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ORIGINAL ARTICLE

Evaluation of machining process by integrating 3D manufacturing dispersions, functional constraints, and the concept of small displacement torsors

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Abstract This paper presents a 3D formalization of manufacturing tolerancing which associates the concept of small displacements, the functional constraints, and manufacturing process capability. This approach would make it possible to establish relations which limit the surface variations of the production mechanism by the functional specifications. These relations have a number of requirements imposed by designer and are sufficient to evaluate the manufacturing process. The variations of the manufacturing process are defined in relation to variations measured by 3D measuring.

Keywords 3D tolerancing · Functional tolerancing · Manufacturing tolerancing · Small displacement torsors

1 Introduction

During the process of industrialization of a product, the manufacturing tolerancing belongs to the definition step of the machining. In particular, it makes it possible to validate the choice of the processes by the characterization of the acceptable geometrical variations on the workpiece to respect the functional constraints, the manufacturing requirements, and the geometrical specifications of the workpiece during machining while ensuring an optimal manufacture (minimal cost or minimal time).

The 3D manufacturing tolerancing remains scarcely approached on the level of the research activities. However, in the machining field, the phenomena which cause variations of the machined surfaces compared to their nominal position are known (machine errors, material deformations, tools wear, datum, etc.), but the quantification of these variations is commonly established starting from an unidirectional approach like the 1D tolerance chains approach [1] and the Δl method implemented by Bourdet [2] before being extended by Anselmetti and Bourdet [3] to calculate the machined cost and by Bouzid et al. [4] to optimize the variations on machined surfaces and to minimize the rough dimension in the three directions of workpiece. Nevertheless, the 1D approaches do not take into account the influence of the surfaces orientation variations.

To calculate variations on machined surfaces, some works existed in the literature have proposed 3D tolerancing approaches. Bourdet and Ballot [5] have defined the concept of small displacement torsors used for the representation of the geometrical deviation in the manufacturing process. Many theories were derived from this concept. Some works have been derived from this concept:

Bourdet and Ballot [6] have proposed the ΔTol tolerancing method with the concept of the small displacement torsors to describe the variations between the machined surfaces and surfaces of the nominal workpiece.

Villeneuve et al. [7] have proposed a 3D approach of specification which integrates the functional constraints of a mechanism which are related to the workpiece manufacturing process.

Anselmetti et al. [8, 9] have analyzed the surface variations influence on the contact between the parts of a mechanism, on the geometrical variations of machined surface, and thereafter on the tolerances of the dimensions limited by these surfaces to evaluate the machining process by integrating manufacturing variation and functional constraints.

Laifa et al. [10] have explored the small displacement torsors approach to propose a 3D formalization of functional tolerance which associates with the definition according to the ISO standard of dimensional and geometrical functional

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specifications. This modeling provides a geometrical tool which will be useful for the development of 3D tolerance models and verifying the validity of manufacturing process.

Zhang and Qiao [11] have presented an approach to optimize the values of tolerance zone considering the dimension tolerances, the orientation, and position tolerances. This approach employs convex set to describe the uncertainties of 3D tolerance zone variations and utilizes the reliability index to define the safety degree of 3D tolerance zone.

Xu and Keyser [12] have presented a geometric method of tolerance analysis that provides the geometric approximation of the tolerance zone as well as numeric boundaries.

Pasquale et al. [13] have proposed a graphic approach mainly composed of two different analyses to validate the global consistency of a 3D tolerance specification set. The first analysis is aimed to verify, at assembly level, the adequacy of a tolerance specification set by determining the assembly subsets of parts which influence each assembly key characteristic. The second analysis is aimed to verify the consistency of the tolerance specification set by using rules mainly based on the TTRS theory.

Anselmetti [14] have presented a statistical method to analyze the influence of the geometrical variations in a mechanism (manufacturing variations of surfaces and gaps) on the reliability of a product. The authors have analyzed the influence of geometrical variations on the probability of having a possible assembly.

This paper's contribution is the association of the small displacement torsors concept, the functional constraints, and the manufacturing process to establish relations which limit the mechanism feature variations by the functional specifications in order to verify the validity of the manufacturing process.

Quantification of variations is the great handicap. In this paper, we have developed the work of Radouani and Ballu et al. [15] to formalize an empirical model to estimate the values of the gap variations.

2 Modeling of variations in a machining process

One compares the machining process to a mechanism made up primarily of the machine (*M*), the part holder (*A*), its holder surfaces (*Ai*), the workpiece (*P*), its datum surfaces (*Fi*), and its machined surfaces (*Si*). The variations of these parts and their surfaces compared to their nominal positions will be modeled by a small displacement torsors (Fig. 1).

Small displacement torsor characteristics of these variations are defined as follows:

$T^n_{A/M}$: Small displacement torsor of the part holder *A* compared to its nominal position in setup *n*.

$T^n_{Ai/M}$: Small displacement torsors of holder surfaces variations *Ai* compared to their nominal positions in setup *n*.

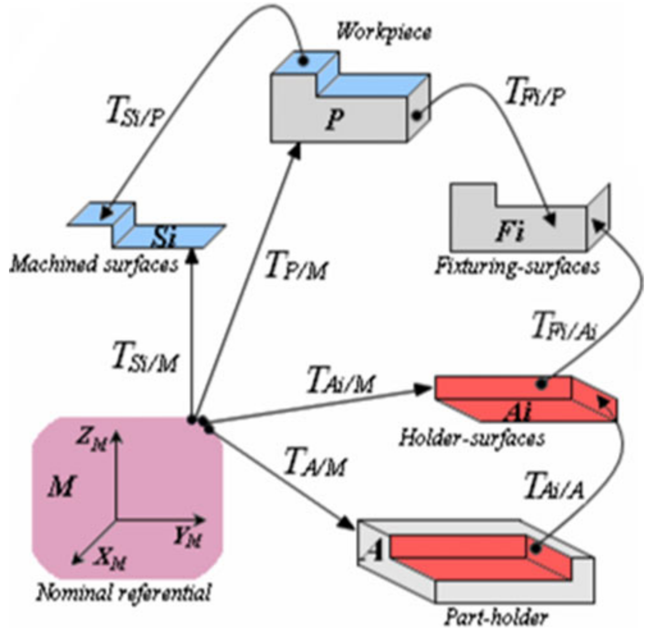

Fig. 1 Modeling of the various variations in a machining setup

$T^n_{P/M}$: Small displacement torsor of the workpiece in setup *n* compared to its nominal position in datum *M*.

$T_{Si/M}$: Variation torsors of the machined surfaces *Si* compared to their nominal position in datum *M* (variations linked to the machining).

$T^n_{Ai/A}$: Variation torsors of the *Ai* surfaces compared to their nominal positions on the part holder in setup *n*. This variation defines the geometric variations of the part holder.

$T^n_{Fi/Ai}$: Gap torsors that define the characteristics of the joint between the workpiece and the part holder at the level of the joint *Fi*/*Ai* in setup *n*.

$T_{Di/P}$ and $T_{Si/P}$: Variation torsors of workpiece surfaces compared to their nominal positions on the workpiece datum P (*Si*: machined surfaces; *Di*: datum surfaces in functional requirements).

The variations of the surface machined in a setup *n* compared to its nominal position in the workpiece datum P can be decomposed as follows:

$$T_{Si/P} = T_{Si/M} - T^n_{P/M}. \quad (1)$$

Workpiece variations in setup *n* compared to its nominal position in datum *M* can be decomposed into:

$$T^n_{P/M} = T^n_{Ai/M} + T^n_{Fi/Ai} - T^n_{Fi/P}.$$

Relation (1) becomes:

$$T_{Si/P} = T_{Si/M} - T^n_{Fi/Ai} - T^n_{Ai/M} + T^n_{Fi/P}.$$

Let us consider now two machined surfaces Sa and Sb in two setups, respectively, m and n. The variations of the one compared to the other can be modeled by the small displacement torsor $T_{Sb/Sa}$ which can be decomposed into:

$$T_{Sb/Sa} = T_{Sb/P} - T_{Sa/P}. \tag{2}$$

The development of relation (2) gives:

$$T_{Sb/Sa} = \left(T^n_{Sb/M} - T^n_{Fi/Ai} - T^n_{Ai/M} + T^n_{Fi/P}\right) - \left(T^m_{Sa/M} - T^m_{Fi/Ai} - T^m_{Ai/M} + T^m_{Fi/P}\right).$$

This torsor is reduced to the difference of the manufacturing variations of two surfaces Sa and Sb ($T_{Sb/Sa}=T_{Sb/M}-T_{Sa/M}$) if two surfaces are machined in the same setup n. The calculation of this torsor will be more difficult when datum surfaces are machined in the anterior setups.

$T_{Si/M}$ and $T^n_{Ai/M}$ can be measured directly on the machine with a sensor in several points whereas the gap torsors $T^n_{Fi/Ai}$ can be evaluated by the measuring of deviations on the datum supports or it can be estimated by using the empirical models.

3 Integration: manufacturing processes–functional constraints

Respecting the constraints imposed by the designer requires taking into account of the process imperfections and their evolution during stages of machining. This imposes limitation on the manufacturing variations by the tolerances relating to the functional requirements.

3.1 Parallelism requirement

On the basis of the definition, according to the ISO tolerancing standards of the geometrical requirement, one calculates dispersions on such a dimension of given direction. Figure 2 shows an example of a parallelism constraint.

In the workpiece referential (P), one models the variations of the datum (D) and variations of the toleranced surface (S), respectively, by the torsors:

$$\{T_{D/P}\}_O = \begin{Bmatrix} \alpha_{D/P} & 0 \\ \beta_{D/P} & 0 \\ 0 & w_{D/P} \end{Bmatrix}_O, \left\{T_{S/P}\right\}_O = \begin{Bmatrix} \alpha_{S/P} & 0 \\ \beta_{S/P} & 0 \\ 0 & w_{S/P} \end{Bmatrix}_O.$$

The variation of the surface (S) compared to the plan (D) is:

$$\{T_{S/D}\}_O = \{T_{S/P}\}_O - \{T_{D/P}\}_O = \begin{Bmatrix} \alpha_{S/P}-\alpha_{D/P} & 0 \\ \beta_{S/P}-\beta_{D/P} & 0 \\ 0 & w_{S/P}-w_{D/P} \end{Bmatrix}_O.$$

The variations between toleranced surface (S) and its nominal position in the referential related to the datum (D) is defined by the displacement of any point of the toleranced surface M_S compared to the point corresponding M_D (Fig. 3). These variations, depending only on the rotation variations (geometrical orientation tolerance), are calculated by the relation:

$$\rightarrow M_D M_S = \begin{pmatrix} \alpha_{S/P}-\alpha_{D/P} \\ \beta_{S/P}-\beta_{D/P} \\ 0 \end{pmatrix} \wedge \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{pmatrix} \left(\beta_{S/P}-\beta_{D/P}\right) z_i \\ -(\alpha_{S/P}-\alpha_{D/P}) y_i \\ -\left(\beta_{S/P}-\beta_{D/P}\right) x_i + (\alpha_{S/P}-\alpha_{D/P}) y_i \end{pmatrix}.$$

To check the geometrical condition of parallelism, it should be checked that all the points of toleranced surface are in the tolerance zone. It is necessary checked that:

$$\overrightarrow{M_D M_S} \cdot \overrightarrow{n_D} \le IT. \tag{3}$$

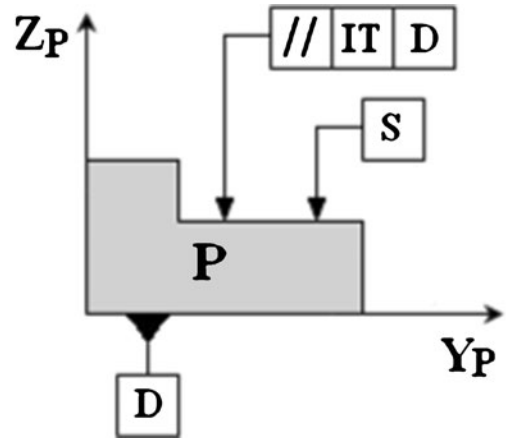

Fig. 2 Example of a parallelism constraint

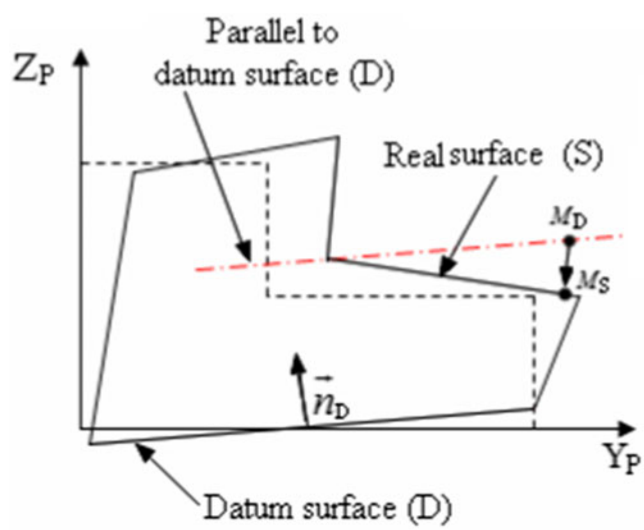

Fig. 3 Variations between toleranced surface and datum surface

$$\overrightarrow{n_D} = \begin{pmatrix} \beta_{D/P} \\ -\alpha_{D/P} \\ 1 \end{pmatrix}$$: Normal vector to the plan (D).

By neglecting the terms of the second order (small displacements) relation (3) becomes:

$$\left(\beta_{D/P}-\beta_{S/P}\right)x_i + \left(\alpha_{S/P}-\alpha_{D/P}\right)y_i \le IT.$$

3.2 Perpendicularity requirement

This geometrical constraint depends only on the variations of rotation. It is enough to check that all the points of the associated surface lie between two parallel plans distant of the value of the perpendicularity tolerance interval and parallel to the element of situation (plan theoretically perpendicular to the specified reference). Figure 4 gives an example of a perpendicularity requirement.

In the workpiece referential (P), one models the variations of the datum (D) and the variations of the toleranced surface (S), respectively, by the torsors:

$$\{T_{D/P}\}_O = \begin{Bmatrix} \alpha_{D/P} & 0 \\ \beta_{D/P} & 0 \\ 0 & w_{D/P} \end{Bmatrix}_O, \{T_{S/P}\}_O = \begin{Bmatrix} \alpha_{S/P} & 0 \\ 0 & v_{S/P} \\ \gamma_{S/P} & 0 \end{Bmatrix}_O.$$

By using the property of the SDT (0+a=0), the variation of the surface (S) compared to the surface (D) is written:

$$\{T_{S/D}\}_O = \{T_{S/P}\}_O - \{T_{D/P}\}_O = \begin{Bmatrix} \alpha_{S/P}-\alpha_{D/P} & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_O.$$

The variations between toleranced surface (S) and its nominal position in the datum (D) is defined by the displacement of any point of the toleranced surface M_S compared to the point corresponding to M_D of the situation plan perpendicular to the datum (D) (Fig. 5). These variations, depending only on

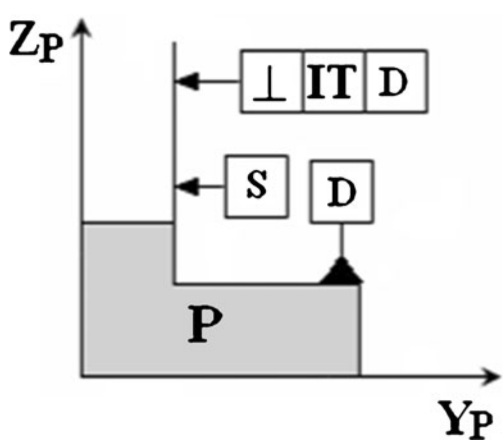

Fig. 4 Example of a perpendicularity requirement

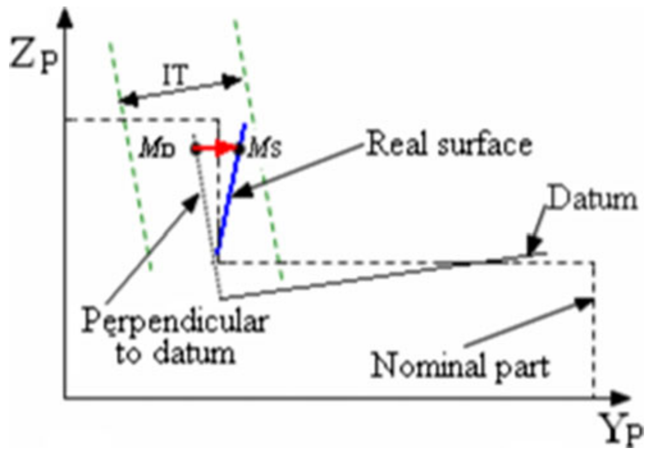

Fig. 5 Variations between toleranced surface and situation plan

the rotation variations (geometrical orientation tolerance), are calculated by the relation:

$$\overrightarrow{M_D M_S} = \begin{pmatrix} \alpha_{S/P}-\alpha_{D/P} \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{pmatrix} 0 \\ -\left(\alpha_{S/P}-\alpha_{D/P}\right)z_i \\ \left(\alpha_{S/P}-\alpha_{D/P}\right)y_i \end{pmatrix}.$$

To check the geometrical condition of perpendicularity, it should be checked that all the points of toleranced surface are in the tolerance zone. It is necessary checked that:

$$\overrightarrow{M_D M_S} \cdot \overrightarrow{n_D} \le IT. \quad (4)$$

$$\overrightarrow{n_D} = \begin{pmatrix} \alpha_{D/P} \\ 1 \\ -\gamma_{D/P} \end{pmatrix}$$: Normal vector to the situation plan perpendicular to (D).

By neglecting the terms of the second order (small displacements), relation (4) becomes:

$$\left(\alpha_{D/P}-\alpha_{S/P}\right)z_i \le IT.$$

3.3 Location requirement

According to the ISO tolerancing standards, the tolerance zone of this specification is limited by two parallels plans, distends of the tolerance interval, and lays out symmetrically compared to the theoretically exact position of considered surface. Toleranced surface, in all its extent, must be within the tolerance zone. Figure 6 gives an example of a location requirement.

As the tolerance of parallelism, the variations between toleranced surface (S) and its nominal position in the datum (D) are defined by the displacements of any point of the toleranced surface M_S compared to the corresponding point M_D (Fig. 3). These variations, dependent on the rotation and

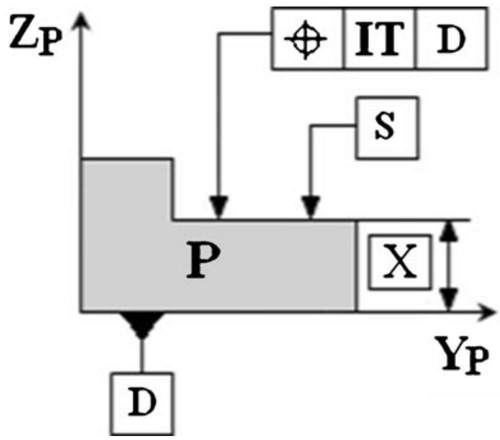

Fig. 6 Example of a location requirement

translation variations (geometrical position tolerance), are calculated by the relation:

$$\rightarrow M_D M_S = \begin{pmatrix} 0 \\ 0 \\ w_{S/P} - w_{D/P} \end{pmatrix} + \begin{pmatrix} \alpha_{S/P} - \alpha_{D/P} \\ \beta_{S/P} - \beta_{D/P} \\ 0 \end{pmatrix} \wedge \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -\left(\beta_{S/P} - \beta_{D/P}\right) x_i + \left(\alpha_{S/P} - \alpha_{D/P}\right) y_i \\ + \left(w_{S/P} - w_{D/P}\right) \end{pmatrix}.$$

To check the geometrical condition of location, it should be checked that all the points of toleranced surface are in the tolerance zone. It is necessarily checked that:

$$\left| \overrightarrow{M_D M_S} \cdot \overrightarrow{n_D} \right| \leq \frac{IT}{2}. \tag{5}$$

By neglecting the terms of the second order (small displacements), relation (5) becomes:

$$\left| \left(\beta_{D/P} - \beta_{S/P}\right) x_i + \left(\alpha_{S/P} - \alpha_{D/P}\right) y_i + \left(w_{S/P} - w_{D/P}\right) \right| \leq \frac{IT}{2}.$$

3.4 Dimensional requirement

According to the ISO tolerancing standards, linear dimensional requirements limit only local dimensions. Consequently, the linear dimensional requirements will be carried only between surfaces having a local dimensions mainly cylinders, spheres, and parallel plans having sufficient matter in opposite. Each real local dimension must be respected independently of other local real dimensions.

The condition with two limits requires that the dimension of the part lies between the limiting values of the functional dimension.

The variations of the two surfaces (*D*) and (*S*) are modeled, respectively, by the small displacement torsors:

$$\left\{T_{D/P}\right\}_O = \begin{Bmatrix} \alpha_{D/P} & 0 \\ \beta_{D/P} & 0 \\ 0 & w_{D/P} \end{Bmatrix}_O, \quad \left\{T_{S/P}\right\}_O = \begin{Bmatrix} \alpha_{S/P} & 0 \\ \beta_{S/P} & 0 \\ 0 & w_{S/P} \end{Bmatrix}_O.$$

The variations between toleranced surface (*S*) and its nominal position in the datum (*D*) are defined by the displacement of any point of the toleranced surface M_S compared to the corresponding point M_D. These variations, dependent on the rotation and translation variations, are calculated by the relation:

$$\rightarrow M_D M_S = \begin{pmatrix} 0 \\ 0 \\ w_{S/P} - w_{D/P} \end{pmatrix} + \begin{pmatrix} \alpha_{S/P} - \alpha_{D/P} \\ \beta_{S/P} - \beta_{D/P} \\ 0 \end{pmatrix} \wedge \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \left(\beta_{D/P} - \beta_{S/P}\right) x_i + \left(\alpha_{S/P} - \alpha_{D/P}\right) y_i \\ + \left(w_{S/P} - w_{D/P}\right) \end{pmatrix}.$$

To check the dimensional requirement, it should be checked that all the points of toleranced surface are in the tolerance zone (two parallel plans with the datum and distant of IT). It is necessary to check that:

$$\overrightarrow{M_D M_S} \cdot \overrightarrow{n_D} \leq IT. \tag{6}$$

By neglecting the terms of the second order (small displacements), relation (6) becomes:

$$\left(\beta_{D/P} - \beta_{S/P}\right) x_i + \left(\alpha_{S/P} - \alpha_{D/P}\right) y_i + \left(w_{S/P} - w_{D/P}\right) \leq IT.$$

4 Application

The studied workpiece is defined by a design drawing specified with the ISO tolerancing standards (Fig. 7). The machining process is described by Fig. 8. The machined surfaces are noted (1, 2, 3…) and rough surfaces are noted Bi.

4.1 Parallelism constraint

This geometrical requirement of orientation imposes that toleranced surface (surface 2) lies between two parallel plans distant of the parallelism tolerance interval (IT=0.1mm) and parallel to the specified datum (surface 1). So that toleranced surface is in the tolerance zone, it is necessary that the variations of these two surfaces obey the following preset relation:

$$\left(\beta_{1/P} - \beta_{2/P}\right) x + \left(\alpha_{2/P} - \alpha_{1/P}\right) y \leq 0.1. \tag{7}$$

The expanse of surface (2) is defined by dimensions according to $\left(\overrightarrow{X}\right)$ and $\left(\overrightarrow{Y}\right)$ directions, respectively, by Lx= 30mm and Ly=40mm. Relation (7) becomes:

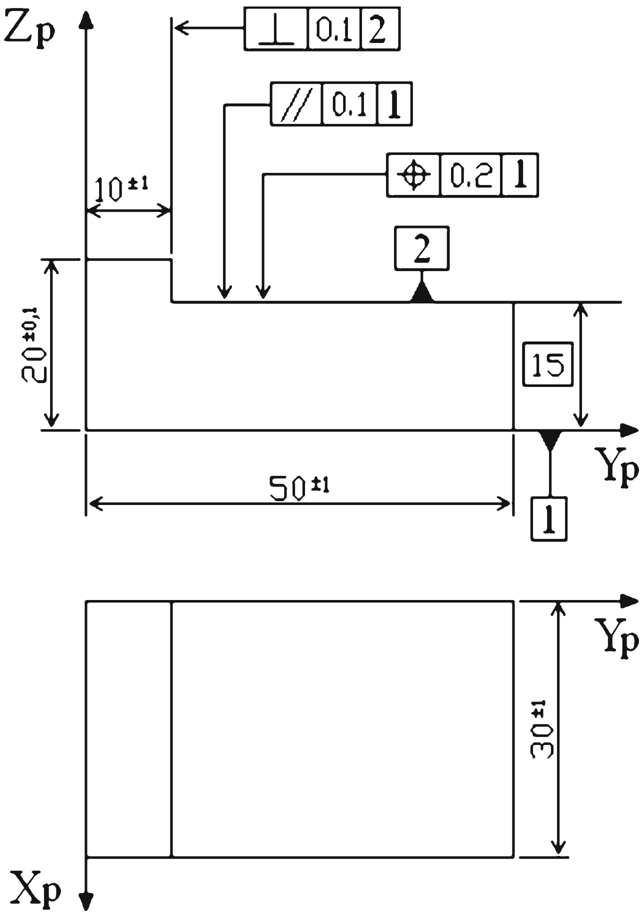

Fig. 7 Workpiece geometry

$$30\left(\beta_{1/P}-\beta_{2/P}\right)+40\left(\alpha_{2/P}-\alpha_{1/P}\right)\leq 0.1. \quad (8)$$

Limit surfaces of parallelism tolerances (1) and (2) are machined, respectively, in setup (10) and (20). In this last setup, surface (1) is a principal datum. By applying relation (4), the variations of toleranced surface (2) compared to the datum (1) are written:

$$\left\{T_{2/1}\right\}_O=\left\{T_{2/P}\right\}_O-\left\{T_{1/P}\right\}_O.$$

The variation torsor of toleranced surface compared to nominal workpiece can be developed according to the measurable variations:

$$\begin{aligned}\left\{T_{2/P}\right\}_O &= \left\{T_{2/M}\right\}_O-\left\{T^{20}_{Di/Ai}\right\}_O-\left\{T^{20}_{Ai/M}\right\}_O+\left\{T^{20}_{Di/P}\right\}_O\\ &=\left\{\begin{matrix}\alpha_{2/M} & 0\\ \beta_{2/M} & 0\\ 0 & w_{2/M}\end{matrix}\right\}_O-\left\{\begin{matrix}\alpha^{20}_{1/A_1} & u^{20}_{B_5/A_{B_5}}\\ \beta^{20}_{1/A_1} & v^{20}_{B_1/A_{B_1}}\\ \gamma^{20}_{B_5/A_{B_5}} & w^{20}_{1/A_1}\end{matrix}\right\}_O\\ &\quad-\left\{\begin{matrix}\alpha^{20}_{A_1/M} & u^{20}_{A_{B_5}/M}\\ \beta^{20}_{A_1/M} & v^{20}_{A_{B_1}/M}\\ \gamma^{20}_{A_{B_5}/M} & w^{20}_{A_1/M}\end{matrix}\right\}_O+\left\{\begin{matrix}\alpha_{1/P} & u_{B_5/P}\\ \beta_{1/P} & v_{B_1/P}\\ \gamma_{B_5/P} & w_{1/P}\end{matrix}\right\}_O\end{aligned}.$$

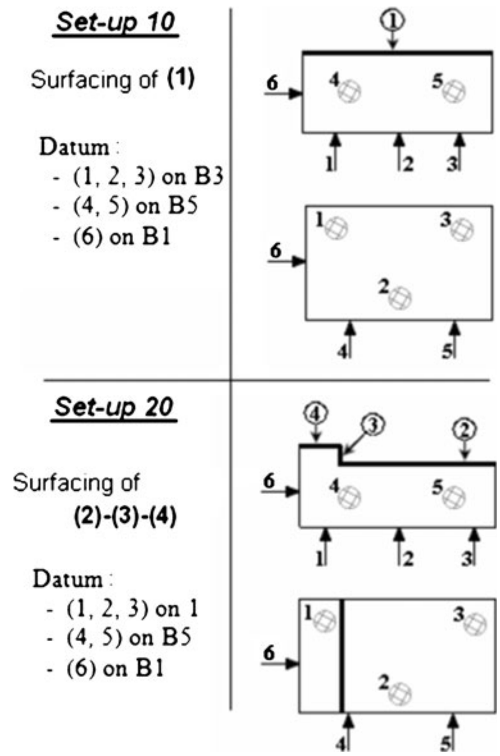

Fig. 8 Machining process

By using the SDT property (0+a=0), this torsor is then written1:

$$\begin{aligned}\left\{T_{2/P}\right\}_O &=\left\{\begin{matrix}\alpha_{2/M} & 0\\ \beta_{2/M} & 0\\ 0 & w_{2/M}\end{matrix}\right\}_O-\left\{\begin{matrix}\alpha^{20}_{1/A_1} & 0\\ \beta^{30}_{1/A_1} & 0\\ 0 & w^{30}_{1/A_1}\end{matrix}\right\}_O\\ &\quad-\left\{\begin{matrix}\alpha^{20}_{A_1/M} & 0\\ \beta^{30}_{A_1/M} & 0\\ 0 & w^{30}_{A_1/M}\end{matrix}\right\}_O+\left\{\begin{matrix}\alpha_{1/P} & 0\\ \beta_{1/P} & 0\\ 0 & w_{1/P}\end{matrix}\right\}_O\end{aligned}.$$

With:

$$\left\{\begin{matrix}\alpha_{1/P} & 0\\ \beta_{1/P} & 0\\ 0 & w_{1/P}\end{matrix}\right\}_O=\left\{T_{P_1/P}\right\}_O.$$

So, the variations torsor of the toleranced surface (2) related to the surface (1) is:

$$\left\{T_{2/1}\right\}_O=\left\{\begin{matrix}\alpha_{2/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M} & 0\\ \beta_{2/M}-\beta^{20}_{1/A_1}-\beta^{20}_{A_1/M} & 0\\ 0 & w_{2/M}-w^{20}_{1/A_1}-w^{20}_{A_1/M}\end{matrix}\right\}_O.$$

Relation (8) therefore becomes:

$$-30\left(\beta_{2/M}-\beta^{20}_{1/A_1}-\beta^{20}_{A_1/M}\right)+40\left(\alpha_{2/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M}\right)\leq 0.1.$$

4.2 Perpendicularity constraint

This geometrical constraint of orientation requires that the toleranced surface (surface 3) lies between two parallel plans distant of the perpendicularity tolerance interval (IT=0.1mm) and perpendicular to the specified datum (surface 2). So that toleranced surface is in the tolerance zone, it is necessary that the variations of these two surfaces respect the following preset relation:

$$\left(\alpha_{2/P}-\alpha_{3/P}\right)z\leq 0.1. \tag{9}$$

The expanse of surface (4) is defined by dimensions according to $\left(\overrightarrow{X}\right)$ and $\left(\overrightarrow{Z}\right)$ directions, respectively, by Lx= 30mm and Lz=5mm. Relation (9) becomes:

$$5\left(\alpha_{2/P}-\alpha_{3/P}\right)\leq 0.1. \tag{10}$$

Toleranced surface (3) is positioned compared to the datum surface (2); these two surfaces are machined in the same setup (20). The variations torsor of toleranced surface compared to the datum surface is reduced to:

$$\{T_{3/2}\}_O = \{T_{3/P}\}_O - \{T_{2/P}\}_O = \{T_{3/M}\}_O - \{T_{2/M}\}_O$$
$$= \left\{\begin{matrix}\alpha_{3/M} & 0\\ 0 & v_{3/M}\\ \gamma_{3/M} & 0\end{matrix}\right\}_O - \left\{\begin{matrix}\alpha_{2/M} & 0\\ \beta_{2/M} & 0\\ 0 & w_{2/M}\end{matrix}\right\}_O = \left\{\begin{matrix}\alpha_{3/M}-\alpha_{2/M} & 0\\ 0 & 0\\ 0 & 0\end{matrix}\right\}_O.$$

Relation (10) becomes:

$$5\left(\alpha_{2/M}-\alpha_{3/M}\right)\leq 0.1.$$

4.3 Location constraint

This geometrical constraint of position requires that the toleranced surface (surface 2) will be limited by two parallel plans, distant of the tolerance interval (IT=0.2mm), and arranged symmetrically compared to the theoretically exact disposition from considered surface [15mm of the reference surface (surface 1)]. So that toleranced surface is in the tolerance zone, it is necessary that the variations of these two surfaces check the preset following relation:

$$\left|\left(\beta_{1/P}-\beta_{2/P}\right)x+\left(\alpha_{2/P}-\alpha_{1/P}\right)y+w_{2/P}-w_{1/P}\right|\leq 0.1. \tag{11}$$

The expanse of surface (2) is defined by dimensions according to $\left(\overrightarrow{X}\right)$ and $\left(\overrightarrow{Y}\right)$ directions, respectively, by Lx= 30mm and Ly=40mm. Relation (11), accordingly, becomes:

$$\left|30\left(\beta_{1/P}-\beta_{2/P}\right)+40\left(\alpha_{2/P}-\alpha_{1/P}\right)+w_{2/P}-w_{1/P}\right|\leq 0.1. \tag{12}$$

Limit surfaces of location requirement (1) and (2) are machined, respectively, in setup (10) and (20). In this last setup, surface (1) is a principal datum surface. By applying relation (4), the variations of toleranced surface (2) compared to the datum (1) are written:

$$\{T_{2/1}\}_O = \{T_{2/P}\}_O - \{T_{1/P}\}_O.$$

The variations torsor of toleranced surface compared to nominal part can be developed according to the measurable variations, into:

$$\{T_{2/P}\}_O = \{T_{2/M}\}_O - \left\{T^{20}_{Di/Ai}\right\}_O - \left\{T^{20}_{Ai/M}\right\}_O + \{T_{Di/P}\}_O$$
$$= \left\{\begin{matrix}\alpha_{2/M} & 0\\ \beta_{2/M} & 0\\ 0 & w_{2/M}\end{matrix}\right\}_O - \left\{\begin{matrix}\alpha^{20}_{1/A_1} & u^{20}_{B_5/A_{B_5}}\\ \beta^{20}_{1/A_1} & v^{20}_{B_1/A_{B_1}}\\ \gamma^{20}_{B_5/A_{B_5}} & w^{20}_{1/A_1}\end{matrix}\right\}_O$$
$$-\left\{\begin{matrix}\alpha^{20}_{A_1/M} & u^{20}_{A_{B_5}/M}\\ \beta^{20}_{A_1/M} & v^{20}_{A_{B_1}/M}\\ \gamma^{20}_{A_{B_5}/M} & w^{20}_{A_1/M}\end{matrix}\right\}_O + \left\{\begin{matrix}\alpha_{1/P} & u_{B_5/P}\\ \beta_{1/P} & v_{B_1/P}\\ \gamma_{B_5/P} & w_{1/P}\end{matrix}\right\}_O.$$

By using the SDT property (0+a=0), this torsor is written then:

$$\{T_{2/P}\}_O = \left\{\begin{matrix}\alpha_{2/M} & 0\\ \beta_{2/M} & 0\\ 0 & w_{2/M}\end{matrix}\right\}_O - \left\{\begin{matrix}\alpha^{20}_{1/A_1} & 0\\ \beta^{20}_{1/A_1} & 0\\ 0 & w^{20}_{1/A_1}\end{matrix}\right\}_O$$
$$-\left\{\begin{matrix}\alpha^{20}_{A_1/M} & 0\\ \beta^{20}_{A_1/M} & 0\\ 0 & w^{20}_{A_1/M}\end{matrix}\right\}_O + \left\{\begin{matrix}\alpha_{1/P} & 0\\ \beta_{1/P} & 0\\ 0 & w_{1/P}\end{matrix}\right\}_O.$$

With:

$$\left\{\begin{matrix}\alpha_{1/P} & 0\\ \beta_{1/P} & 0\\ 0 & w_{1/P}\end{matrix}\right\}_O = \{T_{1/P}\}_O.$$

So, the variations torsor of the toleranced surface (2) related to the surface (1) is:

$$\{T_{2/1}\}_O = \left\{\begin{matrix}\alpha_{2/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M} & 0\\ \beta_{2/M}-\beta^{20}_{1/A_1}-\beta^{20}_{A_1/M} & 0\\ 0 & w_{2/M}-w^{20}_{1/A_1}-w^{20}_{A_1/M}\end{matrix}\right\}_O.$$

Therefore, relation (12) becomes:

$$\left|\begin{matrix}-\ 30\left(\beta_{2/M}-\beta^{20}_{1/A_1}-\beta^{20}_{A_1/M}\right)+\\ 40\left(\alpha_{2/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M}\right)+\left(w_{2/M}-w^{20}_{1/A_1}-w^{20}_{A_1/M}\right)\end{matrix}\right|\leq 0.1.$$

4.4 Dimensional constraint $CD_1=20^{\pm 0.1}$

The nominal plans: surface (1) and surface (4) limiting this condition are perpendicular to the direction $\left(\overrightarrow{Z}\right)$. So that toleranced surface is in the tolerance zone (IT=0.2mm), it is necessary that the variations of these two surfaces check the following preset relation:

$$\left(\beta_{4/P}-\beta_{1/P}\right)x+\left(\alpha_{1/P}-\alpha_{4/P}\right)y+w_{1/P}-w_{4/P}\leq 0.2. \qquad (13)$$

The expanse of surface (4) is defined by dimensions according to $\left(\overrightarrow{X}\right)$ and $\left(\overrightarrow{Y}\right)$ directions, respectively, by Lx=30mm and Ly=10mm. Relation (13) becomes:

$$30\left(\beta_{1/P}-\beta_{4/P}\right)+10\left(\alpha_{4/P}-\alpha_{1/P}\right)+\left(w_{4/P}-w_{1/P}\right)\leq 0.2. \qquad (14)$$

Limiting surfaces (1) and (4) of the dimensional requirement are machined, respectively, in setups (10) and (20). In this last setup, surface (1) is a principal datum. By applying relation (4), the variations torsor of the toleranced surface compared to the datum surface is written:

$$\{T_{4/1}\}_O=\{T_{4/P}\}_O-\{T_{1/P}\}_O.$$

The variations torsor of the toleranced surface (4) compared to nominal workpiece can be developed to measurable variations:

$$\begin{aligned}\{T_{4/P}\}_O&=\{T_{4/M}\}_O-\left\{T^{20}_{Di/Ai}\right\}_O-\left\{T^{20}_{Ai/M}\right\}_O+\{T_{Di/P}\}_O\\&=\begin{Bmatrix}\alpha_{4/M} & 0\\ \beta_{4/M} & 0\\ 0 & w_{4/M}\end{Bmatrix}_O-\begin{Bmatrix}\alpha^{20}_{1/A_1} & u^{20}_{B_5/A_{B_5}}\\ \beta^{20}_{1/A_1} & v^{20}_{B_1/A_{B_1}}\\ \gamma^{20}_{B_5/A_{B_5}} & w^{20}_{1/A_1}\end{Bmatrix}_O\\&\quad-\begin{Bmatrix}\alpha^{20}_{A_1/M} & u^{20}_{A_{B_5}/M}\\ \beta^{20}_{A_1/M} & v^{20}_{A_{B_1}/M}\\ \gamma^{20}_{A_{B_5}/M} & w^{20}_{A_1/M}\end{Bmatrix}_O+\begin{Bmatrix}\alpha_{1/P} & u_{B_5/P}\\ \beta_{1/P} & v_{B_1/P}\\ \gamma_{B_5/P} & w_{1/P}\end{Bmatrix}_O.\end{aligned}$$

By using the SDT property, this torsor is written, so:

$$\begin{aligned}\{T_{4/P}\}_O&=\begin{Bmatrix}\alpha_{4/M} & 0\\ \beta_{4/M} & 0\\ 0 & w_{4/M}\end{Bmatrix}_O-\begin{Bmatrix}\alpha^{20}_{1/A_1} & 0\\ \beta^{20}_{1/A_1} & 0\\ 0 & w^{20}_{1/A_1}\end{Bmatrix}_O\\&\quad-\begin{Bmatrix}\alpha^{20}_{A_1/M} & 0\\ \beta^{20}_{A_1/M} & 0\\ 0 & w^{20}_{A_1/M}\end{Bmatrix}_O+\begin{Bmatrix}\alpha_{1/P} & 0\\ \beta_{1/P} & 0\\ 0 & w_{1/P}\end{Bmatrix}_O.\end{aligned}$$

With:

$$\{T_{1/P}\}_O=\begin{Bmatrix}\alpha_{1/P} & 0\\ \beta_{1/P} & 0\\ 0 & w_{1/P}\end{Bmatrix}_O.$$

So, the variations torsor of the toleranced surface (4) related to the surface (1) is:

$$\{T_{4/1}\}_O=\begin{Bmatrix}\alpha_{4/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M} & 0\\ \beta_{4/M}-\beta^{20}_{1/A_1}-\beta^{20}_{A_1/M} & 0\\ 0 & w_{4/M}-w^{20}_{1/A_1}-w^{20}_{A_1/M}\end{Bmatrix}_O.$$

Relation (14) therefore becomes:

$$\begin{aligned}&30\left(-\beta_{4/M}+\beta^{20}_{1/A_1}+\beta^{20}_{A_1/M}\right)+10\left(\alpha_{4/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M}\right)\\&\quad+\left(w_{4/M}-w^{20}_{1/A_1}-w^{20}_{A_1/M}\right)\leq 0.2.\end{aligned}$$

By applying the same step, one established the relation relating to the dimensional constraint $CD_2=10^{\pm 1}$.

5 Inequalities system

Finally, we obtain these relations relating to the constraints imposed by functional requirement. To check the system of equations and then the validity of the manufacturing process, we must quantify the deviations.

Parallelism

$$-30\left(\beta_{2/M}-\beta^{20}_{1/A_1}-\beta^{20}_{A_1/M}\right)+40\left(\alpha_{2/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M}\right)\leq 0.1.$$

Perpendicularity

$$5\left(\alpha_{2/M}-\alpha_{3/M}\right)\leq 0.1.$$

Location

$$\left|\begin{aligned}&-30\left(\beta_{2/M}-\beta^{20}_{1/A_1}-\beta^{20}_{A_1/M}\right)+\\&40\left(\alpha_{2/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M}\right)+\left(w_{2/M}-w^{20}_{1/A_1}-w^{20}_{A_1/M}\right)\end{aligned}\right|\leq 0.1.$$

Dimensional $CD_1=20^{\pm 0.1}$

$$\begin{aligned}&30\left(-\beta_{4/M}+\beta^{20}_{1/A_1}+\beta^{20}_{A_1/M}\right)+10\left(\alpha_{4/M}-\alpha^{20}_{1/A_1}-\alpha^{20}_{A_1/M}\right)\\&+\left(w_{4/M}-w^{20}_{1/A_1}-w^{20}_{A_1/M}\right)\leq 0.2.\end{aligned}$$

Dimensional $CD_2 = 10^{\pm 1}$

$$
\begin{aligned}
&-5\begin{pmatrix} \alpha_{3/M} - \alpha^{20}_{1/A_1} - \alpha^{20}_{A_1/M} + \alpha_{1/M} - \alpha^{20}_{B_3/A_{B_3}} - \alpha^{20}_{A_{B_3}/M} \\ + \quad \alpha_{B_3/M} - \quad \alpha_{B_1/M} \end{pmatrix} \\
&+ 30 \left(\gamma_{3/M} - \gamma^{20}_{B_5/A_{B_5}} - \gamma^{20}_{A_{B_5}/M} + \gamma_{B_5/M} - \gamma_{B_1/M}\right) \\
&+ \left(v_{3/M} - v^{20}_{B_1/A_{B_1}} - v^{20}_{A_{B_1}/M}\right) \quad \leq \quad 2 \quad .
\end{aligned}
$$

6 Quantification of 3D deviations

The evaluation of the deviations can result either from models of behavior or from experimental measurements. This last require many experimental tests.

The respect of the functional specifications between the geometrical entities of a part passes by the limitation of the relative deviations of these entities in accordance to the relation:

$$
\begin{aligned}
T_{Sb/Sa} = &\left(T_{Sb/M} - T^n_{Fi/Ai} - T^n_{Ai/M} + T^n_{Fi/P}\right) \\
&-\left(T_{Sa/M} - T^m_{Fi/Ai} - T^m_{Ai/M} + T^m_{Fi/P}\right).
\end{aligned}
$$

This relation evokes the four following torsors:

$T^n_{Fi/P}$: Variation of reference surface compared to nominal part. This torsor will be broken up according to the three others as much as necessary since surfaces of references in the phase considered were machined in former phases.

$T^n_{Ai/M}$ and $T_{Si/M}$: respectively are variations of holder surfaces and machined surfaces. The components of these torsors can be measured directly on the machine with a sensor mounted on the spindle of the machine.

$T^n_{Fi/Ai}$: Torsor expressing the characteristics of the gap between the part and the part holder. The estimated values of the components of this torsor are based on the development of an experimental study [16] which considers when stacking two parts A and B; the flatness defects of the contact surfaces can introduce a significant gap from nominal positions of the surfaces in contact (Fig. 9).

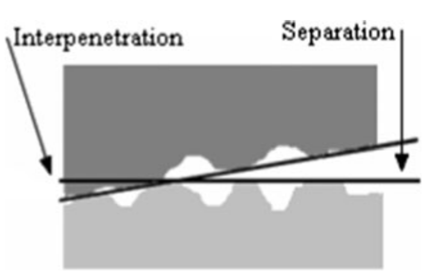

Fig. 9 Influence of flatness defects on the gap between two parts [16]

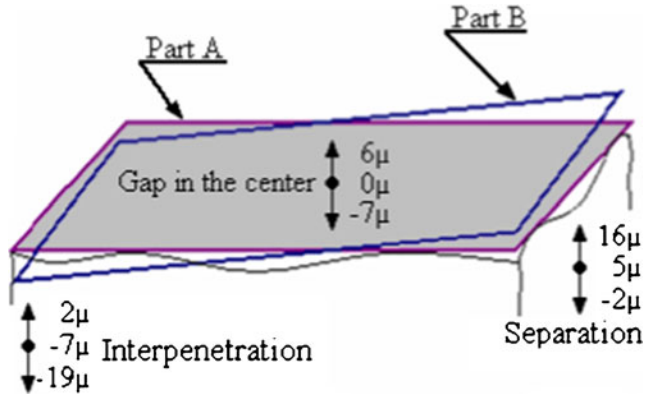

Fig. 10 Statistical behavior of plane-plane contact [16]

The principle of the study is to compare, in various points, the size of the assembly of two parts and the sum of the dimensions of the two parts.

This study yielded the following results (Fig. 10):

- The gap in the center of the surface is centered in the zero value.
- The maximum interpenetration is smaller than the sum of the flatness tolerance intervals of the two surfaces in contact: Interpenetration maxi=$IT^{⏥}_A + IT^{⏥}_B$.
- The maximum separation is smaller than the sum of the flatness tolerance intervals of the two surfaces in contact: Separation maxi=$IT^{⏥}_A + IT^{⏥}_B$.

Based on the results of this study, we consider that the relative translation of two surfaces in contact is null and deviations are type rotations. In the worst case, the maximum variation of two planes is equal to the sum of the interpenetration and separation, i.e., twice the sum of the flatness tolerance intervals of the two surfaces in contact:

$$\Delta l = 2 \times \left(IT^{⏥}_A + IT^{⏥}_B\right).$$

Using the principle of uniform distribution of deviations, we can calculate the deviations of the two plans in contact via the relationships:

$$\alpha = 2\left(IT^{⏥}_A + IT^{⏥}_B\right)/Y \text{ and } \beta = 2\left(IT^{⏥}_A + IT^{⏥}_B\right)/X.$$

Note that the interval flatness tolerances can be determined by experimental measurements or empirical models based on manufacturing processes.

7 Conclusion

This approach makes it possible to evaluate the manufacturing process by the limitation of the variations which occur at the various production setups.

To verify the validity of the manufacturing process, simply verify the inequalities for the constraints imposed by the designer, taking into account the imperfections of the manufacturing process. In this study, we proposed a model that evokes only entities directly quantifiable by measuring deviations, with sensor that mount directly to the machine tool or by estimation with an empirical model developed and validated by an experimental study.

These variations, converted into small displacements appraisable in experiment by 3D measuring, must remain lower than the dimensional and geometrical requirements imposed by designer by separately respecting the relations relating to each requirement.

This analysis model will have to validate by an experimental study which consists in measuring the various variations of each setup and checking that the variations obtained on the workpiece at the end of the setup are good within the imposed limits.

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