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ORIGINAL ARTICLE

A dynamic multi-commodity inventory and facility location problem in steel supply chain network design

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Abstract Logistics network design is a major strategic issue due to its impact on the efficiency and responsiveness of the supply chain. This paper focuses on strategic and tactical design of steel supply chain (SSC) networks. Ever-increasing demand for steel products enforces the steel producers to expand their production and storage capacities. The main purpose of the paper includes preparing a countrywide production, inventory, distribution, and capacity expansion plan to design an SSC network. The SSC networks consist of iron ore mines as suppliers, raw steel producer companies as producers, and downstream steel companies as customers. Demand is assumed stochastic with normal distribution and known at the beginning of planning horizon. To achieve the service level of interest, a potential production capacity along with two kinds of safety stocks including emergency and shared safety stocks are suggested by the authors. A mixed integer nonlinear programming (MINLP) model and a mixed integer linear programming (MILP) model are presented to design dynamic multi-commodity SSC networks. To evaluate the performance of the MILP model, a real case of SSC network design is solved. Furthermore, solving two proposed models by using a commercial solver for a set of numerical test cases shows that the MILP model outperforms MINLP in medium- and large-scale problems in terms of computational time. Finally, the complexity of the linear model is investigated by relaxing some major assumptions.

Keywords Steel supply chain network · Facility location · Stochastic demand · Shared safety stock · Potential production capacity · Blending problem

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1 Introduction

Nowadays, to more easily supply products to customers, many production companies tend to create a network of close and well-organized communications called supply chain due to the new situations and changes in technology [1]. From a general point of view, a supply chain consists of all stages involved, directly or indirectly, in fulfilling customers' demand. In fact, supply chain is a network of facilities consisting of suppliers, producers, assembly lines, and distribution centers where materials, information, and financial flows interconnect them [2]. It can be inferred that a supply chain beside production and distribution tasks consists of assembly, storage, and retail actions [3].

Supply chain management (SCM) is the process of planning, implementing, and controlling the operations of the supply chain in an efficient way [4]. Decision making in SCM can be categorized into three levels based on the planning horizon: strategic, tactical, and operational [3]. In strategic level, the company plans the configuration of its supply chain for next few years. Strategic decisions include determining tasks to be accomplished in the organization, and tasks to be outsourced, as well as decision about production and storage capacities, and transportation modes. Planning horizon of the tactical level is from seasons to 1 year. In tactical level, the company decides about transportation planning and inventory handling regarding the decisions that made in strategic level, in advance.

In this paper, two mathematical models are proposed for designing steel supply chain networks (SSCNs) along with facility location, production planning, and inventory holding. The proposed models focus on both strategic and tactical decisions. In strategic level, location and capacity of new raw steel producers (if required) among a set of potential sites are selected. Moreover, capacity expansion plans for existing producers are determined. The model locates and expands the producers in proper points where costs of the entire SSCN are minimized. Nevertheless, in tactical level, decisions on iron ore supplier selection, production capacity assignment, safety stocks (SS) handling, and import/export raw steel products are made. The approach proposed in this paper is general such that it can be used to design new SSCNs and to improve existing ones. More precisely, the following are the major questions that would be replied through solving the proposed models:

1. How would be the expansion plans of producers in an SSCN?
2. How would the producers procure their required iron ore from the mines?
3. How would be the production plans of producers to satisfy customers' demands?
4. How would be the inventory holding system to afford demand fluctuations?

In remaining of the paper, a brief literature review is reported in Section 2. The problem and its assumptions are described in details in Section 3. Section 4 presents the proposed mixed integer nonlinear programming (MINLP) model for the problem and describes its linearization method. As the main contribution of the paper, the proposed mixed integer linear programming (MILP) model of the problem is also presented in Section 4. Results of a real case in Iran SSCN and some numerical test cases are described in Section 5. Moreover, discussions about the obtained results and sensitivity analysis are described in this section. Finally, a summary of the research, the conclusions, and future research directions are listed in Section 6.

2 Literature review

SCM involves finding the best possible configuration of the supply chain. In supply chain design, facility location problem, production, inventory, transportation, and routing have been considered. Many researchers have studied transportation and facility location problem in the past decade. Melo et al. [4] defined facility location problem as a set of spatially distributed customers and a set of facilities to serve customers' demands. Jayaraman and Pirkul [5] studied multi-commodity transportation and facility location problem with limited capacity under static conditions. They developed an MILP model for the problem solved by using a heuristic method. Although their model determined product flows from suppliers to producers, the model did not decide about location of the suppliers. Their proposed model makes decisions on suppliers' layer, which is rarely considered by the researchers.

Erlebacher and Meller [6] proposed an MINLP model for location-allocation problem of distribution centers (DCs). Moreover, inventory level in DCs has been determined using their model. They considered stochastic demand during lead-time, which is involved using a portion of standard deviation of total demand in lead-time. The model proposed cycle and safety stocks to cope with demand fluctuations. They modeled the problem based on average demand and holding costs; however, the model did not consider any shortage or backorder costs.

Syam [7] considered inventory, transportation, and facility location problem in a multi-commodity supply chain with dynamic planning horizon. He proposed to use the advantages of group transportation in product flows and routing, group storage, and other advantages of clustering into the proposed model. Melo et al. [8] modeled the same problem and considered dynamic location and relocation of facilities. They assumed that increasing/decreasing of the production capacity is possible in the facilities. In addition, it is also possible to shift production capacity between every pair of facilities in two succeeding time periods. Furthermore, Cordeau et al. [9] studied this problem in a countrywide environment. The number and location of new facilities, production technology selection, and production and storage capacities are the decision variables of their proposed model. Moreover, decisions about supplier selection, transportation mode selection, and product flows in supply chain are made through the model.

Gabor and Ommeren [10] studied transportation, inventory, and facility location problem with stochastic demand. They assumed that customers' demands are independent and follow a Poisson Process. There is a SS to satisfy demands with service level of $(1-\alpha)$; however, α has been considered as a decision variable. Since their proposed problem is NP-hard, an approximate heuristic algorithm was developed to solve it. In a more recent work, Thanh et al. [11] deliberated transportation, inventory, and facility location problem in a four-layer supply chain with dynamic planning horizon. They modeled a flexible framework in which the products could be dispatched not only between producers but also from producers to the customers. They assumed discounted rates on fixed purchasing costs for those who purchase more than one product. Moreover, a minimum rate of utilization for each facility was assumed.

Blending is another problem which has been considered since the 1950s and recently many researchers have interested in this problem. In supplying raw materials (e.g., iron ore, wheat, soil, and coal), suppliers provide it with specific amounts of various components. Especially, in the SSCNs, each mine provides iron ore with a specific amount of chemical components (CCs) including but not limited to Fe, Fe_3O_4, SiO_2, Al_2O_3, S, P, CaO, and MgO, which may vary during the time. The producers decide about procuring and mixing various types of iron ore from various mines in order to achieve their desired range of CCs. Liu and Sherali [12] developed a mathematical model for transportation and blending problem of coal. Their proposed supply chain consists of three layers, in which seaports are the central layer. They supposed that coal is not stored in the central layer and is only divided into smaller batches and dispatched to the customers. Bilgen [13] proposed an MILP model for maritime transportation and blending problem in a real case in Turkey. Movafagh and Farahani [14] proposed an MILP model to determine the optimal location of wheat storage facilities (i.e., silos). Their model guarantees that a predetermined minimum of components of products is delivered to the customers. They assigned a predefined level of components to the silos. This level of requirements is satisfied through shipping wheat from various suppliers and silos to the target silo. Import and export are allowed in their proposed model. Moreover, they proposed an innovative model to support both strategic and tactical management decisions which is relatively scarce in the literature.

Melo et al. [4] provided a comprehensive review of researches on facility location problem and SCM. They stated that while most of the literature focused on solution methods in multilayer supply chains, intra-layer flows have not been considered well in reviewed literature. They concluded that in spite of developing too many models for multilayer supply chains, integrating stochastic demands and facility location problem in SCM models needs more efforts. Furthermore, as Arabani and Farahani [15] mentioned, there exist some potential applications of dynamic facility location problems which have not received enough attention so far.

A comparative literature survey is presented in Table 1. The model proposed by the authors is compared to existing models in this table. The reviewed researches are classified based on the following criteria:

1. Decisions made in the model (location, production, inventory, and transportation),
2. Type of planning horizon (static or dynamic),
3. Modeling approach (deterministic, stochastic, or fuzzy),
4. Type and number of products (single, multi or bulk material),
5. Whether the problem includes blending decisions or not,
6. The number of supply chain echelon (single or multiple echelon),
7. Solution methods.

There are many models proposed by researchers to design three-layer supply chains; nevertheless, models including blending and location are rarely found in the literature. Complex facility location constraints until recently have limited most of research in this area to static and deterministic problems. Moreover, there is no model which integrates the facility location, production, inventory and blending problem in a real SSCN with intra-layer product flows and stochastic demands.

3 Problem settings and description

In this paper, a configuration of SSCN design has been studied, which consists of three main layers, namely iron ore mines, producers, and customers. The products of producers are raw steel (e.g., Billet, Bloom, and Slab). The customers are downstream steel companies who roll the raw steel products into a number of different final steel products (e.g., sheet products, rods, and h-beams). Furthermore, steel-making process is considered as an internal supply chain in the producers' sites. The steel-making process includes three stages: melting by either electric arc furnace (EAF) or blast furnace (BF), refining by ladle furnace, and continuous casting. However, in this paper, both smelting and refining stages are merged and considered as one intra-layer denoted by smelting stage. Each producer has a number of EAFs or BFs in smelting stage which feed some parallel casting lines. Moreover, there is an exclusive casting line to produce each product. The raw steel products can be either stored in warehouses of the producers or delivered to the customers. Generally, the internal supply chain includes three intra-layers namely smelting, casting, and warehouse. A schematic example of the SSCN is illustrated in Fig. 1. The capacity of smelting stage is usually less than total capacity of casting lines due to flexibility in steel-making process. For more information on steel-making process, readers would refer to Atighechian et al. [30].

In this paper, dynamic multi-commodity inventory and facility location problem with stochastic customers' demand in SSCN design is modeled. It is assumed that beside existing raw steel producers, there are some potential sites to establish new ones. At the beginning of planning horizon, each existing producer has an initial capacity in smelting, casting, and warehouse. It is obvious that the initial capacities of all potential sites are zero. Capacity expansion of existing producers and potential sites are modular. Moreover, developing modules will remain in operation until the end of the planning horizon. The model locates new modules in proper points where costs of the entire SSCN are minimized. First, the model considers some modules with different capacities as input. Then, it decides in each potential site or existing producer which modules should be established to minimize total costs and satisfy demands. In addition, the production (i.e., smelting and casting) and inventory levels in all producers are determined and the customers are allocated to the producers. Commodity flows from each layer (or intra-layer) to the consequent one are also determined in this approach. The

Table 1 Review of some existing models in supply chain network design (beyond 2000)

	Decision variables[a]	Planning horizon[b]	Model type[c]	Commodity[d]	Blending	Echelon[e]	Solution method	Other aspects
Authors' approach	L, P, I, T	D	S	B, M	Yes	M	Standard Solver	Intra-layer flows, capital budgeting
Jayaraman and Pirkul [5]	L, P, T	S	D	M	No	M	Heuristic and Lagrangian	–
Erlebacher and Meller [6]	L, P, I, T	S	S	S	No	M	Heuristic	Case study
Syam [7]	L, P, I, T	D	D	M	No	M	Lagrangian and meta-heuristic	Mode selection
Melo et al. [8]	L, P, I, T	D	D	M	No	M	Standard solver	Flexible supply chain
Cordeau et al. [9]	L, P, I, T	S	D	M	No	M	B and B and decomposition	Mode selection
Gabor and Ommeren [10]	L, I, T	S	S	S	No	S	Heuristic	–
Thanh et al. [11]	L, P, I, T	D	D	M	No	M	Standard solver	Discount
Liu and Sherali [12]	T	S	D	B	Yes	M	Heuristic + standard solver	Case study
Bilgen [13]	T	D	D	B	Yes	S	Heuristic + standard solver	–
Movafagh and Farahani [14]	L, I, T	D	D	B	Yes	M	Meta heuristic + standard solver	Mode selection
Boudia and Prins [16]	P, I, T	D	D	S	No	S	Meta-heuristic	Vehicle routing
Liang and Cheng [17]	P, I, T	D	F	M	No	S	Heuristic	MODM, time delivery
Gendron and Semet [18]	L, T	S	D	S	No	M	Standard solver	Mode selection
Hinojosa et al. [19]	L, P, I, T	D	D	M	No	M	Lagrangian + decomposition	–
Kutanoglu and Lohiya [20]	I, T	S	S	S	No	S	Standard solver	Mode selection
Eksioglu et al. [21]	P, I, T	D	D	M	No	S	Lagrangian + decomposition	Scheduling
Snyder et al. [22]	L, I, T	S	S	S	No	M	Lagrangian + decomposition	Scenario planning
Bilgen and Ozkarahan [23]	I, T	D	D	B	Yes	S	Standard solver	Mode selection
Vila et al. [24]	L, P, I, T	D	D	M	No	M	Standard solver	Case study
Persson and Lundgren [25]	I, T	D	S	B	Yes	S	Heuristic	Mode selection
Lee and Kim [26]	P, I, T	D	D	M	No	S	Heuristic + simulation	Scheduling
Nozick and Turnquist [27]	L, P, I, T	S	S	S	No	M	–	Case study
Sadjady and Davoudpour [28]	L, P, I, T	S	D	M	No	M	Lagrangian-based heuristic	Mode selection
Zhang et al. [29]	P, I	D	D	S	No	M	Meta heuristic	Case study

[a] *L* location, *P* production, *I* inventory, *T* transportation

[b] *S* static, *D* dynamic

[c] *D* deterministic, *S* stochastic, *F* fuzzy

[d] *S* single, *M* multiple, *B* bulk material

[e] *S* single echelon, *M* multiple echelon

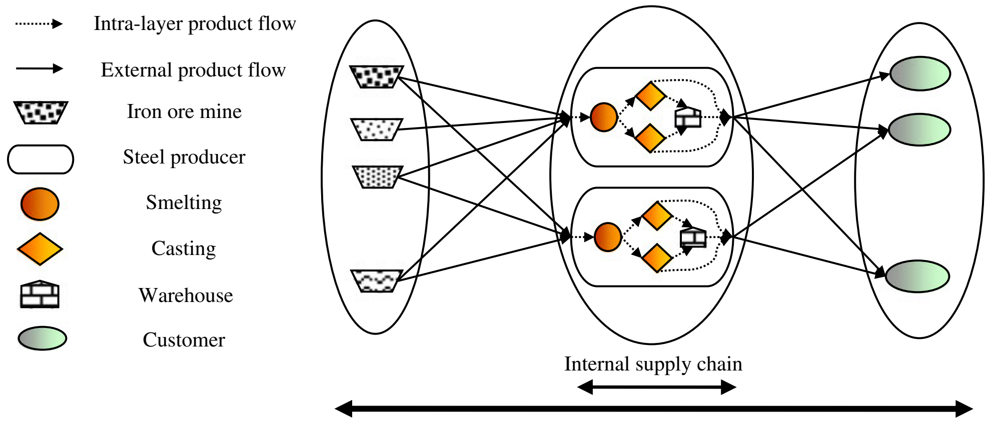

Fig. 1 Example of an SSCN with an internal supply chain in producers' sites

planning horizon is divided into a set of consecutive and integer time periods which have equal length.

In SSCNs, costs are divided into two categories: business costs or operating costs, and investment costs for establishing new facilities and expanding capacity of existing producers, which are constrained by the available capital. Although the available capital is predetermined and limited in each time period, unlimited borrowing and lending are allowed. It is assumed that lending interest rate is less than borrowing one. Moreover, non-invested capital available in a time period is subject to an interest rate and the returned value can be used in the subsequent time periods for reinvestment. The lending and borrowing levels are decided in the model. One of the most important issues in SSCN design is capital budgeting because there is a trade-off between expanding capacities via locating new facilities and satisfying demands by import raw steel products due to capital limitations.

The customers are allowed to purchase steel products from an outside producer (i.e., to import steel products) beside domestic producers. On the other hand, the producers are allowed to sell some products to an outside customer (i.e., to export steel products). But, in each time period, total quantity of imported/exported steel products is limited due to national policies. In order to guarantee a minimum level of benefit for the producers with respect to their investment size, a lower bound on the ratio of their production level to production capacity is defined as the minimum acceptable utilization rate.

In steel production process, among the main chemical components of iron ore, Fe and Fe_3O_4 appear as purity of iron metal, and the other CCs appear as impurity or slag. Therefore, to produce a unit of steel product, a special quantity of iron ore is required, which directly related to the CCs of that iron ore. This is known as the production yield of the iron ore. More precisely, the more amounts of Fe and Fe_3O_4 and the less amounts of SiO_2, Al_2O_3, S, P, CaO, and MgO in iron ore result in the more productivity in steel-making process. Therefore, each producer needs a given quantity of qualified iron ore with a predetermined acceptable range of CCs which can be provided by mixing different kinds of iron ore with an appropriate ratio. The blending problem which involves in the SSCN is to determine the quantity of iron ore of each mine that should be supplied for a producer to minimize total purchasing and transportation costs without exceeding the available capacities and violating the acceptable range of CCs. Moreover, a managerial decision in the SSCN is to select new technologies of enriching iron ore. The enriched iron ore is used as raw materials in steel production process. Enrichment of iron ore results in decreasing the production costs, energy and firebrick consumption, and volume of slag on top of molten steel in EAFs or BFs. Hence, enrichment the CCs of iron ore through applying new facilities is included in the model.

It is assumed that the customers' demands are independent normally distributed random variables, each one has a given average and variance that increase through the time. Consequently, total demand for each product is a normally distributed random variable, with its average being the sum of all averages and its variance is the sum of all variances (i.e., the square of standard deviation is the sum of squared standard deviations), which can easily be justified using characteristic functions.

To yield a service level of interest, holding a specific amount of safety stock is frequent among the producers. In some cases, the producers do not cooperate with each other in the same supply chain, and each one holds the SS, to satisfy their own customers' demands. In other occasions, all the producers in the same supply chain collaborate to share a common SS to satisfy total demand of all customers in the supply chain. The amount of SS is directly related to the standard deviation of the customers' demand [6]. Thus, it is obvious that the amount of common SS for the whole supply chain is less than total amount of SS held by the producers individually. Argue is that, standard deviation of the summation of some normal random variables is less than the summation of standard deviations of these normal random variables. This is a general consequence whenever the customers' demands follow the normal distribution.

In order to decrease the safety stock level and its holding costs on the entire SSCN, it is suggested that the producers cooperate with each other in holding SS for those customers who require the same product. Indeed, based on total demand, a common SS is held to achieve service level of the SSCN (i.e., $1-\alpha$). On the other hand, the business experience says that in order to decrease the transportation costs of the SS through the second echelon of SSCN, the best location for holding the common SS is a possible median point among customers with respect to their standard deviations. Instead, in order to increase responsiveness, it is more effective to hold enough SS in a location near each customer. So, to create a balance between responsiveness and transportation costs of SS, the safety stock is divided into emergency SS (ESS) and shared SS (SSS). Furthermore, a potential production capacity (PPC) beside ESS and SSS is suggested to reach service level of interest. In other words, the common SS is covered by ESS, SSS, and PPC.

The ESS is exclusively held for each customer in the nearest producer(s) and is used to respond customer's demand as quickly as possible when it exceeds the average demand. The second purpose to store ESS is to satisfy demand until either the SSS is procured from the farther producer(s) or the PPC is applied to produce extra products beyond SS. However, the SSS is commonly held and the PPC is commonly reserved for customers. The PPC is a part of the production capacity not necessarily used in every time period, but just to keep the service level. Note that the PPC is proposed in cases that demands exceed safety stocks, holding costs are too high, and/or a single product with different grades is demanded by some customers. This approach may result in decreasing total holding costs.

Since the demand is uncertain empirical finding, the optimal location(s) to hold SSS and to reserve PPC is really difficult. However, single (multiple) facility location problem or capacitated p-median problem can be used to find the optimal location(s). In the proposed model, some producers near to each customer are allowed to hold ESS and some producers in median points are allowed to hold SSS.

4 Problem formulation

In this section, dynamic multi-commodity inventory and facility location problem is formulated as mathematical models. Notations, parameters, and decision variables are described in advance. MINLP model of the problem as well as some real-world constraints are described later. Furthermore, an MILP model based on the proposed MINLP one is also presented. It is supposed that all relative data (such as costs, capacities, and so on) were collected by using appropriate methods, e.g., forecasting methods and/or business analysis of the company prior to modeling the problem. The following notations (indices of the sets) are used in the models presented in this paper:

I: Set of iron ore mines, $i \in \{1,2,\ldots,I\}$.
J: Set of raw steel producers, $J = J^c \cup J^o$, $j \in \{1,2,\ldots,J\}$.
J^c: Set of existing steel producers, $J^c \subset J$, $j^c \in \{1,2,\ldots,J^c\}$
J^o: Set of potential sites for establishing new steel producers, $J^c \subset J$, $j^o \in \{1,2,\ldots,J^o\}$
J_m: Set of smelting modules, $j_m \in \{1,2,\ldots,J_m\}$.
J_c: Set of casting modules for each product, $j_c \in \{1,2,\ldots,J_c\}$.
J_w: Set of warehouse modules, $j_w \in \{1,2,\ldots,J_w\}$.
K: Set of customers, $k \in \{1,2,\ldots,K\}$.
T: Set of time periods, $t \in \{1,2,\ldots,T\}$.
P: Set of products, $p \in \{1,2,\ldots,P\}$.
H: Set of CCs of iron ore, $.h \in \{1,2,\ldots,H\}$

Moreover, the main parameters used in the models are:

$\overline{CS_i^t}$: Maximal supply capacity of iron ore mine i in time period t.
$\overline{CM_j}$: Initial capacity of smelting in producer j.
$\overline{CC_{jp}}$: Initial capacity of casting of product p in producer j.
$\overline{CW_j}$: Initial storage capacity of in-site warehouse of producer j.
CAM_{j_m} : Capacity of smelting module type j_m.
$CAC_{j_c p}$: Capacity of casting module type j_c for product p.
CAW_{j_w} : Capacity of warehouse module type j_w.
VAL_h: Improvement in enriching the CC of h in iron ore when respective new technology is applied.
$\underline{U_j}$: Minimal acceptable utilization rate of producer j (independent of t).
$\overline{I}^t$: Maximal acceptable ratio of imported products to total domestic production in time period t.
$\underline{E}^t$: Minimal acceptable ratio of exported product to total domestic production in time period t.
Qs_{ih}^t: Ratio of CC of h in iron ore of mine i in time period t.
Qp_{jh}^t: Minimal or maximal admissible ratio of CC of h in iron ore required for producer j in time period t.
PY_i^t: Production yield, the quantity of iron ore of mine i in time period t required to produce a unit of steel product (it is estimated based on the quality of iron ore).
SC_{ij}: Unit transportation cost of iron ore from mine i to producer j.
PC_{jp}: Unit production cost of product p made by producer j.

NTC_h: Installation cost of respective new technology to enrich the CC of h in iron ore.

$MIC_{j\,j_m}$: Installation cost of smelting module type j_m in producer j.

$CIC_{j\,j_c p}$: Installation cost of casting module type j_c for product p in producer j.

$WIC_{j\,j_w}$: Installation cost of warehouse module type j_w in producer j.

TC_{jkp}: Unit transportation cost of product p from producer j to customer k.

D^t_{kp}, σ^t_{kp}: Average and standard deviation demand of customer k for product p in time period t, respectively.

$1-\alpha^t_p$: Service level of interest for product p in time period t.

$Z_{\alpha^t_p}$: Confidence coefficient of product p in time period t (i.e., a value of standard normal distribution that the area under the curve after that point is α^t_p).

$0.5+\beta^t_{kp}$: Emergency service level of interest for product p of customer k in time period t.

$Z_{0.5-\beta^t_{kp}}$: Emergency confidence coefficient of product p for customer k in time period t.

Ci_{kp}: Unit import cost of product p for customer k.

Ce_{jp}: Unit export net income of product p by producer j.

H_{jp}: Unit holding cost of product p at warehouse of producer j in each time period.

B^t: Capital available in time period t.

R_l, R_b: Interest rate of lending and borrowing, respectively.

$$E_{jkp} = \begin{cases} 1 & \text{If producer } j \text{ is allowed to store ESS of product } p \text{ for customer } k, \\ 0 & \text{Otherwise.} \end{cases}$$

$$S_{jp} = \begin{cases} 1 & \text{If producer } j \text{ is allowed to store SSS of product } p, \\ 0 & \text{Otherwise.} \end{cases}$$

$$K_h = \begin{cases} +1 & \text{If CC of } h \text{ in iron ore is } Fe \text{ or } Fe_3O_4, \\ -1 & \text{Otherwise.} \end{cases}$$

The following decision variables are used in formulation of the SSCN design models.

x^t_{ij}: Quantity of iron ore shipped from mine i to producer j in time period t.

y^t_{jp}: Quantity of product p produced by producer j in time period t.

z^t_{jkp}: Quantity of product p shipped from producer j to customer k in time period t.

ppc^t_{jp}: Potential production capacity of product p reserved by producer j in time period t.

x^t_{jp}: Quantity of product p held in warehouse of producer j in time period t.

ess^t_{jkp}: Quantity of product p held as ESS in warehouse of producer j for customer k in time period t.

sss^t_{jp}: Quantity of product p held as SSS in warehouse of producer j in time period t.

imp^t_{kp}: Quantity of product p imported by customer k in time period t.

exp^t_{jp}: Quantity of product p exported by producer j in time period t.

b^t: Amount of capital borrowed from outside of SSCN from time period t to time period $t+1$.

l^t: Lending amount of capital (invested outside of SSCN) from time period t to time period $t+1$.

$$m^t_{j\,j_m} = \begin{cases} 1 & \text{If a smelting module type } j_m \text{ is established in producer } j \text{ in time period } t, \\ 0 & \text{Otherwise.} \end{cases}$$

$$c^t_{j\,j_c\,p} = \begin{cases} 1 & \text{If a casting module type } j_c \text{ for product } p \text{ is established in producer } j \text{ in time period } t, \\ 0 & \text{Otherwise.} \end{cases}$$

$$w^t_{j\,j_w} = \begin{cases} 1 & \text{If a warehouse module type } j_w \text{ is established in producer } j \text{ in time period } t, \\ 0 & \text{Otherwise.} \end{cases}$$

$$q^t_{j\,h} = \begin{cases} 1 & \text{If new technology to enrich the CC of } h \text{ is utilized by producer } j \text{ in time period } t, \\ 0 & \text{Otherwise.} \end{cases}$$

4.1 Proposed mathematical model

With reference to the above notations, parameters, and variables, the MINLP model for the SSCN design is presented as follows.

($\mathbf{M_0}$) Minimize

$$\begin{aligned}&\sum_t\sum_i\sum_j SC_{ij}.x_{ij}^t+\sum_t\sum_j\sum_p PC_{jp}.\left(y_{jp}^t+ppc_{jp}^t\right)+\sum_t\sum_j\sum_p H_{jp}.inv_{jp}^t\\&\quad-\sum_t\sum_j\sum_p Ce_{jp}.exp_{jp}^t+\sum_t\sum_k\sum_p Ci_{kp}.imp_{kp}^t\\&\quad+\sum_t\sum_j\sum_k\sum_p TC_{jkp}.Z_{jkp}^t\\&\quad+\sum_t\sum_j\left(\sum_{j_m} MIC_{j\,j_m}.m_{j\,j_m}^t+\sum_{j_c}\sum_p CIC_{jj_cp}.c_{jj_cp}^t\right.\\&\quad\left.+\sum_{j_w} WIC_{j\,j_w}.w_{j\,j_w}^t\right)+\sum_t\sum_j\sum_h NTC_h.q_{jh}^t+b^T-l^T\end{aligned}\tag{1}$$

Subject to:

$$\sum_j z_{jkp}^t+imp_{kp}^t\ge D_{kp}^t\qquad\forall t,k,p\tag{2}$$

$$y_{jp}^t+inv_{jp}^{t-1}=inv_{jp}^t+\sum_k z_{jkp}^t+exp_{jp}^t\qquad\forall t,j,p\tag{3}$$

$$\sum_i\frac{x_{ij}^t}{PY_j^t}\ge\sum_p\left(y_{jp}^t-ppc_{jp}^{t-1}+ppc_{jp}^t\right)\qquad\forall t,j\tag{4}$$

$$\sum_i\left(Qs_{ih}^t+VAL_h\sum_{l=1}^t q_{jh}^l\right)K_hx_{ih}^t\ge Qp_{jh}^t.\sum_i K_hx_{ij}^t\qquad\forall t,j,h\tag{5}$$

$$\sum_t q_{jh}^t\le 1\qquad\forall j,h\tag{6}$$

$$\sum_j x_{ij}^t\le CS_i^t\qquad\forall t,i\tag{7}$$

$$\sum_p x_{jp}^t\ge\underline{U}_j\left(\overline{CM}_j+\sum_{j_m}\left(CAM_{j_m}\sum_{l=1}^t m_{jj_m}^l\right)\right)\qquad\forall t,j\tag{8}$$

$$\sum_p\left(y_{jp}^t+ppc_{jp}^t\right)\le\overline{CM}_j+\sum_{j_m}\left(CAM_{j_m}\sum_{l=1}^t m_{jj_m}^l\right)\qquad\forall t,j\tag{9}$$

$$\sum_{j_m} m_{jj_m}^t\le 1\qquad\forall t,j\tag{10}$$

$$y_{jp}^t+ppc_{jp}^t\le\overline{CC}_{jp}+\sum_{j_c}\left(CAC_{j_cp}\sum_{l=1}^t c_{j\,j_c\,p}^l\right)\qquad\forall t,j,p\tag{11}$$

$$\sum_{j_c} c_{j\,j_c\,p}^t\le 1\qquad\forall t,j,p\tag{12}$$

$$\sum_p\sum_{j_c} c_{j^o\,j_c\,p}^t\le P.\sum_{j_m}\sum_{l=1}^t m_{j^o\,j_m}^l\qquad\forall t,j^o\tag{13}$$

$$\sum_p inv_{jp}^t\le\overline{CW}_j+\sum_{j_w}\left(CAW_{j_w}\sum_{l=1}^t w_{j\,j_w}^l\right)\qquad\forall t,j\tag{14}$$

$$\sum_{j_w} w_{j\,j_w}^t\le 1\qquad\forall t,j\tag{15}$$

$$\sum_{j_w} w_{j^o\,j_w}^t\le\sum_{l=1}^t\sum_{j_m} m_{j^o\,j_m}^l\qquad\forall t,j^o\tag{16}$$

$$\sum_k\sum_p imp_{kp}^t\le\overline{I}^t\sum_j\sum_p y_{jp}^t\qquad\forall t\tag{17}$$

$$\sum_j\sum_p exp_{jp}^t\ge\underline{E}^t\sum_j\sum_p y_{jp}^t\qquad\forall t\tag{18}$$

$$\begin{aligned}&\sum_j\sum_{j_m} MIC_{j\,j_m}.m_{j\,j_m}^t+\sum_j\sum_{j_c}\sum_p CIC_{jj_c}.c_{j\,j_c\,p}^t\\&\quad+\sum_j\sum_{j_w} WIC_{j\,j_w}.w_{j\,j_w}^t+\sum_j\sum_h NTC_h.q_{jh}^t\qquad\forall t\\&\quad+(1+R_b)b^{t-1}-b^t-(1+R_l)l^{t-1}+l^t\le B^t\end{aligned}\tag{19}$$

$$inv_{jp}^t = \sum_k E_{jkp}.ess_{jkp}^t + S_{jp}.sss_{jp}^t \quad \forall t, j, p \tag{20}$$

$$\sum_j E_{jkp}.ess_{jkp}^t \geq z_{0.5-\beta_{kp}^t}.\sigma_{kp}^t \quad \forall t, k, p \tag{21}$$

$$\sum_j \left(S_{jp}.sss_{jp}^t + ppc_{jp}^t\right) \geq z_{\alpha_p^t}.\sqrt{\sum_k \left(\sigma_{kp}^t\right)^2 - \sum_j \sum_k E_{jkp}.ess_{jkp}^t} \quad \forall t, p \tag{22}$$

$$x_{ij}^t, y_{jp}^t, z_{jkp}^t, ppc_{jp}^t, imp_{kp}^t exp_{jp}^t, inv_{jp}^t, ess_{jkp}^t, sss_{jp}^t, b^t, l^t \geq 0 \quad \forall t, i, j, k, p \tag{23}$$

$$m_{jj_m}^t, c_{jj_cp}^t, w_{jj_w}^t, q_{jh}^t \in \{0, 1\} \quad \forall t, j, j_m, j_c, j_w, p, h \tag{24}$$

The objective function (1) minimizes total costs including transportation costs of iron ore from mines to producers, production costs, inventory holding costs, attained net income by export (with minus sign), import costs, transportation costs of raw steel products from producers to customers, fixed opening costs associated with establishing smelting, casting and warehouse modules, new technology implementation costs, and total financial liabilities of the SSCN at the end of planning horizon (i.e., $b^T–l^T$). Note that T denotes end of the planning horizon, b^T denotes total debt of SSCN at the end of planning horizon, and l^T denotes total capital invested outside of SSCN at the end of planning horizon.

Constraints (2) ensure that average demand of every customer is satisfied. Equations (3) impose the product flows conservation of producers while fulfilling the average demand of customers. Constraints (4) guarantee that every producer receives enough quantity of iron ore from all mines to produce raw steel products. This is calculated based on total quantity of products that should be produced and total potential production capacities that should be reserved in time period $t-1$ and t by the producer. Actually, the quantity of iron ore provided for potential production capacities in time period $t-1$ is transferred to time period t as initial inventory. Note that, if producer j purchases x_{ij}^t tons of iron ore from mine i in time period t, it can produce x_{ij}^t/PY_i^t tons of raw steel products.

Nonlinear inequalities (5) assure that each producer receives different kinds of iron ore with an appropriate ratio to be able to provide its own qualified iron ore in each time period. Note that, if there is a violation of the admissible range of CCs, the producer can apply respective new technology to enrich iron ore, and to meet its blending requirements. As an example, by using desulfurization technology, a producer can decrease the sulfur (S) content which means yielding better quality raw material. Besides increasing the lifetime of catalysts in reformer of reduction plant results in decreasing production costs. As mentioned in Section 3, the CCs of Fe and Fe_3O_4 appear as purity and other CCs appear as impurity in steel production process. Thus, Fe and Fe_3O_4 need minimum levels (e.g., Fe≥67.5 % and Fe_3O_4≥40 %), while other CCs need maximum levels (e.g., SiO_2≤1.3 %, Al_2O_3≤ 0.4 %, S≤0.04 %, P≤0.035 %, CaO≤1 %, and MgO≤0.5 %) in the admissible ranges of CCs. The parameter K_h is used to distinguish between purity and impurity of CC of h.

Constraints (6) state that new technology to enrich CCs of iron ore can be installed for the producers only once during planning horizon. The maximum capacity of the mines is observed in constraints (7). Minimum acceptable production levels and maximum production capacities of smelting are controlled via constraints (8) and (9), respectively.

Constraints sets (10), (12), and (15) state that in each time period, it is possible to install at most one new expansion module for smelting, casting for each product, and warehouse in each producer, respectively. Maximum casting capacity for all producers is controlled through constraint (11). This constraint states that sum of production quantity and potential production capacity must not exceed the casting capacity. Constraints (13) guarantee that any casting module in potential sites would not be established unless at least one smelting module has already been installed there. Similarly, constraints (16) ensure that warehouse modules would be established in potential sites if there is at least one installed smelting module. Furthermore, constraints (14) impose maximum storage capacity on warehouses of the producers.

Constraints sets (17) and (18) state that the quantity of products imported/exported in each time period should not be more/less than a given maximum/minimum level. Constraints (19) impose capital budgeting limitations according to Weingartner's horizon model [31, 32] into the model. Therefore, constraints (19) and the last two terms of objective function (1) (i.e., b^T-l^T) guarantee that the total financial liabilities are minimized at the end of planning horizon.

Equations (20) calculate total inventory of each product in every time period as the summation of emergency and shared safety stocks. As mentioned in Section 3, in order to decrease the SS level on the entire SSCN, the producers cooperate with each other in holding SS, and in order to increase responsiveness, an ESS is exclusively held for each customer near it. The ESS held for each customer is determined based on its emergency service level and its standard deviation (i.e., $z_{0.5-\beta_{kp}^t}.\sigma_{kp}^t$) by constraints (21). Emergency service level (β_{kp}^t) is typically defined as the ratio of "demand filled by ESS/total demand." Therefore, the probability of filling up demand of customer k by ESS is $100\times\beta_{kp}^t$%. On the other hand, to achieve the service level of interest, the common SS regarding the standard deviation of total demand and confidence coefficient is calculated as $z_{\sigma_p^t}.\sqrt{\sum_k \left(\sigma_{kp}^t\right)^2}$. Thus, to cover the common SS, appropriate levels of SSS and PPC are determined through constraints (22). Both service level ($1-\alpha_p^t$) and emergency service level (β_{kp}^t) could be estimated for example by simulation methods. In this case, some scenarios can be considered, and then the best one is chosen such that sum of shortage, holding and transportation costs of SS is minimized.

It is of interest to notice that, the regular inventory (the part above the common SS) can be included into the SSS because it may be a cost-saving decision compared to the production capacity expansion. Finally, constraints set (23) and (24) state the non-negativity and binary restrictions on the decision variables, respectively.

4.2 Model refinement

In real applications of the problem, there are many features which are not observed in the proposed MINLP model (M_0). For instance, it is not possible to purchase iron ore in small batches, e.g., Chadormalo mine in Iran SSCN would contract to supply at least 300,000 tons of iron ore per year [33]. In order to confine the continuous variable x_{ij}^t such that whether it takes zero or a value greater than or equal to K, new binary variable u_{ij}^t is and following constraints set are introduced:

$$x_{ij}^t \le M u_{ij}^t \qquad \forall t, i, j \tag{25}$$

$$x_{ij}^t \ge K u_{ij}^t \qquad \forall t, i, j \tag{26}$$

$$u_{ij}^t \in \{0, 1\} \qquad \forall t, i, j \tag{27}$$

In which K is the minimum admissible batch size, and M is a positive large value which should be at least greater than the sum of all supply capacities of the iron ore mines. Thus, to observe the above limitation, constraints (25) to (27) are appended to model M_0.

The iron ore purchased from various mines would be supplied in fine or lump form. Fine iron ore could be directly passed to pellet-making plant, whereas lump iron ore needs to be milled in advance. Maximum milling capacity of the producers in each time period can be expressed in following constraints:

$$\sum_i C_j . x_{ij}^t \le L_j \qquad \forall t, j \tag{28}$$

Where L_j is the capacity limit of milling in producer j in each time period and C_i is a binary parameter denoting whether mine i provides lump iron ore or not. Considering all the aforementioned constraints set (e.g., (25) to (28)) in model M_0 results in a more realistic MINLP model of the problem:

(M_1) Minimize
Objective function (1)
Subject to
Constraints set (2) to (28).

4.3 Linearization of proposed model

Both models M_0 and M_1 of the problem are nonlinear due to the term of $\sum_{i=1}^{I} \sum_{l=1}^{t} VAL_h q_{jh}^l x_{ij}^t$ in constraints (5), while it contains the multiplication of two decision variables. To avoid complexity of solving MINLP model, the authors suggested linearizing the model, using a heuristic lemma proposed by Li and Sun [34].

Lemma 1 [34]: *The quadratic phrase q.x in which q is a binary variable and x is a continuous variable can be substituted with a continuous variable f. Afterwards, the new variable is restricted by following linear constraints:*

$$f \le M\, q \tag{29}$$

$$x - M(1-q) \le f \le x \tag{30}$$

$$f \ge 0 \tag{31}$$

In which, M is a positive large value. Therefore, if $q=1$ then $f=x$, and if $q=0$ then $f=0$.

In order to apply above lemma in the proposed model, the quadratic phrase $q_{jh}^l . x_{ij}^t$ is substituted by f_{jh}^{lt}, constraints (32) are replaced with constraints (5), and constraints sets (33) to (35) are appended to the model M_1.

$$\sum_i \left(Qs_{ih}^t . x_{ij}^t + VAL_h \sum_{l=1}^{t} f_{ijh}^{lt} \right) K_h \ge Qp_{jh}^t \sum_i K_h x_{ij}^t \quad \forall t, j, h \tag{32}$$

$$f_{ijh}^{lt} \le M . q_{jh}^l \qquad \forall i, j, h, t, l | l \le t \tag{33}$$

$$x_{ij}^t - M\left(1 - q_{ij}^t\right) \le f_{ijh}^{lt} \le x_{ij}^t \qquad \forall i, j, h, t, l | l \le t \tag{34}$$

$$f_{ijh}^{lt} \ge 0 \qquad \forall i, j, h, t, l | l \le t \tag{35}$$

More precisely, after applying lemma 1, an MILP model of the problem is obtained as follows.

(M_2) Minimize
Objective function (1)
Subject to
Constraints set (2) to (4),
Constraints set (6) to (28),
Constraints set (32) to (35).

5 Results and discussion

A real test case of Iran SSCN is designed to evaluate the proposed model of inventory and facility location problem. Furthermore, performance of the proposed MINLP and MILP

Table 2 Percentage of CCs comprised in domestic iron ore mines

Mine	Chemical compounds							
	Fe	Fe_3O_4	SiO_2	Al_2O_3	CaO	P	S	MgO
Chadormalo	68	41.6	1.3	0.46	0.39	0.03	0.027	0.19
Golgohar	67.75	67	1.5	0.3	0.3	0.04	0.5	1.2
Choghart	66.83	35	1.8	0.99	0.75	0.022	0.02	0.51

models in the same running time is compared in a set of numerical test cases. Finally, sensitivity analysis of the more effective assumptions and parameters in the model is done in the numerical test cases. All test cases were solved by using the branch-and-bound (B-and-B) solver of LINGO 8.0 on a Pentium IV Core 2 Duo with 2.00 GHz processor and 1 GB RAM.

5.1 A real case: Iran SSCN

To assure the correct performance of constraints and objective function, the model is tested on a realistic scenario in Iran SSCN design where the customers' demands are stochastic. In this case, Chadormalo, Golgohar, and Choghart are considered as domestic iron ore mines, and Samarco, Kudremukh, Carajas, CVRD, Ferteco, and MBR as abroad ones [33]. As mentioned in Section 3, each of these mines supplies iron ore with specific CCs. Using the historical data for CCs of each mine, a regression model can be applied to forecast the CCs for the future. For instance, the CCs of iron ore in domestic mines in 2007 are presented in Table 2.

The producers' layer of Iran SSCN consists of three nationwide raw steel producers, namely Khouzestan Steel Co. (KSC), Mobarakeh Steel Co. (MSC), and Esfahan Steel Co. (ESC). Moreover, a fictitious producer in abroad (OUT) is assumed to import products from whenever any of the customers cannot be supplied from any domestic producers. The customers' layer comprises of (1) MSC, (2) ESC, (3) Kavian Steel Co., (4) National Industrial Steel Group Co., (5) Ahwaz Pipe and Rolling Mills Co., (6) Khouzestan Oxin Steel Co., (7) Azerbayjan Steel Co., (8) Khorasan Steel Co., (9) Amir Kabir Khazar Steel Co., (10) Iran Alloy Steel Co., and (11) a fictitious outside customer to export products. Note that, both MSC and ESC produce not only raw steel products, but also final steel ones, whereas KSC produces only raw ones. It is assumed that there is no potential site to establish new raw steel producers.

Table 3 Demand for Billet in the first time period in Iran SSCN

Demand	#Customer									
	1	2	3	4	5	6	7	8	9	10
Average (×1,000 tons)	600	1,080	300	360	0	0	120	240	420	300
Stand. Dev. (×1,000 tons)	21	60	19	26	0	0	3	7	36	26

The product flows from the first layer to the second one are iron ore with different CCs and from the second layer to the third are Billet, Bloom, and Slab. In this case, the average demand of customer was estimated using moving average based on the available historical data on its order quantities in last 2 years. Due to the functional and strategic role of steel in economy, the demand for raw steel products can be estimated with a high reliability [35]. It is assumed that KSC is allowed to store all three kinds of products, MSC is only allowed to store Slab, and ESC is allowed to store both Billet and Bloom as SSS. The standard deviation of demand is considered as a percentage of the average demand ranging from 0 to 10 %. Moreover, Iran national policy stated that by next 5 years, the ratio of imported raw steel products to total domestic steel production should be decreased from 35 to 15 %, and simultaneously the ratio of exported products to total domestic steel production should be increased from 10 to 30 % with an annual growth rate of 5 %. It is supposed that the service level of interest is 0.95 (i.e., confidence coefficient is 1.645) for all products in each time period and the emergency service level is 0.1 (i.e., emergency confidence coefficient is 0.25) for all customers and products in each time period. Customers' demands for Billet in the first time period is represented as a normal distribution, where their averages and standard deviations are shown in Table 3.

The producers have a number of melting furnaces (EAF in both KSC and MSC, and BF in ESC), which feed some parallel casting lines. At present, MSC is only able to produce Slab and ESC produces both Billet and Bloom. Moreover, KSC and OUT can produce all three kinds of raw steel products. Table 4 presents the initial capacities of producers in smelting, casting, and warehouse. As noted earlier, due to flexibility in production, the capacity of smelting stage is less than total capacity of casting lines. For example, KSC has six EAFs with total capacity of 2.4 million tons per year, while the Billet, Bloom, and Slab casting lines could produce 0.8, 1, and 1.2 million ton(s) per year, respectively. Since this is the dominant scenario in raw steel producers, smelting stage is recognized as the bottleneck of steel-making process.

The strategic objective of the model for nationwide test case of Iran SSCN design is to suggest an efficient guideline for importing and exporting raw steel products, and expanding the capacity of smelting stage, casting lines, and warehouse of producers. Furthermore, the model is able to not only estimate the capital needed for expansion plans but also propose the policies for selecting appropriate iron ore mines for long-term contracts. On the other hand, the main tactical objectives of the model are safety stocks handling, transportation planning,

Table 4 Initial capacities of raw steel production in Iran SSCN in 2009

Producer	Capacity (million ton(s) per year)				
	Smelting	Casting			Warehouse
		Billet	Bloom	Slab	
KSC	2.4	0.8	1.0	1.2	0.15
MSC	5.4	0.0	0.0	6.0	0.5
ESC	1.8	1.0	1.2	0.0	0.1
OUT	∞	∞	∞	∞	0.0

allocation of customers to the producers, and assigning the potential production capacities to the producers.

After solving the model, some noteworthy results are obtained which show the promising performance of proposed model. Based on solution obtained from the Iran SSCN model, a number of solution attributes are schematically shown in Figs. 2, 3, 4, 5, and 6. The plans for capacity expansion of smelting stages and Slab's casting lines by next 5 years are illustrated in Figs. 2 and 3, respectively. It is concluded that total raw steel production capacity in Iran SSCN should expand from 9.6 million tons in 2009 to 18.6 million tons in 2014 (see Fig. 2). Furthermore, Fig. 3 illustrates that by the next 5 years, total capacity of Slab's casting lines needs to be increased from 7.2 to 12.7 million tons per year. The production plan to produce three kinds of steel products in the next 5 years and assignment of customers to the producers in the first time period are shown in Figs. 4 and 5, respectively. Regarding Fig. 4, KSC should produce 3.9 million tons raw steel products including 779 thousand tons of Billet, 421 thousand tons of Bloom, and 2.7 million tons of Slab in the first time period. Furthermore, with reference to Fig. 5, KSC should supply MSC (customer #1), Kavian Steel Co. (customer #3), and National Industrial Steel Group Co. (customer #4) by 55, 300 and 360 thousand tons of Billets in the first time period, respectively.

Table 3 illustrates that total demand for Billet in the first time period is a normal random variable with average 3.42 million tons and standard deviation of 84 thousand tons. Therefore, to achieve service level of 0.95 for Billet, the

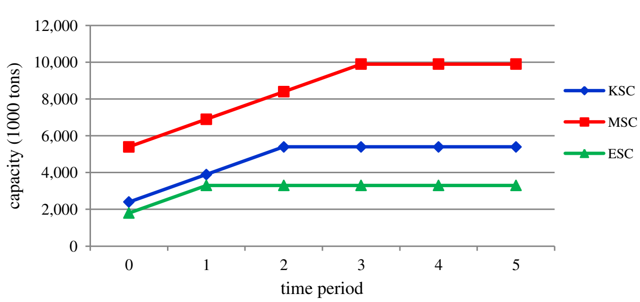

Fig. 2 Capacity expansion plans for smelting stages in Iran SSCN

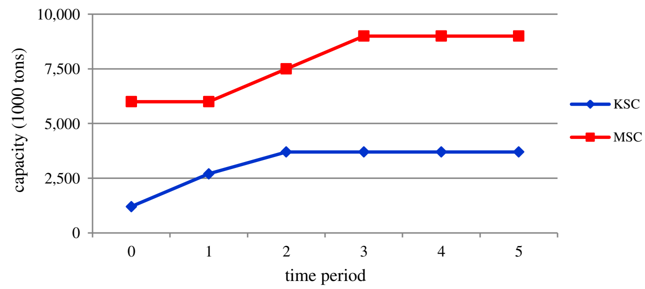

Fig. 3 Capacity expansion plans for Slab's casting lines in Iran SSCN

producers should hold or reserve at least 139 thousand tons of Billets (i.e., ≈ 1.645×84) as ESS, SSS, or PPC in the first time period. The SSS of Billet in the warehouse of producers is depicted in Fig. 6. As illustrated in this figure the level of SSS in the first time period is 89 thousand tons. On the other hand, to reach emergency service level of 0.1, the producers should hold at least 50 thousand tons of Billets (i.e., ≈ 0.25× 198 in which 198 is the sum of standard deviations of demands) as ESS in the first time period. As a result, the PPC reserved by producers of Billet in the first time period are zero. Because there is a trade-off between holding SS and reserving PPC in each time period, and the SSS along with ESS of Billet (i.e., 89+50) are enough to reach the service level of interest in the first time period. So, based on this solution, it seems reserving PPC is not as cost-saving as holding SS. Argue is that, if the demand not exceed the SS in a time period, the unused SS will be transferred to the next time period as an initial inventory (like regular inventory), but the unused PPC is not possible to be applied in the next time period(s). However, to keep the service level in the last time period, reserving PPC is more reasonable than holding SSS regarding the solution (see Fig. 6). Furthermore, as can be seen in Fig. 6, the level of SSS in the fourth time period is 236 thousand tons that is too high. Conceptually, this is due to regular inventory that appeared into the SSS.

5.2 Numerical test cases

To evaluate the complexity of proposed models for inventory and facility location problem in SSCN design, 18 numerical test cases are designed. The test cases are divided into six

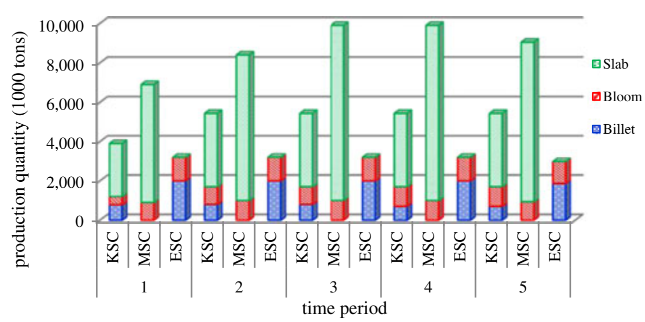

Fig. 4 Raw steel production plan in Iran SSCN

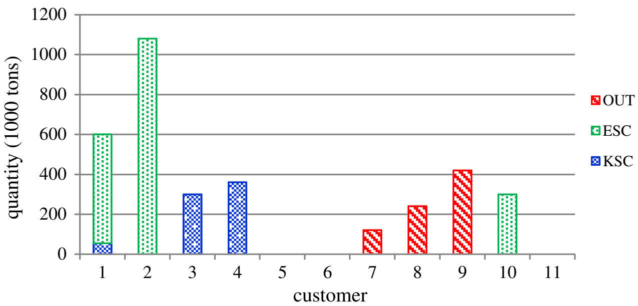

Fig. 5 Proposed plan to assign customers of Billet to the producers in the first time period

Table 5 The main characteristics of six categories of test cases

Category	No. of suppliers (I)	No. of producers (J)	No. of customers (K)	No. of periods (T)	No. of products (P)
A	9	3	10	5	3
B	10	4	12	5	4
C	12	5	15	5	4
D	10	5	12	5	5
E	15	6	20	5	3
F	12	6	18	5	5

various categories based on the number of supplier, producer, customer, time period, and product. Table 5 presents the characteristics of the SSCN for the categories. Each category involves three test cases of modules with various capacity levels, which are described in Table 6. In order to eliminate the effect of data structure, other parameters used in the test cases were generated randomly using uniform distribution by specific range. In fact, each test case was generated via a macro that recorded in Visual Basic for Applications 6.30 in Excel environment.

To evaluate the performance of models of the problem in SSCN design, namely the MINLP of M_1 and the MILP of M_2, comparative results are reported in Table 7. All test cases were solved by two models at the same running times. Optimality gap between the objective function values (OFV) of two models is calculated using the formula (36).

$$\%\text{Gap} = \frac{\text{OFV}(M_2) - \text{OFV}(M_1)}{\text{OFV}(M_1)} \times 100\% \qquad (36)$$

With reference to Table 7, in 9 test cases out of 18, the MILP model found better objective function values in compare to the MINLP model at the same running times. However, in three test cases, MINLP model found better solutions than MILP. Furthermore, in six test cases, MINLP model could not find any feasible solution within 2 h of running time, while in the same cases, MILP model obtained admissible solutions. In the cases that MILP model shows superior performance, the solutions found by the MILP model are averagely 5.79 % better than the MINLP solutions regarding the optimality gaps. On the other hand, in same running times, wherever MINLP outperforms MILP, solutions obtained by MINLP model are averagely 4.96 % better than MILP solutions. Furthermore, the MILP model outperforms MINLP especially in large-scale test cases belonging to categories E and F because the MINLP model could not find any feasible solution in four test cases of these categories. All test cases were solved via B-and-B solver of LINGO8.0, with global width branching strategy for nodes expansion. It is important to note that the results may change if other solution methods were applied to solve the models.

To evaluate the effects of some of the major assumptions on computational complexity of the problem, the authors formulated the problem in five various scenarios. The following assumptions are considered in sensitivity analysis of the problem:)a) minimum acceptable batch size of iron ore,)b) decisions on new technology selection, and (c) capital budgeting constraints. The first scenario is same as MILP model of M_2, the second to forth scenarios are abbreviated as M_2-L, M_2-T and M_2-B in which the assumptions)a),)b), and)c) are ignored, respectively. In the fifth scenario (i.e., M_2-LTB), all the aforementioned assumptions are ignored.

The test case number 3 of all categories which have the most number of binary variables in their own categories (see Table 7) were selected to evaluate the complexity of the problem. The computational time was limited to an hour for all test cases. The objective function value of the best known feasible solution found in this time interval was recorded as the best solution.

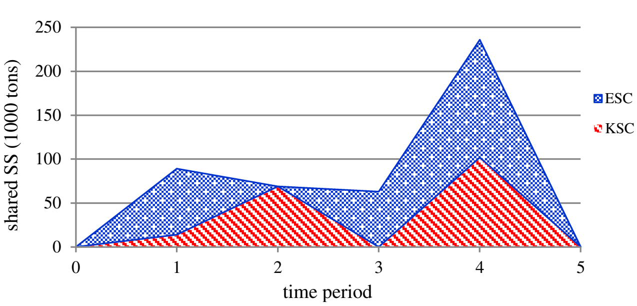

Fig. 6 Shared safety stock (SSS) of Billet in warehouse of producers

Table 6 Number of capacity levels in the test cases

Test cases no.	No. of smelting capacity levels (J_m)	No. of casting capacity levels (J_c)	No. of warehouse capacity levels (J_w)	No. of CCs in iron ore (H)
1	4	3	3	8
2	5	3	4	8
3	5	5	3	8

Table 7 Summary of test results for comparison of MINLP and MILP formulations

Category	Test cases no.	MINLP model of M_1		MILP model of M_2		Running time (s)	Gap (%)
		No. of nonlinear terms	OFV	No. of binary variables	OFV		
A	1	225	67,702	495	67,802	1,800	+0.15
	2	225	90,456	525	83,891	1,800	−7.16
	3	225	97,632	600	89,280	1,800	−8.55
B	1	320	132,748	740	118,850	3,600	−10.47
	2	320	No feasible	780	127,018	3,600	–
	3	320	160,266	920	147,314	3,600	−8.08
C	1	450	133,160	975	142,252	7,200	+6.83
	2	450	185,419	1,025	181,219	7,200	−2.27
	3	450	No feasible	1,200	191,758	7,200	–
D	1	400	192,800	1,000	190,383	7,200	−1.25
	2	400	158,151	1,050	142,336	7,200	−10.02
	3	400	173,858	1,275	187,584	7,200	+7.89
E	1	630	157,728	1,170	151,096	7,200	−4.21
	2	630	139,123	1,230	138,945	7,200	−0.13
	3	630	No feasible	1,380	151,094	7,200	–
F	1	540	No feasible	1,260	227,808	7,200	–
	2	540	No feasible	1,320	301,278	7,200	–
	3	540	No feasible	1,590	260,266	7,200	–

The optimality gap as an efficient indicator to measure the computational complexity is calculated via the formula (37), in which the best solution is compared to the lower bound of feasible solutions and potential solutions corresponding to the unfathomed branches found so far. The "Lower Bound" is limit on how far the solver will be able to improve the objective function. It is noted that the "Best Solution" is a descending function, while the Lower Bound is an ascending function over the time such that the Best Solution can never exceed the Lower Bound. The fact that these two values are close indicates that the algorithm progresses well and the current best solution is very close to the optimal one. These two values coincide to each other in the optimal solution. So, the scenario with large values of minimum, average, and maximum of optimality gap is more complicated.

Table 8 reports results of the experiments, in which "non-zero entries density" is defined as the ratio of the total number of non-zero entries to the total number of entries in the constraints matrix. The non-zero entries density is used to evaluate the sparsity of constraints matrix. In the field of numerical analysis, a sparse matrix is a matrix populated primarily with zeros [36]. A large amount of memory is consumed when dealt with a large sparse matrix. Moreover, "normality of constraints matrix" is specified as the ratio of the total number of decision variables to constraints. The minimum, average, and maximum of these two ratios for all the six test cases as well as the number of binary variables and the optimality gap have been calculated.

$$\text{Optimality Gap\%} = \frac{\text{Best Solution} - \text{Lower Bound}}{\text{Lower Bound}} \times 100\% \tag{37}$$

Table 8 Summary of test results to evaluate complexity of formulations within an hour

	Scenarios				
	M_2	M_2-L	M_2-T	M_2-B	M_2-LTB
No. of binary variables:					
Min.	600	465	480	600	345
Average	1161	878	967	1161	685
Max.	1,590	1,230	1,350	1,590	990
Non-zero entries density:					
Min.	0.000097	0.000103	0.001075	0.000010	0.001493
Average	0.000170	0.000178	0.001667	0.000168	0.002404
Max.	0.000307	0.000323	0.002866	0.000303	0.004044
Normality of constraints matrix:					
Min.	0.60	0.58	2.96	0.60	3.48
Average	0.66	0.65	3.62	0.66	4.45
Max.	0.72	0.70	4.21	0.72	5.17
Optimality gap (%):					
Min.	3.22	5.01	0.00	1.26	0.00
Average	12.81	9.95	5.46	10.69	0.24
Max.	21.25	17.63	11.42	28.53	0.65

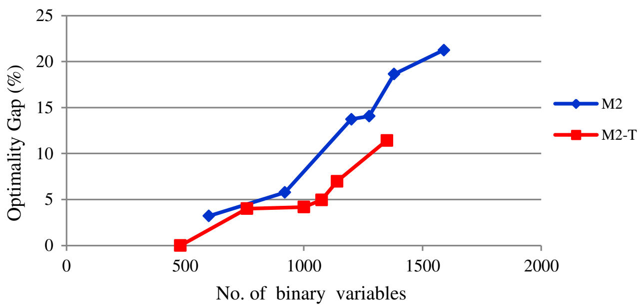

Fig. 7 Optimality gaps as functions of the number of binary variable

Regarding the results presented in Table 8, assumptions (b), (a), and (c) influenced computational complexity in descending order. The average number of binary variables reduced by 17 % (i.e., ≈ 100×(1,161–967)/1,161 %) if assumption (b) is omitted. Also, the average normality of constraints matrix and the average non-zero entries density are almost five times and nine times greater in scenario M_2-T compared to original model M_2, respectively. This result is supported by the average optimality gaps, which decreased from 12.81 % for M_2 to 5.46 % for M_2-T at the same running time. On the whole, it seems that a large portion of computational complexity of MILP model is in relation with making decisions on new technology selection to enrich the CCs of iron ore.

There is high variation in optimality gaps of the test cases solved under the same scenario. Figure 7 illustrates that optimality gap for the test cases solved under scenarios M_2 and M_2-T follows almost ascending trend when the number of binary variable is increasing. This is due to various sizes of SSCN and the nature of other data available for the problem. As can be seen in Table 8 and Fig. 7, all the test cases solved under scenario M_2 have an optimality gap greater than 3 %.

6 Conclusion and future research

Because of the increasing importance of network costs in business supply chains, this paper presents two general innovative models for dynamic multi-commodity inventory and facility location problem in SSCN design. The proposed approach can be applied to not only design new SSCN but also improve existing ones. Moreover, the models support multiple capacity levels for each facility and also consider cost-savings associated with cooperation of the producers in holding safety stocks. More precisely, the authors considered following criteria which add more novelty to the research: (a) dynamic stochastic customer demand, (b) intra-layer product flows, (c) blending problem, (d) new technology selection to enrich iron ore, and (e) capital budgeting. To reach the service level of interest, the authors proposed the concept of emergency and shared safety stocks which create a balance between responsiveness and transportation costs of SS on the whole SSCN. Furthermore, the potential production capacity would decrease the inventory holding costs for the producers. To reduce the complexity of the proposed MINLP model, the model is linearized by defining a new variable and adding some constraint to the model. The logical solution obtained for a real case in Iran SSCN design indicates the acceptable performance of the proposed MILP model. Moreover, the solutions found by B-and-B solver of LINGO8.0 illustrates that MILP model outperforms MINLP one, especially in large-scale test cases. The sensitivity analysis for the effects of the major assumptions shows that a large portion of computational complexity of the MILP model is due to new technology selection.

Indeed, solving large-scale test cases results in large optimality gaps; thus, applying more efficient solution methods like hybrid evolutionary algorithms would be an interesting area for future researches. In addition, an MILP model has been proposed only for forward SSCN design in this paper, while to reproduce raw steel products from scrap iron, the reverse SSCN design may influence the locations and capacities of raw steel producers. So, to avoid sub-optimality caused by separate design of forward and reverse SSCN, integrated forward/reverse SSCN design needs more efforts.

In this paper, only one transportation mode was considered between layers of the SSCN, and there was no constraint on delivery time to improve the responsiveness of the SSCN. Furthermore, a given number of downstream steel companies (i.e., final steel producers) have been considered as the customers. However, in the real SSCNs, the capacity of final steel production should be expanded proportional to the capacity of raw steel production by establishing new downstream steel companies and expansion of existing ones. So, dealing with this issue in a four-layer SSCN leads to more realistic model. Whereas, steel substitute products (e.g., composites) threaten to invest in the SSCNs, integrated design and optimization of logistics network for steel and its substitute products with dependent demands is a capable area for research.

References

1. Shavandi H, Bozorgi B (2012) Developing a location–inventory model under fuzzy environment. Int J Adv Manuf Technol 63:191–200
2. Sabri EH, Beamon BM (2000) A multi-objective approach to simultaneous strategic and operational planning in supply chain design. Omega 28:581–598
3. Chopra S, Meindl P (2007) Supply chain management: strategy, planning, and operation, 3rd edn. Prentice-Hall, New York
4. Melo MT, Nickel S, Saldanha da Gama F (2009) Facility location and supply chain management—a review. Eur J Oper Res 196:401–412

5. Jayaraman V, Pirkul H (2001) Planning and coordination of production and distribution facility for multiple commodities. Eur J Oper Res 133:394–408
6. Erlebacher SJ, Meller RD (2000) The interaction of location and inventory in designing distribution systems. IIE Trans 32:155–166
7. Syam SS (2002) A model and methodologies for the location problem with logistical components. Comput Oper Res 29:1173–1193
8. Melo MT, Nickel S, Saldanha da Gama F (2005) Dynamic multicommodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning. Comput Oper Res 33:181–208
9. Cordeau JF, Pasin F, Solomon MM (2006) An integrated model for logistics network design. Ann Oper Res 144:59–82
10. Gabor AF, Van Ommeren JCW (2006) An approximation algorithm for a facility location problem with stochastic demands and inventories. Oper Res Lett 34:257–263
11. Thanh PN, Bostel N, Peton O (2008) A dynamic model for facility location in the design of complex supply chains. Int J Prod Econ 113:678–693
12. Liu CM, Sherali HD (2000) A coal shipping and blending problem for an electric utility company. Omega 28:433–444
13. Bilgen B (2006) An iterative fixing variable heuristic for solving a combined blending and distribution planning problem. Proceedings of the 6th International Conference on Numerical Methods and Applications, pp 231–238
14. Movafagh Pour MA, Farahani RZ (2007) Designing and solving a model for facility location, inventory and transportation in bulk material supply chain. Proceedings of the 5th International Industrial Engineering Conference, July 2007, Tehran, Iran
15. Arabani AB, Farahani RZ (2012) Facility location dynamics: an overview of classifications and applications. Comput Indu Eng 62:408–420
16. Boudia M, Prins C (2009) A memetic algorithm with dynamic population management for an integrated production-distribution problem. Eur J Oper Res 195:703–715
17. Liang TF, Cheng HW (2009) Application of fuzzy sets to manufacturing/distribution planning decisions with multi-product and multi-time period in supply chain. Expert syst appl 36:3367–3377
18. Gendron B, Semet F (2009) Formulations and relaxations for a multi-echelon capacitated location–distribution problem. Comput Oper Res 36:1335–1355
19. Hinojosa Y, Kalcsics J, Nickel S, Puerto J, Velten S (2008) Dynamic supply chain design with inventory. Comput Oper Res 35:373–391
20. Kutanoglu E, Lohiya D (2008) Integrated inventory and transportation mode selection: a service parts logistics system. Trans Res Part E 44:665–683
21. Eksioglu SD, Eksioglu B, Romeijn HE (2007) A Lagrangian heuristic for integrated production and transportation planning problem in a dynamic, multi-item, two-layer supply chain. IIE Trans 39:191–201
22. Snyder LV, Daskin MS, Chung-Piaw T (2007) The stochastic location model with risk pooling. Eur J Oper Res 179:1221–1238
23. Bilgen B, Ozkarahan I (2007) A mixed-integer linear programming model for bulk grain blending and shipping. Int J Prod Econ 107:555–571
24. Vila D, Martel A, Beauregard R (2006) Designing logistics network in divergent process industry: a methodology and its application to the lumber industry. Int J Prod Econ 102:358–378
25. Persson JA, Lundgren MG (2005) Shipment planning at oil refineries using column generation and valid inequalities. Eur J Oper Res 163:631–652
26. Lee YH, Kim SH (2002) Production–distribution planning in supply chain considering capacity constraints. Comput Ind Eng 43:169–190
27. Nozick LK, Turnquist MA (2001) Inventory, transportation, service quality and the location of distribution centers. Eur J Oper Res 129:362–371
28. Sadjady H, Davoudpour H (2012) Two-echelon, multi-commodity supply chain network design with mode selection, lead-times and inventory costs. Comput Oper Res 39:1345–1354
29. Zhang J, Liu X, Tu YL (2011) A capacitated production planning problem for closed-loop supply chain with remanufacturing. Int J Adv Manuf Technol 54:757–766
30. Atighechian A, Bijari M, Tarkesh H (2009) A novel hybrid algorithm for scheduling steel-making continuous casting production. Comput Oper Res 36:2450–2461
31. Weingartner HM (1963) Mathematical programming and the analysis of capital budgeting problems. Prentice-Hall, New York
32. Park CS, Sharp-Bette GP (1990) Advanced engineering economics. Wiley, New York
33. Sabzevari Zadeh A, Sahraeian R (2010) A general model for production–transportation planning in steel supply chain. Int J Iron and Steel Society of Iran 7(1):11–16
34. Li D, Sun X (2006) Non-linear integer programming. Springer, New York
35. Fisher ML (1997) What is the right supply chain for your product. Harvard Business Review, Boston
36. Stoer J, Bulirsch R (2002) Introduction to numerical analysis, 3rd edn. Springer, New York