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ORIGINAL ARTICLE

Stability criteria based on argument principle of a general dynamical system in cutting process

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Abstract Stability prediction is important to avoid chatter and improve production efficiency in cutting process. Many methods including analytical, experimental, and numerical ones have been proposed. In this work, a stability criteria method using argument principle is proposed for a general dynamical systems. The method needs only to evaluate the characteristic function on a straight segment on the imaginary axis and the argument on the boundary of a bounded half circular region. The method is applied to three milling models in cutting process. Examples which show the evaluation of stability criteria proposed in the paper is simple and valid compared with full-discretization method.

Keywords Cutting · Argument principle · Stability criteria

1 Introduction

Machining is one of the most common manufacturing processes in industry due to its high flexibility and ability to produce parts with excellent quality. Chatter, a type of self-excited vibrations arising in metal cutting operations, is a major limitation in machining resulting in poor quality and reduced productivity. Under certain conditions, the cutting process may become unstable yielding oscillations with high amplitudes and cutting forces. Feed rate optimization, cutting force prediction, and stability prediction are all important research problems for milling process [1, 2]. However, stability prediction is the key problem to optimal selection of spindle speed and cutting depth to avoid chatter and improve production efficiency. Many methods including analytical, experimental, and numerical ones have been proposed.

Smith and Tlusty [3] presented a method to generate stability lobes by time domain simulations of the chatter vibrations in milling process. Altintas and Budak [4] presented an analytical method (ZOA method) for predicting milling stability lobes based on the mean of the Fourier series of the dynamical milling coefficients. The temporal finite element analysis (TFEA) for milling process simulation was presented by Bayly et al. [5]. Some experimental methods are utilized to get the stability boundaries [6–8]. Stability of systems could be evaluated numerically applying the semi-discretization method for periodic delay differential equations [9]. The semi-discretization was used in [10] to obtain approximate solutions for retarded functional differential equations (RFDEs). The essence of the method is that the delayed and the time-dependent terms are approximated by piecewise constant values (zeroth-order approximation), and, consequently, the RFDE is approximated by a series of ordinary differential equations (ODEs). The solutions of these ODEs lead to a finite-dimensional discrete map approximation of the RFDE. The semi-discretization method can effectively be used for analyzing cutting processes, like the milling process for which the governing RFDE has time periodic coefficients [11], the turning process with varying spindle speed for which the time delay itself is also time dependent in the governing RFDE [12], or feedback control systems [13, 14]. The method was recently refined in ref. [10, 15]. Convergence proof for the semi-discretization method can be found in ref. [16]. The methods reviewed above have their advantages and disadvantages,

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respectively [17]. In ref. [17], a full-discretization method based on the direct integration scheme was presented for prediction of milling stability. The fundamental mathematical model of the dynamical milling process considering the regenerative effect is expressed as a linear time periodic system with a single discrete time delay, and the response of the system is calculated via the direct integration scheme with the help of discretizing the time period. Then, the Duhamel term of the response is solved using the full-discretization method. The full-discretization method has high computational efficiency without loss of any numerical precision compared with semi-discretization.

Delay differential equations (DDEs) describe systems where the present rate of change of state depends on a past value (or history) of the state. The theory of DDEs is a generalization of the theory of ODE into infinite dimensional phase spaces. In cutting process, delay differential equations were used to describe dynamical systems. The pioneering orthogonal chatter stability models were introduced by Tlusty and Polacek [18], and Tobias and Fiswick [19] almost at the same period but independent of each other. The dynamical orthogonal cutting system is the form by Altintas [20]

$$m\ddot{x} + c\dot{x} + kx = F_r = K_r a[h_0 - (x(t) - x(t-\tau))] \quad (1.1)$$

Here, m is a lumped mass, k is stiffness and c is damping at the cutting point, F_r is cutting force, K_r is the radial cutting coefficients, a is width of cut, h_0 is static chip thickness, and $x(t)$ and $x(t-\tau)$ are the present and past vibration amplitudes in the radial direction, respectively. After linearizing the cutting force variation $\triangle F$ at some nominal chip thickness, the linearized equation of motion of classical regenerative chatter becomes (see [21])

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{k_1}{m}(x - x_\tau) \quad (1.2)$$

Here, ω_n is the natural angular frequency of the undamped free oscillating system, $\zeta = c/(2m\omega_n)$ is the so-called relative damping factor, x_τ denotes the delayed value of x, and the cutting force coefficient k_1 is the slope of the cutting force at the nominal chip thickness. In nondimensional form

$$\ddot{x} + 2\zeta\dot{x} + x = p(x - x_\tau) \quad (1.3)$$

where $p = \frac{k_1}{m\omega_n^2}$. If we let $X = (x, \dot{x})^{\mathrm{T}}$, state-space form of the Eq. (1.3) is

$$\dot{X} = Ax(t) + Bx(t-\tau) \quad (1.4)$$

where $A = \begin{pmatrix} 0 & 1 \\ -(1-p) & -2\zeta \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ -p & 0 \end{pmatrix}$.

Another dynamical equation of a single DOF milling model [22, 23] is

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = -\frac{wh(t)}{m_t}(x - x_\tau) \quad (1.5)$$

where w is the depth of cut and m_t is the modal mass of the tool. The time delay τ is equal to the tool passing period $60/(N\mathbf{\Omega})$, where N is the number of the cutter teeth and $\mathbf{\Omega}$ is the spindle speed in revolutions per minute. $h(t)$ is the cutting force coefficient which is defined as

$$h(t) = \sum_{j=1}^{N} g(\phi_j(t))\sin(\phi_j(t))[K_t\cos(\phi_j(t)) + K_n\sin(\phi_j(t))] \quad (1.6)$$

where K_t and K_n are the tangential and the normal linearized cutting force coefficients, respectively, and $\phi_j(t)$ is the angular position of the jth tooth defined by

$$\phi_j(t) = (2\pi\mathbf{\Omega}/60)t + (j-1)2\pi/N. \quad (1.7)$$

The function $g(\phi_j(t))$ is defined as

$$g(\phi_j(t)) = \begin{cases} 1, & \text{if } \phi_{st}(t) < \phi_j(t) < \phi_{ex}(t) \\ 0, & \text{otherwise} \end{cases} \quad (1.8)$$

where $\phi_{st}(t)$ and $\phi_{ex}(t)$ are the start and exit angles of the jth cutter tooth. Through some simple transformations, the state-space form of the single DOF milling model (1.5) can be represented as

$$\dot{X} = Ax(t) + Bx(t-\tau) \quad (1.9)$$

where $A = \begin{pmatrix} -\zeta\omega_n & \frac{1}{m_t} \\ m_t(\zeta\omega_n)^2 - m_t\omega_n^2 - wh(t) & -\zeta\omega_n \end{pmatrix}$,

$B = \begin{pmatrix} 0 & 0 \\ wh(t) & 0 \end{pmatrix}$.

Dynamical models deal with concentrated forces acting on the tool. These forces are the components of the resultant of the distributed force system along the rake face of the tool. Stépán derived a model which takes the distributed characteristic of the cutting force into account by means of a shape function $\omega(\theta)$. The equation of motion is the linear DDE [24]

$$\ddot{x} + 2\zeta\dot{x} + x = p\int_0^h x(t-\theta)\omega(\theta)\mathrm{d}\theta - p\int_{-\tau-h}^{-\tau} x(t+\theta)\omega(\tau+\theta)\mathrm{d}\theta \quad (1.10)$$

Equation (1.10) could be equivalently expressed as

$$\ddot{x} + 2\zeta\dot{x} + x = p\int_{-h}^0 x(t+\theta)\omega(-\theta)\mathrm{d}\theta - p\int_{-h}^0 x(t-\tau+\theta)\omega(\theta)\mathrm{d}\theta \quad (1.11)$$

Let $X = (x, \dot{x})^{\mathrm{T}}$ and Eq. (1.11) is transformed into

$$\dot{X} = \begin{pmatrix} 0 & 1 \\ -1 & -2\zeta \end{pmatrix}\begin{pmatrix} x \\ \dot{x} \end{pmatrix} + p\int_{-h}^0 \begin{pmatrix} 0 & 0 \\ \omega(-\theta) & 0 \end{pmatrix}\begin{pmatrix} x(t+\theta) \\ \dot{x}(t+\theta) \end{pmatrix}\mathrm{d}\theta \quad (1.12)$$

$$-p\int_{-h}^0 \begin{pmatrix} 0 & 0 \\ \omega(-\theta) & 0 \end{pmatrix}\begin{pmatrix} x(t-\tau+\theta) \\ \dot{x}(t-\tau+\theta) \end{pmatrix}\mathrm{d}\theta$$

In this paper, dynamical systems, similar to Eqs. (1.4), (1.9), and (1.12), could be generally unified to express in the following state-space form

$$\dot{X} = AX + BX(t-\tau) + p\int_{-h}^{0} K(\theta)X(t+\theta)\mathrm{d}\theta \\ -p\int_{-h}^{0} H(\theta)X(t-\tau+\theta)\mathrm{d}\theta \quad (1.13)$$

with initial function

$$x(t) = \varphi(t), \quad -h < \theta \le 0 \quad (1.14)$$

where $A, B, K, H \in \mathbb{R}^{n\times n}$ are corresponding matrices and τ stands for delay.

Recently, several stability criteria for system bounded regions have been investigated via the characteristic function [25–29]. Asymptotic stability of linear neutral systems with distributed delay are presented using the argument principle [30]. This work is motivated by [30]. Then stability criteria for dynamical system (1.13) in cutting process is obtained in Section 2. In Section 3, the method is applied to three milling models in cutting processes. In Section 4, conclusions with a brief discussion are presented.

2 Stability criteria of system (1.13)

Assume that initial condition $\varphi(t)$ is continuously differentiable

$$\sup_{-h\le\theta\le 0} |\varphi(t)| < \infty, \quad \sup_{-h\le\theta\le 0} |\dot{\varphi}(t)| < \infty \quad (2.1)$$

Matrices of system (1.13) $K(\theta) = (k_{ij}), H(\theta) = (h_{ij})$ are functions of bounded variation and with bounded first moments

$$\int_{-h}^{0} |\theta||k_{ij}|\delta\theta < \infty, \quad \int_{-h}^{0} |\theta||h_{ij}|\delta\theta < \infty, \quad i, j = 1, 2, ...n \quad (2.2)$$

Under the conditions (2.1) and (2.2), system (1.13) has the unique solution $x(t, \varphi)$ and there exists a Laplace transform of the solution. We now apply Laplace transform to system (1.13), the characteristic equation is

$$P(z) = \det[zI - A - e^{-z\tau}B - \int_{-h}^{0} e^{z\theta}K(\theta)\delta\theta \\ - \int_{-h}^{0} e^{z\theta}e^{-z\tau}H(\theta)\delta\theta] = 0 \quad (2.3)$$

where I is a unit matrix. Let

$$\bar{B} = e^{-z\tau}B, \quad \bar{K}(z) = \int_{-h}^{0} e^{z\theta}K(\theta)\delta\theta, \quad \bar{H}(z) = \int_{-h}^{0} e^{z\theta}e^{-z\tau}H(\theta)\delta\theta$$

Characteristic equation yields

$$P(z) = \det[zI - A - \bar{B} - \bar{K}(z) - \bar{H}(z)] = 0 \quad (2.4)$$

whose root is called a characteristic root.

Lemma 2.1 *(Argument principle [31]) Suppose that*

(i) a function $G(s)$ is analytic throughout the domain D except for poles, the domain D is interior to a positively oriented simple closed counter;
(ii) $G(s)$ is analytic and nonzero on;
(iii) counting multiplicities, Z is the number of zeros and Y is the number of poles of $G(s)$ inside γ.

Then

$$\frac{1}{2\pi}\triangle_\gamma \arg G(s) = Z - Y \quad (2.5)$$

where change of the argument of $G(s)$ along the closed line γ is defined by

$$\triangle_\gamma \arg G(s) = \arg G(\gamma_2) - \arg G(\gamma_1) \quad (2.6)$$

where γ_1, γ_2 stand for the starting point and final point of γ, respectively.

Lemma 2.2 *(Kolmanovskii and Myshkis [32]) Let characteristic* Eq. (2.4) *has no zeros in the half plane $\Re z \ge 0$ and kernels K and H satisfy condition* (2.2)*. Then system* (1.13) *is asymptotically stable.*

The main results of the present paper will be derived.

Theorem 2.3 *Assume that condition* (2.2) *holds, there exists a positive constant r and matrices satisfy*

$$\|A\| + \|B\| + \|\bar{K}\| + \|\bar{H}\| \le r, \quad (2.7)$$

where $\|\cdot\|$ denotes norm of matrices. Let z be a characteristic root of Eq. (2.4) *with $\Re z \ge 0$, then a bounded half circular region in the complex plane $|z| \le r$ includes all characteristic roots of* Eq. (2.4).

Proof Since z is a characteristic root of Eq. (2.4) with $\Re z \ge 0$, it has

$$P(z) = \det[zI - A - \bar{B} - \bar{K}(z) - \bar{H}(z)] = 0 \quad (2.8)$$

Let $W = A+B+\bar{K}+\bar{H}$, there exists a integer j, $1 \le j \le n$, such that

$z = \lambda_j(W)$.

Hence,

$$|z| = |\lambda_j(W)| \le \|A+B+\bar{K}+\bar{H}\| \le \|A\|+\|B\|+\|\bar{K}\|+\|\bar{H}\| \le r.$$

The proof is completed. □

Here, the following definitions are necessary for obtaining main results.

Definition 2.4 Let (ρ, θ) be polar coordinates. Let l_a be the straight segment which is on the imaginary axis, whose two terminal points are $d_1 = (r, -\pi/2)$ and $d_2 = (r, \pi/2)$, respectively. Let l_b be the half circumference on the right half plane defined by

$$l_b = \{(\rho, \theta) : \rho = r, -\pi/2 \le \theta \le \pi/2\} \tag{2.9}$$

Furthermore, let $l = l_a \cup l_b$ and D stand for the set of a bounded half circular region surrounded by l. The boundary of D is l and $\bar{D} = D \cup l$.

Definition 2.5 On the complex plane, we take two points $q_1 = (r + \varepsilon, -\pi/2)$ and $q_2 = (r + \varepsilon, \pi/2)$, respectively, where any $\varepsilon > 0$. Take l'_a as the straight segment on the imaginary axis $l'_a = q_1 q_2$ and

$$l'_b = \{(\rho, \theta) : \rho = r + \varepsilon, -\pi/2 \le \theta \le \pi/2\}. \tag{2.10}$$

$l' = l'_a \cup l'_b$, Q stands for the set of a bounded half circular region surrounded by l' and $\bar{Q} = Q \cup l'$.

Domains $\bar{D}$ and $\bar{Q}$ can be seen Fig. 1.

Corollary 2.6 *Under conditions of Theorem 2.3, let z be the characteristic roots of characteristic* Eq. (2.4) *with $\Re z \ge 0$, then $z \in \bar{D}$.*

Theorem 2.7 *Under conditions of Theorem 2.3, system* (1.13) *is asymptotically stable if and only if*

$$P(z) \ne 0 \qquad z \in l \tag{2.11}$$

and

$$\triangle_l \arg P(z) = 0 \tag{2.12}$$

where $\triangle_l \arg P(z)$ is change of the argument of $P(z)$ along the closed half circle l.

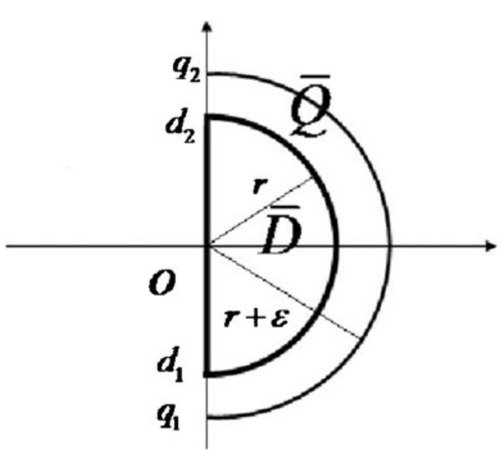

Fig. 1 The regions $\bar{D}$ and $\bar{Q}$

Proof Suppose system (1.13) is asymptotically stable. All zeros of $P(z)$ are on the left half plane. Hence, while $z \in l$, $P(z) \ne 0$ holds and the change of argument of $P(z)$ should be also zero along the closed half circle l by argument principle.

Conversely, assume that Eqs. (2.11) and (2.12) hold. Since $P(z)$ is an entire function, it has at most a finite number zeros in any bounded region. According to Lemma 2.2, system (1.13) is asymptotic stability while $P(z) \ne 0$ in the half plane $\Re z \ge 0$. And from Corollary 2.6, if $P(z) = 0$ in the half plane $\Re z \ge 0$, then $z \in \bar{D}$. Hence, we need to check whether $P(z) = 0$ for $z \in \bar{D}$. Using argument principle, if the change of argument of $P(z)$ is equal to zero along the closed half circle l, then $P(z) \ne 0$. Therefore, system (1.13) is asymptotic stability. □

If the closed circle l is replaced by l', then Theorem 2.7 is obvious and straightforward . Since matrices A, B, $\bar{K}$, and $\bar{H}$ are real and the characteristic roots of Eq. (2.4) are symmetric with respect to the real axis, it is sufficient to only check whether $P(z) = 0$ for $z \in d_2 o$, the upper half part of $d_1 d_2$. Stability criterion Theorem 2.7 could be simplified as follows.

Theorem 2.8 *Under conditions of Theorem 2.3, system* (1.13) *is asymptotically stable if and only if*

$$P(z) \ne 0 \qquad z \in d_2 o \tag{2.13}$$

and

$$\triangle_{l'} \arg P(z) = 0 \tag{2.14}$$

where o stands for the origin, i.e. $o = (0, 0)$.

The proof is similar to Theorem 2.7. Theorem of unstable criteria could be obtained.

Theorem 2.9 *Under conditions of Theorem 2.3, system* (1.13) *is unstable. If*

$$P(z) \ne 0 \qquad z \in d_2 o \tag{2.15}$$

and

$$\frac{1}{2\pi} \triangle_{l'} \arg P(z) = Z, \tag{2.16}$$

where Z is number of unstable characteristic roots.

Remark 2.10 Theorem 2.9 not only provided a unstable criteria for system (1.13), but also gave the number of unstable characteristic roots by the argument principle if there are no characteristic roots on the imaginary axis.

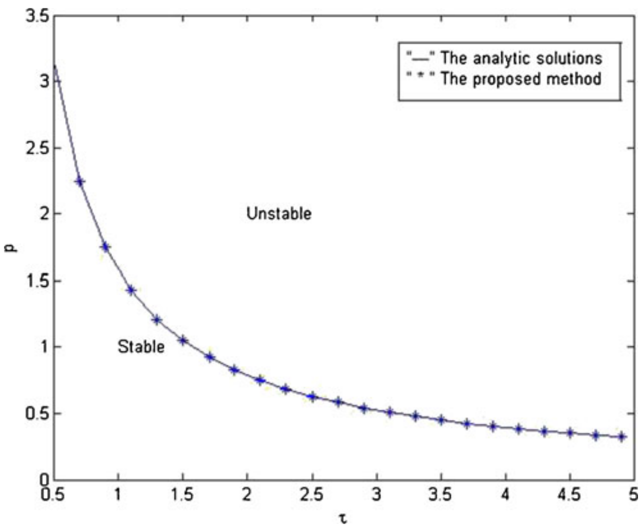

Fig. 2 Stability chart of Eq. (3.1)

3 Experiments

In this section, the method presented by this paper would be applied to three milling models in cutting processes.

Example 3.1 Firstly, let us consider a simple milling model [33]

$$\dot{x} = -px(t-\tau), \quad p > 0 \tag{3.1}$$

It is easy to know $p = \frac{\pi}{2\tau}$ is the so-called stability curve on the parameter plane (τ, p). Figure 2 shows this stability curve (see [33]). Points below the curve correspond to stable behavior, while upper ones correspond to unstable behavior.

In this paper, apply Laplace transform to Eq. (3.1), then characteristic equation is obtained

$$P(z) = z + pe^{-z\tau} = 0 \tag{3.2}$$

Using stability criteria Theorem 2.8 and 2.9, stable behaviors at some different parameters p and τ would be obtained by the symbol "*" in Fig. 2. It clearly shows a good agreement with stability curve.

Here, phase curves of $P(z)$ at two parameters $p = 1.5, \tau = 1$ and $p = 1, \tau = 3$ are provided respectively in Fig. 3 for understanding the method presented in the paper. In Fig. 3a, since change of the argument $\triangle_{l'} \arg P(z) = 0$, system (3.1) is stable at $p = 1.5, \tau = 1$. However, in Fig. 3b, change of the argument $\frac{1}{2\pi}\triangle_{l'} \arg P(z) = 2$, so system (3.1) is unstable at $p = 1, \tau = 3$.

Example 3.2 Let us consider a classical milling model (1.5) and its state-space form Eq. (1.9) [22, 23]. Hence, characteristic equation of (1.9) is obtained by

$$\det\left[\begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix} - \begin{pmatrix} -\zeta\omega_n & \frac{1}{m_t} \\ m_t(\zeta\omega_n)^2 - m_t\omega_n^2 - wh(t) & -\zeta\omega_n \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ wh(t) & 0 \end{pmatrix}\right] = 0 \tag{3.3}$$

Where tool has two fluted cutter, the natural frequency is $\omega_n/(2\pi) = 922$ Hz, the relative damping is $\zeta = 0.011$, the modal mass is $m_t = 0.03993$ kg, the cutting force coefficients are $K_t = 6 \times 10^8 \mathrm{N/m^2}$ and $K_n = 2 \times 10^8 \mathrm{N/m^2}$. In Fig 4, curve "-" shows stability lobes of $\boldsymbol{\Omega}$ versus w on Eq. (1.9) provided by full-discretization method [17]. The symbol "*" show stable behaviors at some different parameters using the proposed method in the paper.

Subsequently, we would consider another milling model (1.11) which takes the distributed characteristic of the cutting force into account by means of a shape function [24].

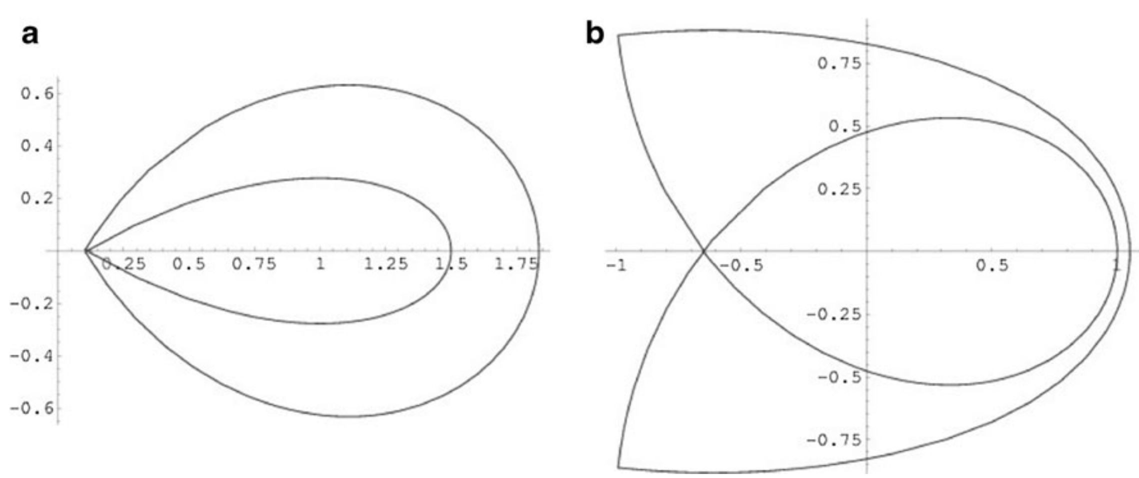

Fig. 3 Phase curves $P(z)$ of Eq. (3.2). **a** $p = 1.5, \tau = 1(stable)$, **b** $p = 1, \tau = 3(unstable)$

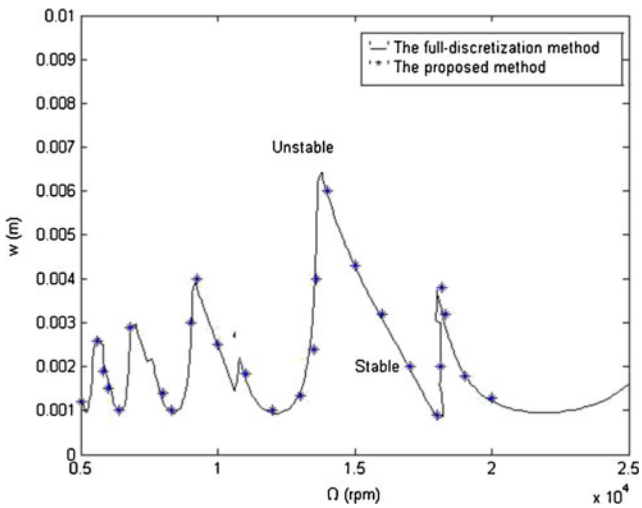

Fig. 4 Stability chart of Eq. (3.3)

Example 3.3 If the shape $\omega(\theta)$ of delay differential Eq. (1.11) is approximated by the exponential function [24]

$$\omega(\theta) = \frac{1}{q_0\tau} e^{\frac{\theta}{q_0\tau}}$$

where q_0 is the ratio of the short and regenerative delay, then Eq. (1.11) can be transformed into a third-order system

$$q_0\tau\dddot{x}+(1+2\zeta q_0\tau)\ddot{x}+(2\zeta+q_0\tau)\dot{x}+(1+p)x(t)-px(t-\tau)=0. \tag{3.4}$$

Hence, characteristic equation of delay differential Eq. (3.4) is obtained

$$P(z)=q_0\tau z^3+(1+2\zeta q_0\tau)z^2+(2\zeta+q_0\tau)z+1+p-pe^{-z\tau}=0, \tag{3.5}$$

where $p = \frac{k_1}{m\omega_n^2}$ and k_1 is cutting force coefficient [24]. Take the values $m = 50$ kg, $\zeta = 0.05$, $\omega_n = 775$ rad/s, and $q_0 = 0.01$. Using the proposed method in the paper, the stability charts in the parameters plane of cutting force coefficient k_1 versus angular velocity $\boldsymbol{\Omega}$ is shown in Fig. 5.

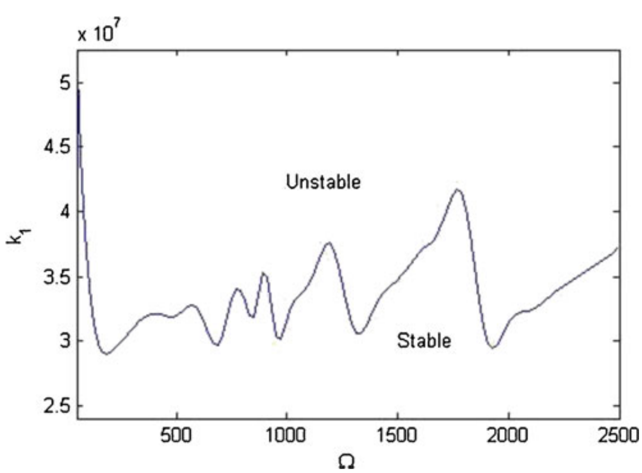

Fig. 5 Stability chart of Eq. (1.11)

4 Conclusions

In this work, a stability criteria method using argument principle is proposed for a general dynamical systems (1.13). The stability criteria method need only to evaluate the characteristic function on a straight segment on the imaginary axis and the argument on the boundary of a bounded half circular region. At different parameters, if phase curves $P(z)$ have no intersection points on imaginary axis and lie in right half plane (see Fig. 3), then the dynamic system is in stable behavior. Otherwise, the system is unstable. In Section 3, the method is applied to three milling models in cutting processes. Examples 3.1 and 3.2 show a good agreement with stability curve obtained by analytical solution or full-discretization method. Example 3.3 shows that the presented method could be also applied to linear milling models which described differential-integral delay equations. Results show that the evaluation of stability criteria method proposed in the paper is very simple and valid.

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