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ORIGINAL ARTICLE

Robustness of thermal error compensation modeling models of CNC machine tools

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Abstract In order to achieve effective control of thermal error compensation of computer numerical control (CNC) machine tools, the prediction accuracy and robustness of the compensation model is particularly important. In this paper, the temperature of sensitive points and thermal error of the spindle in Z direction are measured. Using a combination of fuzzy clustering analysis and gray correlation method to select temperature-sensitive points and then using multiple linear regression of least squares and least absolute estimation methods, distributed lag model, and support vector regression machine to establish prediction models of the relationship between temperature of sensitive points and the thermal error. Also, the temperature values of sensitive points and the thermal error in the experimental conditions of different ambient temperatures and different spindle speeds are measured. By comparing the prediction accuracy of various prediction models under different experimental conditions verify the robustness of the models. Experimental results show that when the modeling data are less, the prediction accuracy of multiple linear regression of least squares and least absolute estimation methods and distributed lag model is declined, and their robustness are poor, while support vector regression model has good prediction accuracy and its robustness remains strong when changing the experimental conditions. However, when modeling data are rich, the prediction accuracy of various algorithms is improved, but the robustness of support vector regression model is volatile. The robustness analysis of different models provides a useful reference for the thermal error compensation model, selection of CNC machine tools, and has good engineering applications.

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Keywords Machine tools · Thermal error · Multiple linear regression model · Distributed lag model · Support vector regression model · Robustness

1 Introduction

In various sources of error in computer numerical control (CNC) machining accuracy, thermal error has been the major one [1]. Reducing thermal error is the key to improve CNC machining accuracy. In thermal error compensation, modeling technology is emphasized. As the CNC thermal error depends largely on the processing conditions, the processing cycle, the usage of coolant, and surrounding environment, etc. and its nonlinear and interaction, it is quite difficult to establish an accurate mathematical model of thermal error just with theoretical analysis [2]. The most common method of thermal error modeling is experimental modeling method, which analyzes thermal error and temperature based on statistical theory [3]. Jianguo Yang et al. put forward CNC packet optimization modeling. According to the relationship between variable temperatures, this method divides the temperature into groups, then permutates and combines with thermal error, and selects temperature-sensitive points for modeling [4–6]. In South Korea, SK Kim et al. establish temperature field of CNC ball screw system with finite element method [7]. At the University of Michigan, S. Yang et al. establish thermal error model with the use of cerebella model arithmetic computer neural network [8]. Huanglin Zeng et al. use rough artificial neural network for thermal error analysis and modeling [9]. Chen Cheng et al. select the temperature-sensitive points with clustering analysis theory, measure the temperature with PT100 and thermal error with laser interferometer, and establish a multivariate linear model [10]. Since these modeling methods are offline and pre-modeled, the modeling data measured in a certain period of time and the thermal error mathematical model, based on these methods, lack robustness and cannot forecast thermal error correctly in a long term with the change of seasons [11]. In recent years, support vector machine, a machine learning theory just for small sample, is developed as the best theory in small sample statistics and forecast [12]. Support vector machine was based on VC dimension, and it reduced structural risk. It effectively solves the problem between model selection and owe-learning, over-learning, small simple, nonlinear, local optimum, and dimension disasters with a simple structure, thus greatly improving generalization ability [13, 14]. In this paper, measuring thermal error of Leaderway V-450 of CNC machining center, selecting temperature-sensitive points with the combination of fuzzy clustering and gray correlation theory, establishing compensation model separately with multiple linear regression model, distributed lag model, and support vector machine, and estimating the multiple linear regression model with the least squares and the least absolute deviation are the objectives of this study. By analysis of robustness of these models, a reference of thermal error compensation modeling is provided, and it has actual engineering application value.

2 Thermal error modeling models

2.1 Multiple linear regression model

Multiple linear regression model is a statistical method to seek relationship between multi-input and single-output. The temperature increments of temperature-sensitive points are as independent variable and thermal error as dependent variable to establish the multiple linear regression model of machine tools. The general equation is as follows:

$$y_i = b_0 + b_1 x_{i1} + b_2 x_{i2} + \cdots + b_k x_{ik} + e_i (i = 1, 2, .., n) \quad (1)$$

From Eq. (1), $(x_{i1}, x_{i2}, \cdots, x_{ik})$ are temperature increments of the sensitive points, $b_0, b_1, \cdots, b_k$ are the coefficient of temperature variables, y_i is the measurement value of thermal deformation, and e_i is the deviation of y_i.

At the same time, using the least absolute deviation and least squares estimation criteria, we calculated the multiple linear regression model. The least squares method is more mature in terms of methods and more perfect in terms of theory, and a commonly used method. It is widely used in many practical problems in the field of science and technology and also applied in modeling technique of machine tools. The least absolute deviation method has small influence of outliers, and its robustness is better than the least squares method. But the least absolute deviation method is nondifferentiable and has great difficulty on calculation. In this paper, the algorithm theory and program of the least absolute deviation method use the reference [15].

The least squares criterion is as follows:

$$\sum_{i=1}^{n} [y_i - (b_0 + b_1 x_{i1} + b_2 x_{i2} + \cdots + b_k x_{ik})]^2 = \min \quad (2)$$

The least absolute deviation criterion is as follows:

$$\sum_{i=1}^{n} |y_i - (b_0 + b_1 x_{i1} + b_2 x_{i2} + \cdots + b_k x_{ik})| = \min \quad (3)$$

2.2 Distributed lag model

If the dependent variable is not only related to one or more current value of explanatory variables but also related to its certain lag value, this is called distributed lag model and denoted by:

$$y_t = \alpha_0 + \sum_{j=1}^{u} \sum_{i=0}^{n} \beta_{j,i} x_{j,t-i} + \varepsilon_t, \varepsilon_t \sim IID\left(0, \sigma^2\right) \quad (4)$$

From Eq.(4), n is the maximum lag order, a_0 is a constant, u is the number of exogenous variables, y_t is the dependent variable, $\beta_{j,i}$ is the coefficient, and $x_{j,t-i}$ is the ith lag order value of the jth independent variable.

Due to the great amount of experimental data, using the simple expedient estimation method determined the lag order n. That is to take n=1,2…i, and then using the least squares fits data in different conditions of i. When the lag variable regression coefficients become statistically insignificant or there is a variable coefficient change sign, the i-1 is the final lag order.

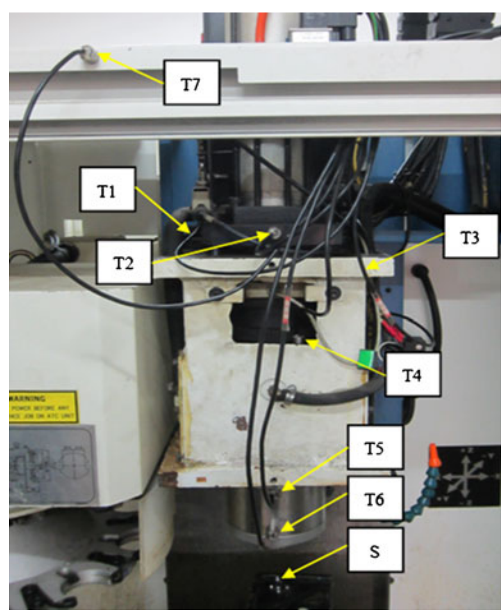

Fig. 1 Measurement experiment of thermal error

Table 1 The installation locations and functions of sensors

Sensor	Location	Function
T1, T2	Z-axis motor	Temperature measurement of motor
T3, T4	Spindle sleeve	Temperature measurement of spindle
T5, T6	Front bearing of Spindle	Temperature measurement of spindle
T7	Machine casing	Ambient temperature measurement
S	Under the spindle	Thermal deformation measurement of Z direction of spindle

2.3 ε − support vector regression model

Statistical learning theory established by Vapnik is a specialized theory which researches the machine learning law in the case of finite sample theory. The support vector machine is a new classification and regression tool on the basis of this theory [13]. Support vector machine improves the generalization ability through structural risk minimization principle, and it can solve the practical problems better, such as small sample size, nonlinear, high dimension, and local minima point. It has been applied in pattern recognition, signal processing, function approximation, and other fields.

2.3.1 The principle of ε − SVR

The corresponding vector machine is called support vector machine, when insensitive function ε is introduced, and its constrained optimization problem can be expressed as follows:

$$\left.\begin{array}{ll} \min & \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{l}\left(\xi_i+\xi_i^*\right) \\ \text{s.t.} & w\cdot x_i + b - y_i \le \varepsilon + \xi_i \\ & y_i - w\cdot x_i - b \le \varepsilon + \xi_i^* \qquad i=1,2,\cdots,l \\ & \xi_i \ge 0 \\ & \xi_i^* \ge 0 \end{array}\right\} \tag{5}$$

Table 2 The results of Gray Correlation

Sensors	T1	T2	T3	T4	T5	T6
Correlation γ	0.611	0.589	0.569	0.576	0.602	0.634

From Eq. (5), w is the weight vector, C is the penalty parameter, l is the number of inputs, ξ_i,ξ_i^* are the slack variables, x_i are inputs, b is the deviation, and y_i are target values.

Introducing Lagrange function will convert it to the dual problem, and it can be expressed as follows:

$$\left.\begin{array}{ll} \min & \frac{1}{2}\sum_{i,j=1}^{l}\left(a_i^*-a_i\right)\left(a_j^*-a_j\right)\left(x_i\cdot x_j\right) + \varepsilon\sum_{i=1}^{l}\left(a_i^*+a_i\right) \\ & -\sum_{i=1}^{l} y_i\left(a_i^*-a_i\right) \\ \text{s.t.} & \sum_{i=1}^{l}\left(a_i^*-a_i\right) = 0 \\ & 0\le a_i^*\le C,\ 0\le a_i\le C \quad i=1,2,\cdots,l \end{array}\right\} \tag{6}$$

From Eq. (6), a,a^* are Lagrange multipliers.

According to the Mercer conditions [13] and using the kernel function $K(x_i,x)$, the decision function of ε − support vector regression (SVR) machine can be expressed as follows:

$$f(x) = \sum_{i=1}^{l}\left(a_i^*-a_i\right)K(x_i,x) + b \tag{7}$$

From Eq. (7), a^*,a,b are calculated by Eq. (6).

The common kernel functions that satisfy the Mercer condition are linear kernel function, polynomial kernel

Fig. 2 The temperature values of sensors

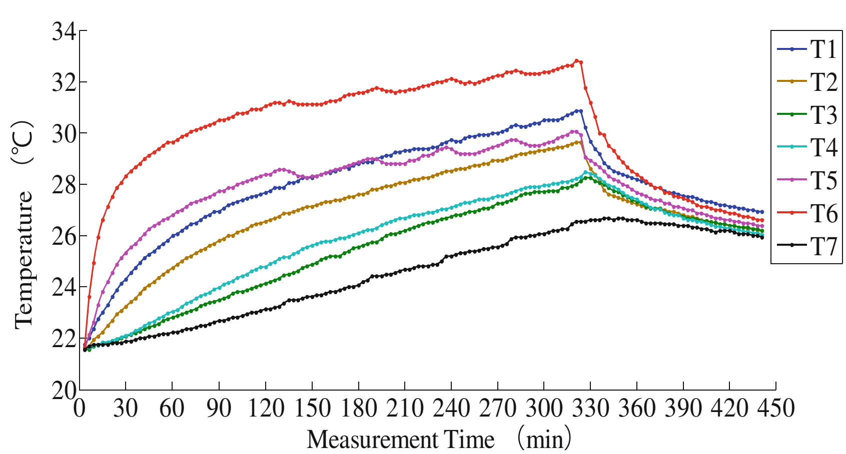

Table 3 The spindle speed and ambient temperature of experiments

Batches	Spindle Speed (rpm)	Ambient Temperature (°C)
$K1_{2000}$	2,000	5.6~8.1
$K1_{4000}$	4,000	6.2~10.6
$K1_{6000}$	6,000	6.5~10.0
$K2_{2000}$	2,000	7.0~9.4
$K2_{4000}$	4,000	9.4~12.4
$K2_{6000}$	6,000	8.3~12.5
$K3_{2000}$	2,000	23.9~26.1
$K3_{4000}$	4,000	24.9~31.7
$K3_{6000}$	6,000	27.3~31.5

function, radial basis function (RBF), Fourier kernel function, and so on.

2.3.2 *The parameter selection of* ε − SVR

In this paper, selecting the RBF kernel function is as follows:

$$K\left(x,x^{'}\right)=\exp\left(-\left\|x-x^{'}\right\|^{2}/\sigma^{2}\right) \tag{8}$$

From Eq. (8), σ is the width parameter of function and it controls the radial range of function, and $1/\sigma^2$ is recorded as parameter g. Usually, we use Gaussian RBF kernel function for modeling since it has high training speed and high precision [16]. In this paper, the Gaussian RBF kernel function is used for thermal error modeling; the training prediction accuracy and robustness depend on the values of g and C. The parameter selection methods of support vector machine typically includes homing-based network traversal algorithm, cross-validation method, and so on. This paper chooses the cross-validation method [17] to optimize the model parameters because of its relatively simple application, and it has good practicability.

Table 4 The fitting standard deviation S of models

Models	LS-MLR	LA-MLR	DL	SVR
S (um)	1.5577	1.8402	0.8644	0.4729

3 Thermal error measurement experiment and temperature sensitive point selection

3.1 Experimental design

Experimental system of Leaderway V-450 CNC machine tool is shown in Fig. 1. It is used to measure temperature and thermal error of Z direction of spindle. The installation locations and functions of temperature sensors and displacement sensor are shown as Table 1.

3.2 The temperature-sensitive point selection

Reducing the complexity of the model is conducive to the application, so it is better to minimize the number of independent variables to establish the model. In this paper, the combination of fuzzy clustering [18] and gray correlation degree are used to select temperature-sensitive points [19].

3.2.1 *Fuzzy clustering*

Fuzzy clustering method is based on fuzzy matrix to classify all of the research objects; the objects in the same cluster are very similar, while objects in different clusters have large dissimilarity. Assumption domain $U=\{u_i|i=0,1,\cdots N\}$, where u_i is the temperature point. In order to classify U, it is necessary to calculate the correlation coefficient r_{ij} of the statistics relationship between objects which are classified and then determine the fuzzy matrix. By the transitive closure method, the fuzzy matrix is transferred into an equivalent matrix, and the fuzzy classification is finished.

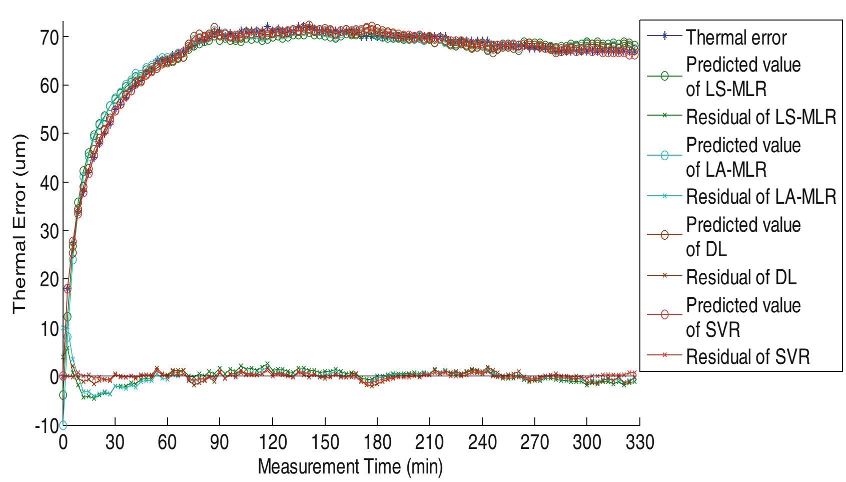

Fig. 3 The fitting effect of $K1_{6000}$

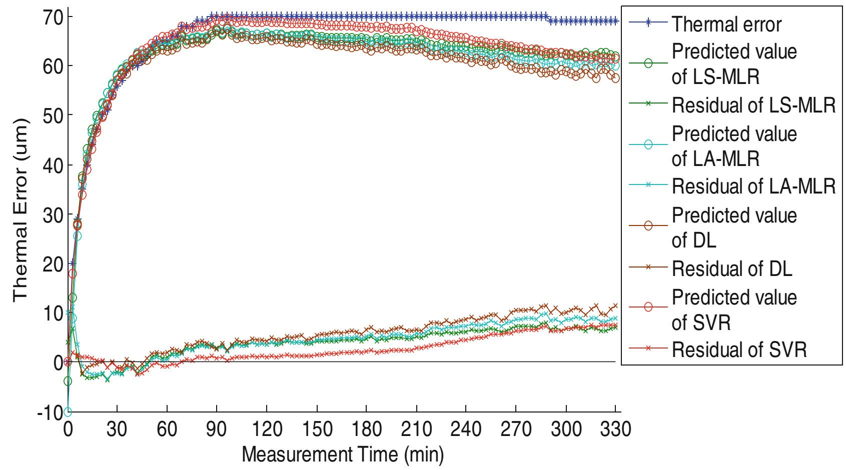

Fig. 4 The prediction effect of $K2_{6000}$

Setting up $u_i \in U, u_{ik}(k=1,2,\cdots n)$ is the kth measurement value of u_i, the correlation coefficient is as follows:

$$r_{ij} = \frac{\sum_{k=1}^{n}\left(u_{ik}-\bar{u}_i\right)\left(u_{jk}-\bar{u}_j\right)}{\sqrt{\left(u_{ik}-\bar{u}_i\right)^2\left(u_{jk}-\bar{u}_j\right)^2}} i = 0, 1, \cdots N \quad (9)$$

From Eq. (9), $u_i = \frac{1}{n}\sum_{k=1}^{n} u_{ik}, \ \ u_j = \frac{1}{n}\sum_{k=1}^{n} u_{jk}$.

So, the fuzzy matrix is $R=(r_{ij})_{N\times N}$, by square method; the fuzzy matrix R is transferred into equivalent fuzzy matrix $t(R)$. The fuzzy clustering is done for U with $t(R)$. Where $\overline{R} = t(R)$ is the cut set at $\lambda(0\leq\lambda\leq1)$ of $t(R)$, note $\overline{R} = \left(\overline{r}_{ij}\right)_{N\times N}$. When $\overline{r}_{ij} = 1$, u_i, and u_j are the same class.

3.2.2 Gray correlation degree

Gray system theory proposes the concept of gray correlation analysis for the various subsystems and seeks numerical relationship between the various factors in the system by a certain method. The equation of gray correlation degree is as follows:

$$\gamma(x_0, x_i) = \frac{1}{n}\sum_{i=1}^{n} r\left(x_0(k), x_i(k)\right) \quad (10)$$

$$r\left(x_0(k), x_i(k)\right) = \frac{\min\limits_i \min\limits_k \left|x_0(k)-x_i(k)\right| + \rho \max\limits_i \max\limits_k \left|x_0(k)-x_i(k)\right|}{\left|x_0(k)-x_i(k)\right| + \rho \max\limits_i \max\limits_k \left|x_0(k)-x_i(k)\right|} \quad (11)$$

From Eq. (11), ρ is the resolution coefficient, $\rho\in[0,1]$.

According to the above equations, the correlation degree $\gamma(x_0,x_i)$ between thermal error of CNC machine tool and the various temperature measurement data are calculated. The greater the correlation degree, the greater impact on thermal error, and this temperature point can be regarded as the modeling point.

The combination of fuzzy clustering and gray correlation degree are used to select temperature-sensitive points. The temperature values collection are shown as Fig. 2.

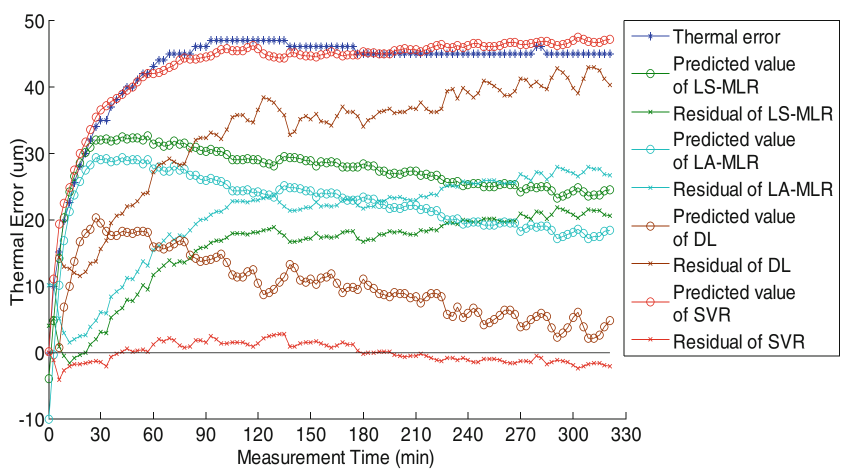

Fig. 5 The prediction effect of $K3_{6000}$

Table 5 The prediction standard deviation S of models

Models	LS-MLR	LA-MLR	DL	SVR
S of $K2_{6000}$ (um)	4.9317	5.8966	6.9904	3.7914
S of $K3_{6000}$ (um)	16.8501	21.7091	35.2822	1.4765

The equivalent fuzzy matrix of the temperature points T1~T7 is as follows:

$$R = \begin{bmatrix} 1.0000 & 0.9978 & 0.9875 & 0.9875 & 0.9898 & 0.9898 & 0.9596 \\ 0.9978 & 1.0000 & 0.9875 & 0.9875 & 0.9898 & 0.9898 & 0.9596 \\ 0.9875 & 0.9875 & 1.0000 & 0.9953 & 0.9898 & 0.9898 & 0.9531 \\ 0.9875 & 0.9875 & 0.9953 & 1.0000 & 0.9898 & 0.9898 & 0.9531 \\ 0.9898 & 0.9898 & 0.9898 & 0.9898 & 1.0000 & 0.9885 & 0.9643 \\ 0.9898 & 0.9898 & 0.9898 & 0.9898 & 0.9885 & 1.0000 & 0.9643 \\ 0.9596 & 0.9596 & 0.9531 & 0.9531 & 0.9643 & 0.9643 & 1.0000 \end{bmatrix}$$

Set λ=0.98 and the result of classification are as follows:

$$e(R) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

According to the results of fuzzy clustering analysis, the sensor T1, T2, T3, T4, T5, and T6 are classified as Group I, and T7 as Group II. The correlation degree between each sensor temperature value of Group I and thermal error are calculated, and the results are shown as Table 2.

From Table 2, the gray correlation degree between sensor T6 and thermal error S is the highest, so T6 of Group I is selected as sensitive point. So the sensor T6 and T7 are selected as temperature-sensitive points to modeling.

Similarly, the results of temperature-sensitive points selection of other sample data are the same with the above, and sensors T6 and T7 are used for modeling.

3.3 Experimental scheme

The thermal error measurement experiment is done nine times in different seasons (different ambient temperature) and different spindle speeds of CNC machining center. The ambient temperature and spindle speed of every experiment is shown as Table 3.

From Table 3, the meaning of Kn_m is the *n*th test data measured in meters per revolution per minute of spindle speed. For example, $K1_{2000}$ is the first test data measured in 2,000 rpm of spindle speed, $K2_{2000}$ is the second test data measured in 2,000 rpm of spindle speed, and $K3_{2000}$ is the third test data measured in 2,000 rpm of spindle speed.

4 The robustness analysis of modeling models

Robustness means there are some gaps in the model and the actual object; the model still has a satisfactory performance of simulation and prediction. In this paper, a multiple linear regression of least squares (LS-MLR) and least absolute (LA-MLR) estimation models, distributed lag (DL) model, and SVR model are used to establish prediction model of the data $K1_{6000}$; the fitting accuracy of every model is analyzed, and then the prediction model is used to forecast other batches of data to determine the robustness of every model. At the same time, according to the modeling data sources characteristics, each algorithm robustness analysis is given.

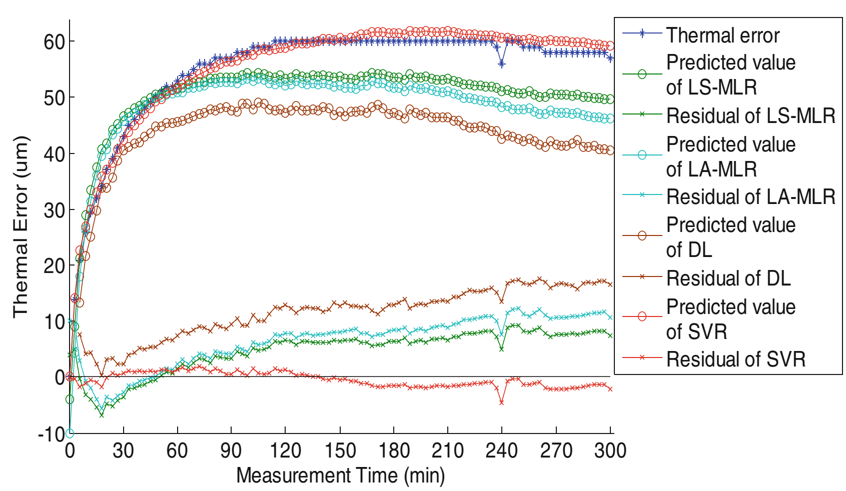

Fig. 6 The prediction effect of $K1_{4000}$

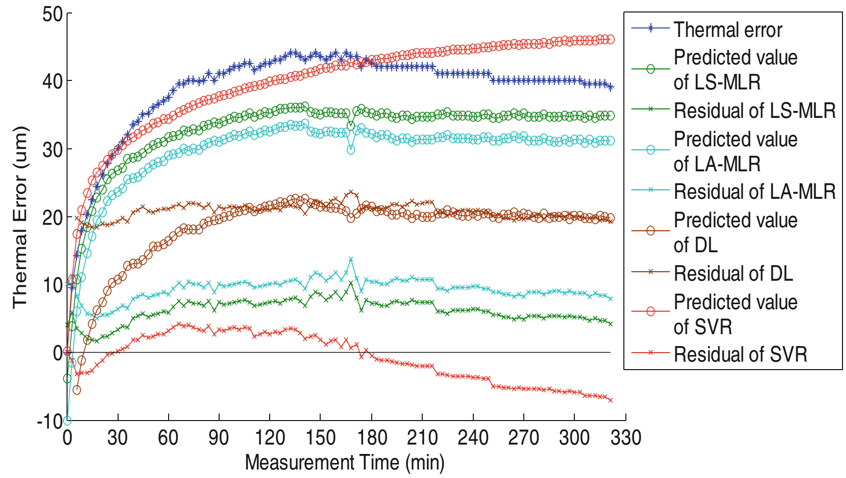

Fig. 7 The prediction effect of $K1_{2000}$

4.1 The fitting accuracy of different models

The equations of models are shown as follows:

The equation of LS-MLR model:

$$y = -3.9304 + 4.9095T_6 - 7.1656T_7 \tag{12}$$

From Eq. (12), T_6 is the temperature increments of sensor T6, and T_7 is the temperature increments of sensor T7.

The equation of LA-MLR model:

$$y = -10.025 + 5.4606T_6 - 8.7339T_7 \tag{13}$$

From Eq. (13), T_6 is the temperature increments of sensor T6, and T_7 is the temperature increments of sensor T7.

The equation of DL model (the lag order of DL model is 2 by expediency estimation method):

$$\begin{aligned} y_t = &-34.4798 + 9.5056T_{6t} - 0.4002T_{6t-1} - 2.0641T_{6t-2} \\ &-5.0974T_{7t} - 3.1364T_{7t-1} - 2.5486T_{7t-2} \end{aligned} \tag{14}$$

From Eq. (14), $T_{6\ t}$ is the temperature increments of sensor T6, $T_{6\ t\text{-}1}$ is the lag 1-order temperature increments of sensor T6, $T_{6\ t\text{-}2}$ is the lag 2-order temperature increments of sensor T6, $T_{7\ t}$ is the temperature increments of sensor T7, $T_{7\ t\text{-}1}$ is the lag 1-order temperature increments of sensor T7, and $T_{7\ t\text{-}2}$ is the lag 2-order temperature increments of sensor T7.

The equation of SVR model:

$$y = f(x) = \sum_{i=1}^{l} \left(a_i^* - a_i\right) \exp\left(-0.0026\|x_i - x\|^2\right) + 49.551 \tag{15}$$

From Eq. (15), by cross-validation method to optimize the parameters, sensitivity function g=0.0026 and punishment coefficient C=24,834. By using LIBSVM in Matlab, support vector, support vector corresponding coefficient ($a_i^* - a_i$), and constant b=49.551 are obtained; l is the amount of data (the number is 110 of this batch data).

The fitting standard deviations of models for $K1_{6000}$ are shown as Table 4 and fitting effect as Fig. 3.

From Table 4, the fitting accuracy of SVR is the best, DL is the second, and the LA-MLR is the worst.

In order to analyze the robustness of each model, every prediction model is used to forecast other batches of data according to the same spindle speed at different temperatures (ambient temperature vary greatly), the same temperature (ambient temperature vary small) in different spindle speeds, and the different temperatures and different spindle speeds.

Table 6 The prediction standard deviation S of models

Models	LS-MLR	LA-MLR	DL	SVR
S of $K1_{4000}$ (um)	6.1024	7.9943	12.7225	1.3758
S of $K1_{2000}$ (um)	6.3962	9.5209	21.4321	3.6063

Table 7 The prediction standard deviation S of models

Models	LS-MLR	LA-MLR	DL	SVR
S of $K2_{4000}$(um)	3.0338	4.0081	8.8666	4.3273
S of $K2_{2000}$(um)	4.3945	7.0063	17.0858	3.6476
S of $K3_{4000}$(um)	25.2375	32.9404	52.1711	4.6328
S of $K3_{2000}$(um)	10.2685	14.8759	31.6701	3.5465

Table 8 The fitting standard deviation S of models

Models	LS-MLR	LA-MLR	DL	SVR
S (um)	2.7329	2.7664	2.6638	0.6355

4.2 The same spindle speed at different ambient temperature

The prediction model established by $K1_{6000}$ is used to forecast the data of $K2_{6000}$ and $K3_{6000}$, and the prediction effects are shown as Figs. 4 and 5. The prediction standard deviations of models are shown as Table 5.

According to the analysis of prediction standard deviations of models, it is known that spindle speed remains unchanged and ambient temperature increases a little; the prediction accuracy of models remain better, but as ambient temperature increases, the prediction effects of LS-MLR, LA-MLR, and DL models are worse; LS-MLR model is relatively good, and DL model is the worst. In addition, SVR model maintains good prediction accuracy.

4.3 The same ambient temperature in different spindle speeds

The prediction model established by $K1_{6000}$ is used to forecast the data of $K1_{4000}$ and $K1_{2000}$, and the prediction effects are shown as Figs. 6 and 7. The prediction standard deviations of models are shown as Table 6.

According to the analysis of prediction standard deviations of models, it is known that ambient temperature remains unchanged, and spindle speed is gradually reduced. LS-MLR and LA-MLR models still have certain prediction accuracy; the prediction accuracy of DL model is getting worse, and SVR model always maintain good prediction accuracy. The stability of the pros and cons of models are followed by SVR, LS-MLR, LA-MLR, and DL.

4.4 The different ambient temperature and different spindle speeds

The prediction model established by $K1_{6000}$ is used to forecast the data of $K2_{4000}$, $K2_{2000}$, $K3_{4000}$, and $K3_{2000}$, and the prediction standard deviations of models are shown as Table 7.

According to the analysis of prediction standard deviations of models, it is known that ambient temperature changes small, and spindle speed is gradually reduced; LS-MLR model and SVR model have high prediction accuracy, the prediction accuracy of LA-MLR model is gradually reduced, and the prediction effect of DL model is gradually worse; ambient temperature vary greatly (over 10 °C) and spindle speed is gradually reduced, only the prediction accuracy of SVR model remains better, others are poor. The stability of the pros and cons of models are followed by SVR, LS-MLR, LA-MLR, and DL.

4.5 Model robustness analysis of modeling data source characteristics change

In the above analysis, the fitting accuracy of DL model is higher, while its robustness is poor. Taking the model algorithm differences into account, especially the multicollinearity problem of DL model, making the algorithm prediction accuracy will be greatly affected in the case of comprehensive data information. Because the establishment of the above models are based on sample data at 6,000 rpm under certain conditions, the ambient temperature and spindle speed changes have a greater impact on the variation of the thermal error of CNC machine tools, and it is difficult to reflect these effects in a single batch sample data and greatly affects the robustness of DL model. In order to improve this situation, it can be increased to give a solution that contains a variety of modeling sample data under the different

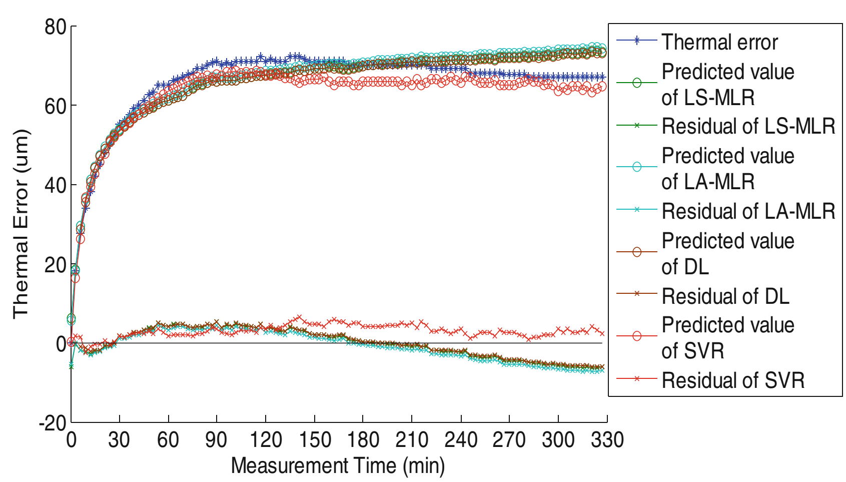

Fig. 8 The prediction effect of $K1_{6000}$

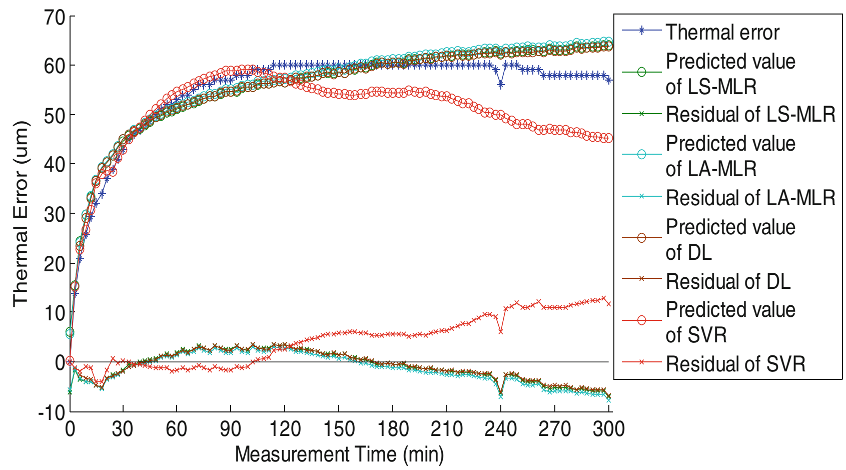

Fig. 9 The prediction effect of $K1_{4000}$

conditions which consist of the more influential factors, i.e., as much as possible, consolidated different speeds and in different ambient temperature conditions of the data modeling. These data contain relatively comprehensive information to overcome the multicollinearity problem in the DL model.

The difference of ambient temperature between the first batch data and the second batch data are smaller, so the second batch and the third batch data are used to establish prediction model to forecast the first batch data. The prediction model established by comprehensive data of $K2_{6000}$, $K3_{6000}$, $K2_{4000}$, $K3_{4000}$, $K2_{2000}$, and $K3_{2000}$ is used to forecast the data of $K1_{6000}$, $K1_{4000}$, and $K1_{2000}$. According to the analysis of prediction accuracy of each model, the robustness of models established by different algorithms is determined.

The equations of prediction models established by the test data of $K2_{6000}$, $K3_{6000}$, $K2_{4000}$, $K3_{4000}$, $K2_{2000}$, and $K3_{2000}$ are shown as follows:

The equation of LS-MLR model:

$$y = 6.053 + 3.5585T_6 - 0.8386T_7 \tag{16}$$

From Eq. (16), T_6 is the temperature increments of sensor T6, and T_7 is the temperature increments of sensor T7.

The equation of LA-MLR model:

$$y = 5.6132 + 3.6287T_6 - 0.8565T_7 \tag{17}$$

From Eq. (17), T_6 is the temperature increments of sensor T6, and T_7 is the temperature increments of sensor T7.

The equation of DL model (the lag order of DL model is 2 by expediency estimation method):

$$\begin{aligned} y_t = 6.6099 + 3.531T_{7t} - 0.4715T_{7t-1} + 0.4653T_{7t-2} \\ +0.5654T_{9t} + 0.3749T_{9t-1} - 1.8522T_{9t-2} \end{aligned} \tag{18}$$

From Eq. (18), $T_{6\ t}$ is the temperature increments of sensor T6, $T_{6\ t\text{-}1}$ is the lag 1-order temperature increments of sensor T6, $T_{6\ t\text{-}2}$ is the lag 2-order temperature increments of sensor T6, $T_{7\ t}$ is the temperature increments of sensor T7, $T_{7\ t\text{-}1}$ is the lag 1-order temperature increments of sensor T7, and $T_{7\ t\text{-}2}$ is the lag 2-order temperature increments of sensor T7.

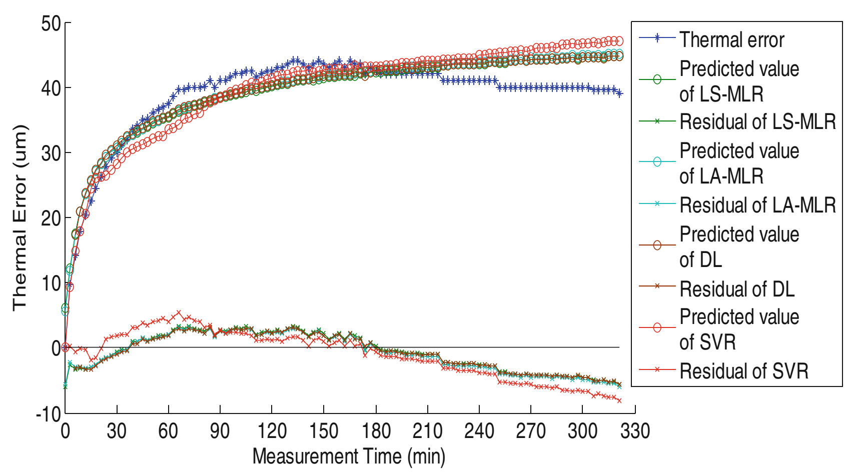

Fig. 10 The prediction effect of $K1_{2000}$

Table 9 The prediction standard deviation S of models

Models	LS-MLR	LA-MLR	DL	SVR
S of $K1_{6000}$ (um)	3.6668	3.8228	3.6205	3.2881
S of $K1_{4000}$ (um)	3.1649	3.3663	3.0717	6.6150
S of $K1_{2000}$ (um)	2.9429	3.0080	2.9186	3.7922

The equation of SVR model:

$$y = f(x) = \sum_{i=1}^{l} \left(a_i^* - a_i\right) \exp\left(-0.6598\|x_i - x\|^2\right) - 39.911 \quad (19)$$

From Eq. (19), by cross-validation method to optimize the parameters, sensitivity function g=0.6598, and punishment coefficient C=97.0059. By running the software LIBSVM in Matlab, support vector, support vector corresponding coefficient $(a_i^* - a_i)$, and constant b=39.911 are obtained; l is the amount of data (the number is 679 of this batch data).

The fitting standard deviations of prediction models are shown as Table 8. The prediction models are used to forecast the data of $K1_{6000}$, $K1_{4000}$, and $K1_{2000}$, and the prediction effects are shown as Figs. 8, 9, and 10. The prediction standard deviations of models are shown as Table 9.

The prediction models are established by comprehensive data of $K2_{6000}$, $K3_{6000}$, $K2_{4000}$, $K3_{4000}$, $K2_{2000}$, and $K3_{2000}$. The fitting accuracy and prediction accuracy of LS-MLR model, LA-MLR model, and DL model have been greatly improved, and their robustness has been markedly enhanced. The fitting accuracy of SVR model remains high, but comparing with other algorithms, the prediction accuracy is volatile and the robustness is declined.

4.6 Applicability analysis of the models

The robustness analysis of thermal error compensation models are only for the thermal error of Z direction of Leaderway V-450 CNC machining center spindle. The thermal error of X, Y, and Z directions of spindle are shown as Fig. 11.

From Fig. 11, the thermal error variation of Z direction is significantly different with thermal error variation of X and Y directions, so the thermal error compensation models of Z direction cannot be applied to the modeling of X and Y directions. Also, the thermal error of X and Y directions are all much smaller than that of Z direction; only by compensating the thermal error of Z direction can the modeling requirements of high precision and high robustness be met.

5 Conclusions

By the long-term measuring of thermal error and the temperature in the sensitive point, multi batches of experiment data are obtained, and the prediction models with various algorithms are established. Aiming at the three conditions of different ambient temperature with same spindle speed, different spindle speeds with same ambient temperature, and different spindle speeds with different ambient temperature, the accuracy and stability of prediction models are analyzed. According to the analysis of experiment results, when modeling for one batch data, DL model has good fitting accuracy, but its robustness is poor. The robustness of LA-MLR model is not superior but worse than LS-MLR model. The view that the robustness of LA-MLR model is superior to LS-MLR model is based on the management of abnormal data. However, the probability of abnormal data in the thermal error measurement of CNC machine tool is very small. So, the advantage of LA-MLR model is not embodied. Because of the large quantity of data in the thermal error sample and the complexity of LA-MLR model, the effect of LA-MLR model in practical application in thermal error modeling is relatively worse than LS-MLR model. SVR model has high fitting accuracy, good retaining ability in prediction effect, and strong robustness; it has relatively good engineering application in the thermal error compensation modeling of CNC machine tools. When using

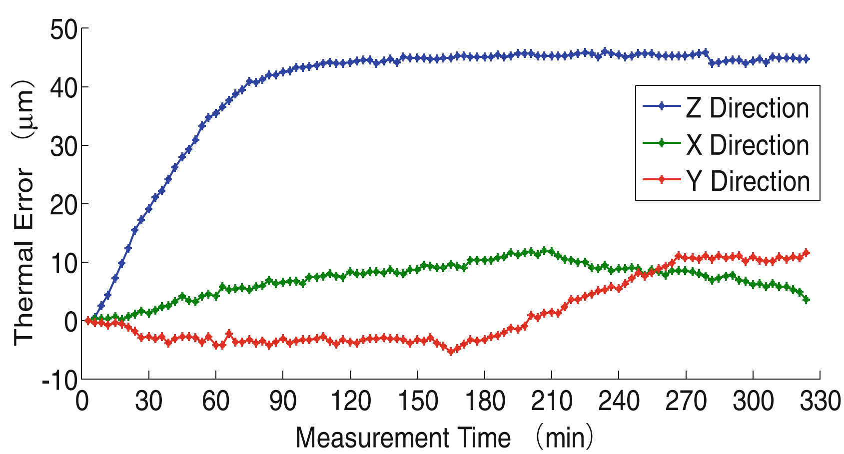

Fig. 11 The thermal error of X,Y,Z directions

multiple batches of data to establish prediction models, the fitting accuracy and prediction accuracy of LS-MLR model, LA-MLR model, and DL model have been improved a lot, and their robustness have obviously enhanced. However, comparing with the former model, the prediction accuracy of SVR model is volatile and its robustness is declined.

For the selection of the thermal error compensation modeling of CNC machine tools, according to the feature of data source and the theory of modeling, different models on different conditions would change a lot on accuracy and robustness. In this paper, the accuracy and robustness of some modeling methods which are just referred to different spindle speeds and ambient temperature have been analyzed. The factors of cutting conditions and other modeling algorithms are out of consideration, and these details need further research.

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