### ORIGINAL ARTICLE

# A mathematical model and a heuristic approach for periodic material delivery in lean production environment

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Abstract One of the important design elements for a good production system is material handling. In cases where it is not well-designed, it can be the bottleneck in the system. Moreover, it can cause a lot of wastes such as waiting time, idle time, and excessive transportation and cost. In this study, material handling in lean-based production environments is taken into account. Depending on the lean structure of the production systems such as being pull-based, smooth, and repetitive, delivering the materials to the stations periodically becomes important. At this point, milk-run trains are highly used in real applications since they enable the handling of required amount of materials on a planned basis. With this study, it is aimed to develop a specific model for milk-run trains which travel periodically in the production environment on a predefined route in equal cycle times with the aim of minimizing work-in-process and transportation costs. Since the milk-run trains having equal cycle times start their tours at the same time intervals, it becomes simple to manage them. For this reason, they are used in lean production systems where level scheduling is performed. The developed model is based on mixed-integer linear programming, and since it is difficult to find the optimum solution due to the combinatorial structure of the problem, a novel heuristic approach is developed. A numerical example is provided so as to show the applicability of the mathematical model and the heuristic approach.

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#### 1 Introduction

Choosing the most proper material handling equipment (MHE) and using it effectively are very important for eliminating material handling waste which is regarded as one of the seven wastes in the lean production system whose objectives are to improve quality, reduce time and cost, and increase organizational effectiveness [[1,](#page-15-0) [2](#page-15-0)]. Hiregoudar and Reddy [[3\]](#page-15-0) state that material handling constitutes 25 % of the workers, 55 % of the factory area, and 87 % of the production time. In case a good material handling system design is accomplished, it is indicated that the cost will decrease by between 10 % and 30 % [[4\]](#page-15-0).

There is not a specific material handling system which is valid for all the production systems. According to the properties of the production environment, the most proper equipment can change. The approaches used for choosing the most proper equipment are mainly deterministic, probabilistic, and knowledge-based approaches [\[5](#page-15-0)]. There are a lot of studies dealing with choosing the most proper equipment using various decision making techniques [[6](#page-15-0)–[9](#page-15-0)]. The selection is made among the main material handling equipment types such as conveyors, cranes, industrial trucks, automated guided vehicles (AGVs), storage/retrieval systems, and industrial robots [\[10](#page-15-0)]. Moreover, specific studies can also be seen about routing and scheduling of these material handling systems such as AGVs and automatic storage and retrieval systems [\[11](#page-15-0)–[14\]](#page-15-0).

This study is not dealing with selecting the most proper MHE but deals with using a specific material handling equipment effectively in special conditions. A lean production environment is taken into consideration, and depending on the properties of such a production environment, such as smooth and repetitive production, it is seen that milk-run trains which are shown in Fig. 1 have started to get important in the real applications. Although milk-run approach is widely used for inbound and outbound logistics, there are few applications for production environment [\[15](#page-15-0)].

Milk-run trains can be regarded within periodically moving manual systems, and as Chase et al. [\[16](#page-15-0)] state, such systems provide a just in time working environment for the production systems. By using the milk-run trains in the lean production areas, as seen in the studies of Akillioglu et al. [\[17](#page-15-0)] and Domingo et al. [[18\]](#page-15-0), an important improvement in the material handling system is provided with respect to work in process (WIP) quantity, transportation cost, and utilization rate.

Besides other sectors, milk-run train applications especially take place in the sector of automotive industries [\[19](#page-15-0)–[21](#page-15-0)]. Most of the observed cases in real production environments specifically in the automotive industries show that the milk-run trains have equal cycle times. That is, all of the milk-run trains in the production environment make their tours at the same time periods which provide an easier management. In this study, for the design of such a specific milk-run train system, a mixed-integer linear programming model is developed aiming to minimize mainly the WIP and transportation costs. Proper routes and time period are investigated. Because of the combinatorial structure of the problem, a novel heuristic approach is proposed as well. For showing the applicability of the mathematical model and the heuristic approach, a numerical example already in literature [[15\]](#page-15-0) is used.

The remainder of the article is organized as follows. Literature review is provided in "Section 2." "Section [3](#page-2-0)" includes the mixed integer linear programming model for milk-run trains having equal cycle times. In Section [4](#page-5-0), the heuristic approach is provided. Numerical example is presented in Section [5](#page-8-0), and finally, the conclusion part is included in Section [6](#page-12-0) with the references following.



Fig. 1 Milk-run trains in the plant

#### 2 Literature review

Since material handling is regarded as an integral part of the facility layout [\[22](#page-15-0), [23](#page-15-0)], they are mostly considered together. Moreover, the design of milk-run distribution systems in the plants is also related with vehicle routing problems (VRP). There are so many studies about VRP, but few of them include routing problems in the production environment.

Regarding the VRP literature, it seems that there is a similarity between the milk-run problem in the production environment and the classical VRPs outside the plant in some ways. For instance, since both deliveries and pickups occur and the milk-run train is capacitated, this problem can be regarded as capacitated vehicle routing problem with pickups and deliveries. However, since it includes determining the quantities to be delivered, there seems a similarity with inventory routing problems. Despite some common points with other problems, Vaidyanathan et al. [\[24](#page-15-0)] state that it is different from the classical VRPs by having some distinctive properties such as the amounts required by the demand points are the functions of the vehicle feeding them. That is, the cycle time of the vehicles determines the quantity to be delivered. Moreover, the inventory related with cycle time, the relationship between the stations, limited stock areas, and multiple stock areas are the other factors that make the differences. Due to the explained differences, it is hard to model the milk-run problem in production environment with the existing methods used for inbound and outbound logistics [\[15](#page-15-0)].

Since the scope of this article is limited with in-plant logistics, the studies about the logistics including the VRPs outside the plant are not regarded. The selected studies which are especially related with in-plant logistics are as follows:

Rajagopalan and Heragu [[25\]](#page-15-0) presented a research framework in the areas of facility layout and discrete material handling system design for the aim of observing the trend in the literature. In their study, they tried to merge the fields of facility layout and material flow network designs. Sinriech and Samakh [\[26](#page-15-0)] developed a genetic approach for the pickup/delivery station location problem in segmented flow-based material handling systems. Vaidyanathan et al. [\[24](#page-15-0)] studied the VRP in just in time systems. Aiming to minimize the total trip time of the vehicles, they developed a nonlinear programming model. Due to the nonlinearity of the problem, they improved a heuristic solution procedure and a lower bound algorithm for comparing the results of the heuristic. Another heuristic procedure was developed by Hwang [[27\]](#page-15-0) to find the minimum number of transporters in a fixed facility layout with predetermined material flow paths. Similar to Vadiyanathan et al. [[24\]](#page-15-0), Akillioglu et al. [\[17](#page-15-0)] developed a model for lean production environment. With the mixed-integer linear programming model they

<span id="page-2-0"></span>developed, improvements in some of the lean metrics such as transportation and WIP costs are provided. In addition to the model, they also made a simulation study for the production environment. Domingo et al. [[18\]](#page-15-0) constructed a milk-run distribution system in a real lean manufacturing environment. Their application results showed that dock-todock time and cycle times were decreased, and moreover, unnecessary inventories, excessive transportation, and idle times were reduced. Costa et al. [\[28](#page-15-0)] presented a material delivery system simulation using a logistics train in an electronic company. According to the experimental results, it was shown that periodically moving train had financial advantages. Kilic et al. [\[15](#page-15-0)] classified the milk-run problems in plants and developed models for one specific category in their classification scheme. Within the category, they developed models for one routed and multiple routed vehicles traveling in determined time periods. They provided a numerical example to show the applicability of the models. Satoglu and Sahin [\[20](#page-15-0)] proposed a non-linear mathematical model for a just-in-time periodical material supply system and developed a heuristic algorithm for the related model. Golz et al. [[19\]](#page-15-0) proposed a heuristic solution approach to minimize the required number of shuttle drivers at high variant mixed model assembly lines. The proposed heuristic approach consists of two stages. Within the first stage, transportation orders are obtained from the given assembly sequence and in the second stage, the assignment of these orders to the tours of shuttle system is performed by considering constraints such as transportation capacity, due dates, and tour scheduling.

Regarding the literature, it is concluded that the studies are limited about the modeling of periodically moving vehicles in lean production systems. With this study, differently from the existing studies, a mathematical model is developed for a specific type of milk-run problem. The developed model consists of milk-run trains having equal cycle times. Moreover, a novel heuristic approach is developed for the first time for the related type of problem.

# 3 A mixed-integer linear programming model for milk-run trains having equal cycle times

The milk-run system in the plants consists of periodically moving trains in the production environment. The train starts its tour from the material depot and visits the assigned stations on its route in specified time intervals. The train not only drops off the materials but can also pick up the materials from the stations and take it to the related station or assembly lines. At the end of its tour, it returns the material depot again and repeats the same movement periodically. The required loading–unloading activities are performed at the depots and stations. It is aimed to minimize the cost

including WIP and transportation which are the two main resources of waste. By reducing the WIP, the capital spared for it will be free and less floor space will be used. At the same time, the problems hiding with high WIP can begin surfacing. These are the opportunities to improve the systems. On the other hand, delivering the materials less frequently will decrease the transportation costs. The optimum values minimizing the total cost are searched via the proposed model. However, a tradeoff occurs between these two cost variables as shown in Fig. 2, that is, if the number of tours decreases, quantity of WIP increases and vice versa.

To apply milk-run distribution system, the production environment must be smooth and repetitive. That is, it must be lean. To construct a milk-run distribution system, two main design parameters are needed. These are the routes of the vehicles and time periods of the vehicles' tours. In this study, a mathematical model is developed for the milk-run trains having equal tour periods. As seen in some of the lean production applications, milk-run trains having the same tour cycle time enable an easier management. While developing the mathematical model, it is benefited from the studies of Kilic et al. [[15\]](#page-15-0) and Akillioglu et al. [[17\]](#page-15-0). Similar indices and abbreviations indicated in the study of Kilic et al. [[15\]](#page-15-0) are used in the proposed mathematical model which is originally developed according to the specific conditions mentioned in this study. Mixed-integer linear programming model which is widely used also in different fields [\[29](#page-15-0)] is used for choosing the routes and their time periods. For this aim, first of all, probable routes are determined mainly regarding the physical conditions and product flow. The steps for determining the routes are explained in detail in the further parts within the procedure for route construction. Among these probable routes, the most proper ones optimizing the objective function and satisfying the constraints are chosen. Assumptions and the model are as follows:

#### Assumptions

- & A pull-based lean production environment is considered.
- Production is repetitive and smooth.
- No change is made to the layout. The system design is made according to the existing layout.



Fig. 2 Relationship between WIP and transportation cost



(Including fixed trip time of the vehicle, total loading-unloading time and idle time) vehidle $r$  The idle time of the vehicle "r"

Sets

- A The set of stock areas in the material depot, assembly station, and product warehouse (a, b)
- B The set of stock points related with each other (pulling or pushing) (i, j)
- C The stock areas in the same cell (a, b)
- $N_r$  The set of stock points that can be assigned to the route "r"
- $N_a$  The set of stock points in the stock area "a"

### Parameters

 $VT_r$  The fixed trip time of the route/vehicle "r" (only the trip time not including loadingunloading or other times)



Objective function

$$
\begin{aligned}\n\sum_{r} \sum_{i} D_{ir} W_i f/2 + \sum_{r} \sum_{i} S_{ir} W_i f + \\
\text{Min } z &= \sum_{t} \sum_{r} \sum_{r} C_r Z_{tr} . (TWT/(tt_t, TI)) + \\
\sum_{r} \text{vcost} . route_{r}\n\end{aligned} \tag{1}
$$

1. In the objective function, it is aimed to minimize mainly two cost resources. The first one is to minimize the WIP cost consisting of the demand of the stock points within the time period and safety stock. The second one is cost-related, with the vehicle cost consisting of the tour numbers of the milk-run trains and the fixed costs.

**Constraints** 

$$
\sum_{r} X_{ir} = 1 \quad \forall i \tag{2}
$$

$$
\sum_{r} Y_{ar} = 1 \quad \forall a, a \notin A \tag{3}
$$

$$
\sum_{i \notin Nr} X_{ir} = 0 \quad \forall r \tag{4}
$$

$$
X_{ir} = X_{jr} \quad \forall (i, j) \in B, \forall r
$$
 (5)

 $M.X_{ir} \ge D_{ir} \quad \forall i, \forall r$  (6)

 $M.X_{ir} \geq S_{ir} \quad \forall i, \forall r$  (7)

 $M.X_{ir} \geq L_{ir} \quad \forall i, \forall r$  (8)

 $L_{ir} = D_{ir}.l_i \quad \forall i, \forall r$  (9)

 $D_{ir} + M.(1-X_{ir}) \ge$ vehcycle<sub>r</sub>.d<sub>i</sub> ∀i, ∀r (10)

 $S_{ir} = D_{ir} \cdot P \quad \forall i, \forall r$  (11)

$$
\sum_{i} \sum_{r} (D_{ir} + S_{ir}).v_i \le vol_a \ \ i \in N_a, \forall a \tag{12}
$$

$$
M.Y_{ar} \ge \sum_{i} X_{ir} \quad i \in N_a, \forall r, \forall a, a \notin A
$$
 (13)

 $Y_{ar} = Y_{br} \quad \forall (a, b) \in C, \forall r$  (14)

 $[TI.t<sub>t</sub>–M.(1–Z<sub>tr</sub>)] \le$ vehcycle<sub>r</sub> ∀r, ∀t (15)

$$
[TI.tt_t + M.(1-Z_{tr})] \ge \text{velocity.} \quad \forall r, \forall t \tag{16}
$$

$$
\sum_{t} Z_{tr} \le 1 \quad \forall r \tag{17}
$$

$$
\sum_{r} \text{route}_{r} = \sum_{r} \sum_{t} Z_{tr} \tag{18}
$$

 $M.\text{route}_r \geq Z_{tr} \quad \forall r, \forall t$  (19)

$$
M.\text{route}_r \ge \sum_i X_{ir} \quad \forall r \tag{20}
$$

$$
route_r = \sum_t Z_{tr} \quad \forall r \tag{21}
$$

$$
\sum_{r} \text{route}_{r} \le \text{vehnum} \tag{22}
$$

$$
M.\text{route}_r \ge \text{ vehicle}_r \quad \forall r \tag{23}
$$

$$
vchilde_r \leq max vehicle_r \quad \forall r \tag{24}
$$

$$
M.\text{route}_r \ge D_{ir} \quad \forall r, \forall i \tag{25}
$$

$$
M.\text{route}_r \ge S_{ir} \quad \forall r, \forall i \tag{26}
$$

$$
M. route_r \ge L_{ir} \quad \forall r, \forall i \tag{27}
$$

$$
velocity_{r} = \sum_{i} L_{ir} + route_{r} \cdot VT_{r} + vehicle_{r} \quad \forall r \qquad (28)
$$

$$
\sum_{r} Z_{tr} \leq M \cdot v v_{t} \quad \forall t \tag{29}
$$

$$
\sum_{r} route_{r} - \sum_{r} Z_{tr} \leq M.(1 - \nu \nu_{t}) \quad \forall t \tag{30}
$$

$$
\sum_{ss_r=1}^{s_r} \sum_i D_{ir} \cdot v_i \cdot seq_{ss_r i} \cdot sign_i \le cap \quad \forall r, \forall s_r
$$
 (31)

$$
X_{ir}, Z_{tr}, Y_{ar}, \text{route}_r, \nu v_t \in \{0, 1\} \quad \forall i, \forall a, \forall r, \forall t
$$
  
others  $\geq 0$  (32)

2. Each stock point is assigned to only one route.<br>3. Each stock area except the ones in the mat

- Each stock area except the ones in the material depot, assembly station, and product warehouse is assigned to only one route.
- 4. The stock points which cannot be assigned to the route due to the flow and precedence constraints are not assigned to the related route.
- <span id="page-5-0"></span>5. The dependent stock points which push or pull each other are assigned to the same route.
- 6. The demand of the stock points which are not assigned to the route becomes zero.
- 7. The safety stock of the stock points which are not assigned to the route becomes zero.
- 8. The total loading–unloading time of the stock points that are not assigned to the route becomes zero.
- 9. For each stock point, the total loading–unloading time is determined.
- 10. The demand of a stock point is determined by the multiplication of the demand rate of a stock point and the cycle time of the vehicle.
- 11. The safety stock is determined as a ratio "P" of the demand at the related stock point.
- 12. The volume of the total demand and safety stock of the stock points in a stock area cannot be more than the volume of that stock area.
- 13. All the stock points in a stock area (except the ones in the material depot, assembly station, and product warehouse) are assigned to only one route.
- 14. All the stock areas in the same station are assigned to the same route.
- 15, 16. The time period of the chosen route is determined.<br>17. Each route is assigned to, at most, one time period.
- 17. Each route is assigned to, at most, one time period.<br>18. The number of chosen routes and time periods are
- The number of chosen routes and time periods are equal.
- 19. Time period is not assigned to an unselected route.<br>20. Any stock point is not assigned to an unselected
- Any stock point is not assigned to an unselected route.
- 21. Maximum one time period is assigned to a selected route.
- 22. The maximum number of vehicles in the system is limited.
- 23. No idle time is assigned to an unselected route.<br>24. Each vehicle's maximum idle time is restricted.
- 24. Each vehicle's maximum idle time is restricted.<br>25. No demand is assigned to an unselected route.
- 25. No demand is assigned to an unselected route.<br>26. No safety stock is assigned to an unselected ro
- 26. No safety stock is assigned to an unselected route.<br>27. No loading-unloading time is assigned to an unse-
- No loading–unloading time is assigned to an unselected route.
- 28. One tour cycle time of the route "r." The cycle time of the vehicle consists of the total loading– unloading time, the fixed tour time, and the idle time of the vehicle.
- 29, 30. For each time period, the total number of selected routes is made to be either equal to zero or equal to the total number of selected routes assigned to the related time period for providing the equally timed periods.
- 31. At each assignment of stock point "i," the capacity of the vehicle is controlled.
- 32. The decision variables are either 0–1 integer variables or positive real numbers.

#### 4 A heuristic approach for the model

Depending on the combinatorial structure of the mathematical model proposed, it is difficult to find the optimal solution for large-sized problems having a huge number of variables. So as to overcome this drawback, a heuristic approach is proposed.

It is mainly aimed to find the routes and time period of the routes in the proposed mathematical model. Since the time period of the milk-run trains is equal, there is only one time period to be found. The time period can be defined as the total time that all the related activities of the milk-run train must be completed in. Within the related activities, there are mainly the fixed cycle time of the train and total loading–unloading time of the materials carried. The time period also determines the quantity to be delivered to the stock areas. For instance, if the time period of the trains is 1 h, this means that the trains will carry 1 h demand of all stock areas. This also affects the loading–unloading time of the materials carried. So, depending on the importance of the time period in the model, it is mainly focused in the heuristic approach. For each time period between minimum and maximum values, the best routes are searched and the most suitable stations are assigned to the routes regarding some procedures. The cost is calculated for each time period, and finally, the time period having the least cost is selected. The proposed heuristic algorithm is shown in Fig. [3.](#page-6-0) Some of the steps of the algorithm including procedures need further explanations. The procedures included in the algorithm are procedure for route construction, procedure for determining minimum and maximum time periods, procedure for initial route selection, procedure for general route selection, and lastly, procedure for station selection. Moreover, further explanation is also provided for time period control step.

As shown in Fig. [3,](#page-6-0) within the first step of the proposed heuristic algorithm, the data about the properties of material, vehicle, layout, and station are collected. Some of the such data are the demand rate of the materials, the monetary values of the materials, the capacity of the milk-run train, the velocity of the milk-run train, the distances on the layout, the position of the stations, the physical area of stations, and so on. All of the parameters are given in detail within the numerical example.

After obtaining the related parameters of the system, the probable routes are constructed with respect to the procedure for route construction. The parameters such as the fixed cycle time of the probable routes are then computed by dividing the distance of the route to the velocity of the milk-run train.

After the procedure for route construction, for minimizing the solution time and unnecessary operations, minimum and maximum time period values are obtained with respect <span id="page-6-0"></span>Fig. 3 Steps of the heuristic algorithm



to the procedure for determining minimum and maximum time periods. The obtained minimum time value is taken as the first time value. Then, the initial routes are found by applying the initial route selection procedure, and assignments are made to the chosen route starting from the stations being passed by the least number of routes (station selection procedure).

While assigning stations (stock areas) to the routes, the capacity of the vehicle and time period is checked. If any of them is violated, that station is skipped. If there is no problem about the capacity and the time period, the related station is assigned to the route. Assignment to the related route continues until there is no problem about the capacity and the time period.

If there are unassigned stations and it is impossible to assign any station to the related route, then by applying the procedure for general route selection, a new route is chosen, and similar steps are applied until there are no unassigned stations. If there are still unassigned stations after trying all the routes, the related initial route is skipped and remaining initial routes are tried. When it is provided that there are no unassigned stations, the cost is calculated for the related initial route and the time period. After trying all the initial routes for the same time period, the time period value is increased by one unit for the next iteration. Cost is calculated for each time period until it is reached to the maximum time period. After the last iteration, the solution having the minimum cost is chosen. The time period of the best solution gives the cycle time of the system. The related procedures used within the algorithm are as follows:

## Procedure for route construction

A big number of probable routes can be defined regarding the layout of the factory. However, taking into consideration all the probable routes can cause unnecessary workload and long solution times. For this reason, the following points should be considered while constructing the probable routes.

- The route combinations are formed having the start and end at the same point (material depot).
- The routes having a physical constraint regarding the layout and being opposite to the work flow are eliminated.
- The routes not passing by at least one of the related (dependent) workstations are eliminated.
- The routes passing by a workstation more than twice are eliminated.

# Procedure for determining minimum and maximum time periods

In the presented mathematical model, the most suitable time periods are searched and assigned to the routes. With the proposed procedure, the lower and

upper bounds for time periods are obtained and the number of time periods that will be regarded is decreased. For the minimum time period, the route having the least fixed cycle time is determined. The minimum time period is the minimum value that is greater than that value. Then, the minimum time period value  $(tmin)$ can be indicated as in the Eq. 33.

Minimum time period  $\geq$  The route having the least fixed cycle time  $(33)$ 

For obtaining the maximum time period, it is benefited from the vehicle capacity and stock areas at the stations.

Since the number of maximum vehicles that can be used in the layout is known, maximum time value can be obtained. When the time period of the vehicle increases, proportionally, the quantity that will be carried also increases, and this affects the capacity usage rate of the vehicle. In that case, the time value satisfying the Eq. 34 gives the probable maximum time period value  $(t)$ .

 $(maximum number of vehicles*the capacity of vehicle)$  $=$  ( $t^*$ demand of all materials to be carried per time)

$$
(34)
$$

On the other hand, when the stock areas at the workstations are taken into consideration, the space that is occupied by the materials must not be more than the workstation area. Regarding also the safety stock quantities, the time value satisfying the following Eq. 35 for each of the stock areas is determined, and the smallest of them becomes the other probable maximum time period value  $(t)$ .

 $(1 + \text{safety stock ratio})^*(\text{demand of materials at the  
stock area per time**t*) = stock area$  (35) stock area per time\*t) = stock area

The smallest of the probable maximum time period values  $(t)$  obtained from the vehicle capacity constraint and the stock area constraint is regarded as the maximum time period (tmax).

#### Procedure for initial route selection

While choosing the initial route or routes, the following steps are applied and the initial route set is formed.

- & The stations are ranked from smallest to largest with respect to the number of routes passed by them.
- Regarding the station/stations being passed by the smallest number of routes, the related routes are selected.
- <span id="page-8-0"></span>Among the selected routes, first of all the one passing by the highest number of stations is selected. If there is more than one route satisfying the condition, the one or ones having the least fixed cycle time are selected and added to the initial route set.
- Secondly, within the selected routes, the route having the least fixed cycle time is selected, and if there is more than one route satisfying the condition, the one or ones passing by the highest number of stations is selected and added to the initial route set.

#### Procedure for general route selection

After selecting the initial route, general route selection procedure is applied for selecting the other routes. Similar to initial route selection, the following steps are applied.

- The number of routes passing by the unassigned stations is determined.
- & Regarding the unassigned station/stations being passed by the smallest number of routes, the related routes are selected.
- & Among the selected routes, the one passing by the highest number of unassigned stations is selected. If there is more than one route satisfying the condition, the one or ones having the least fixed cycle time is selected.

#### Station selection procedure

While assigning a station to a route, it is started from the stations being passed by the least number of routes. Precedence relations are also considered while selecting a station. That is, if one station is related with other station, both of them are chosen at the same time.

#### Time period control

For each assignment of a station to a route, time period should be checked. Since loading–unloading take place whenever a station is assigned to a route, the total time required for the travelling of the vehicle should be controlled. For checking the suitability of the time period, the inequality (Eq. 36) must be satisfied for each selected route:

(The fixed traveling time of the related route  $+$  total loading−unloading times of the related stations assigned to the route  $\leq$ The considered time period $(t)$ 

 $(36)$ 

For making the heuristic approach clearer, pseudocode is provided in Fig. 4. The notations used are as follows:



1	<b>Begin</b> Milk-run heuristic
$\overline{c}$	Set the parameters (The properties of material, vehicle, layout and station)
3	Run procedure for route construction (Obtain $r=1$ to $rr$ )
$\overline{4}$	Run procedure for determining minimum and maximum time periods (Obtain $t_{min}$ and
	$t_{max}$ )
5	Run procedure for initial route selection $(r_{initial} = 1,,k)$
6	For $t=t_{min}$ to $t_{max}$
7	For $r_{initial} = 1$ to k
8	$r = r_{initial}$
9	Run procedure for station selection (Obtain the assignment order of
	unassigned stations for router)
10	For each station s with respect to the assignment order
11	If $load(r) \leq cap$ and $cycle(r) \leq t$ (when station s is assigned) Then
12	Assign station s to route r
13	Else
14	Skip station s
15	End if
16	<b>Next</b> station s
17	If there are unassigned stations and all routes are not triedThen
18	Run procedure for general route selection (Obtain $r_{new}$ )
19	$r = r_{new}$
20	Go to 9
21	<b>Else if</b> there are unassigned stations and all putes are tried <b>Then</b>
22	Go to 27
23	Else
24	Compute the cost for the related $r_{initial}$ and t; $[cost(r_{initial}, t)]$
25	Record cost $(r_{initial}, t)$ to the solution set
26	End if
27	Next $r_{initial}$
28	Next $t$
29	Select the least cost( $r_{initial}$ , t) from the solution set
30	<b>End</b> Milk-run heuristic

Fig. 4 Pseudocode for the heuristic algorithm



The pseudocode for the heuristic algorithm is shown as in Fig. 4.

#### 5 A numerical example

For proving the applicability of the mathematical model and the heuristic approach, the numerical example in the study of Kilic et al. [[15\]](#page-15-0) is used with some modifications such as considering the monetary values of WIP. There are material depot, 16 cells, assembly station, and a product warehouse as seen in Fig. [5.](#page-9-0) Totally, there are 98 stock points and 36 stock areas consisting of stock points (sp) in the system. Each cell has two stock areas: input and output. The arrows between the cells indicate that there is a material flow between the cells.

As mentioned before, the first step is to determine the probable routes regarding the layout, flow of the products,

<span id="page-9-0"></span>Fig. 5 The layout and the probable routes in the numerical example



ROUTE 1  $\longrightarrow$   $\longrightarrow$  ROUTE 9  $\cdot \cdot \cdot$  OTHER ROUTES  $\longrightarrow$  RELATED CELLS

and physical conditions. Although more routes could be defined, for the easiness of the problem, the number is limited with nine routes. The probable routes and the cells that can be assigned to the routes are shown as in Table 1.

The parameters can have different values depending on the conditions of the application area. In this numerical example, values of some parameters are determined small for the easiness of operations. The demand rates of each stock point are defined as cases per minute, and the capacity of the vehicles and stock areas are also defined as cases.

### 5.1 Parameters

The parameters and their values are as follows:

The related cells: There is a material flow between some of the cells. These cells are called related cells; Cell 3-Cell 4, Cell 6-Cell 7, Cell 12–Cell 13, and Cell 15–Cell 16 are related cells.

The time for the vehicles to traverse the routes: The average velocity of the vehicles is 60 m/min. So the time to traverse each route is: route 1, 4.7 min; route 2, 4.7 min; route 3, 4.7 min; route 4, 4.7 min; route 5, 4.2 min; route 6, 4.2 min; route 7, 4.7 min; route 8, 4.2 min; route 9, 3.7 min.

The average cost of one-cycle tour of each route: The transportation cost mainly depends on the energy type used by the material handling equipment and the material to be carried. So there is a wide range of price. Cost per kilometer is regarded as \$0.25/km for all the vehicles, and the cost of 1 cycle of each route is: route 1, \$0.0705; route 2, \$0.0705; route 3, \$0.0705; route 4, \$0.0705; route 5, \$0.063; route 6, \$0.063; route 7, \$0.0705; route 8, \$0.063; and route 9, \$0.055.

Total working time: Total working time is determined as 8 h (480 min).

Time interval value: The time interval is determined as 2 min.

CELL 16  $\leftarrow$  CELL 15  $\leftarrow$  CELL 14  $\leftarrow$  CELL 13  $\rightarrow$  CELL 12

The number of time intervals: The total number of time intervals is 120.

The monetary value of the WIP: The monetary value of one case of each stock point "i" is shown in Table [2](#page-10-0).

The quantity of safety stock: One cycle demand is the safety stock for each stock point.

The costs related with the milk-run train: Regarding the depreciation and the driver cost of the vehicle, the daily cost is assumed to be \$60 for one milk-run train.

The loading-unloading time of the material cases: It is assumed to be 0.3 min for one case. But for the materials in the material depot, taking into account of the searching time, it is assumed to be 0.45 min for one case.

The capacity of the train: The train has four wagons, and each wagon can carry 16 unit cases.

The volume of the stock areas: Each stock area has the capacity of ten unit cases.





 $\sim$ 



<span id="page-10-0"></span>



The demand or supply quantity of stock points per time: According to the stock point type (demand or supply), the quantity per time and the sign showing the type of the stock point (supply=1 or demand=−1) are shown in Table [3](#page-11-0).

#### 5.2 Application of the mathematical model

The mixed-integer linear programming model is coded and run in GAMS optimization program with CPLEX solver. Depending on the structure of the problem type, it is difficult to find the optimum solution in such kinds of problems. However, the model was run for approximately 3 days in a computer having a 4 GB RAM and a 2.93 GHz processor to get a good feasible solution and the cost is found as \$226.54. The cycle time of the system is obtained as 18 min. That is, in every 18 min, three milk-run trains will travel in the system for material delivery by using the routes 1, 4, and 9. The summary of the results indicating the routes chosen, cycle times of the routes, and the cells assigned to the routes are shown in Table [4.](#page-11-0)

#### 5.3 Application of the heuristic approach

The developed heuristic approach is applied in the numerical example. The feasible solutions between minimum (tmin) and maximum (tmax) time values are provided.

Within the first steps of the heuristic algorithm, the related parameters about material, vehicle, layout, and station are gathered, and nine routes are constructed with respect to the procedure for route construction. With respect to the minimum and maximum time periods procedure, the minimum time value (tmin) is regarded as 4 min which is the smallest value greater than the least fixed cycle time of all the routes. Since one time interval is regarded as 2 min. All the increase in time units will be the multiples of 2 min. The maximum time value (*tmax*) is obtained by regarding the stock areas and found as 20 min due to the limited area of the cell 1. For the selection of initial routes, the related procedure is applied, and firstly, the cells being passed by least number of routes are obtained as cell 5, cell 6, cell 7, cell 9, cell 12, and cell 13 as shown in Table [5.](#page-12-0)

Three routes pass by each of these cells. The related routes are route 1, route 2, route 3, route 4, and route 7. With respect to the steps of the procedure, among these routes firstly the one passing by the largest number of cells are selected and considered as initial route, and this is the route 1 passing by 15 cells. Secondly, among these five routes, the one having the smallest fixed cycle time is selected. But since the fixed cycle times of all the routes are equal, the one or ones passing by the highest number of routes is selected and that is again route 1. So there is only one initial route and that is the route 1. After determining the initial route, cells are assigned to the initial route with

Stock point (sign)	Case/min	Stock point (sign)	Case/min	Stock point (sign)	Case/min	Stock point (sign)	Case/min	Stock point (sign)	Case/min
1(1)	0.1	21(1)	0.05	41 $(-1)$	0.05	61(1)	0.05	$81(-1)$	0.05
2(1)	0.1	22(1)	0.05	42(1)	0.05	$62(-1)$	0.05	$82(-1)$	0.05
3(1)	0.05	23(1)	0.05	43 $(1)$	0.05	$63(-1)$	0.05	$83(-1)$	0.10
4(1)	0.05	24(1)	0.05	$44(-1)$	0.10	64(1)	0.05	$84(-1)$	0.10
5(1)	0.05	25(1)	0.10	$45(-1)$	0.10	$65(-1)$	0.05	$85(-1)$	0.10
6(1)	0.05	$26(-1)$	0.1	46(1)	0.10	66 (1)	0.05	$86(-1)$	0.10
7(1)	0.05	$27(-1)$	0.1	$47(-1)$	0.10	$67(-1)$	0.05	$87(-1)$	0.05
8(1)	0.05	$28(-1)$	0.05	48 $(-1)$	0.10	$68(-1)$	0.05	$88(-1)$	0.05
9(1)	0.05	29(1)	0.05	49(1)	0.10	69(1)	0.05	$89(-1)$	0.05
10(1)	0.10	30(1)	0.05	$50(-1)$	0.10	$70(-1)$	0.05	$90(-1)$	0.05
11(1)	0.10	$31(-1)$	0.05	$51(-1)$	0.10	71(1)	0.05	$91(-1)$	0.05
12(1)	0.10	$32(-1)$	0.05	52(1)	0.10	72(1)	0.05	$92(-1)$	0.05
13(1)	0.10	33(1)	0.05	53 $(1)$	0.10	$73(-1)$	0.05	93(1)	0.10
14(1)	0.10	34(1)	0.05	$54(-1)$	0.10	$74(-1)$	0.10	94(1)	0.10
15(1)	0.10	$35(-1)$	0.05	$55(-1)$	0.05	75(1)	0.05	95(1)	0.10
16(1)	0.05	36(1)	0.05	56(1)	0.10	$76(-1)$	0.05	$96(-1)$	0.10
17(1)	0.05	$37(-1)$	0.05	$57(-1)$	0.05	$77(-1)$	0.05	$97(-1)$	0.10
18(1)	0.05	$38(-1)$	0.05	58 $(1)$	0.05	$78(-1)$	0.05	$98(-1)$	0.10
19(1)	0.05	39(1)	0.05	$59(-1)$	0.05	$79(-1)$	0.05		
20(1)	0.05	$40(-1)$	0.05	$60(-1)$	0.05	$80(-1)$	0.05		

<span id="page-11-0"></span>Table 3 The supply/demand quantity of each stock point per time [\[15\]](#page-15-0)

respect to the station selection procedure. The assignments are made starting from the selected cells 5, 6, 7, 9, 12, and 13. But feasible assignments could not be made for the time periods 2 (4 min) and 3 (6 min). For the time period 4 (8 min), a feasible assignment is made which does not violate the constraints. For all the other time periods between minimum and maximum time values, the assignments are performed regarding the steps of the heuristic approach, and finally, the solutions for each feasible assignment are provided. The solution for the time period 10 (20 min) which is also the best solution obtained is shown in Table [6](#page-12-0).

Solutions for other time periods are provided in the [Appendix](#page-13-0). In the tables, there are the routes in the rows and the cells (C1 to C16) in the columns. When an assignment of a cell is made to a route, it is indicated as "1," since each cell is assigned to one route. There is only one "1" in each column except for the assembly column abbreviated by "As." It can be served by more than one route each of which can carry one of the related parts from assembly station to product warehouse. In the last row, the total cost is given, and in the last column, the total time of each route including fixed tour time of the route and total loading–unloading times are provided. This total time can be regarded as the cycle time of each route without the idle times. As can be noticed, the total time value

of each route is not greater than the related time period (the cycle time of the system). The difference between the related time period and the total time gives the idle time of the vehicle on that route. Consequently, the results indicate that, in every 20 min, three milk-run trains using the routes 1, 4, and 9 will deliver the required parts with the lowest cost which is \$230.60.

Since there are no alternative heuristic approaches in the literature for the related type of problem, it is not possible to compare the results. But when the results gathered from the solution of the heuristic approach and the mathematical model are compared, it is seen that the deviation of the heuristic approach value from the mathematical model value is approximately 1.79 %.

Table 4 The results of the mathematical model

Chosen routes	Cycle time (min)	Cells
r1	18	C <sub>5</sub> , C <sub>6</sub> , C <sub>7</sub> , C <sub>12</sub> , C <sub>13</sub> , C <sub>14</sub>
r4	18	C <sub>1</sub> , C <sub>3</sub> , C <sub>4</sub> , C <sub>9</sub> , C <sub>10</sub> , C <sub>11</sub>
r <sub>9</sub>	18	C <sub>2</sub> , C <sub>8</sub> , C <sub>15</sub> , C <sub>16</sub>

<span id="page-12-0"></span>



Ro. route

#### 6 Conclusion

Milk-run trains travelling periodically on predefined paths provide a standard parts feeding system for the lean production systems which are repetitive and smooth. A special case of milk-run trains in lean production environment is taken into consideration in this study. Within this special case, the cycle times of all the milk-run trains are equal. As observed in real life applications, since it provides simplification of milk-run train management, it has been chosen by the firms. Although milk-run trains having equal cycle times are only mentioned conceptually in the study of Kilic et al. [\[15\]](#page-15-0), this study differs from the existing studies by presenting a mathematical model and a novel heuristic approach.

Since it is difficult to find the optimum solution with the mathematical model when the number of variables increases, a novel heuristic approach is developed to overcome this drawback. To prove the applicability of the proposed model and the heuristic algorithm, a numerical example already in literature is used with few modifications. The mathematical model and the heuristic approach are applied, and solutions are obtained.

This study also shows that there is need for heuristic approaches depending on the difficulty for finding the optimum solution in such kinds of in-plant logistics problems. Meta-heuristic algorithms such as genetic algorithm, antcolony optimization algorithm and simulated annealing algorithm can be applied, and the solutions can be compared. Moreover, a lower-bound algorithm can be developed to provide a comparison basis for the developed heuristic approaches. Finally, the proposed model and the heuristic approach can be applied in real production environment, and new models and solution approaches can be developed for in-plant logistics design regarding various situations of production. Furthermore, multiple item situations in the real environment may be incorporated within the context of this study by modifying the parameters of the papers by [[30](#page-15-0)–[33\]](#page-15-0).

**Table 6** The solution table for the time period " $t=20$  min"

Ro.	C1	C <sub>2</sub>	C <sub>3</sub>	C4	C <sub>5</sub>	C <sub>6</sub>	C7	C8	C9	C10	C11	C12	C13	C14	C15	C <sub>16</sub>	As.	Total time
1					$\mathbf{1}$													19.85
2																		
3																		
4		$\mathbf{1}$						$\mathbf{1}$				$\mathbf{1}$	1					19.55
5																		
6																		
$7\phantom{.0}$																		
8																		
9															$\mathbf{1}$		$\mathbf{1}$	18.40
		Total $cost = $230.60$																

# <span id="page-13-0"></span>Appendix

The heuristic approach solution tables for the time periods from  $t=8$  min to  $t=18$  min.



Ro.	C1	C <sub>2</sub>	C <sub>3</sub>	C4	C <sub>5</sub>	C <sub>6</sub>	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	As.	Total time
																		8
$\overline{2}$																		
3																		8
$\overline{4}$																		7.94
5																	1	7.62
6																	$\perp$	4.68
$7\phantom{.0}$																		
8																		
9																		7.84
		Total $cost = $401.93$																

**Table 8** The solution table for the time period " $t=10$  min"

Ro.	C1	C2	C <sub>3</sub>	C4	C5	C <sub>6</sub>	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	As.	Total time
$\mathbf{1}$																		9.5
2																		
3																		
$\overline{4}$																		9.8
5																		9.9
6																	1	4.8
$7\phantom{.0}$																		
8																		
9																	1	9.85
		Total $cost = $338.42$																

**Table 9** The solution table for the time period " $t=12$  min"



# **Table 10** The solution table for the time period " $t=14$  min"



Table 11 The solution table for the time period " $t=16$  min"



**Table 12** The solution table for the time period " $t=18$  min"

Ro.	C1	C <sub>2</sub>	C <sub>3</sub>	C4	C <sub>5</sub>	C <sub>6</sub>	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	As.	Total time
$\mathbf{1}$										-1								17.66
$\overline{2}$																		
$\mathfrak{Z}$																		
$\overline{4}$												$\mathbf{1}$	$\mathbf{1}$	1				17.39
5 <sup>5</sup>																	$\mathbf{1}$	5.28
6																		
$\tau$																		
8																		
9																1		17.20
		Total $cost = $288.22$																

#### <span id="page-15-0"></span>References

- 1. Ohno T (1988) Toyota production system: beyond large-scale production. Productivity Press, Cambridge
- 2. Zhou B (2012) Lean principles, practices, and impacts: a study on small and medium-sized enterprises (SMEs). Ann Oper Res. doi[:10.1007/s10479-012-1177-3](http://dx.doi.org/10.1007/s10479-012-1177-3)
- 3. Hiregoudar C, Reddy BR (2007) Facility planning & layout design: an industrial perspective. Technical Publications Pune, India
- 4. Drira A, Pierreval H, Hajri-Gabouj S (2007) Facility layout problems: a survey. Annu Rev Control 31:255–267
- 5. Heragu SS (2008) Facilities design. CRC Press, Boca Raton, FL, USA
- 6. Kusiak A, Heragu SS (1988) KBSES: a knowledge-based system for equipment selection. Int J Adv Manuf Technol 3(3):97–109
- 7. Chakraborty S, Banik D (2006) Design of a material handling equipment selection model using analytic hierarchy process. Int J Adv Manuf Technol 28:1237–1245
- 8. Onut S, Kara SS, Mert S (2009) Selecting the suitable material handling equipment in the presence of vagueness. Int J Adv Manuf Technol 44:818–828
- 9. Mahdavi I, Shirazi B, Sahebjamnia N (2011) Development of a simulation-based optimisation for controlling operation allocation and material handling equipment selection in FMS. Int J Prod Res 49(23):6981–7005
- 10. Sule D (1994) Manufacturing facilities. PWS Publishing Company, Boston
- 11. Sari Z, Saygin C, Ghouali N (2005) Travel-time models for flowrack automated storage and retrieval systems. Int J Adv Manuf Technol 25(9–10):979–987
- 12. Lau HYK, Zhao Y (2007) Integrated scheduling of handling equipment at automated container terminal. Ann Oper Res 159:373–394
- 13. Fazlollahtabar H, Rezaie B, Kalantari H (2010) Mathematical programming approach to optimize material flow in an AGVbased flexible jobshop manufacturing system with performance analysis. Int J Adv Manuf Technol 51(9–12):1149–1158
- 14. Popovic D, Vidovic M, Bjelic N (2012) Application of genetic algorithms for sequencing of AS/RS with a triple-shuttle module in classbased storage. Flex Serv Manuf J. doi[:10.1007/s10696-012-9139-2](http://dx.doi.org/10.1007/s10696-012-9139-2)
- 15. Kilic HS, Durmusoglu MB, Baskak M (2012) Classification and modeling for in-plant milk-run distribution systems. Int J Adv Manuf Technol 62(9–12):1135–1146
- 16. Chase RB, Aquilano NJ, Jacobs FR (1998) Production and operations management-manufacturing and services. Irwin McGraw-Hill, USA
- 17. Akıllıoğlu AH, Baydoğan MG, Bolatlı Y, Canbaz D, Halıcı A, Sezgin Ö, Özdemirel NE, Türkcan A (2006) Pull-based milk-run

distribution system design for a firm producing diesel injectors (Turkish: Dizel Enjektör Üretimi Yapan Bir Şirket İçin Fabrika İçi Çekme Esaslı Tekrarlı Dağıtım Sistemi Tasarımı). Industrial Engineering Journal (Turkish: Endüstri Mühendisliği Dergisi) 17(3):2–15

- 18. Domingo R, Alvarez R, Pena MM, Calvo R (2007) Materials flow improvement in a lean assembly line: a case study. Assembly Autom 27(2):141–147
- 19. Golz J, Gujjula R, Günther HO, Rinderer S, Ziegler M (2012) Part feeding at high-variant mixed-model assembly lines. Flex Serv Manuf J 24:119–141
- 20. Satoglu SI, Sahin IE (2012) Design of a just-in-time periodic material supply system for the assembly lines and an application in electronics industry. Int J Adv Manuf Technol. doi[:10.1007/](http://dx.doi.org/10.1007/s00170-012-4171-7) [s00170-012-4171-7](http://dx.doi.org/10.1007/s00170-012-4171-7)
- 21. Kilic HS, Durmusoglu MB (2012) Design of kitting system in lean-based assembly lines. Assembly Autom 32(3):226–234
- 22. Aiello G, Enea M, Galante G (2002) An integrated approach to the facilities and material handling system design. Int J Prod Res 40(15):4007–4017
- 23. Stephens MP, Meyers FE (2010) Manufacturing facilities: design & material handling. Pearson, USA
- 24. Vaidyanathan BS, Matson JO, Miller DM, Matson JE (1999) A capacitated vehicle routing problem for just in time delivery. IIE Trans 31:1083–1092
- 25. Rajagopalan S, Heragu SS (1997) Advances in discrete material handling system design. Decision Sciences and Engineering Systems Department, USA
- 26. Sinriech D, Samakh E (1999) A genetic approach to the pickup/ delivery station location problem in segmented flow based material handling systems. J Manuf Syst 18(2):81–99
- 27. Hwang HS (2004) Heuristic transporter routing model for manufacturing facility design. Comput Ind Eng 46:243–251
- 28. Costa B, Dias LS, Oliveira JA, Pereira G (2008) Simulation as a tool for planning a material delivery system to manufacturing lines. Engineering Management Conference IEEE International (978-1- 4244-2288-3)
- 29. Kilic HS (2013) An integrated approach for supplier selection in multi-item/multi-supplier environment. Appl Math Modell. doi[:10.1016/j.apm.2013.03.010](http://dx.doi.org/10.1016/j.apm.2013.03.010)
- 30. Sharma S (2007) Interchange of the holding/shortage costs in multiproduct manufacture. Proc IME B J Eng Manufact 221(1):135–140
- 31. Sharma S (2008) Theory of exchange. Eur J Oper Res 186:128–136
- 32. Sharma S (2009) Extending Sanjay Sharma's theory of exchange. Int J Appl Manage Sci 1(4):325–339
- 33. Sharma S (2009) Single/multiple parameter swapping in the context of Sanjay Sharma's theory of exchange. Int J Adv Manuf Technol 40(5–6):629–635