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ORIGINAL ARTICLE

Closed-form solutions for multi-objective tolerance optimization

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Abstract Component tolerances have important influence on the cost and performance of products. In order to obtain suitable component tolerances, multi-objective tolerance optimization model is studied, in which the combined polynomial and exponential functions are used to model manufacturing cost. In this paper, analytical methods are proposed to solve the multi-objective optimization model. In this model, the objective function is not a monotone function, and it is possible that the assembly tolerance constraint, including worst-case method and root sum square method, is inactive. Therefore, two closed-form solutions are proposed for each component tolerance in terms of the Lambert W function. When the assembly tolerance constraint is not considered, the component tolerances are obtained and named as the initial closed-form solutions. If the initial solutions satisfy assembly tolerance constraint, it is the final value of optimal tolerances. Otherwise, constrained optimization model is established and Lagrange multiplier method is applied to obtain the new closed-form solution of component tolerances as the final value of optimal tolerances. Several simulation examples are used to demonstrate the proposed method.

Keywords Tolerance optimization · Closed-form solution · Lambert W function · Multi-objective optimization

1 Introduction

As an important part of product development, tolerance design affects many aspects of product lifecycle. In recent years, tolerance design has received the attention of many researchers.

Firstly, tolerance specifications have important influence on manufacturing cost. Tight tolerances can result in complicated manufacturing process, while low tolerances may mean low manufacturing cost and poor product performance. Therefore, there are close relationship between manufacturing cost and tolerance, which should be established. In the past decades, many manufacturing cost-tolerance models have been proposed [1–3]. These models can be divided into four categories. The first categories of manufacturing cost-tolerance models are reciprocal power functions. The second categories are exponential functions. The third categories are polynomial functions. The fourth categories are hybrid models, such as combined reciprocal power and exponential function, combined polynomial and exponential function, reciprocal power and exponential hybrid function, etc.

In earlier years, only manufacturing cost was included in the objective function [1–5]. But in recent years, quality loss has received the attention of many researchers and become a part of objective function [6–15]. Sivakumar et al. [6] proposed two evolutionary optimization techniques, which were elitist nondominated sorting genetic algorithm (NSGA-II) and multi-objective particle swarm optimization (MOPSO), to solve a multi-objective tolerance allocation model, in which the assembly tolerance, manufacturing cost, and quality loss were all included in the objective function. Geetha et al. [7] applied genetic algorithm to solve a multi-objective tolerance optimization problem, which simultaneously considered the following objective functions: manufacturing cost, quality loss, machining time, and machine overhead/idle time cost.

After tolerance optimization models are established, proper methods should be employed to solve the optimization models. In the past years, various numerical optimization

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methods have been proposed to determine optimal tolerance, including genetic algorithm [4–8], particle swarm optimization [8], nonlinear programming method [9–11], simulated annealing [12], ants colony algorithm [13], game theoretic approach [14], etc.

Although numerical methods are widely used to obtain optimal tolerance, the Lagrange multiplier method, as a classical method for constrained optimal problem, should be the first choice as it can yield the closed-form solution [16]. Because of its high efficiency and accuracy, Lagrange multiplier method has been applied by a few researchers to calculate optimal tolerance [1, 2, 5, 16–21]. Using Lagrange multiplier method, the nonlinear equations are established, and it is possible to obtain closed-form solutions. If the closed-form solutions can be established, the optimal tolerances can be calculated quickly and accurately. Whether the closed-form solutions can be established depends on the following aspects: (1) manufacturing cost-tolerance model, (2) quality loss cost, and (3) assembly tolerance model. Some of the researches about optimal tolerance based on the Lagrange multiplier method are collected in Table 1. Chase et al. [2], Singh et al. [16], and Kumar et al. [17] studied reciprocal power manufacturing cost-tolerance models. Singh et al. [5] and Kumar et al. [18] researched exponential cost function and worst-case assembly constraint. Although the closed-form solution of optimal worst-case tolerances with exponential cost function can be calculated using general method, optimal statistical tolerance allocation with exponential cost function is a tough problem since a transcendental equation with exponential coefficient need to be calculated [19]. In fact, the exponential functions appear in many fields and can be solved using the Lambert W function [22]. But there are few researches about the application of the Lambert W function on tolerance allocation. Employing the Lambert W function, Cheng et al. [19, 20] solved statistical tolerance allocation with exponential cost function and obtained the closed-form solution of optimal tolerances. But in Cheng's studies, only manufacturing cost was considered and quality loss was not included in the objective function. Although Govindaluri et al. [23] and Shin et al. [24] considered quality loss and applied Lambert W function to obtain the optimal tolerances, assembly constraint was not considered. Liu et al. [21] applied the Lagrange multiplier method and Lambert W function to obtain closed-form optimal tolerance.

In this paper, the Lagrange multiplier method is applied to solve multi-objective tolerance allocation models, and unified closed-form solutions of optimal tolerances are established. In the models, the combined polynomial and exponential functions, which have less modeling errors than exponential functions, are used to model manufacturing cost. Both manufacturing cost and quality loss are included in the objective function.

2 Problem definition

2.1 Assembly tolerance constraint

Two most widely used tolerance analysis methods, including worst-case method and root-sum-square method, are considered in this paper. Based on the worst-case criteria, the assembly tolerance for nonlinear assembly can be represented as:

$$\sum_{i=1}^{n} \xi_i t_i \leq t_0 \tag{1}$$

where $\xi_i = |\partial Y / \partial x_i|$, Y is the assembly function, n is the number of component tolerances, t_i is the tolerance of component i, and t_0 is the specified functional tolerance. Based on the root-sum-square method, the assembly tolerance for nonlinear assembly is established as:

$$\sum_{i=1}^{n} \xi_i^2 t_i^2 \leq t_0^2 \tag{2}$$

Table 1 Closed-form solution to optimal tolerance allocation based on the Lagrange multiplier method

Researches	Cost-tolerance function	Quality loss	Assembly constraint	Closed-form solution
Chase [2]	Reciprocal power function	Not included	Root-sum-square	Obtained
Singh [5]	Exponential function	Not included	Worst-case	Obtained
Singh [16]	Reciprocal power function	Not included	Worst-case, root-sum-square	Obtained
Kumar [17]	Reciprocal power function	Not included	Worst-case	Obtained
Kumar [18]	Exponential function	Not included	Worst-case (complex assembly)	Not obtained
Cheng [19]	Exponential function	Not included	Root-sum-square	Obtained
Cheng [20]	Reciprocal power function, Exponential function	Not included	Worst-case, root-sum-square	Obtained
Liu [21]	Exponential function	Included	Worst-case, root-sum-square	Obtained

2.2 Manufacturing cost

In this paper, the combined polynomial and exponential functions is studied, and the total manufacturing cost for the assembly may be expressed as follows:

$$c_{M_1} = \sum_{i=1}^{n} \left(a_i + b_i e^{-m_i t_i}\right) \text{ (Exponential functions)} \tag{3}$$

$$c_{M_2} = \sum_{i=1}^{n} \left(a_i + b_i t + c_i e^{-m_i t_i}\right) \text{ (Combined linear and exponential functions)} \tag{4}$$

$$c_{M_3} = \sum_{i=1}^{n} \left(a_i + b_i t_i + c_i t_i^2 + d_i e^{-m_i t_i}\right) \text{ (Combined squared and exponential functions)} \tag{5}$$

where a_i, b_i, c_i, d_i, and m_i are manufacturing cost coefficients for the component i.

2.3 Quality loss

According to Taguchi's standpoints, the expected quality loss cost for normal distribution can be calculated as follows:

$$C_L = \frac{A}{9T^2}\sum_{i=1}^{n} t_i^2 \tag{6}$$

Where A is quality loss coefficient and T is the single side functional tolerance stack up limit for the assembly dimension.

3 Problem formulation

In the tolerance optimization model, the manufacturing cost and quality loss are included in the objective function, which can be written as follows:

$$C_{T_1} = C_{M_1} + C_L = \sum_{i=1}^{n} \left(a_i + b_i e^{-m_i t_i}\right) + \frac{A}{9T^2}\sum_{i=1}^{n} t_i^2 \tag{7}$$

$$C_{T_2} = C_{M_2} + C_L = \sum_{i=1}^{n} \left(a_i + b_i t + c_i e^{-m_i t_i}\right) + \frac{A}{9T^2}\sum_{i=1}^{n} t_i^2 \tag{8}$$

$$C_{T_3} = C_{M_3} + C_L = \sum_{i=1}^{n} \left(a_i + b_i t_i + c_i t_i^2 + d_i e^{-m_i t_i}\right) + \frac{A}{9T^2}\sum_{i=1}^{n} t_i^2 \tag{9}$$

The upper objective functions can be rewritten in the following unified form:

$$C_T = \sum_{i=1}^{n} \left(A_i + B_i t_i + C_i t_i^2 + D_i e^{-m_i t_i}\right) \tag{10}$$

4 Calculation of optimal tolerances

In this paper, analytical method is applied to solve the tolerance optimization model, and the following procedure is proposed.

4.1 Unconstrained optimal solution

Because the objective function shown in Eq. 10 is not a monotone function, the assembly tolerance constraint is firstly neglected and unconstrained optimization model is established to calculate optimal tolerance. The objective function is minimized by setting the first derivative of Eq. 10 equal to zero:

$$B_i + 2C_i t_i - m_i D_i e^{-m_i t_i} = 0 \quad i = 1, 2, \cdots, n \tag{11}$$

The solution of Eq. 11 is calculated as follows:

$$t_i^* = r_i + \frac{1}{m_i} lambertw\left(m_i e^{-m_i r_i}/l_i\right) \quad i = 1, 2, \cdots, n \tag{12}$$

where

$$r_i = -\frac{B_i}{2C_i}$$

$$l_i = \frac{2C_i}{m_i D_i}$$

lambertw is the Lambert W function [22], which is involved in many mathematical software packages.

Although t_i^* is obtained, it is not necessary that t_i^* satisfy assembly tolerance constraint. If t_i^* $(i=1,2,\cdots,n)$ cannot satisfy assembly tolerance constraint, constrained optimization model should be established and solved as follows.

4.2 Constrained optimal solution

4.2.1 Worst-case tolerance constraint

The Lagrange multiplier method is adopted and the following equation is established by combining Eqs. 1 and 10:

$$\sum_{i=1}^{n} \left(A_i + B_i t_i + C_i t_i^2 + D_i e^{-m_i t_i}\right) + \lambda\left[\sum_{i=1}^{n} \xi_i t_i - t_0\right] \tag{13}$$

where λ is the Lagrange multiplier. Setting the first derivatives of Eq. 13 equal to zero:

$$B_i + 2C_i t_i - m_i D_i e^{-m_i t_i} + \lambda \xi_i = 0 \quad i = 1, 2, \cdots, n \tag{14}$$

λ can be eliminated from Eq. 14 and the following relationship between t_1 and other tolerances is obtained:

$$e^{-m_i t_i} = \frac{2C_i}{m_i D_i} t_i - \frac{2\xi_i C_1 t_1 + \xi_i B_1 - \xi_i m_1 D_1 e^{-m_1 t_1} - \xi_1 B_i}{\xi_1 m_i D_i} \quad i = 2, \cdots, n \tag{15}$$

The solution of Eq. 15 is

$$t_i = R_i + \frac{1}{m_i} lambertw\left(m_i e^{-m_i R_i} / L_i\right) \quad i = 2, \cdots, n \tag{16}$$

where

$$R_i = \frac{2\xi_i C_1 t_1 + \xi_i B_1 - \xi_i m_1 D_1 e^{-m_1 t_1} - \xi_1 B_i}{2\xi_1 C_i}$$

$$L_i = \frac{2C_i}{m_i D_i}$$

Substituting Eq. 16 into the worst-case tolerance constraint, the following equation is obtained:

$$\xi_1 t_1 + \sum_{i=2}^{n} \xi_i \left(R_i + \frac{1}{m_i} lambertw\left(m_i e^{-m_i R_i} / L_i\right) \right) = t_0 \tag{17}$$

In Eq. 17, t_1 is the only unknown variable and can be calculated iteratively. After t_1 is obtained, other component tolerances t_i can be calculated using Eq. 16.

4.2.2 Root-sum-square tolerance constraint

Combining Eqs. 2 and 10, the following augmented equation is established:

$$\sum_{i=1}^{n} \left(A_i + B_i t_i + C_i t_i^2 + D_i e^{-m_i t_i}\right) + \lambda \left[\sum_{i=1}^{n} \xi_i^2 t_i^2 - t_0^2 \right] \tag{18}$$

where λ is the Lagrange multiplier. Letting the first derivatives of Eq. 18 equal to zero:

$$B_i + 2C_i t_i - m_i D_i e^{-m_i t_i} + 2\lambda \xi_i^2 t_i = 0 \quad i = 1, 2, \cdots, n \tag{19}$$

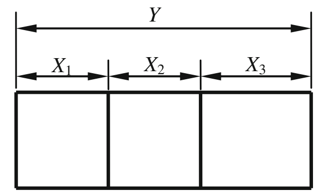
λ can be eliminated from Eq. 19 and the following equation is derived:

$$e^{-m_i t_i} = \frac{\xi_i^2 m_1 D_1 e^{-m_1 t_1} + 2\xi_1^2 C_i t_1 - 2\xi_i^2 C_1 t_1 - \xi_i^2 B_1}{\xi_1^2 m_i D_i t_1} t_i + \frac{B_i}{m_i D_i} \quad i = 2, \cdots, n \tag{20}$$

The solution of Eq. 20 is

$$t_i = R_i + \frac{1}{m_i} lambertw\left(m_i e^{-m_i R_i} / L_i\right) \quad i = 2, \cdots, n \tag{21}$$

Fig. 1 Example A



where

$$R_i = -\frac{\xi_1^2 B_i t_1}{\xi_i^2 m_1 D_1 e^{-m_1 t_1} + 2\xi_1^2 C_i t_1 - 2\xi_i^2 C_1 t_1 - \xi_i^2 B_1}$$

$$L_i = \frac{\xi_i^2 m_1 D_1 e^{-m_1 t_1} + 2\xi_1^2 C_i t_1 - 2\xi_i^2 C_1 t_1 - \xi_i^2 B_1}{\xi_1^2 m_i D_i t_1}$$

Substituting Eq. 21 into the root-sum-square tolerance constraint, the following equation is obtained:

$$\xi_1^2 t_1^2 + \sum_{i=2}^{n} \xi_i^2 \left(R_i + \frac{1}{m_i} lambertw\left(m_i e^{-m_i R_i} / L_i\right) \right)^2 = t_0^2 \tag{22}$$

Solving Eq. 22, t_1 can be obtained, and other component tolerances t_i can be calculated using Eq. 21.

4.3 Determining the optimal component tolerances

In order to obtain the optimal component tolerances, the following optimization process is proposed: Firstly, Eq. 12 is used to obtain the initial component tolerances t_i^* $(i=1,2,\cdots,n)$. If t_i^* satisfy the assembly tolerance constraint, it is the optimal component tolerances. If t_i^* do not satisfy the constraint, Lagrange multiplier method is adopted, and Eqs. 16 and 17 are solved to obtain optimal worst-case

Table 2 Cost-tolerance data for example A

Dimensions	Process	Parameters of cost function		
		a	*b*	*c*
X_1	1	5.0	34.2245	765
	2	4.7	39.9819	782
	3	4.36	45.0974	790
X_2	1	6.05	53.1921	975
	2	5.62	60.0065	995
	3	5.29	149.5845	986
X_3	1	5.38	72.6260	1,386
	2	5.31	96.5270	1,412
	3	5.22	82.8130	1,400

Table 3 Comparison of calculating results for example A

Techniques	Dimensions	Process	Tolerance	Objective function			
				f_1	f_2	f_3	f_c
Analytical method	X_1	1	0.0014490	0.005	43.337016	0.0373588	1.734027
	X_2	1	0.0018015				
	X_3	1	0.0017495				
NSGA-II [6]	X_1	1	0.001500	0.005	40.856503	0.042082	1.677145
	X_2	1	0.001900				
	X_3	1	0.001900				
MOPSO [6]	X_1	1	0.001424	0.005	43.3628	0.0374	1.735023
	X_2	1	0.001828				
	X_3	1	0.001745				

tolerances, or Eqs. 21 and 22 are solved to obtain statistical tolerances.

5 Numerical examples

The examples proposed by Sivakumar et al. [6] are applied to demonstrate the effectiveness of the method proposed in this paper. In [6], a multi-objective tolerance optimization models is proposed, which has three objectives, including worst-case assembly tolerance (f_1), manufacturing cost of the assembly (f_2), and quality loss (f_3). Then, the following combined objective function (f_c) can be established:

$$\min: \quad f_c = \frac{W_1}{N_1} f_1 + \frac{W_2}{N_2} f_2 + \frac{W_3}{N_3} f_3 \tag{23}$$

where

$$f_1 = \sum_{i=1}^{n} \xi_i t_i$$

$$f_2 = \sum_{i=1}^{n} \left(a_i + b_i e^{-m_i t_i}\right)$$

$$f_3 = \frac{A}{9T^2} \sum_{i=1}^{n} t_i^2$$

W_1, W_2, and W_3 are weight coefficients of f_1, f_2, and f_3, respectively. The equal weights are given and three weight coefficients are W_1=W_2=W_3=0.333. N_1, N_2, and N_3 are normalized parameters for f_1, f_2, and f_3, respectively. In these examples, the selection of manufacturing process from the alternatives for each component is also considered.

5.1 Example A

As shown in Fig. 1, the assembly function of example A is expressed as:

$$Y = X_1 + X_2 + X_3$$

The assembly tolerance constraint is as follows:

$$t_1 + t_2 + t_3 \le 0.005$$

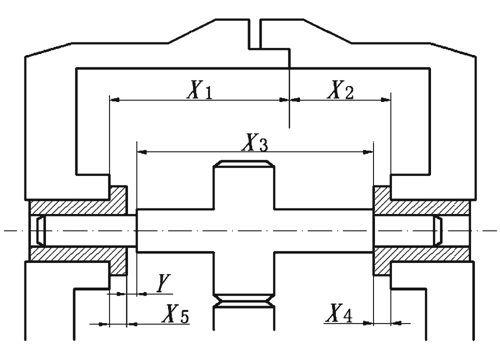


Fig. 2 Gearbox assembly

Table 4 Cost-tolerance data for example B (gearbox assembly)

Dimensions	Process	Parameters of cost function		
		a	b	c
X_1, X_2	1	18.50	71.25	214.56
	2	20.82	68.44	208.68
	3	19.05	69.32	211.05
	4	18.32	73.56	220.73
X_3	1	42.50	30.254	82.566
	2	39.20	33.443	86.688
	3	38.05	34.322	79.005
X_4, X_5	1	32.50	28.25	82.45
	2	29.20	30.43	86.70
	3	28.05	31.42	80.05
	4	29.32	34.16	78.82

Table 5 Comparison of calculating results for example B (gearbox assembly)

Techniques	Dimensions	Process	Tolerance	Objective function			
				f_1	f_2	f_3	f_c
Analytical method	X_1, X_2	4	0.014995	0.081223	161.061371	0.002178	0.635905
	X_3	3	0.017559				
	X_4, X_5	3	0.016837				
NSGA-II [6]	X_1, X_2	4	0.015138	0.085751	159.019974	0.002518	0.641941
	X_3	3	0.024135				
	X_4, X_5	3	0.01567				
MOPSO [6]	X_1, X_2	4	0.014416	0.0863	158.5301	0.0025	0.639893
	X_3	3	0.024018				
	X_4, X_5	3	0.016713				

The cost-tolerance data for example A are shown in Table 2. The combined objective function is as follows:

$$f_c = \frac{W_1}{N_1} f_1 + \frac{W_2}{N_2} f_2 + \frac{W_3}{N_3} f_3$$

where $W_1 = W_2 = W_3 = 0.333$, $N_1 = 0.01$, $N_2 = 10$, and $N_3 = 0.10$ [6].

The optimal tolerances are obtained by the analytical method proposed in this paper and compared with those obtained by NSGA-II and MOPSO [6], which are given in Table 3. In order to obtain the optimal tolerance, firstly, Eq. 12 is used, and the initial optimal tolerances are calculated as follows: $t_1^* = 0.0026764$, $t_2^* = 0.0027748$, and $t_3^* = 0.0024860$. Because $t_1^* + t_2^* + t_3^* > 0.005$, the initial optimal tolerances do not satisfy assembly tolerance constraint. Then, Eqs. 16 and 17 are used to obtain the optimal component tolerances: $t_1 = 0.0014490$, $t_2 = 0.0018015$, and $t_3 = 0.0017495$.

Table 3 demonstrates that the optimal component tolerances obtained by NSGA-II method, which are 0.001500, 0.001900, and 0.001900, violate the constraint. Both analytical method and MOPSO method obtain valid calculating results. Table 3 shows that the results of the analytical method are reasonable, and the combined objective function f_c is minimized.

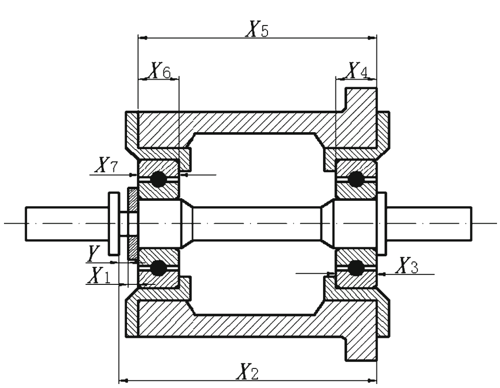


Fig. 3 Shaft and housing assembly

5.2 Example B (gearbox assembly)

Example B is gearbox assembly shown in Fig. 2, and the assembly function is expressed as:

$$Y = X_1 + X_2 - X_3 - X_4 - X_5$$

The assembly tolerance constraint is as follows:

$$t_1 + t_2 + t_3 + t_4 + t_5 \leq 0.26$$

The cost-tolerance data for the gearbox assembly are shown in Table 4, and the combined objective function is as follows:

$$f_c = \frac{W_1}{N_1} f_1 + \frac{W_2}{N_2} f_2 + \frac{W_3}{N_3} f_3$$

where $W_1 = W_2 = W_3 = 0.333$, $N_1 = 1.0$, $N_2 = 100$, and $N_3 = 0.01$ [6].

Table 6 Tolerance cost data for example C (shaft and housing assembly)

Dimensions	Process	Parameters of cost function		
		a	b	c
X_1	Vendor supplied (fixed tolerance $t_1 = 0.0381$; $C_1 = 5.00$)			
X_2	1	5.34	66.43	2.738
	2	5.12	62.22	2.340
X_3, X_7	Vendor supplied (fixed tolerance $t_3 = 0.0635$; $C_3 = 50.00$)			
X_4, X_6	1	15.34	69.43	2.728
	2	15.12	65.22	2.340
	3	14.85	66.87	2.112
X_5	1	11.34	72.43	2.738
	2	11.12	68.22	2.340
	3	10.85	69.87	2.112

Table 7 Comparison of calculating results for example C (shaft and housing assembly)

Techniques	Dimensions	Process	Tolerance	Objective function			
				f_1	f_2	f_3	f_c
Analytical method	X_1		0.0381	0.278216	395.613180	0.009645	1.442156
	X_2	2	0.024710				
	X_3, X_7		0.0635				
	X_4, X_6	2	0.028292				
	X_5	2	0.031820				
NSGA-II [6]	X_1		0.0381	0.291699	393.666779	0.010314	1.442392
	X_2	2	0.026594				
	X_3, X_7		0.0635				
	X_4, X_6	2	0.03				
	X_5	2	0.040005				
MOPSO [6]	X_1		0.0381	0.2894	393.9548	0.0103	1.442539
	X_2	2	0.020013				
	X_3, X_7		0.0635				
	X_4, X_6	2	0.030171				
	X_5	2	0.043949				

The optimal tolerances are calculated by analytical method and given in Table 5. Firstly, using Eq. 12, the initial optimal tolerances are calculated as follows: t_1^*=0.014995 , t_2^*= 0.014995 , t_3^*=0.017559, and t_4^*=0.016837, t_5^*=0.016837. Because $t_1^*+t_2^*+t_3^*+t_4^*+t_5^*<0.26$, the initial optimal tolerances satisfy assembly tolerance constraint. Therefore, the optimal component tolerances are obtained and Eqs. 16 and 17 do not need to be solved. Table 5 indicates that an improvement of combined objective function is obtained by the analytical method proposed in this paper.

5.3 Example C (shaft and housing assembly)

Example C is shaft and housing assembly shown in Fig. 3, and assembly function is expressed as:

$$Y = -X_1 + X_2 - X_3 + X_4 - X_5 + X_6 - X_7$$

The assembly tolerance constraint is as follows:

$$t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 \leq 0.3831$$

The cost-tolerance data for example C are shown in Table 6. Example C involves a few vendor supplied components, such as X_1, X_3, and X_7, the tolerances of which are predetermined. Therefore, only t_2, t_4, t_5, and t_6 need to be determined.

The combined objective function is as follows:

$$f_c = \frac{W_1}{N_1} f_1 + \frac{W_2}{N_2} f_2 + \frac{W_3}{N_3} f_3$$

where W_1=W_2=W_3=0.333, N_1=1.0, N_2=100, and N_3=0.10 [6].

The optimal tolerances are obtained using analytical method proposed in this paper and given in Table 7. Firstly, Eq. 12 is used to obtain the initial optimal tolerances: t_2^*= 0.024710 , t_4^*=0.028292 , t_5^*=0.028292, and t_6^*=0.031820. Because the initial optimal tolerances satisfy assembly tolerance constraint, the optimal component tolerances are determined. Table 7 indicates that a smaller combined objective function value is obtained using the method proposed in this paper.

6 Conclusion

Multi-objective tolerance allocation is researched, and analytical method is applied to obtain the closed-form solutions for the optimal tolerance in order to minimize the combined objective function. Because the objective function is not a monotonic function, the assembly constraint is not necessarily active. In this paper, the Lambert W function is applied and two forms of closed-form solutions are obtained. Comparing with other method, the proposed method has lower computation complexity, and can be applied easily by scholars and engineers.

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