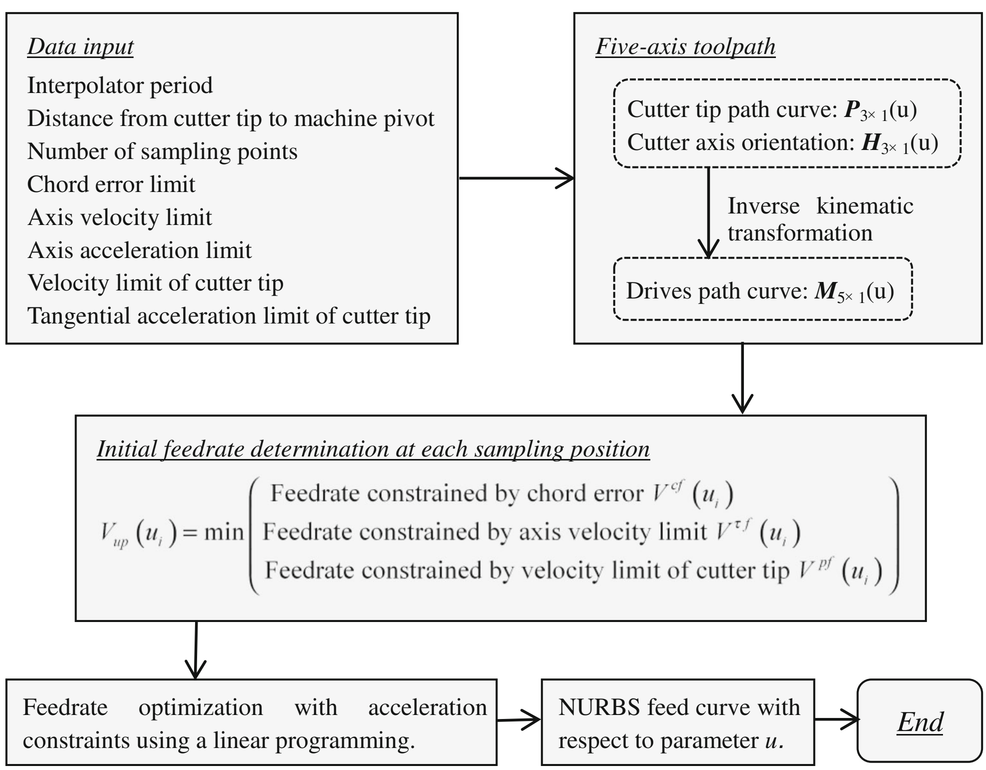

Fig. 1 Proposed feed scheduling method for five-axis machining

formats, and further the constraints in the model can be converted into linear patterns.

The rest of this paper is organized as follows. Section 2 generalizes the proposed adaptive feed interpolation method. The analytic relations between the geometric parameters of tool path and the kinematic performances of machine and cutter, the feed optimization model, constraint conditions, and concrete algorithms are all given in detail. In Section 3, the final results are provided by illustrative examples. Section 4 concludes the paper.

2 The proposed feedrate scheduling method

2.1 Description of toolpath

In five-axis NC machining, cutter location data (CL data) consists of cutter position data and tool orientation data, whether it is expressed in the form of discrete points or dual parametric curve. If the original CL data is assumed as the former, namely a sequence of discrete positions along the toolpath in workpiece coordinate system, then according to the original CL data the trajectory of cutter tip and the orientation function of cutter axis can also be further fitted to a well-developed dual parametric curve with a normalized arc length parameter. Also, combined with the inverse kinematic transformation, the original CL data can be converted into the position coordinates of each axis or can be further fitted to axis spline with the same parameter as that of in tool path. Therefore, the initial given CL data is defined here as parametric format.

As the specific parametric curve to describe the tool path, NURBS curve is usually adopted due to that NURBS itself offers great flexibility and precision for handling both analytic and modeled shapes. The NURBS spline used as the description of five-axis toolpath is expressed as follows.

$$\begin{aligned}\boldsymbol{P}(u) &= \left[P_x(u), P_y(u), P_z(u)\right] \\ &= \sum_{i=0}^{n} N_{i,k}(u)\omega_i \boldsymbol{p}_i \Big/ \sum_{i=0}^{n} N_{i,k}(u)\omega_i \\ \boldsymbol{H}(u) &= \left[H_x(u), H_y(u), H_z(u)\right] \\ &= \sum_{i=0}^{n} N_{i,k}(u)\omega_i \boldsymbol{h}_i \Big/ \sum_{i=0}^{n} N_{i,k}(u)\omega_i \quad u\in[0,1]\end{aligned} \tag{1}$$

where the NURBS curve $\boldsymbol{P}(u)$ describes the cutter tip path, and $\boldsymbol{H}(u)$ stands for the unit cutter axis orientation. $\boldsymbol{p}_i$ and $\boldsymbol{h}_i$ represent the control points, and $n+1$ is the number of control points. ω_i is the weight factor, and κ is the degree of the NURBS spline. Besides, u is defined as the normalized arc length parameter in cutter tip path curve $\boldsymbol{P}(u)$, the real arc length at parameter position u can be expressed as $s=\kappa u$, where κ is the total length of the path curve $\boldsymbol{P}(u)$. $N_{i,k}(u)$ is the B-spline basis function with the following recursive formulas.

$$\begin{aligned} &N_{i,0}(0) = \begin{cases} 1 & u_i \le u \le u_{i+1} \\ 0 & otherwise \end{cases} \\ &N_{i,k}(u) = \frac{u-u_i}{u_{i+k}-u_i} N_{i,k-1}(u) + \frac{u_{i+k+1}-u}{u_{i+k+1}-u_{i+1}} N_{i+1,k-1}(u) \end{aligned} \tag{2}$$

where $[u_0, \cdots, u_{n+k+2}]$ represents the knot vectors.

Thus, for the discrete five-axis cutter location data, they can be fitted by two NURBS spline with the same parameter u for ensuring that the motion of cutter tip and the tool axis is synchronized. For each cutter location data in workpiece coordinate system, there also exists a corresponding postprocessing data in machine coordinate system through inverse kinematic transformation. Similarly, the sequence of discrete positions of drive axes can also be fitted by five NURBS axis splines with the same parameter u in formula (1). They are

$$\boldsymbol{M}(u) = [X(u), Y(u), Z(u), A(u), C(u)] \tag{3}$$

Without losing generality, a spindle-rotating/tilting type machine tool is adopted to implement the proposed feedrate scheduling method as an example. The coordinate system of spindle rotating/tilting AC-type configuration is shown in Fig. 2. Through inverse kinematic transformation, the displacements of drive axes can be obtained on the basis of the given dual NURBS toolpath, and they are expressed as

$$\begin{aligned} &A(u) = \cos^{-1}(\mathrm{H}_z(u)) \\ &C(u) = -\tan^{-1}\left(\mathrm{H}_x(u)/\mathrm{H}_y(u)\right) \\ &X(u) = P_x(u) + L\sin(A(u))sin(C(u)) \\ &Y(u) = P_x(u) - L\sin(A(u))\cos(C(u)) \\ &Z(u) = P_z(u) + L\cos(A(u)) - L \end{aligned} \tag{4}$$

2.2 Feedrate optimization model

Generally, feed optimization mainly aims at minimizing the machining time under the given constraints. The key difference among diverse feed optimization models is in the difference of constraint conditions. In feed optimization model, the minimum time optimal objective function can be described as the following form.

$$\mathrm{Min}\left(T_{\Sigma}\right) = \min_{\dot{s}} \int_0^s \frac{ds}{\dot{s}} = \min_{\dot{u}} \int_0^1 \frac{du}{\dot{u}} \tag{5}$$

From the above formula, it is easy to deduce that the machining time is minimum means the feedrate is maximum at any parameter u under the premise of satisfying preset constraints. In other words, the quadratic sum of the feedrate at all sampled positions along the toolpath should reach their maximum limit. So, for discrete cutter location data, it is

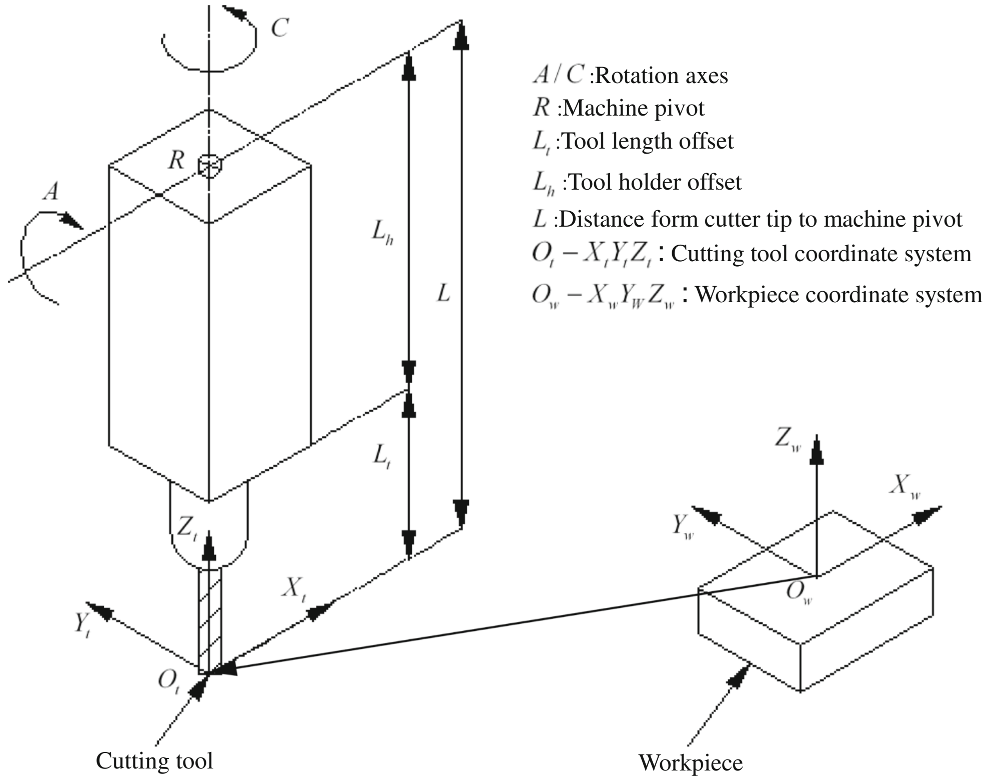

Fig. 2 Coordinate system of spindle rotating/tilting AC type configuration

reasonable to make a substitution of the feedrate optimization objective function in formula (5) with the following expression.

$$\begin{aligned} &Max\ \ \Gamma = \sum_{i=1}^{n} \xi_i \quad \boldsymbol{\xi} = (\xi_1, \xi_2, \cdots, \xi_n)^{\mathrm{T}} \quad \xi_i = V(u_i)^2 \\ &s.t. \quad \boldsymbol{A\xi} \le \boldsymbol{b} \\ &\qquad\quad \xi_i \ge 0 \end{aligned} \tag{6}$$

where $V(u_i)$ is the feedrate of cutter tip at the sampled parameter position u_i, and n is the number of the sampled positions. The matrix inequality $\boldsymbol{A\xi} \le \boldsymbol{b}$ makes the optimal feedrate under the predefined constraints. The main purpose of using the substitution is to convert nonlinear constraints as linear ones, and thus the constrained optimization model can be simplified.

In the proposed feedrate scheduling method, the constraints in formula (6) mainly include the limits of geometric error, drive characters, and cutting performance. These constraints are all converted into linear constraints. Compared with formula (5), optimization model (6) avoids integral computation, and establishes a linear relation among all predefined constraints, instead of the complex, interactional, and nonlinear relations. Also, it is easy to know that model (6) is suitable for discrete CL format only if the parametric toolpath is sampled with a suitable sampling interval.

2.3 Geometric and kinematical constraints

Mathematical formalism of geometric constraint, drive constraint, and cutting performance constraint will be derived in this section. These constraints, including maximum chord error, maximum velocities, and accelerations of five-drive axes, maximum feedrate, and feed acceleration of cutter tip are given for the purpose of ensuring machining accuracy, the kinematic performance of machine tool, and the stability of cutting process.

2.3.1 Geometric constraint

Geometric constraint is here referred to as chord error. According to the relation between the chord error and the kinematic characters and geometric characters of the path curve, it is known that the chord error will get larger with the increase of feedrate, and therefore the chord error is most likely to exceed its limit at large-curvature regions of the path curve. Therefore, the feedrate has to be limited by the chord error constraint to ensure the machining accuracy. As shown in Fig. 3, the computation formula of the chord error can be derived as follows

$$\varepsilon_i = \rho_i - \sqrt{\rho_i^2 - \left(\frac{V(u_i)T_s}{2}\right)^2} \tag{7}$$

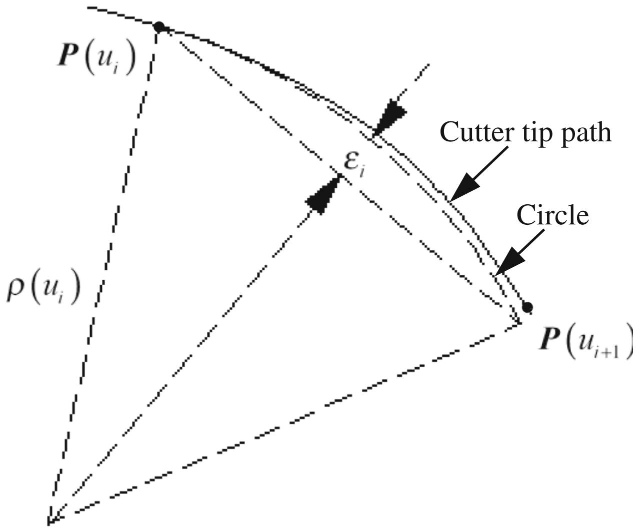

Fig. 3 Chord error

where T_s is the interpolator period of the CNC system and ρ_i is the radius of curvature at the parameter position u of the path curve.

Given a maximum chord error $\delta_{\max}$, the feasible feedrate with chord error constraint can be obtained according to the following formula.

$$V(u_i) \le 2\sqrt{\rho_i^2 - (\rho_i - \delta_{\max})^2}/T_s \tag{8}$$

Neglecting the second-order small qualities, formula (8) can be further simplified as

$$V(u_i) \le 2\sqrt{2\rho_i\delta_{\max}}/T_s \tag{9}$$

where ρ_i can be calculated with the following formula.

$$\rho(u) = \frac{\|\boldsymbol{P}_u(u)\|^3}{\|\boldsymbol{P}_u(u) \times \boldsymbol{P}_{uu}(u)\|} \tag{10}$$

2.3.2 Drive constraints

In five-axis machining, if the velocities and acceleration of drive axes exceed their physical saturation limits, the mechanical structure of the machine will probably get damaged, and the machine motion will also probably become unstable which will seriously affect the machining precision. In a word, the drive constraints in the feedrate scheduling for five-axis CNC machining are absolutely necessary.

In general, the feedrate of cutter tip is defined as

$$V(u) = \left\|\frac{d\boldsymbol{P}(u)}{dt}\right\| = \left\|\frac{d\boldsymbol{P}(u)}{du}\frac{du}{dt}\right\| = \left\|\frac{d\boldsymbol{P}(u)}{du}\right\|\frac{du}{dt} \tag{11}$$

As defined in the previous section, the normalized path parameter u and the total path length κ have such relation that $s = \kappa u$.

$$\left\|\frac{d\boldsymbol{P}(u)}{du}\right\| = \left\|\frac{d\boldsymbol{P}(u(s))}{ds}\frac{ds}{du}\right\| = \kappa \tag{12}$$

Combined with the above two formulas, the following equation can be obtained.

$$\frac{du}{dt} = \frac{V(u)}{\kappa} \tag{13}$$

Then, take the derivative of formula (12) with respect to time t.

$$\frac{d^2u}{dt^2} = \frac{d(V(u)/\kappa)}{dt} = \frac{a(u)}{\kappa} \tag{14}$$

Using the above formulas (11)–(14), the analytical relation between the motion of cutter and five-drive axes can be built via inverse kinematic transformation. Thus, according to the concrete expressions of five-axis splines as illustrated in formula (3), the velocities and accelerations of five drive axes can be defined as follows.

$$\left.\begin{aligned} V_\tau(u) &= \frac{d\boldsymbol{M}^\tau(u)}{dt} = \boldsymbol{M}_u^\tau\frac{du}{dt} = \boldsymbol{M}_u^\tau\frac{V(u)}{\kappa} \\ A_\tau(u) &= \frac{dV_\tau(u)}{dt} = \boldsymbol{M}_{uu}^\tau\left(\frac{V(u)}{\kappa}\right)^2 + \boldsymbol{M}_u^\tau\frac{a(u)}{\kappa} \end{aligned}\right\} \tag{15}$$

where τ is the symbol of each drive axis, $\tau = x, y, z, a, c$. By means of formula (15), the relations between the feedrate and acceleration of cutter tip and those of five drive axes are established so that the kinematic characters can be evaluated according to the kinematic characters of cutter.

In the feed optimization model (6), the path curve needs to be continuously sampled with a suitable parameter interval. Hence, the acceleration of each sample position is usually obtained by numerical calculation. For a given parameter position u_i, the tangential acceleration of cutter tip can be approximated by the following formula.

$$a(u_i) = \left(V(u_{i+1})^2 - V(u_i)^2\right)/(2\kappa(u_{i+1} - u_i)) \tag{16}$$

Substitute $a(u_i)$ into formula (15), then the drive constraints can be written as the following form

$$\left.\begin{aligned} &-V_{\max}^\tau \le \boldsymbol{M}_u^\tau(u_i)\frac{V(u_i)}{\kappa} \le V_{\max}^\tau \\ &-A_{\max}^\tau \le \boldsymbol{M}_{uu}^\tau(u_i)\left(\frac{V(u_i)}{\kappa}\right)^2 + \frac{\boldsymbol{M}_u^\tau(u_i)\left(V(u_{i+1})^2 - V(u_i)^2\right)}{2\kappa^2(u_{i+1} - u_i)} \le A_{\max}^\tau \end{aligned}\right\} \tag{17}$$

where $V^{\tau}_{\max}$ and $A^{\tau}_{\max}$ stand for the maximum feedrate and acceleration of each drive axis, $\tau = x, y, z, a, c$, and the formula (17) can be further arranged as

$$\left.\begin{array}{l} 0 \leq \left(\dfrac{\boldsymbol{M}^{\tau}_{u}(u_i)}{\kappa}\right)^2 V\ (u_i)^2 \leq \left(V^{\tau}_{\max}\right)^2 \\ -A^{\tau}_{\max} \leq \dfrac{\boldsymbol{M}^{\tau}_{u}(u_i)}{2\kappa^2(u_{i+1}-u_i)} V\ (u_{i+1})^2 + \\ \left(\dfrac{\boldsymbol{M}^{\tau}_{uu}(u_i)}{\kappa^2} - \dfrac{\boldsymbol{M}^{\tau}_{u}(u_i)}{2\kappa^2(u_{i+1}-u_i)}\right) V(u_i)^2 \leq A^{\tau}_{\max} \end{array}\right\} \tag{18}$$

2.3.3 Cutting performance constraints

In five-axis machining, the kinematic transformation between the Cartesian space and joint space is nonlinear because of the existence of two rotational axes. That means the feedrate of cutter tip may change abruptly, even under the satisfaction of drive constraints. In order to alleviate the feed fluctuation of cutter tip to maintain constant cutting load and ensuring machining quality, it is necessary to take into account the constraints of kinematic characters of cutter tip, namely maximum feedrate and tangential acceleration of cutter tip. The corresponding expressions are as follows.

$$\left.\begin{array}{l} V\ (u_i) \leq {}_{V}\max \\ -a_{\max} \leq \left(V(u_{i+1})^2 - V(u_i)^2\right) / \left(2\kappa(u_{i+1}-u_i)\right) \leq a_{\max} \end{array}\right\} \tag{19}$$

After that, analytical formulas for all preset constraints in feed optimization have been established including formulas (9), (18), and (19). Through the three formulas, the constraint matrix of the linear programming issue in formula (6) can be easily obtained.

2.3.4 Detailed algorithm

For the proposed method, in feed optimization the first key issue is to obtain the matrix $\boldsymbol{A}$ and $\boldsymbol{b}$ in formula (6). Generally, the matrixes can be calculated directly according to the inequity formulas (9), (18), and (19), but the computation complexity can be further simplified in practice. At each sampled position u, the maximum velocity of each axis has relation with the feedrate of current cutter tip and related geometric information of this point. In addition, feedrate constrained by chord error is also only related to the given geometric information. Therefore, at each sampled parameter position u, the maximum feasible feedrate satisfying the chord error limit, the feedrate limits of cutter tip and the maximum velocity limits of five drive axes, can be easily obtained by performing a simple comparison with the following expression.

$$V_{ub}(u_i) = \min\begin{bmatrix} V^{cf}(u_i) \\ V^{pf}(u_i) \\ V^{\tau f}(u_i) \end{bmatrix} = min\begin{bmatrix} \dfrac{2\sqrt{2\rho_i\delta_{\max}}}{T_s} \\ V_{max} \\ \dfrac{\kappa V^{\tau}_{\max}}{\left|\boldsymbol{M}^{\tau}_{u}(u_i)\right|} \quad \tau = \mathrm{x, y, z, a, c.} \end{bmatrix} \tag{20}$$

Taking V_{ub} as the up boundary of feedrate in the feed optimization model solved by a linear programming algorithm, it can reduce the computation time dramatically. Then, the feedrate scheduling problem can be finally arranged as

$$\begin{array}{l} \text{Max } \Gamma = \displaystyle\sum_{i=1}^{n} V(u_i)^2 \\ s.t. \begin{cases} -a_{\max} \leq \left(V(u_{i+1})^2 - V(u_i)^2\right) / \left(2\kappa(u_{i+1}-u_i)\right) \leq a_{\max} \\ -A^{\tau}_{\max} \leq \dfrac{\mathbf{M}^{\tau}_{u}(u_i)}{2\kappa^2(u_{i+1}-u_i)} V\ (u_{i+1})^2 \\ +\left(\dfrac{\mathbf{M}^{\tau}_{uu}(u_i)}{\kappa^2} - \dfrac{\mathbf{M}^{\tau}_{u}(u_i)}{2\kappa^2(u_{i+1}-u_i)}\right) V(u_i)^2 \leq A^{\tau}_{\max} \quad \tau = x, y, z, a, c. \\ 0 \leq V\ (u_i)^2 \leq V_{ub}(u_i)^2 \end{cases} \end{array} \tag{21}$$

Thus, from the above formula, it can be seen that the feedrate optimization is transformed as a standard linear programming problem, and it can be solved by the existing well-developed algorithm. The procedure of computing the maximum feasible feedrate at sampling positions is summarized as follows

Input: the original CL data, the preset values of all constraints, interpolator period, the distance from cutter tip to machine pivot, and the number of sampling positions.

- According to the geometric expressions of cutter tip and the unit orientation function of cutter axis, build the spline function of each drive axis using the inverse kinematic transformation with the same parameter as the normalized arc length parameter in cutter tip path curve.
- Divide parameter u of the path curve with a suitable parameter interval, and then calculate the first and second derivatives of the path curve with respect to parameter u, as well as the curvatures associated with all sampled positions of the cutter tip path.
- Using related formulas (9), (18), and (19), calculate the matrix of the optimization objective function and the constrained conditions.
- Make use of the well-developed algorithm of linear programming to get the optimal feedrate at each sampled position, and then fit NURBS spline to the sampled feedrate data with respect to parameter u.

Table 1 Predefined constraints for each illustrative example

Method	C-FI	P-FI	PC-FI
Predefined constraints	Axis velocity limit	Chord error limit Axis velocity limit Axis acceleration limit	Chord error limit Axis velocity limit Axis acceleration limit Velocity limit of cutter tip Tangential acceleration limit of cutter tip

3 Illustrative examples

Illustrative examples are conducted to validate the proposed feedrate scheduling method. Except the one to demonstrate the validness of the new proposed method which is denoted as "PC-FI", the other two comparative examples are also conducted by relaxing the preset constraints. The first comparison is conducted with the case of feedrate scheduling limited by maximum axis velocities, and the second comparison is conducted with the case of feedrate scheduling constrained by geometric error and drive limits. The two algorithms used to compare are denoted as "C-FI" and "P-FI", respectively. In order to distinguish the items of constraints for each example intuitively, the corresponding constraints of each algorithm is listed in Table 1, and the feed interpolation parameters is arranged in Table 2. The given cutter axis surface used in the illustrative examples is shown in Fig. 4, and the simulation results are shown in Fig. 5.

From Fig. 5a, it can be seen that although axis velocity limit is taken into account in C-FI module, the corresponding feed profile varies greatly. In the parameter range $u \in \{[0.05,0.16],[0.59,0.79],[0.96,1]\}$, the cutter tip feedrate derived by C-FI module exceeds its limit value 40 mm/s. Especially at parameter position $u = 0.63$, the cutter tip feedrate reaches its maximum peak value 84 mm/s, which is twice more than the allowable limit value 40 mm/s. Moreover, it is clear that the discontinuities of the tangential direction of the cutter tip feedrate curve occurs at parameter position $u = 0.06$, 0.13, 0.63. Generally, an abrupt change of the feedrate curve of cutter tip also leads to a more rapid change of the tangential acceleration of cutter tip. It can be seen from Fig. 5e that the tangential acceleration of cutter tip changes abruptly at the same parameter position $u = 0.06$, 0.13, 0.63. The maximum value of tangential acceleration of cutter tip reaches 1, 054 mm/s^2, which is far from the preset allowable limit 50 mm/s^2. As shown in Fig. 5c, d, using the C-FI algorithm the velocities of three translation axes and two rotation axes are all constrained within the preset ranges. However, the C-FI algorithm is unable to control the dramatic change of acceleration of each drive axis and cutter tip, which has been verified by Fig. 5f–j. Compared with the preset allowable magnitudes of acceleration of five drive axes, it can be seen that using the C-FI algorithm the magnitudes of acceleration of two rotational axes reach 142 and 200°/s^2, respectively, which are all lager than the allowable limits 80°⁄s^2. Also, From Fig. 5h–j, one can see that the maximum acceleration magnitudes of three translation axes reach 1,029, 1,156, and 706 mm/s^2, respectively, which are all beyond the preset allowable limit 200 mm/s^2. In general, C-FI algorithm has no ability to ensure the cutting performance and the drive performance in terms of axis acceleration constraints.

On the basis of C-FI algorithm, P-FI algorithm considers the chord error constraint and axis acceleration constraints additionally. Obviously, the feedrate curve of cutter tip corresponding to P-FI algorithm in Fig. 5a becomes more flat than that in C-FI, and the maximum value also becomes smaller, despite that it still exceeds the preset allowable limit 40 mm/s. As illustrated in Fig. 5e, the same trend occurs in the tangential acceleration curve of cutter tip. The maximum valve of tangential acceleration of cutter tip is reduced from 1,054 mm/s^2 in C-FI to 134 mm/s^2 in P-FI, although both of them exceed the allowable limit 50 mm/s^2. In addition, from Fig. 5c, d, f, g, h, i and j, it can be seen that the drive performances of five axes in P-FI are constrained within the preset ranges. Compared with P-FI algorithm, PC-FI is further constrained by two additional limits, namely the velocity and tangential acceleration limits of cutter tip. Under these constraints, the feedrate associated with PC-FI algorithm is further reduced in the sensitive regions of cutter tip velocity and acceleration and the feedrate curve becomes most flat compared with other two feedrate curves. From Fig. 5a and e, one can see that using the PC-FI algorithm, both the maximum feedrate and tangential acceleration of cutter tip are

Table 2 Five-axis feed interpolation parameters

Parameter	Value
Interpolator period	0.004 s
Chord error limit	0.0005 mm
Velocity limit of *X*-axis, *Y*-axis, *Z*-axis, *A*-axis, *C*-axis	60 mm/s, 60 mm/s, 60 mm/s, 12°/s, 12°/s
Acceleration limit of *X*-axis, *Y*-axis, *Z*-axis, *A*-axis, *C*-axis	50 mm/s^2, 50 mm/s^2, 50 mm/s^2, 120°/s^2, 120°/s^2
Velocity limit of cutter tip	40 mm/s
Tangential acceleration limit of cutter tip	50 mm/s^2

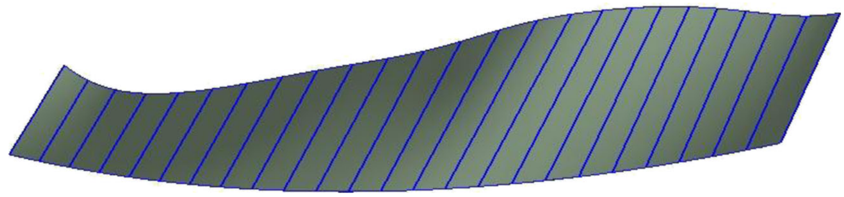

Fig. 4 Cutter axis surface

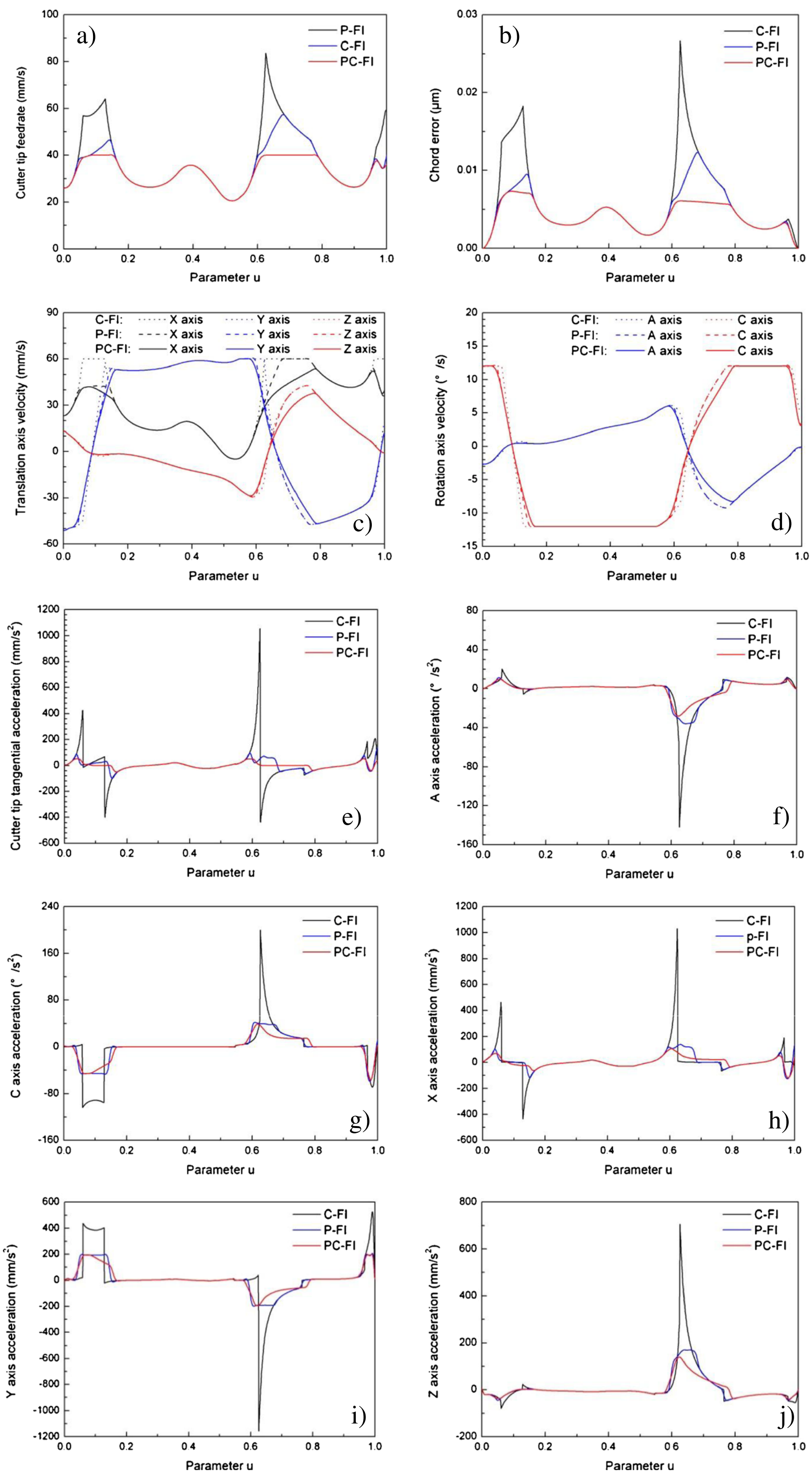

Fig. 5 Simulation results. **a** Cutter tip feedrate, **b** chord error, **c** translation axis velocity, **d** rotation axis velocity, **e** cutter tip tangential acceleration, **f** *A*-axis acceleration, **g** *C*-axis acceleration, **h** *X*-axis acceleration, **i** *Y*-axis acceleration, **j** *Z*-axis acceleration

restrained below the preset values 40 and 60 mm/s^2. The results prove that the proposed PC-FI method is not only able to make the drive constraints under control, but also ensure the cutting performance of cutter tip simultaneously.

Figure 5b show the chord error curves of three feedrate scheduling methods. Although the chord error of each method is within the allowable region, there still exist some obvious differences among three curves. Meanwhile, one can see that the curve shapes of chord error and feedrate of cutter tip have some similarity through the comparison between Fig. 5a and b. The reason is the change of the curvature of cutter tip path is not notable, and thus the chord error gets larger with the increase of the feedrate of cutter tip. The chord error constraint in the illustrated examples has no effect on the final resulting feedrate curve even giving a very small chord error limit. Even so, it is necessary to constrain the chord error in feedrate scheduling for five-axis machining considering the requirements of high-speed machining. Therefore, the proposed PC-FI method still takes chord error limit into account.

In summary, C-FI algorithm which takes axis velocity limit into account is a simple and well-understood way to determine the feedrate in parametric curve interpolator, but its disadvantages are also obvious. It possibly violate machine's axis acceleration limit and the chord error may become overlarge so that it will affect the accuracy of the machined part. At the same time, the cutting performance of cutter tip cannot be well ensured during machining. On the basis of C-FI method, P-FI method adds the axis acceleration constraint and chord error constraint, and therefore the drive performance of the machine has an obvious improvement compared to C-FI, but the cutting performance of cutter tip is still not taken into account, so the overload of cutter probably occurs due to the abrupt change of the feedrate of cutter tip. The proposed PC-FI method simultaneously respect drive constraints and cutting performance of cutter tip in feedrate scheduling, and it can obtain better kinematic characters of both the drive axes and cutter compared with P-FI and C-FI algorithms.

4 Conclusions

Adaptive feedrate scheduling is highly desirable for five-axis machining, especially machining at high speed. However, only drive constraints or geometric constraint are considered in existing scheduling methods. In this paper, an adaptive feedrate scheduling method, which incorporates the chord error limit, drive constraints, and cutting performance constraints of the cutter into the feed optimization model, is proposed for improving the machining quality, accuracy, and reducing the cutter wear. The multiconstraint based feed model is first built and analytic relations between the constraints and geometric characters as well as the kinematic characters of the given path curve are subsequently derived. Through the linearization treatment of constraint conditions, the calculation complexity of the feed optimization model is reduced significantly and thus a linear programming algorithm can be used to solve the model. Illustrative examples are carried out to validate the proposed method. The results show that the proposed method is able to get more smooth motion no matter from the aspect of drive axes or the cutter compared with other methods. The proposed method provides a good solution to the issue of feedrate scheduling for five-axis CNC machining, and it has potential to be applied in five-axis machining of geometrically complex parts. Real-time feedrate scheduling of the proposed method is the future work.

Acknowledgments This work was supported by the National Natural Science Foundation of China (nos. 51075054, 11290143, and 51321004) and the National Basic Research Program of China (no. 2011CB706800).

References

1. Kurt M, Bagci E (2011) Feedrate optimisation/scheduling on sculptured surface machining: a comprehensive review, applications and future directions. Int J Adv Manuf Technol 55:1037–1067
2. Zhang XT, Song Z (2012) An iterative feedrate optimization method for real-time NURBS interpolator. Int J Adv Manuf Technol 62: 1273–1280
3. Lee RS (2000) Real-time surface interpolator for 3-D parametric surface machining on 3-axis machine tools. Int J Mach Tool Manuf 40:1513–1526
4. Ni XY, Wang DH, Li YB (2011) Real-time NURBS curve interpolator based on section. Int J Adv Manuf Technol 54:239–249
5. Yang DCH, Kong T (1994) Parametric interpolator versus linear interpolator for precision surface machining. Comput Aided Des 26(3):225–234
6. Jin YA, He Y, Fu JZ (2013) A look-ahead and adaptive speed control algorithm for parametric interpolation. Int J Adv Manuf Technol 69: 2613–2620
7. Tapie L, Mawussi B, Rubio W, Furet B (2013) Machining of complex-shaped parts with guidance curves. Int Adv Manuf Technol 69:1499–1509
8. Bedi S, Ali I, Quan N (1993) Advanced interpolation techniques for CNC machines. Trans ASME J Eng Ind 115(6):329–336
9. Shpitalni M, Koren Y, Lo CC (1994) Real-time curve interpolators. Comput Aided Des 26:832–838
10. Moetakef Imani B, Ghandehariun A (2011) Real-time PH-based interpolation algorithm for high speed CNC machining. Int J Adv Manuf Technol 56:619–629
11. Wang JB, Yau HT (2013) Universal real-time NURBS interpolator on a PC-based controller. Int Adv Manuf Technol. doi:10.1007/s00170-013-5460-5
12. Sun YW, Jia ZY, Ren F, Guo DM (2008) Adaptive feedrate scheduling for NC machining along curvilinear paths with improved kinematic and geometric properties. Int J Adv Manuf Technol 36:60–68
13. Lai JY, Lin KY, Tseng SJ, Ueng WD (2008) On the development of a parametric interpolator with confined chord error, feedrate, acceleration and jerk. Int J Adv Manuf Technol 37:104–121
14. Yong T, Narayanaswami R (2003) A parametric interpolator with confined chord errors, acceleration and deceleration for NC machining. Comput Aided Des 35:1249–1259

15. Zhang K, Yuan CM, Gao XS (2013) Efficient algorithm for time-optimal feedrate planning and smoothing with confined chord error and acceleration. Int J Adv Manuf Technol 66(9–12):1685–1697
16. Yeh SS, Hsu PL (2002) Adaptive–feedrate interpolation for parametric curves with a confined chord error. Comput Aided Des 34:229–237
17. Sun YW, Zhao Y, Bao YR, Guo DM (2014) A novel adaptive-feedrate interpolation method for NURBS tool path with drive constraints. Int J Mach Tool Manuf 77:74–81
18. Lotfi B, Zhong ZW, Khoo LP (2009) Variable feed rates and variable machine forces for a constant material removal rate and constant cutting force along Pythagorean-hodograph curves. Int J Adv Manuf Technol 40:171–178
19. Tikhon M, Ko TJ, Lee SH, Kim HS (2004) NURBS interpolator for constant material removal rate in open NC machine tools. Int J Mach Tool Manuf 44(2–3):237–245
20. Tsai MS, Nien HW, Yau HT (2011) Development of integrated acceleration/deceleration look-ahead interpolation technique for multi-blocks NURBS curves. Int J Adv Manuf Technol 56:601–618
21. Lee AC, Lin MT, Pan YR, Lin WY (2011) The feedrate scheduling of NURBS interpolator for CNC machine tools. Comput Aided Des 43(6):612–628
22. Dong JY, Ferreira PM, Stori JA (2007) Feed-rate optimization with jerk constraints for generating minimum-time trajectories. Int J Mach Tool Manuf 47:1941–1955
23. Feng JC, Li YH, Wang YH, Chen M (2010) Design of a real-time adaptive NURBS interpolator with axis acceleration limit. Int J Adv Manuf Technol 48:227–241
24. Sekar M, Narayanan VN, Yang SH (2007) Design of jerk bounded feedrate with ripple effect for adaptive NURBS interpolator. Int J Adv Manuf Technol 37:545–552
25. Wang JB, Yau HT (2009) Real-time NURBS interpolator: application to short linear segments. Int J Adv Manuf Technol 41:1169–1185
26. Liang S, Zhao WS, Xi XC (2012) Design of a real-time NURBS interpolator with constant segment length for milling EDM. Int J Adv Manuf Technol 67:427–440
27. Wu JC, Zhou HC, Tang XQ, Chen JH (2011) A NURBS interpolation algorithm with continuous feedrate. Int J Adv Manuf Technol 59:623–632
28. Baek DK, Yang SH, Ko TJ (2012) Precision NURBS interpolator based on recursive characteristics of NURBS. Int J Adv Manuf Technol 65:403–410
29. Dong HT, Chen B, Chen YP, Xie JM, Zhou ZD (2012) An accurate NURBS curve interpolation algorithm with short spline interpolation capacity. Int J Adv Manuf Technol 63:1257–1270
30. Wu JC, Zhou HC, Tang XQ, Chen JH (2012) Fast NURBS interpolation based on the biarc guide curve. Int J Adv Manuf Technol 58(5–8):597–605
31. Ridwan F, Xu X, Ho CL (2012) Adaptive execution of an NC program with feed rate optimization. Int J Adv Manuf Technol 63: 1117–1130
32. Jin YA, He Y, Fu JZ, Chen ZC (2013) An interpolation method for the open CNC system based on EPM. Int J Adv Manuf Technol. doi:10.1007/s00170-013-5044-4
33. Li JG, Zhang TH, Li ZX (2012) An adaptive off-line NURBS interpolator f or CNC machining. Int J Adv Manuf Technol 63: 719–729
34. Heng M, Erkorkmaz K (2010) Design of a NURBS interpolator with minimal feed fluctuation and continuous feed modulation capability. Int J Mach Tool Manuf 50:281–293
35. Tutunea-Fatan OR, Feng HY (2005) Determination of geometry-based errors for interpolated tool paths in five-axis surface machining. J Manuf Sci Eng 127(1):60–67
36. Lavernhe S, Tournier C, Lartigue C (2007) Kinematical performance prediction in multi-axis machining for process planning optimization. Int J Adv Manuf Technol 37(5–6):534–544
37. Li W, Liu YD, Yamazaki KZ, Fujisima MK, Mori MH (2007) The design of a NURBS pre-interpolator for five-axis machining. Int J Adv Manuf Technol 36(9–10):927–935
38. Qiao ZF, Wang TY, Wang YF, Hu M, Liu QJ (2012) Bézier polygons for the linearization of dual NURBS curve in five-axis sculptured surface machining. Int J Mach Tool Manuf 53:107–117
39. Sencer B, Altintas Y, Croft E (2008) Feed optimization for five-axis CNC machine tools with drive constraints. Int J Mach Tool Manuf 48(7–8):733–745
40. Fan W, Gao XS, Yan W, Yuan CM (2012) Interpolation of parametric CNC machining path under confined jounce. Int J Adv Manuf Technol 62(5–8):719–739
41. Fan W, Gao XS, Zhang K (2012) Time-optimal interpolation for five-axis CNC machining along parametric tool path based on linear programming. MM-Prepr 31:21–42
42. Zhang Q, Limit SR (2013) Efficient computation of smooth minimum time trajectory for CNC machining. Int Adv Manuf Technol 68: 683–692
43. Sun YW, Bao YR, Kang KX, Guo DM (2013) An adaptive feedrate scheduling method of dual NURBS curve interpolator for precision five-axis CNC machining. Int J Adv Manuf Technol 68:1977–1987