Int J Adv Manuf Technol (2014) 71:1433–1444
DOI 10.1007/s00170-013-5544-2

ORIGINAL ARTICLE

# Complexity analysis and calculation for sculptured surface in multi-axis CNC machining based on surface subdivision

**Li Li · Bing Chen · Fei Liu · Congbo Li**

Received: 5 April 2013 / Accepted: 9 December 2013 / Published online: 12 January 2014


**Abstract** Complexity of sculptured surfaces has a great influence on multi-axis computer numerical control (CNC) machining performances such as processing efficiency, surface quality, and energy consumption. A term called *surface machining complexity* (SMC) is first presented to describe the complexity level of surface geometrical shape features, and its influence on CNC machining performance. Shape features of sculptured surfaces are classified into seven categories based on surface curvature. An innovative method for quantifying SMC using surface subdivision is proposed. Firstly, representation of sculptured surfaces is introduced. Then, three processes of surface subdivision are presented, which are surface discretization based on iso-parameter line sampling, rough partitioning based on surface shape categories, and region grouping based on two criteria. After that calculation, formulas of SMC including formulas of local SMC and global SMC are developed. The proposed formulas utilize three correction factors to describe the influences of surface size, cutter diameter, grouping order, and mode of different surface shape categories. Finally, the proposed method is applied to calculate SMC for a typical sculptured surface and multi-axis CNC machining experiments to demonstrate the ability of our method, which can form a foundation for further research.

L. Li (✉)
College of Engineering and Technology, Southwest University, Chongqing 400715, People's Republic of China
e-mail: cqulily@163.com

L. Li · F. Liu · C. B. Li
Post-doctoral Study Center of Management Science and Engineering, Chongqing University, Chongqing 400030, People's Republic of China

B. Chen
Dongfang Electric Machinery Co., Ltd., Deyang 618000, People's Republic of China

**Keywords** Machining complexity · Sculptured surface · Surface subdivision · Multi-axis CNC machining · Surface curvature

## 1 Introduction

With the development of high tech, a great variety of products with sculptured surfaces has been found to have broad applications in the aeronautical, automotive, and die/injection mold industries. The applications of sculptured surfaces primarily depend on two aspects: on the one hand, requirements of mechanical properties, functions, and devices with specific performance, which requires products to achieve high-precision mathematical characteristics; on the other hand, it is the need for esthetic appearance to meet the demand for product [1]. Development and implementation of sculptured surface parts depend on complex sculptured surface design and their manufacturing technologies. Therefore, it is necessary to study the relation of the geometrical characteristics of sculptured surfaces to the computer numerical control (CNC) machining performance in terms of the machining time, energy consumption, parts surface roughness, etc. We use a term called *surface machining complexity* (SMC) to describe such relation and complexity level of free-form surface shape characteristics to the CNC machining process.

SMC is quite necessary and beneficial to overall analyze surface shape features, because in the machine cutting system, there are various factors influencing machining process, such as parts surface shape, cutter diameter, cutting machine, parts materials, tool paths, etc. That means surface geometrical characteristics and other factors can be considered together about influences on CNC machining performance. Therefore, it is necessary to study SMC and its specific quantitative methods so as to analyze the influence of surface shape chrematistics in multi-axis CNC machining.

In this work, we first analyze the surface geometrical characteristics and classify surface shape into seven categories according to surface curvature. Then, an innovative method of quantifying SMC is presented. Our method includes two stages. One is surface subdivision, which is carried out by three steps: surface discretization based on iso-parametric line sampling, rough subdivision based on surface shape categories, and region grouping based on two criteria. The other is calculation method, which is performed through calculation of local SMC and global SMC. In the end of the work, a case study is conducted to illustrate the methodology proposed in this paper, including calculating SMC for a typical sculptured surface so as to show the calculation process of SMC and multi-axis CNC machining experiment for demonstrating the influence of SMC on the machine energy consumption and surface roughness of the parts.

The reminder of the paper is organized as follows. Section 2 discusses previous related works. Section 3 introduces complexity analysis for sculptured surfaces, including the concept of SMC and classification of surface shape features. Section 4 introduces the calculation method of SMC, followed by the representation of sculptured surfaces, surface subdivision, and calculation formulas of SMC. Section 5 gives the experiment results and discussion. The conclusions and future works are given in Section 6.

## 2 Related work

There is a large body of previous research on analyzing and evaluating the complexity of part surface in CNC machining process accurately. The study on part surface is so important that many studies have been carried out for various process purposes such as cutter orientation determination [2], tool path planning [3], accurate, smooth surface machining [4], machining parameters selection [5], cutting force prediction or modeling [6, 7], etc.

NURBS (Nn-Uniform Rational B-Splines) surface is taken as a standard design surface in industrial design field, whose mathematical model can be found in literature [8]. The complex NURBS surface has some typical features which are taken as some specific areas satisfying specific geometrical and machining requirements. Some researchers have presented that surface geometrical characteristics have a great influence on the generation of tool paths during the CNC programming process [9, 10] and then influence the CNC machining performance, such as part surface quality, processing efficiency, energy consumption, etc.

Li et al. [2] presented a method for controlling the tool direction during the sculptured surface machining with the help of five-axis machining center for the purpose of improving the quality and efficiency of CNC processing. Their method gives a definition of surface region and a method of surface region partitioning that the part surface is divided into many regions which have similar shape feature according to normal vector of the surface, and tool direction is controlled by cone mean value of each region. This method can be used to automatically plan tool orientation generation, but there is no surface complexity analysis and quantification method, and the influence of surface complexity on the CNC machining capacity.

Xie et al. [11] proposed a concept of surface curvature distribution eigenvalue to evaluate CNC machining efficiency and machining accuracy of which tool path length and maximum scallop height were regarded as the evaluation index, respectively. This work focused on investigating the influence of curvature distribution value on the tool path length and maximum scallop height and testified that the curvature distribution value could be used to evaluate and predict the CNC machining performance of the sculptured surface. However, there still lacks the work about analysis and calculation of sculptured surface geometry. Lin and Wang [12] proposed an algorithm of free-form surface subdivision method based on the surface curvature and the fuzzy C-means method for improving polishing efficiency of free-form surface of mold. The algorithm have divided free-form surface into a series of surface piece families with similar surface information and technological characteristics. This work does not focus on intensively analysis of surface information and technological characteristics of polishing surface, and application is limited to polish off free-form surface of mold.

Sridharan and Shah [13] proposed a logical classification method of multi-axis milling features from the angle of milling considerations and computational methods for CNC tool path generation. CNC milling features were classified into three broad categories: Cut-Thru, Cut-Around, and Cut-On. Meanwhile, Cut-On features were further classified into three categories: Closed Cut-On, Open-Cavity, Open-Surface. The method is helpful to unify various CAM vendors' categories of machining features which are different from each other. Sunil and Pand [14] developed a system for automatic recognition of features from free-form surface CAD models of sheet metal parts represented in STL format. Features were classified into three major classes, face-based, edge-based, and transitive features, and ten subclasses by studying sheet metal industrial parts. The system is limited to the identification of typical features of sheet metal parts without analysis of the machining significance of typical features, and not convenient to connect with other system.

Cicirello and Regli [15] presented an approach to similarly assess manufacturing of solid models of mechanical parts based on machining features. The method is to perform machining feature extraction to map the solid model to a set of machining features; construct a model dependency graph from the set of machining features; and find the nearest neighbors to the query graph using an iterative improvement search across a database of other models. The method can effectively manage the models that are stored in the digital libraries for large design and manufacturing enterprises. Chen and Wu [16] presented a method of region partitioning and CNC machining of compound surfaces. The method divided compound surfaces into three kinds of regions, viz. elliptical region, hyperbolic region, and parabolic region, and described the correlative mathematical models for defining the cutter parameters, the step-forward length, and the path intervals in the CNC machining of the surfaces based on the feature of the regions. The method provides an effective approach for analysis of a compound surface with several types of region and decision of cutter parameters and the CNC machining parameters.

In summary, the current studies mainly focus on the qualitative analysis of sculptured surfaces features and surface shape recognition, but there still lacks an efficient quantitative method for more accurate analysis and evaluation of the complexity level of the machining surface, and there is no study on the relation of surface complexity to the CNC machining performance. If the complexity of processing objects can be calculated accurately when process planning such us milling tool selection and if cutting parameters optimization is made, it will facilitate process engineers to better understand the geometrical complexity of processing objects and the difficulty of the CNC machining, so as to better develop process planning and select optimal cutters and cutting parameters. For this purpose, this work strives to study *surface machining complexity* (SMC) and its specific quantitative method considering the influence of surface shape chrematistics on CNC machining performance.

## 3 Complexity analysis for sculptured surfaces

### 3.1 Concept of surface machining complexity (SMC)

SMC is used to describe the complexity level of surface geometrical shape features, as well as the influence degree of the surface geometrical shape features on its CNC machining performance in terms of the machining time, energy consumption, surface roughness, etc. When we calculate the value of SMC, the influences of surface geometrical shape features on the CNC machining performance must be considered simultaneity. Therefore, SMC and its quantitative method and calculation value can be used by engineers to have a better knowledge on the geometric features and machinability of sculptured surfaces so as to develop the optimal tool path and select the optimal cutting parameters for its CNC machining. In the machine cutting system, there are various factors influencing machining process, including parts surface shape, cutter, cutting machine, workpiece materials, tool path, etc. Hence, all of the factors as well as SMC can be considered together about the influence on CNC machining performance, and this is the purpose of the study.

### 3.2 Classification of surface shape features

Before quantifying SMC, we first analyze shape features of the complex machining surface. Considering the characteristics of sculptured surfaces in CNC machining process, the machining surface can be divided into different shape features based on two criteria. Stoker [17] has pointed out that surface shape greatly depends on *Gaussian curvature K* and *mean curvature H* of each discrete point, so we select *Gaussian curvature* and *mean curvature* to express the concavity and convexity of the surface, and we classify the machining surface into seven categories, which are plane, convex cylinder, convex ellipsoid, convex saddle, concave cylinder, concave ellipsoid, and concave saddle, as shown in Table 1. *Gaussian curvature K* of the surface $S(u, v)$ at a point $(x, y, z)$ is formulated [18, 19] as

$$K = \frac{LN - M^2}{EG - F^2} \tag{1}$$

where $E$, $F$, and $G$ are the components of *the first matrix of a surface A*:

$$A = \begin{bmatrix} S_u \cdot S_u & S_u \cdot S_v \\ S_v \cdot S_u & S_v \cdot S_v \end{bmatrix} = \begin{bmatrix} E & F \\ F & G \end{bmatrix} \tag{2}$$

and $L$, $M$, and $N$ are the components of *the second matrix of a surface B*:

$$B = \begin{bmatrix} S_{uu} \cdot n & S_{uv} \cdot n \\ S_{vu} \cdot n & S_{vv} \cdot n \end{bmatrix} = \begin{bmatrix} L & M \\ M & N \end{bmatrix} \tag{3}$$

**Table 1** Surface shape categories and the relationship with *Gaussian curvature* and *mean curvature*

| Surface shape categories | Mean curvature $H$/mm$^{-1}$ | Gaussian curvature $K$/mm$^{-1}$ |
|---|---|---|
| Plane ($P$) | 0 | 0 |
| Convex cylinder ($Z^1$) | <0 | 0 |
| Convex ellipsoid ($T^1$) | <0 | >0 |
| Convex saddle ($M^1$) | <0 | <0 |
| Concave cylinder ($Z^2$) | >0 | 0 |
| Concave ellipsoid ($T^2$) | >0 | >0 |
| Concave saddle ($M^2$) | >0 | <0 |

Here, vector $n$ is the unit surface normal at a point. The expression of *mean curvature* $H$ is

$$H = \frac{1}{2}\left(\frac{EN - 2FM + GL}{EG - F^2}\right) \quad (4)$$

We can use formulas (1) and (4) to recognize the surface shape according to the seven categories. The relationship of surface shape categories with *Gaussian curvature* and *mean curvature* is shown in Table 1. The classification of surface shape features is helpful to analyze the machining surface in the following work. The next section will describe the quantization method of SMC based on surface subdivision according to the categories of surfaces.

## 4 Calculation method of SMC

An overview of quantifying SMC is illustrated in Fig. 1. Our methodology includes three stages. The first stage is representation of sculptured surfaces using NURBS surface model. The second stage is surface subdivision, including surface discretization based on iso-parametric line sampling, rough subdivision based on surface shape categories, and region grouping based on two criteria. The third stage is calculation formulas for SMC, including calculation formulas of local SMC and global SMC. Moreover, the flowchart of quantifying SMC is also shown in Fig. 1. The details of the important method employed in the methodology will be described in the following subsections.

### 4.1 Representation of sculptured surfaces

Before quantifying the complexity of sculptured surfaces, a mathematical model of the sculptured surface is required. Considering NURBS surfaces can offer a common mathematical form for representing and designing both standard analytic shapes (conics, quadrics, surfaces of revolution, etc.) and free-form curves and surfaces [8]; therefore, this work uses NURBS surfaces to represent machining surfaces. The surface is represented as

$$S(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} w_{i,j}\mathrm{P}_{i,j}N_{i,p}(u)N_{j,q}(v)}{\sum_{i=0}^{n}\sum_{j=0}^{m} w_{i,j}N_{i,p}(u)N_{j,q}(v)} \quad (5)$$

where $i$=0,1,…,$n$, $j$=0,1,…,$m$, $w_{i,j}$ are the weights, $\mathrm{P}_{i,j}$ form a control net, and $N_{i,p}(u)$ and $N_{j,q}(v)$ are the normalized B-splines of degree $p$ and $q$ in the $u$ and $v$ directions, respectively.

### 4.2 Surface subdivision

The key purpose here is to divide a complex machining surface into an optimal number of surface regions to facilitate the calculation of SMC and conduct of CNC machining experiments. References [20, 21] presented a practical approach for subdividing free-form surfaces. In this paper, we introduce a new method for surface subdivision which includes three steps: surface discretization based on iso-parametric line sampling, rough subdivision based on surface shape categories, and region grouping based on two criteria. The details will be discussed in the following subsections.

#### *4.2.1 Surface discretization based on iso-parametric line sampling*

Surface discretization can be carried out based on iso-parametric line sampling method [22]. That is to say, the mathematical method is first used to generate a series of $u$ directional and $v$ directional iso-parametric lines with a proper density. The density of iso-parametric lines is decided by various factors including surface size, surface complexity, the required precision, computational efficiency, etc. If the lines are too sparse, the complexity of the surface and the requirements of the calculation accuracy cannot be reflected; while the lines are too dense, the calculation efficiency of software and hardware may be affected. In fact, when the density of iso-parametric lines is over a degree, the number of divided regions will change little, so it is reasonable of taking the complexity of the surface and the requirement of accuracy as priority indicators to generate iso-parametric lines according to the actual situation.

Then these iso-parametric lines are used to partition the surface, and the intersections of them are the discrete grid points. The global discretization process is shown in Fig. 2a–c. NURBS surface model is usually formulated by $u$ and $v$ parameters; if $u$ (or $v$) becomes series of constant, which is $u$=$u_0$ (or $v$=$v_0$), then $u$ (or $v$) directional iso-parametric lines set can be got. A detailed description of the formula of iso-parametric lines can be found in Ref. [23]. In addition, to analyze the surface curvature, projecting the discrete grid points to the corresponding position in the surface is necessary. Suppose that $P(x, y, z)$ is a discrete grid point's coordinate, project $P(x, y, z)$ to target surface along the Z axis, and then get the projective point $P'(x', y', z')$ which is located in target surface. In fact, the two points will be in the same position because of the same spatial coordinate values. In this way, all the discrete points can be projected to the target surface, and the surface projection points set are shown in Fig. 2d.

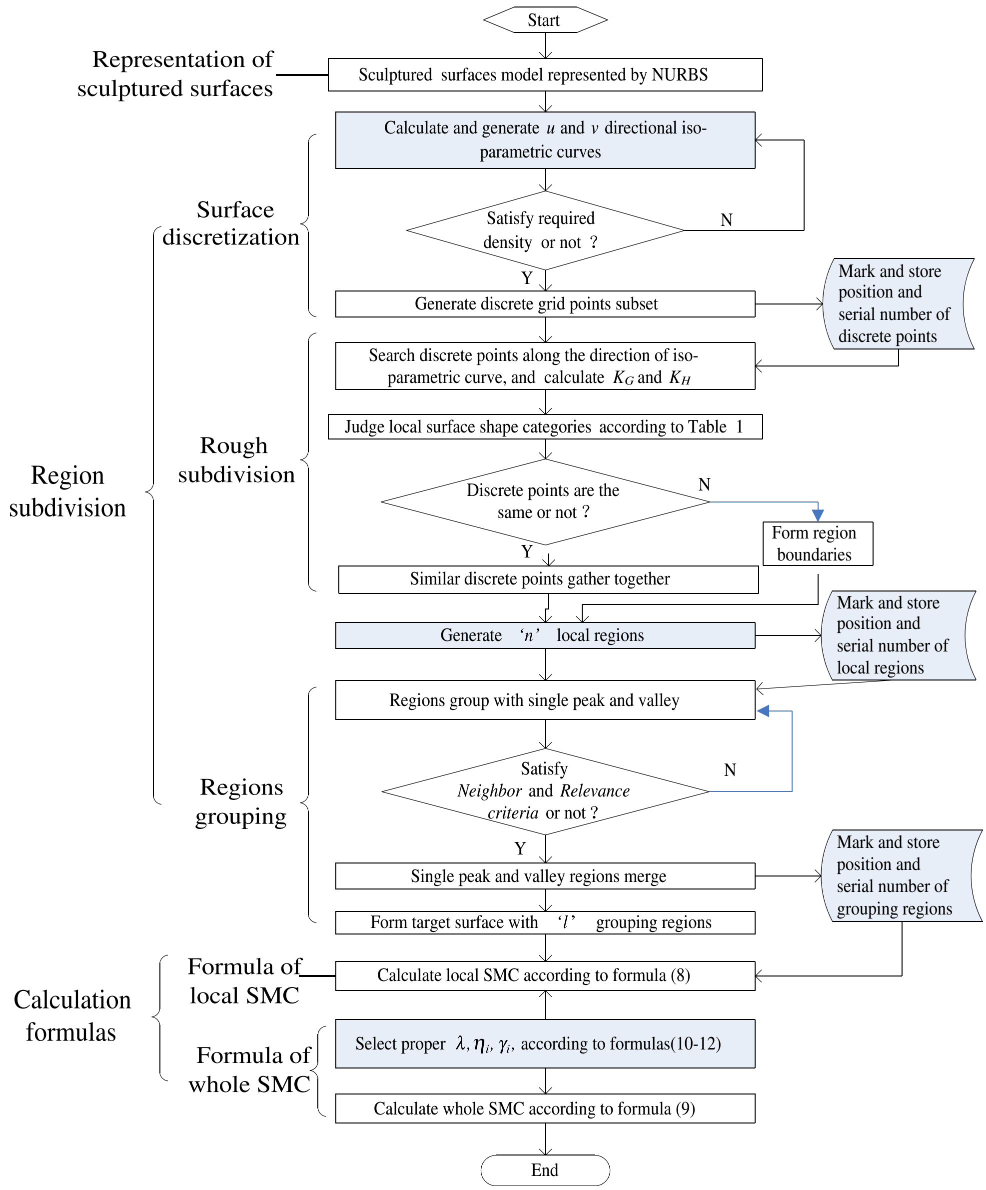


**Fig. 1** An overview and flowchart of our methodology for calculating SMC

### *4.2.2 Rough subdivision based on surface shape categories*

Considering the characteristics of the machining sculptured surface in CNC machining process, the machining surface can be divided into different shape features. Discrete points are sequentially distributed in the iso-parametric line. The approach of dealing with discrete points is to search its neighboring discrete points along the direction of the iso-parametric line, and we use Eqs. (1) and (4) to calculate *Gaussian curvature* and *mean curvature*. If the target point and its neighboring points have the same geometrical features, they will belong to the same cluster. There are some specific segmentation methods which can be used when getting the discrete points clusters [24, 25]. After traversing all the discrete points, they will be allocated into several clusters which can reflect different geometrical features.

Here, a problem of region boundary should be considered carefully. During the traversal processing of discrete points, an available method is to check whether the geometrical shape of newly introduced discrete point changes or not according to *Gaussian curvature* and *mean curvature*. If not, the newly introduced discrete point will be allocated to the former cluster; otherwise, it will be arranged as the boundary point, and the progress will go to the next identification of another discrete points cluster. After processing all discrete points and connecting the boundary points into curves, the surface has been divided into several local regions with different geometrical shapes as shown in Fig. 3.

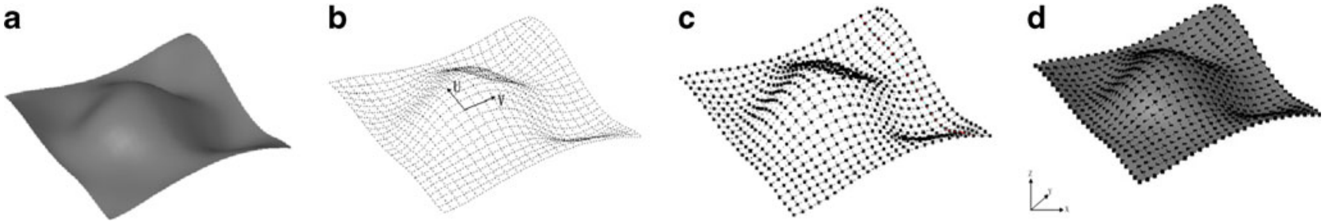


**Fig. 2** The process of surface discretization: **a** prototype surface; **b** *u* and *v* iso-parametric line of the surface; **c** discrete grid points; **d** projection points set of the surface

Supposing that the progress of rough region subdivision has subdivided surface $S$ into $n$ local regions, which contain $p$ planes, $z$ convex cylinder, $t$ convex ellipsoid, $m$ convex saddle, $g$ concave cylinder, $f$ concave ellipsoid, and $q$ concave saddle, where ($n$、$p$、$z$、$t$、$m$、$g$、$f$、$q$) $\in$ N, and all kinds of regions are marked as $P_i$, $Z_i^1$, $T_i^1$, $M_i^1$, $Z_i^2$, $T_i^2$, $M_i^2$, respectively, surface rough subdivision can be described by

$$\begin{cases} n = p + z + t + m + g + f + q \\ S = \sum_{i=1}^{p} P_i + \sum_{i=1}^{z} Z_i^1 + \sum_{i=1}^{t} T_i^1 + \sum_{i=1}^{m} M_i^1 + \sum_{i=1}^{g} Z_i^2 + \sum_{i=1}^{f} T_i^2 + \sum_{i=1}^{q} M_i^2 \end{cases} \tag{6}$$

*4.2.3 Region grouping based on two criteria*

If a grouping region contains single peak and valley, it will well reflect the local complexity of the machining sculptured surface, and the curvature variation of this kind of grouping region can also exhibit significant influence on CNC machining process. So the purpose of this section is to provide a surface region grouping method, which can orderly group and merge two of the seven typical surface shape categories into a target region that contains single peak and valley.

In order to get the optimized surface subdivision, two proposed criteria, *neighbor criterion* and *relevance criterion*, are applied sequentially. *Neighbor criterion* is used to judge whether the discrete points to be grouped are in adjacent positions. *Relevance criterion* refers that there is a grouping priority in the regions after rough subdivision: a convex shape cluster and a concave one are first divided into the same group, especially clusters with the same geometrical property. Then the plane shape cluster is grouped. Matrix $A$ (as shown in formula (7)) which represents the priority and way of cluster grouping is presented according to *neighbor criterion* and *relevance criterion.*

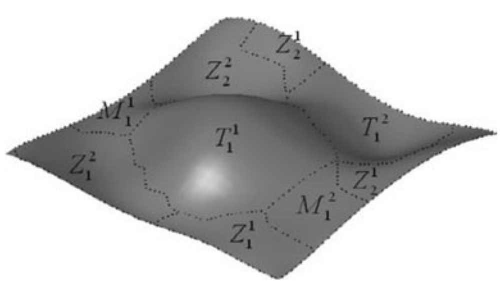


**Fig. 3** Result of rough region subdivision

$$A = \begin{matrix} \\ Z^1 \\ T^1 \\ M^1 \\ P \end{matrix} \begin{matrix} \begin{matrix} Z^2 & T^2 & M^2 & P \end{matrix} \\ \begin{pmatrix} Z^1Z^2 & Z^1T^2 & Z^1M^2 & Z^1P \\ T^1T^2 & T^1Z^2 & T^1M^2 & T^1P \\ M^1M^2 & M^1Z^2 & M^1T^2 & M^1P \\ Z^2P & T^2P & M^2P & PP \end{pmatrix} \end{matrix} \tag{7}$$

Matrix $A$ consists of major and minor geometrical characteristics. Major geometrical characteristics exist in each row, represented by $(Z^1\ T^1\ M^1\ P)^{\mathrm{T}}$, and minor geometrical characteristics exist in each column, represented by $(Z^2\ T^2\ M^2\ P)$. The method of the two-region grouping based on criteria works in the following way: If convex cylinder shape region $Z_i^1$ is first searched, then search a minor geometrical characteristics in its adjacent region which have not been merged. If *neighbor criterion* and *relevance criterion* exist and meet, then merge them and mark the merged region $S_i$. Otherwise, the current geometrical characteristic becomes an independent target region. After all of the major geometrical characteristics are performed, the target surface has been divided into $l$ independent groups. What should be noted is that, during the surface region grouping, if there are several minor geometrical characteristics with similar features, the priority of grouping and merging is to choose the minor geometrical characteristic with the smallest serial number. The result of surface region grouping is shown in Fig. 4.

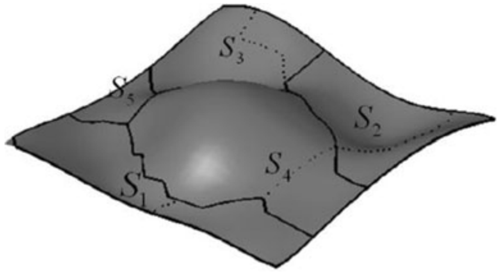


**Fig. 4** Result of surface patch grouping

## 4.3 Calculation formulas

The differences of surface geometrical shapes depend on the variation of surface curvature. In this section, we will calculate the value of SMC through making full use of the local and global curvature variation of the surface; meanwhile, influencing factors which have some influences on CNC machining performance are considered through using correction factors.

### *4.3.1 Formula of local SMC*

Local curvature can reflect the variation of surface geometrical shapes, so it is reasonable to use local curvature variation as the basis of calculating local SMC. In practice, difference of the maximum *mean curvature* $k^i_{\max}$ and the minimal *mean curvature* $k^i_{\min}$ of a grouping region is used as the quantitative value of local SMC.

Each region is defined as $S_i$. For the convenience in analysis and calculation, the *mean curvature* of each discrete point is taken as the required three-dimensional curvature. Then, the local SMC can be expressed as follows:

$$C^i_r = k^i_{\max} - k^i_{\min} = \frac{1}{r^i_p} - \frac{1}{r^i_v} \tag{8}$$

where $C^i_r$ (mm$^{-1}$) represents local SMC, and $k^i_{\max}$, $k^i_{\min}$, $r^i_P$, and $r^i_v$ represent the maximum *mean curvature*, the minimum *mean curvature*, the maximum *mean curvature* radius, and the minimum *mean curvature* radius of a grouping region, respectively. The discrete *mean curvature* can be calculated using the formula (4). Through iteration, the maximum *mean curvature* ($k^i_{\max}=(K_H)_{\max}$) and the minimum *mean curvature* ($k^i_{\min}=(K_H)_{\min}$) can be obtained.

In addition, Ref. [2] points out that the maximum *mean curvature* and the minimum *mean curvature* are separately located at the peak and valley of a local region. Therefore, the *mean curvature* of the peak and valley can stand for the maximum *mean curvature* $k^i_{\max}$ and the minimum *mean curvature* $k^i_{\min}$ in a grouping region; the graph of the grouping surface machining complexity is shown in Fig. 5.

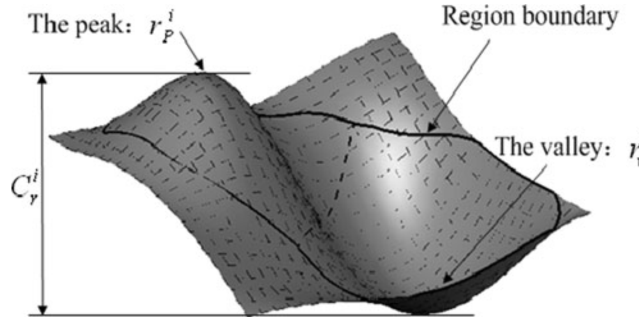


**Fig. 5** The graph of local grouping region

### *4.3.2 Formula of global SMC*

In order to reflect the macro variations of the surface geometrical shape, it is necessary to integrate all of the curvature information of the local machining geometrical characteristics. An effective method is to take the linear superposition of $l$ local SMC when calculating global SMC. Meanwhile, some influencing factors should be considered, such as the size of the processing object, the cutter size, and the group mode of the surface geometry features. Firstly, examining the impact of the size of the processing object is necessary. Even if the surfaces substantially have the same geometrical characteristics, different size models may lead to the fact that the time of tool paths generation and the total length of tool paths are different from each other in the CNC programming, and then result in different processing efficiency of CNC machining. Secondly, examining the impact of the cutter is necessary. For a high-speed precision machining, the cutter size and structure are subject to the state of the machine equilibrium in high-speed operation and the cutting power provided by cutting machine, which will impact part surface quality. Finally, examining the impact of the surface geometry features is necessary. Now most of the tool paths are generated driven by the surface of part, that is to say, the movement of the cutter location point is controlled by the local geometrical information of part surface model, different features of regions lead to difference of cutter parameters and tool path generation methods [16]. In this paper, model size, cutter diameter, and the grouping order and mode of different surface shape categories will be considered through the forms of correction factors. Given that $C_r$ (mm$^{-1}$) represents global SMC, the formula can be defined by

$$C_r = \lambda \sum_{i=1}^{l} \eta_i \gamma_i C^i_r = \lambda \sum_{i=1}^{l} \eta_i \gamma_i \left(k^i_{\max} - k^i_{\min}\right) = \lambda \sum_{i=1}^{l} \eta_i \gamma_i \left(\frac{1}{r^i_p} - \frac{1}{r^i_v}\right) \tag{9}$$

where $\lambda$ is the correction factor of surface size which is decided by the ratio of current surface area to the reference one. Use a projection comparative method to quantify the size correction factor. It means to project the surface model $S$ to XY plane along Z axis (the projected area is $D'$) and then compare projected area $D'$ with the reference surface area $D$ which is decided by the practical situation. Then $\lambda$ can be calculated by the following equation:

$$\lambda = \frac{D'}{D} \tag{10}$$

$\eta=(\eta_1\ \eta_2\dots\eta_l)$ is correction factor of the ratio of cutter diameter $d$ to the minimum curvature radius $r_v^i$ of each grouping region. Cutter diameter has a great influence on the performance of CNC machining. Generally, suppose that path spacing of CNC machining is constant, $\varepsilon_1$ and $\varepsilon_2$ which are determined by the machining condition are constant, and $\varepsilon_2>\varepsilon_1$. Figure 6 shows the comparison between cutter diameter $d$ and local curvature radius $r_v^i$. We can neglect the influence of cutter diameter when machining the surface with quite small local curvature radius and the flat surface since cutter diameter has a little influence on the performance of CNC machining in such circumstance ($2r_v^i/d<\varepsilon_1$), and we can give a smaller value of $\eta_i$. The bigger the local curvature radius is, the greater the influence on surface quality will be. Then $\eta_i$ is a larger value; in the remaining cases, $\eta_i$ is an appropriate intermediate value. When the cutter radius is less than the minimum curvature radius of grouping region, the following formula can be used to determine the value of cutter correction factor $\eta_i$.

$$\eta_{\mathrm{i}} = \frac{\dfrac{d}{2\left|r_v^i\right|}}{\max\left(\dfrac{d}{2\left|r_v^i\right|}\right)} \tag{11}$$

$\gamma=(\gamma_1\ \gamma_2\dots\gamma_l)$ is the correction factor of the way of region grouping. It is necessary to set a correction factor $\gamma_{\mathrm{i}}$ of local surface shape categories, because the local surface shape categories will greatly affect tool path generation when performing CNC machining. Considering the machining condition and the grouping orders and modes of surface shape categories, a correction matrix $K$ is given by

$$K = \begin{array}{c} \\ Z^1 \\ T^1 \\ M^1 \\ P \end{array} \begin{array}{c} \begin{array}{cccc} Z^2 & T^2 & M^2 & P \end{array} \\ \begin{pmatrix} 1.0 & 0.9 & 0.8 & 0.7 \\ 1.0 & 0.9 & 0.8 & 0.7 \\ 1.0 & 0.9 & 0.8 & 0.7 \\ 1.0 & 0.9 & 0.8 & 0.7 \end{pmatrix} \end{array} \tag{12}$$

## 5 Experiment results and discussion

Multi-axis CNC machining experiments are performed to illustrate the methodology proposed in this paper, including calculating SMC for a typical surface analysis so as to show the calculation process of SMC and CNC machining experiment to demonstrate the influence of SMC on the machine energy consumption ($Q$) and surface roughness ($Ra$) of the parts.

### 5.1 Calculating SMC for a typical sculptured surface

Figure 7a shows a typical complex NURBS surface model. As the method discussed above, MATLAB 7.6 which has strong analytical skill is selected to analyze and calculate the typical surface. Then the methods of surface discretization based on iso-parametric line sampling, rough subdivision based on surface shape categories, and region grouping based on two criteria are applied to process this surface. Figure 7b shows the processing result.

This surface is discretized with 48 equidistant $u$ directional and $v$ directional iso-parametric lines considering the surface complexity and computational accuracy. As a result, a discrete point set which contains 1,986 discrete points is got. Then use formulas (1) and (4) to calculate *Gauss curvature* and *mean curvature* of all discrete points, and perform surface rough

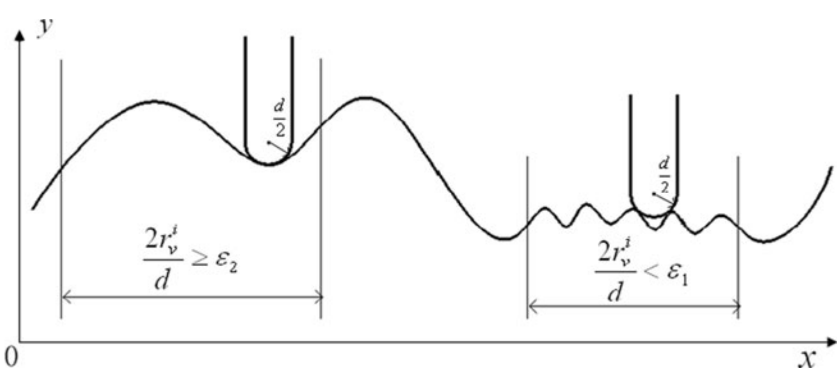


**Fig. 6** Comparison between cutter diameter and local curvature radius

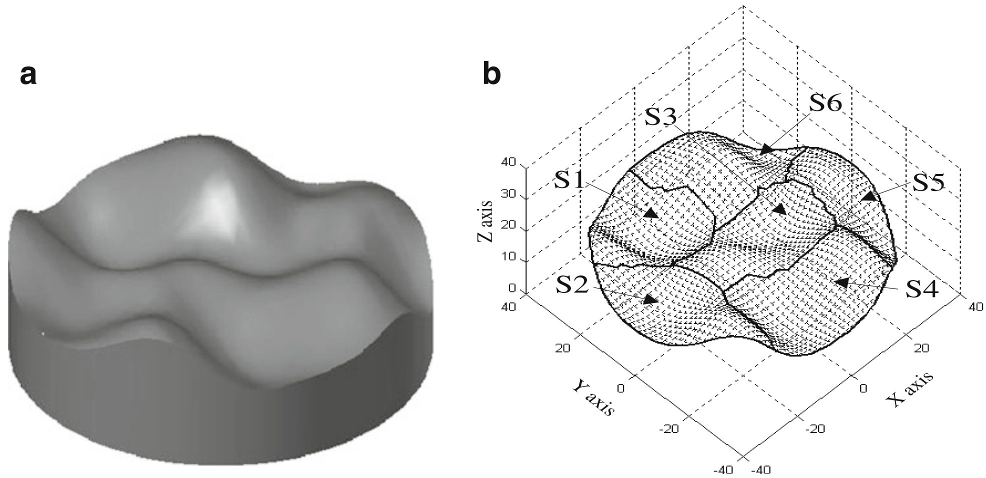


**Fig. 7** Model and grouping results of a typical sculptured surface: **a** typical complex NURBS surface model and **b** grouping result of the typical surface

subdivision according to Table 1. Twelve feature regions are got as shown in Table 2. Using *neighbor criterion* and *relevance criterion* to perform region grouping, six target regions are got. Table 3 shows the basic information of each grouping region which is marked with $S_i$, including grouping modes, the maximum *mean curvature* radius $r^i_P$ and the minimum *mean curvature* radius $r^i_v$ of each grouping region, and correction factors $\lambda$, $\eta_i$, and $\gamma_i$ for calculating the value of SMC.

Here, suppose that the size of typical surface model equals the reference one, so the correction factor of surface area $\lambda$ can be calculated according to formula (10), and the value is 1.0. Moreover, a ball-end mill with cutter radius of 2 mm is chosen. Obviously, if the cutter radius is less than the minimum curvature radius of each grouping region, then cutter correction factor $\eta_i$ can be calculated by formula (11). The value of correction factor of local surface shape features $\gamma_i$ is determined by matrix $A$ and $K$. So, the value of SMC $C_r$ is calculated according to formulas (8) and (9), and the result is 0.847. The calculation method and result will be used in the following five-axis CNC machining experiment.

### 5.2 Five-axis CNC machining experiments

CNC machining experiment is conducted to obtain the machine energy consumption ($Q$) and part surface roughness ($Ra$) which are important in the CNC machining performance evaluation under different values of $C_r$. Choose another four typical surface models as the objects of CNC machining experiment; after surface analysis and calculation as described in the previous sections, the values of $C_r$ are 0.306, 0.426, 0.471, 0.847, and 0.924. Table 4 shows the recorded data and calculation results, where $N$ is the number of discrete points, $n$ is the number of rough regions, and $l$ is the number of grouping regions.

A number of cutting experiments are tested on a carving and milling machine (SmartCNC500, Beijing Jingdiao Co., Ltd., China).

- The carving and milling machine is a five-axis machine, of which the range of spindle speed is from 3,000 to 28,000 rpm and the range of feed rate is from 0 to 6 m/min.
- The cutter (Changfa Hardware Co., Ltd., China) is a solid two-edged ball-end mill with the diameter of 4 mm; the material of the cutter is a tungsten steel.
- The work piece is a 2024 aluminum alloy bar with the diameter of 80 mm and the length of 300 mm.

The cutting experiments were performed in the following cutting conditions.

- The data was collected during the process of surface finish machining.
- Tool path mode was 0° parallel.

**Table 2** Surface rough regions partitioning

| Feature regions | $Z^1$ | $T^1$ | $M^1$ | $Z^2$ | $T^2$ | $M^2$ | $P$ |
|---|---|---|---|---|---|---|---|
| Number | 3 | 2 | 1 | 2 | 1 | 2 | 1 |

**Table 3** Basic information of each grouping region

| Grouping regions | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ |
|---|---|---|---|---|---|---|
| Ways of grouping | $Z^1\ T^2$ | $M^1\ M^2$ | $Z^1\ Z^2$ | $T^1\ Z^2$ | $Z^1\ M^2$ | $T^1\ P$ |
| $r^i_p$/mm | 11.37 | 10.61 | 12.27 | 13.17 | 9.59 | 12.52 |
| $r^i_v$/mm | −10.12 | −9.74 | −11.78 | −9.51 | −9.13 | −13.43 |
| $C^i_r$ | 0.187 | 0.197 | 0.166 | 0.181 | 0.214 | 0.154 |
| $\lambda$ | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| $\eta_i$ | 0.902 | 0.938 | 0.775 | 0.960 | 1.00 | 0.680 |
| $\gamma_i$ | 0.9 | 0.8 | 1.0 | 1.0 | 0.8 | 0.7 |

**Table 4** Calculation results of five sculptured surfaces

| No. | Surface models | Surface subdivision | | | Curvature of grouping regions | | λ | Correction factors for SMC | | SMC | Q/kwh | Ra/um |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $N$ | $n$ | $l$ | $r^i_p$/mm | $-r^i_v$/mm | | $\eta$ | $\gamma$ | $C_r$/mm$^{-1}$ | | |
| 1 |  | 1986 | 7 | 4 | (25.66, 21.82, 24.44, 20.20) | (19.79, 20.53, 22.10, 20.82) | 1.0 | (1.000, 0.965, 0.896, 0.951) | (0.9, 0.7, 1.0, 0.9) | 0.306 | 0.78 | 0.244 |
| 2 |  | 1986 | 9 | 5 | (19.56, 24.04, 16.89, 23.01, 20.85 ) | (16.09, 19.13, 15.34, 18.06, 20.53) | 1.0 | (0.953, 0.802, 1.000, 0.849, 0.747) | (1.0, 0.8, 1.0, 0.9, 0.8) | 0.426 | 0.82 | 1.420 |
| 3 |  | 1986 | 10 | 5 | (17.05, 15.92, 19.55, 16.67, 21.94) | (14.82, 19.95, 13.37, 15.51, 14.75) | 1.0 | (0.902, 0.670, 1.000, 0.862, 0.906) | (0.8, 0.9, 0.9, 0.9, 1.0) | 0.471 | 0.90 | 1.700 |
| 4 |  | 1986 | 12 | 6 | (11.37, 10.61, 12.27, 13.17, 9.59, 12.52) | (10.12, 9.74, 11.78, 9.51, 9.13, 13.43 ) | 1.0 | (0.902, 0.938, 0.775, 0.960, 1.000, 0.680) | (0.8, 0.9, 0.7, 1.0, 0.8, 0.9) | 0.847 | 1.21 | 2.372 |
| 5 |  | 1986 | 13 | 7 | (10.77, 11.31, 13.49, 12.26, 14.18, 11.85 9.23) | (8.98, 10.53, 12.71, 9.21, 13.56, 9.47, 10.31) | 1.0 | (1.000, 0.856, 0.709,0.978, 0.664,0.951, 0.874) | (0.8, 0.9, 0.8, 0.9, 1.0, 0.7, 0.8) | 0.924 | 1.49 | 3.623 |

- Keep spindle speed 1,4000 rpm, feed speed 1.2 m/min, cutting depth 0.08 mm, and path space 0.11 mm.

The experiment adopted RF power analyzer HOKI3390C to measure the machine energy consumption in the global machining process, and roughness instrument TR220 was used to measure surface roughness after CNC machining. Figure 8 shows the CNC milling process.

Figure 9 shows the relationship between SMC ($C_r$) and energy consumption ($Q$) and the relationship between SMC ($C_r$) and surface roughness ($Ra$), respectively. From the two graphs, we can draw a conclusion as follows: (a) the machine energy consumption and the part surface roughness are increasing with the increase of SMC, respectively: and (b) when the value of SMC is small, the increasing extent is small too, and vice versa.

The experiment results demonstrate that, as an important factor influencing the CNC machining, surface shape keeps a close relationship with machining performance. Moreover, the results prove that SMC considering surface shape features can accurately reflect the complexity level of the machining surface, and the calculation method presented in this paper can accurately evaluate the complexity of surface. As a foundation of work for further research, this work will help process engineers to carry out various process decisions better such as cutting parameters selection, cutting tool selection, tool path generation, and so on during the CNC machining.

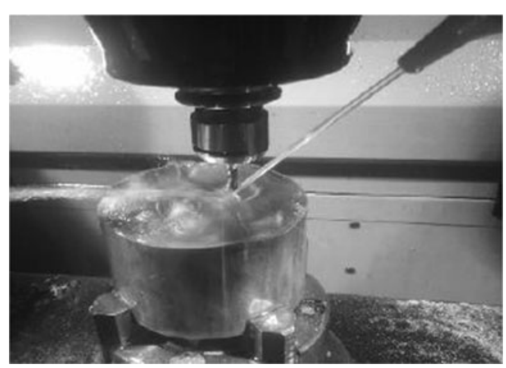

**Fig. 8** Illustration of machined sculptured surface

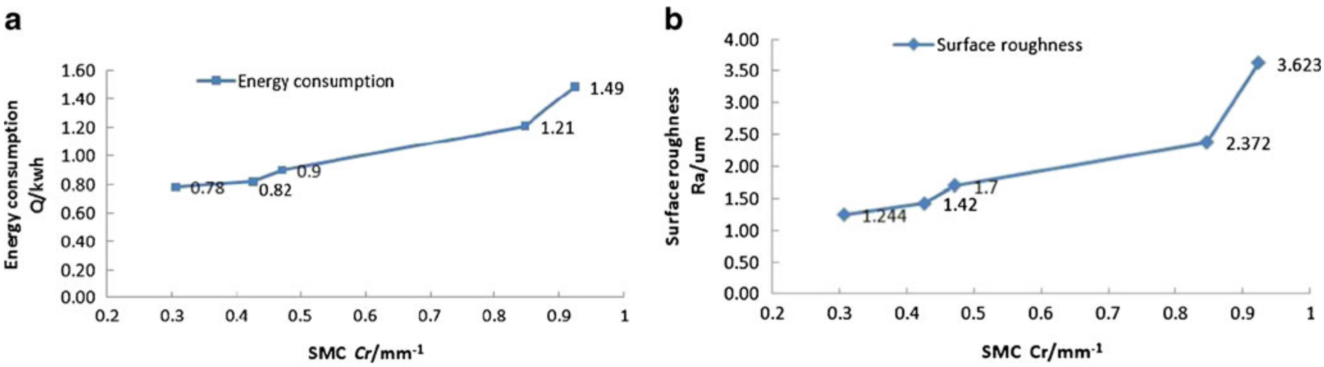


**Fig. 9** The relationship between SMC and energy consumption and surface roughness: **a** relationship between SMC and energy consumption *Q* and **b** relationship between SMC and surface roughness *Ra*

## 6 Conclusions and future works

Sculptured surface is used in geometric modeling to describe all sorts of bendy things like aeroplane wings, car bodies, mobile phones, ship's hulls, and so on which cannot be described by simple curved surfaces such as cylinders and cones. Moreover, sculptured surface machining plays a vital role in the process of bringing new products to the market place. This research is aimed at complexity analysis and calculation for sculptured surface in multi-axis CNC machining based on surface subdivision. Compared to other related works, our approach has the following advantages:

- The concept of *surface machining complexity* (SMC) is first presented to describe the complexity level of surface geometrical shape features, which considers the influence factors of the surface geometrical shape features on its CNC machining performance in terms of the machining time, energy consumption, surface roughness, etc.
- A new method to subdivide a complex sculptured surface into a number of easy-to-calculate regions is presented through surface discretization based on iso-parametric lines sampling, rough subdivision based on surface shape categories, and region grouping using *neighbor criterion* and *relevance criterion.*
- A quantitative method and calculation formulas of SMC based on region subdivision is presented, including quantitative method for local SMC and global SMC, which considers some factors such as model size, cutter diameter, grouping mode of surface shape, etc.

The experiment results demonstrate that the concept and calculation method of SMC proposed in this paper are competent to accurately analyze and evaluate the complexity of surface shape characteristics, which is beneficial for engineers to have knowledge on the geometrical complexity level of processing objects and the difficulty level of CNC machining. As a foundation of research, our approach can be extended in the following research fields, and these are our further research directions.

- *Cutting parameters optimization.* The value of SMC can be considered as one of the influence factors for cutting processes when we implement cutting parameters optimization, such as spindle speed, feed rate, and depth of cut. If the machined workpieces have different values of SMC, their cutting parameters must set different values so as to achieve the shortest processing time, the minimal energy consumption, and the optimal machining surface.
- *Tool path planning.* This research can provide a foundation work for further tool path planning using surface subdivision method and the calculation result of SMC. Surface subdivision in this work is performed according to surface curvature, and this idea can be used to get a number of easy-to-machine patches so as to implement tool path planning according to different patches. Simultaneity, the optimal tool path planning can be achieved according to different calculation results of SMC.
- *Cutter selection and cutter orientation determination.* The optimal cutter can be selected according to different calculation results of SMC, since the value of SMC is calculated based on the surface maximum *mean curvature* and the minimal *mean curvature* of a grouping region. Cutter orientation can be determined through subdivision regions with different surface curvatures.

**Acknowledgments** This research was supported by Fundamental Research Funds for the Central Universities (No. XDJK2014B007 and SWU113086), China Postdoctoral Science Foundation (No. 2012 M521671), and Chongqing Postdoctoral Science Foundation Funded project (No. XM2012007). We are grateful to other people in our team for collaboration and discussion.

## References


1. Zhou ZX, Zhou QY, Ren YH (2010) Complex curve surface design machining equipment current research trend. J Mech Eng 46(17): 105–113
2. Li BL, Wang XL, Hu YJ (2008) Tool orientation control method based on divided-area algorithm. Chin J Mech Eng-En 21(4):452–457
3. Li LL, Zhang YF, Li HY, Geng L (2011) Generating tool-path with smooth posture change for five-axis sculptured surface machining based on cutter's accessibility map. Int J Adv Manuf Technol 53(5–8): 699–709
4. Chen ZC, Khan MA (2013) A new approach to generating arc length parameterized NURBS tool paths for efficient three-axis machining of smooth, accurate sculptured surfaces. Int J Adv Manuf Technol. doi:10.1007/s00170-013-5411-1
5. Li L, Liu F, Chen B, Li CB (2013) Multi-objective optimization of cutting parameters in sculptured parts machining based on neural network. J Intell Manuf. doi:10.1007/s10845-013-0809-z
6. Subrahmanyam KVR, San WY, Soon HG, Sheng H (2010) Cutting force prediction for ball nose milling of inclined surface. Int J Adv Manuf Technol 48:23–32
7. Cao QY, Xue DY, Zhao J, Li Y (2011) A cutting force model considering influence of radius of curvature for sculptured surface machining. Int J Adv Manuf Technol 54(5–8):821–835
8. Piegl L (1991) On NURBS: a survey. IEEE Comput Graph 11(1):55–71
9. Lin RS, Koren Y (1996) Efficient tool-path planning for machining free-form surfaces. J Eng Ind 118(1):20–28
10. Hwang JS (1992) Interference-free tool-path generation in the NC machining of parametric compound surfaces. Comput-Aided Des 24(12):667–676
11. Xie J, ZOU MS, Cui XL (2009) Effect of curvature distribution feature of complex free-form surface on CNC milling performance. J Inst Eng Bangladesh 45(11):158–162
12. Lin JQ, Wang YQ (2007) Subdivision planning of complex free-form surface. J Jilin Univ Eng Tech Edit 37(2):386–390
13. Sridharan N, Shah JJ (2004) Recognition of multi axis milling features: part I-topological and geometric characteristics. J Comput Inf Sci Eng 4(3):242–250
14. Sunil VB, Pand SS (2008) Automat recognition of features from freeform surface CAD models. Comput Aided Des 40(4):502–517
15. Cicirello V, Regli WC (2001) Machining feature-based comparisons of mechanical parts. Proceedings International Conference on Shape Modeling and Applications 176-185.
16. Chen CK, Wu CT (2002) The region division and NC machining of compound surfaces. J Mmater Process Tech 121(1):5–14
17. Stoker JJ (1989) Differential geometry. Wiley-Interscience, Now York
18. Goldman R (2005) Curvature formulas for implicit curves and surfaces. Comput-Aided Geom Des 22(7):632–658
19. Chen ZZC, Dong ZM, Geoffrey WV (2003) Automated surface subdivision and tool path generation for 3 0.5 0.5-axis CNC machining of sculptured parts[J]. Comput Ind 50:319–331
20. Sonthi R, Kunjur G, Gadh R (1997) Shape feature determination using the curvature region representation. Fourth ACM symposium on Solid modeling and applications:285-296.
21. Tuonga NV, Pokornya P (2010) A practical approach for partitioning free-form surfaces. Int J Comput Integ M 23(11):992–1001
22. Song Y, Bronsvoort WF (2005) Fitting and manipulating free-form shapes using templates. J Comput Inf Sci Eng 5(2):86–94
23. Pan RJ (2001) Explicit matrix representation for NURBS curves and surfaces and its algorithm. Chinese J of Comp 24(4):258–288
24. Benko P, Varady T (2004) Segmentation methods for smooth point regions of conventional engineering objects. Comput-Aided Des 36(6):511–523
25. Woo H, Kang E, Wang SY, Kwan H (2002) A new segmentation method for point cloud data. Int J Mach Tool Manu 42(2):167–178