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ORIGINAL ARTICLE

Hybrid prognostic method applied to mechatronic systems

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Abstract Fault detection and isolation, or fault diagnostic, of mechatronic systems has been the subject of several interesting works. Detecting and isolating faults may be convenient for some applications where the fault does not have severe consequences on humans as well as on the environment. However, in some situations, diagnosing faults may not be sufficient and one needs to anticipate the fault. This is what is done by fault prognostics. This latter activity aims at estimating the remaining useful life of systems by using three main approaches: data-driven prognostics, model-based prognostics, and hybrid prognostics. In this paper, a hybrid prognostic method is proposed and applied on a mechatronic system. The method relies on two phases: an offline phase to build the behavior and degradation models and an online phase to assess the health state of the system and predict its remaining useful life.

Keywords Fault detection · Fault diagnostics · Fault prognostics · Remaining useful life · Bond graph modeling

1 Introduction

Fault detection and isolation (FDI) and fault prognostics of industrial systems are two necessary functions as they allow avoiding nondesirable situations and catastrophes. FDI can be applied on both abrupt and incipient faults. Several research and industrial works have been conducted in the domain [6, 7, 16, 19]. The reported methods can be classified in two main categories: qualitative methods and quantitative methods [6, 16]. FDI can be used to do reconfiguration and accommodation and is suitable for systems where the fault does not have severe consequences. For example, detecting and isolating a fault on a valve controlling inflammable liquids may not avoid possible explosions. In this case, the fault is diagnosed a posteriori and thus is undergone.

Contrary to FDI, which is done a posteriori after the appearance of the faults, prognostics aims at anticipating the time of a failure by predicting the remaining useful life (RUL) of the system [1]. Prognostic results can then be used to take appropriate decisions on the system (change of set points, reduce the production load, stop the system, etc.).

Fault prognostic methods can be grouped into three main approaches [5, 7, 17, 18]: data-driven prognostics, model-based prognostics, and hybrid prognostics. Data-driven prognostics is based on the utilization of monitoring data to build behavior models including the degradation evolution, which are then used to predict the RUL [3, 4]. Model-based prognostics, also called physics of failure prognostics, uses models generated from fundamental laws of physics to calculate the RUL [2, 11]. Finally, hybrid prognostics combines both previous approaches and benefits from their advantages (precision and applicability).

This paper presents a hybrid prognostic method with application to mechatronic systems. In this contribution, the behavior model is obtained by using the bond graph (BG) formalism [8, 16] and the degradation models are derived by using the concept of residuals. The degradation of the system's components is supposed to be continuous drifts in the system's parameters. The global model of the mechatronic system (behavior and degradation models) is then used to estimate the current health state of the system, predict its future one, and calculate its RUL.

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The paper is organized as follows: After the introduction, Section 2 presents a brief description of the bond graph formalism and the fault prognostic paradigm. Section 3 gives the framework and details the steps of the proposed method. Section 4 deals with the application of the method on a mechatronic system, where simulation results are presented and discussed. Finally, Section 5 concludes the paper.

2 Bond graph and fault prognostics

2.1 Bond graph modeling

Bond graph tool [8, 16] is a graphical representation of power transfer within a physical system. A bond graph model is situated between the physical model and the mathematical model. It is used in modeling to derive mathematical models in forms of state space and transfer function; in structural analysis of the system's properties like controllability, observability, model reduction, actuator, and sensor placement; and finally in fault detection and isolation.

The generation of a BG model is based on nine BG elements: three passive elements (resistance R, capacitance C, and inertia I), two active elements (source of effort Se and source of flow Sf), and four junction elements (transformer TF, gyrator GY, zero junction 0, and one junction 1). In addition to these nine elements, two detectors representing the sensors are added (effort and flow detectors). These BG elements are proposed to unify the modeling process of multi-physical systems, by using two generalized variables: effort and flow. The product of these two variables is equal to power, which is exchanged between the physical parts of the system.

To obtain a BG model of multi-physical systems in general and of mechatronic systems in particular, it is recommended to start with a word BG. This word BG is obtained by decomposing the whole system into several energy-domain parts (electrical, mechanical, hydraulic, pneumatic, thermal, etc.) and linking these parts between them by using half-arrows representing the power exchanged between them and called bond graph links. Then, for each block, a BG model is derived by using dedicated procedures given in [8]. A BG model of a DC motor, composed of two coupled physical domains (electrical and mechanical), is given in Fig. 1.

2.2 Fault prognostics

Prognostics is a key process of condition-based maintenance [9, 15] (Fig. 2).

Prognostics is defined by the International Organization for Standardization [1] as the estimation of the operating time before failure and the risk of future existence or appearance of one or several failure modes. The time to failure is commonly called remaining useful life (RUL) by the Prognostics and Health Management research community [5, 7, 12, 18]. Figure 3 shows an illustration of a RUL prediction according to a predefined system's performance.

Fault prognostics can be done according to three main approaches: data-driven prognostics, model-based (also called physics of failure) prognostics, and hybrid prognostics. The first approach uses the data provided by sensors (monitoring data) and which capture the degradation evolution of the system. The data are then preprocessed to extract features which are used to learn models for health assessment and RUL prediction [3, 4]. Examples of models are neural networks, regressions, hidden Markov models, support vector regression, etc. The second approach requires a deep understanding of the physical phenomena of the system, including the degradation evolution. This approach uses physical laws to build the global model of the system, which is then used for simulations and predictions to calculate the RUL [2, 11]. Note that the construction of the model is subjected to the availability of a degradation model. Examples of degradation models are those related to crack by fatigue, corrosion, and wear. Finally, the third approach combines both previous approaches. The advantage of the hybrid approach is that it allows doing reliable prognostics at two levels: component-level prognostics and system-level prognostics. The component-level prognostics allows building accurate degradation models which can then be injected in the global model obtained at the system level in order to estimate the remaining useful life of the whole system. Furthermore, the hybrid approach allows modeling the interactions between the components of the system and thus tracking the influence of a degradation in one component on the other components.

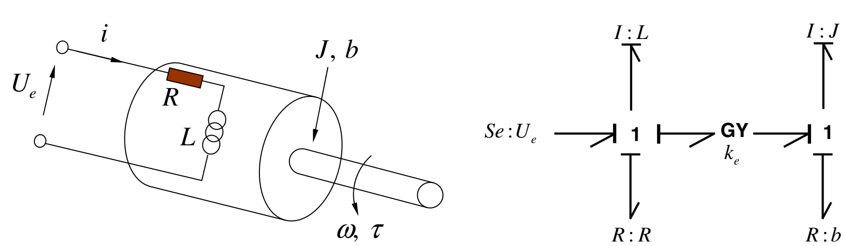


Fig. 1 A DC motor and its BG model

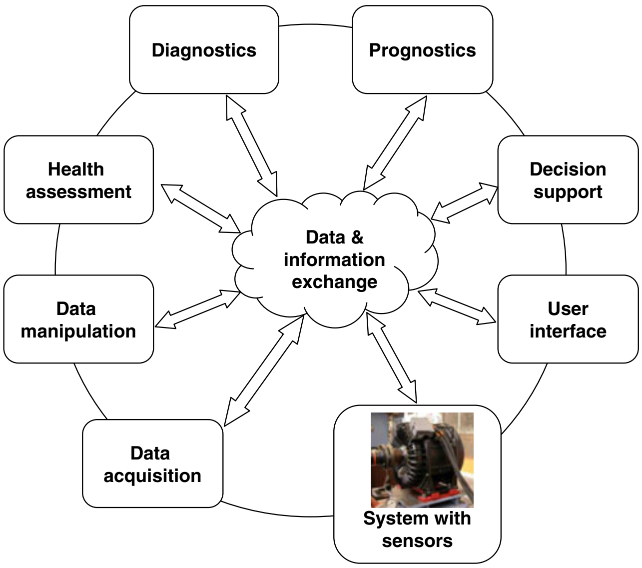


Fig. 2 Steps of a condition-based maintenance

A summary of the advantages and drawbacks of the previous approaches is given in Fig. 4.

Compared to the model-based approach, data-driven methods give less precise prognostics [10, 18], due particularly to the absence of a deterministic behavior model and to the variability of the experimental data needed to learn the degradation model of the physical system. The model-based methods give more precise results, but their implementation on complex physical systems is not trivial because of the difficulty to generate the system's behavior and degradation models. However, these methods can be applied on small system for which the behavior model can be easily obtained. This is the case of mechatronic systems. Nevertheless, even for these systems, it is necessary to have the models of the degradation phenomena before doing prognostics. The degradation models can be learned from experimental data acquired on accelerated life tests done on the system's components or estimated online by using appropriate techniques (residuals, parameter estimation, observers, etc.). Once the degradation models of the components are obtained, they are injected in the behavior model of the system to estimate and predict its health state and calculate its RUL.

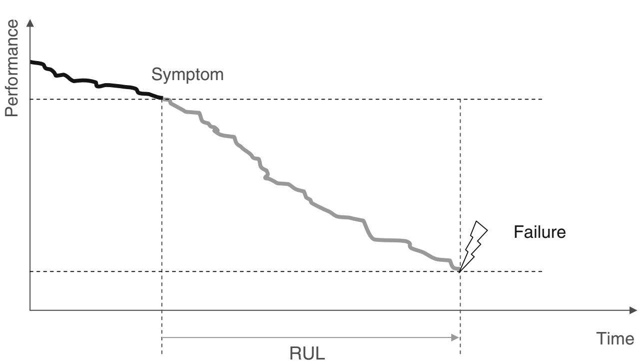


Fig. 3 Illustration of a RUL prediction

The following part of the paper presents a prognostic method applied on mechatronic systems. The method combines both model-based and data-driven approaches. The behavior model of the system is obtained by using the BG tool, whereas the degradation models of the system's components are derived by using the concept of residuals.

3 Fault prognostics of mechatronic systems

The prognostic method proposed in this paper relies on two phases, as shown in Fig. 5: an offline phase to build the dynamic model of the mechatronic system and derive its degradation models and an online phase (or exploitation phase) where the obtained models are used to detect the initiation of the degradation and predict the RUL of the system. Note that, contrary to most reported prognostic works which are component-oriented, the method proposed in this paper is system-oriented. Indeed, the variations (or drifts) in the parameters are propagated to the whole mechatronic system and are taken into account in the global dynamic model for simulations, predictions, and RUL calculation.

Before detailing the steps of each phase shown in Fig. 5, it is necessary to set the framework of the proposed method. This framework is defined by the following assumptions:

1. The sensors are considered to be fault-free and give correct measurements.
2. Only incipient faults are considered (abrupt faults are not taken into account).
3. The faults in the mechatronic system are due to continuous drifts in its parameters.
4. The faults in the actuators are not taken into account.

The first phase of the method includes three steps: the construction of the nominal behavior model of the system, the generation of its degradation model, and the definition of the thresholds (faults' thresholds and system's performance thresholds). The nominal model consists of a set of mathematical equations obtained by using the bond graph formalism [8, 16]. The output of this model is compared to the measurements acquired on the real system to generate residuals, which are then used to derive the degradation models of the system's components. The degradations correspond to changes in the BG elements C, I, and R, as expressed by the following relations:

$$C(t) = C_0 + f(t) \quad (1)$$

$$I(t) = I_0 + g(t) \quad (2)$$

$$R(t) = R_0 + h(t) \quad (3)$$

Fig. 4 Data-driven prognostics vs model-based prognostics

Data-driven prognostics

- Advantages
  - Simplicity of implementation
  - Low cost
- Drawbacks
  - Need of experimental data that represent the degradation phenomena
  - Variability of test results even for a same type of component under same operating conditions
  - Less precision
  - Difficult to take into account the variables operating conditions
  - Component-oriented approach rather than system-oriented approach
  - Difficult to define the failure thresholds

Model-based prognostics

- Advantages
  - High precision
  - Deterministic approach
  - System-oriented approach: propagation of the failure in the whole system
  - The dynamic of the states can be estimated and predicted at each time
  - The failure thresholds can be defined according to the system's performance (stability, precision, …)
  - Possibility to simulate several degradations (drifts on the parameters)
- Drawbacks
  - Need of degradation model
  - High cost of implementation
  - Difficult to apply on complexes systems

C_0, I_0, and R_0 are the nominal values of the BG elements C, I, and R, respectively, and $f(t)$, $g(t)$, and $h(t)$ are the time variations of these elements. For example, a degradation in the stator of an electrical machine can be interpreted as a continuous modification of the electrical resistance of the stator winding. Similarly, a degradation of battery can be explained by a modification of its electrical capacitance.

A residual is a numerical evaluation of analytical redundancy relation (ARR) obtained from an overdetermined system of equations (the number of equations is greater than the number of variables) [13, 14]. An ARR contains only known variables (inputs, outputs, and parameters of the system), and it is represented by the following expression:

$$\text{ARR} : \Phi(K) = 0, \tag{4}$$

where K is the set of known variables. An ARR can represent mass balance, energy balance, etc.

A residual $r(t)$ is a numerical evaluation of an ARR.

$$r(t) = \Phi(K) \tag{5}$$

Residuals are signals which are used to verify the coherence between the nominal and the actual behavior of the system. When the system operates correctly, the residuals' values should be theoretically equal to zero; otherwise, the residuals increase (or decrease) as the system leaves its nominal behavior. Figure 6 shows the principle of a residual signal.

In this contribution, the ARRs and the corresponding residuals are obtained from the BG model by applying the following procedure (more details can be found in [13, 14]):

1. Build the bond graph model in preferred integral causality of the mechatronic system.
2. Put the bond graph model in preferred derivative causality (with inversion of the sensors' causality if necessary).
3. Write the constitutive relation for each junction of the bond graph model in preferred derivative causality.
4. Eliminate the unknown variables from each constitutive relation by covering the causal paths on the bond graph model.

Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be the set of physical parameters of the system which are involved in its dynamic model and in

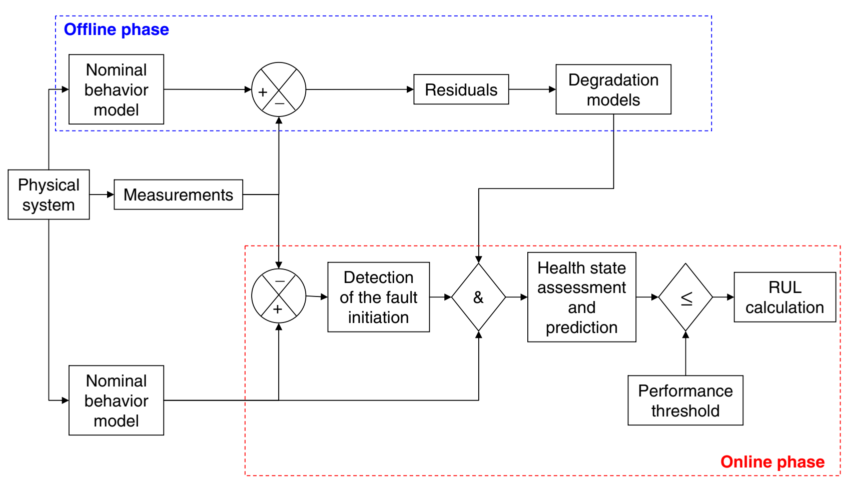


Fig. 5 Overview of the proposed prognostic method

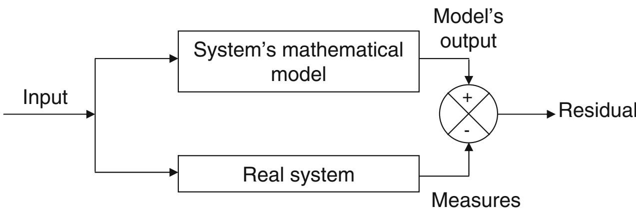


Fig. 6 Principle of residuals

the corresponding residuals. The residual equation given in Eq. 5 can then be rewritten as follows:

$$r(t) = \Phi(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n) \quad (6)$$

Then, the evolution of the degradation can be determined by inverting Eq. 6. For example, in the case where the degradation corresponds to the variation of the parameter α_1, its evolution can be calculated by the following equation:

$$\alpha_1 = \Phi^{-1}(r(t), \alpha_2, \alpha_3, \ldots, \alpha_n) \quad (7)$$

The second phase of the proposed method concerns the exploitation of the models and knowledge obtained in the first phase to assess the health state of the system and calculate its RUL. During this phase, the output of the nominal behavior of the system is continuously compared to the measurements provided by the sensors to detect whether the fault starts to occur or not. If a fault initiation is detected, the process of health assessment and RUL calculation is launched. The detection of a fault initiation is done by continuously evaluating the residuals and by analyzing the corresponding binary fault signature matrix formed by the residuals.

The global model, composed by the nominal model, the degradation model, and the result of the fault detection, is used to assess the health state of the mechatronic system, predict its future one, and calculate its RUL. The RUL is calculated according to a defined performance (which can be related to the precision of the system, its stability, etc.) and by using Eq. 8, which is illustrated in Fig. 7.

$$\mathrm{RUL}(t) = t_f - t_0 \quad (8)$$

Note that during the exploitation of the mechatronic system, several faults can occur. In the case where the faults occur a the same time, the RUL of the system can be calculated from the individual RULs of the components which are failing. The RUL corresponds then to the shortest RUL among the individual RULs.

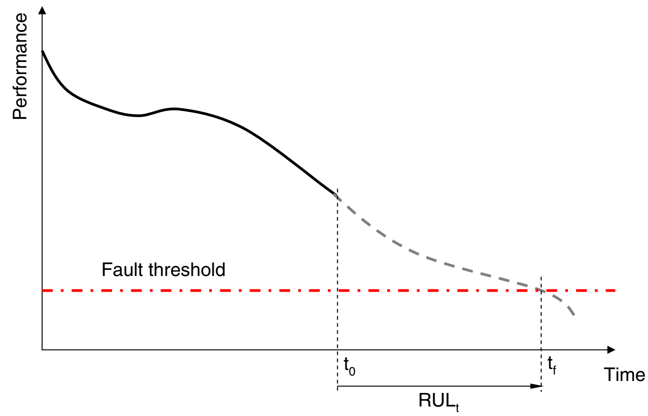


Fig. 7 RUL calculation according to a given performance

4 Case study and simulation results

4.1 Description of the mechatronic system

The mechatronic system considered for the application of the prognostic method described above is shown in Fig. 8.

The main purpose of this system is to position horizontally a load which is situated at the right side of the scheme [8]. The system is composed of a voltage source, which can be a battery, a DC motor providing a rotational movement, and a screw transforming this latter movement to a translational one in order to position the load horizontally.

4.2 Behavior model

The BG model of the system is built by taking into account the following hypotheses:

- The voltage source is constant.
- The electrical part (stator winding) of the DC motor is composed of a resistance and an inductance. Its mechanical part is represented by an inertia and a mechanical friction.
- The translation velocity v_1 is proportional to the rotation velocity ω: $v_1 = S \cdot \omega$.
- The part linking the screw to the mass is not completely rigid and presents some elasticity represented by a stiffness k.
- The load has a mass m and is in friction with the support.

The BG model in integral causality of the mechatronic system presented in Fig. 8 is given in Fig. 9.

The dynamic model of the mechatronic system can be obtained from the BG model in integral causality by following the steps given below:

- Define the inputs, the outputs, and the state space variables.

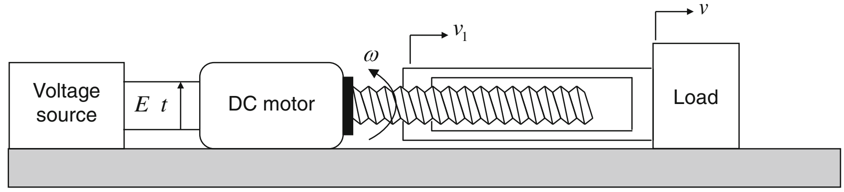


Fig. 8 Scheme of the mechatronic system

- Write the BG equations (junction equations, TF and GY equations, and constitutive elements' equations). These equations must be written by taking into account the causality of the model.
- Combine the above equations to get the differential form of the state space variables and the output variables as functions of state space and input variables.

The equations derived from the junctions "0" and "1," the transformer "TF," and the gyrator "GY" of the bond graph model are given below:

$$\begin{cases} e_2 = e_1 - e_3 - e_4 - e_5 \\ f_1 = f_3 = f_4 = f_5 = f_2 \end{cases} ; \begin{cases} e_5 = k_e \cdot f_6 \\ e_6 = k_e \cdot f_5 \end{cases} ;$$
$$\begin{cases} e_7 = e_6 - e_8 - e_9 \\ f_6 = f_8 = f_9 = f_7 \end{cases} ; \begin{cases} e_9 = S \cdot e_{10} \\ f_{10} = S \cdot f_9 \end{cases} ;$$
$$\begin{cases} e_{10} = e_{12} = e_{11} \\ f_{11} = f_{10} - f_{12} \end{cases} ; \begin{cases} e_{15} = e_{12} - e_{13} - e_{14} \\ f_{12} = f_{13} = f_{14} = f_{15} \end{cases} \quad (9)$$

The constitutive equations of the bond graph elements in Fig. 9 are given in Eq. 10.

$$f_2 = \frac{1}{L_1} \cdot \int e_2 \cdot dt; \quad e_4 = R_1 \cdot f_4; \quad f_7 = \frac{1}{J_1} \cdot \int e_7 \cdot dt; \quad e_8 = b_1 \cdot f_8$$
$$e_{11} = k_1 \cdot \int f_{11} \cdot dt; \quad e_{14} = b_2 \cdot f_{14}; \quad f_{15} = \frac{1}{m} \cdot \int e_{15} \cdot dt \quad (10)$$

From Eqs. 9 and 10, the dynamic model of the mechatronic system can be obtained in the form of state space (Eq. 12). In this equation, the dimension of the state vector x is equal to 4.

$$x = (p_2 \quad p_7 \quad q_{11} \quad p_{15})^T \Rightarrow \dot{x} = (\dot{p}_2 \quad \dot{p}_7 \quad \dot{q}_{11} \quad \dot{p}_{15})^T \quad (11)$$

The variable p_2 stands for the electrical flux of the inductance L_1, p_7 is the momentum of the inertia J_1, q_{11} is the displacement of the part linking the screw to the mass, and p_{15} is the momentum of the mass m. The variable y stands for the output of the mechatronic system and corresponds to the velocity of the mass m.

$$\begin{cases} \dot{x} = \begin{pmatrix} -\frac{1}{L_1} & -\frac{k_e}{J_1} & 0 & 0 \\ \frac{k_e}{L_1} & -\frac{b_1}{J_1} & -S \cdot k_1 & 0 \\ 0 & \frac{S}{J_1} & 0 & -\frac{1}{m} \\ 0 & 0 & k_1 & -\frac{b2}{m} \end{pmatrix} \cdot x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \times E(t) \\ y = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{m} \end{pmatrix} \cdot x \end{cases} \quad (12)$$

In the following part of the section, MATLAB® and Simulink® software tools are used to code and simulate the state space equations including the degradation models and to calculate the remaining useful life of the mechatronic system. The time response of the system to a step input is shown in Fig. 10. From this figure, we can see that the system is stable and its final value corresponding to the mass velocity (without any degradation taken into account) is equal to 0.08 m/s. The stability of the system can be easily verified by calculating the eigenvalues of the state matrix given in Eq. 12. By considering the numerical values of the parameters given in Table 1, the eigenvalues of the system are equal to -275.74, $-033.18+46.99i$, $-033.18-46.99i$, and -12.71.

In the following study, the performance measure according to which the prognostics is done can be the final value of the response, the stability of the system, the precision, etc. This performance will determine the failure threshold and thus the estimation of the RUL of the system. For example, if the stability is considered, the estimation of the RUL will correspond to the time difference between the current time and the limit time for which the system becomes instable. However, in the case where the stability holds despite

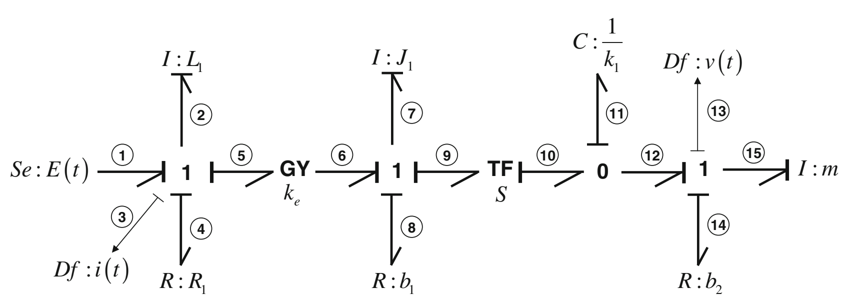


Fig. 9 BG model of the mechatronic system

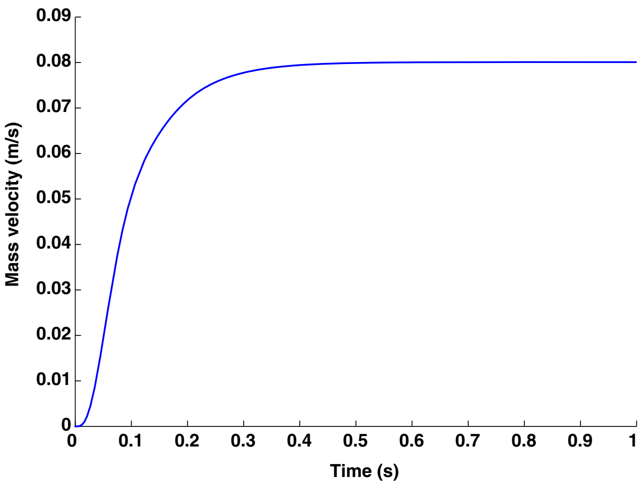


Fig. 10 Step response of the mechatronic system

the degradation, another performance measure will be considered (for example, an acceptable value of the system's response).

4.3 Generation of the residuals

It is necessary to identify the system's components which are subject to degradations and define their degradation models before generating the residuals. In this application, the degradation phenomena which can be taken into account are the drift in the resistance of the DC motor's winding, the magnetic deterioration of the DC motor's permanent magnet, and the bending of the rotating shaft. The degradation of the electrical resistance can be caused by the variation of the resistivity of the winding due to temperature change inside the DC motor. The magnetic degradation concerns the diminution of the magnetic field generated by the permanent magnet of the DC motor. Finally, the bending of the shaft can be induced by overloading the DC motor and by external perturbations.

The ARRs and the corresponding residuals of the mechatronic system shown in Fig. 8 are obtained by using the steps presented in Section 3. By applying these steps, the bond graph model in derivative causality corresponding to the bond graph model given in Fig. 9 is shown in Fig. 11.

Two ARRs can be derived from the bond graph model in derivative causality (one ARR for each sensor). The first ARR is obtained from the junction equation "1" connected to the flow detector $Df : i(t)$.

$$e_3 = e_1 - e_2 - e_4 - e_5 \Leftrightarrow e_1 - e_2 - e_4 - e_5 = 0 \tag{13}$$

By replacing the unknown variables e_1, e_2, e_4, and e_5 by known variables, the following first ARR can be derived:

$$\mathrm{ARR}_1 : E(t) - L_1 \cdot \frac{di(t)}{dt} - R_1 \cdot i(t) - \frac{k_e}{S} \times \left(v(t) + \frac{b_2}{k_1} \cdot \frac{dv(t)}{dt} + \frac{m}{k_1} \frac{d^2 v(t)}{dt^2} \right) = 0 \tag{14}$$

The second ARR is obtained from the junction equation "1" connected to the flow detector $Df : v(t)$.

$$e_{13} = e_{12} - e_{14} - e_{15} \Leftrightarrow e_{12} - e_{14} - e_{15} = 0 \tag{15}$$

By using the same substitutions than for the ARR_1, the second ARR is given below:

$$\mathrm{ARR}_2 : k_e \cdot i(t) - \frac{b_1}{k_e} \cdot \left(E(t) - L_1 \cdot \frac{di(t)}{dt} - R_1 \cdot i(t) \right) - \frac{J_1}{k_e} \cdot \frac{d}{dt} \left(E(t) - L_1 \cdot \frac{di(t)}{dt} - R_1 \cdot i(t) \right) - b_2 \cdot S \cdot v(t) - m \cdot S \cdot \frac{dv(t)}{dt} = 0 \tag{16}$$

The corresponding residuals can be obtained by numerically evaluating the ARRs given in Eqs. 14 and 16.

Table 1 Values of the parameters used for simulation

Symbol	Description	Numerical value
$E(t)$	Voltage source	10 V
k_e	Torque coefficient of the DC motor	0.47 N m/A
S	Coefficient linking the rotation and translation velocities	0.01 m
R_1	Electrical resistance of the DC motor	0.61 Ω
L_1	Inductance of the DC motor	0.0019 H
J_1	Inertia of the rotation part	0.01 N m s^2
b_1	Friction coefficient of the mechanical part of the DC motor	0.3 N s/m
k_1	Stiffness of the linking part between the screw and the mass	3.33×10^{-6} N/m
m	Mass	800 kg
b_2	Friction coefficient of the mass m	3,000 N s/m
α	Predefined parameter related to the degradation	0.01

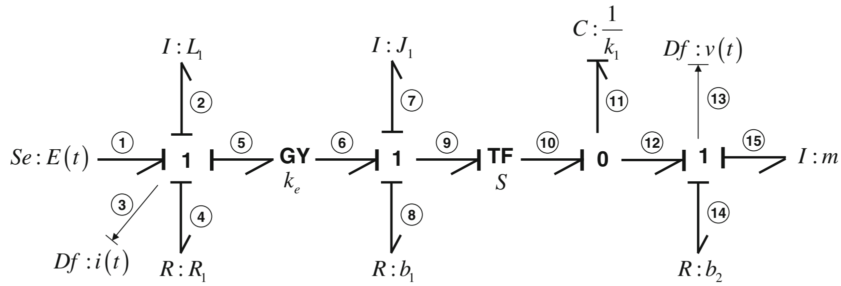


Fig. 11 BG model in derivative causality of the mechatronic system

The two obtained residuals are given by the following expressions:

$$\begin{cases} r_1(t) = E(t) - L_1 \cdot \dfrac{di(t)}{dt} - R_1 \cdot i(t) - \dfrac{k_e}{S} \\ \quad \times \left(v(t) + \dfrac{b_2}{k_1} \cdot \dfrac{dv(t)}{dt} + \dfrac{m}{k_1} \dfrac{d^2 v(t)}{dt^2} \right) \\ r_2(t) = k_e \cdot i(t) - \dfrac{b_1}{k_e} \cdot \left(E(t) - L_1 \cdot \dfrac{di(t)}{dt} - R_1 \cdot i(t) \right) \\ \quad - \dfrac{J_1}{k_e} \cdot \dfrac{d}{dt} \left(E(t) - L_1 \cdot \dfrac{di(t)}{dt} - R_1 \cdot i(t) \right) \\ \quad - b_2 \cdot S \cdot v(t) - m \cdot S \cdot \dfrac{dv(t)}{dt} \end{cases} \tag{17}$$

The advantage of these residuals is that they can be used for fault detection, fault diagnostics, and fault prognostics. In the case of fault detection and diagnostics, the residuals are used as balance equations and they do not need the degradation model (which corresponds to the deviation of the winding's resistance in this case study). Indeed, any change in one or more parameters of the mechatronic system will lead to a variation of the residuals in which the parameters are involved. The relationships between the parameters of the mechatronic system and the residuals can be expressed by a fault signature matrix (more details about the construction of this matrix can be found in [13, 14, 16]). This matrix is a binary one, with each cell i of the matrix containing a value equal to "1" if the parameters of a component is present in the residual r_i and a value equal to "0" otherwise. The "1" value means that the variation of the parameters of a component will induce a variation of the residuals in which these parameters are present. The analysis of this matrix allows to clearly determine which component (or group of components) is faulty.

In the case of fault prognostics, the integration of the degradation should be taken into account in the global model of the system to predict its RUL. In this contribution, the residuals are used to detect the initiation of the degradation on the mechatronic system. For example, in the case of a degradation on the electrical resistance of the motor's winding, the values of both residuals r_1 and r_2 will change. This is because the parameter R_1 related to the electrical resistance is present in the two residuals. The degradation model of the resistance can then be obtained by inverting the residuals' equations. By using the residual r_1, the variation of the resistance can be expressed by the following equation:

$$R_1(t) = \frac{1}{i(t)} \cdot \left[E(t) - L_1 \cdot \frac{di(t)}{dt} - \frac{k_e}{S} \times \left(v(t) + \frac{b_2}{k_1} \cdot \frac{dv(t)}{dt} + \frac{m}{k_1} \frac{d^2 v(t)}{dt^2} \right) - r_1(t) \right] \tag{18}$$

In practice, the degradation models of the system's components can be obtained offline by realizing accelerated life experiments. Then, the derived models are integrated to the behavior model of the system, and the whole model is exploited to assess the system's health state and predict its RUL. The trend of the degradation extracted from Eq. 18 is shown in Fig. 12.

From this figure, one can observe a linear degradation, which can be expressed by the following formula:

$$R(t) = R_1 \cdot (1 + \alpha \cdot t) \tag{19}$$

where R_1 is the nominal value of the resistance (absence of degradation). Indeed, the electrical resistance of the winding can be expressed by the following equation:

$$R = \rho \cdot \frac{L}{S_1} \tag{20}$$

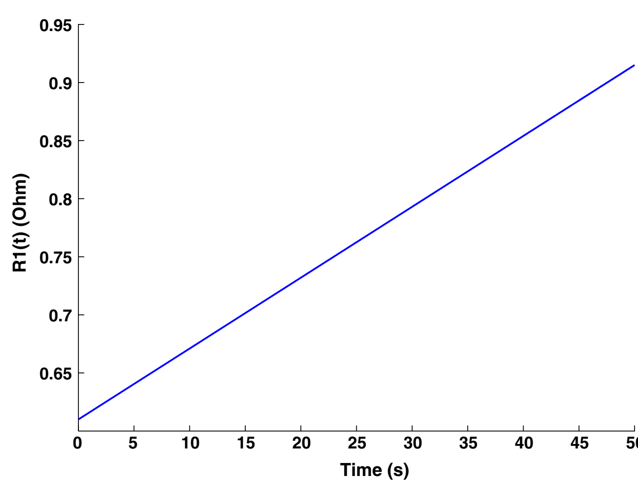


Fig. 12 The linear trend of the degradation

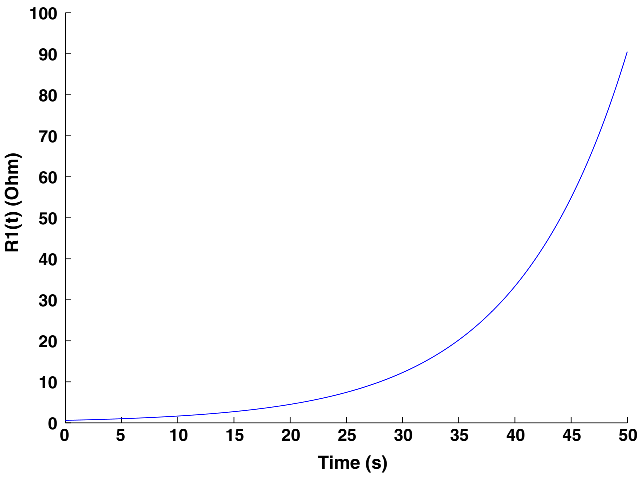


Fig. 13 A nonlinear degradation model

where ρ is the resistivity of the winding, L is its length, and S_1 is its cross section. The degradation of the resistance R_1 is due mainly to the degradation of its resistivity ρ (the variation of the length L is compensated by the variation of the cross section S_1). The variation of the resistivity can be expressed by the following equation:

$$\rho(t) = \rho_0 \cdot (1 + \alpha \cdot t) \tag{21}$$

where ρ_0 is the nominal value of the resistivity. Thus, $R(t) = R_1 \cdot (1 + \alpha \cdot t)$, where $R_1 = \rho_0 \cdot \frac{L}{S_1}$.

In addition to the linear model of the resistance degradation given in Eq. 19, a nonlinear degradation model, with $R(t) = R_1.e^{0.1 \cdot t}$, is simulated (Fig. 13). The results of the simulations related to linear and nonlinear degradations are given in the following subsection.

4.4 RUL estimation

In the absence of failures, the two residuals generated previously should have mean values close to zero. In this case, the residuals are conservative (the algebraic sum of the applied forces on the mechatronic system is equal to zero). The Fig. 14 shows the time evolution of the residuals.

However, in the presence of a degradation in the system, represented in this application by a drift in the electrical resistance of the motor's winding, the residuals affected by this drift will respond and leave their nominal values (which were initially close to zero) to move towards other values depending on the magnitude and the form (or trend) of the degradation. The variations of the two residuals due to the motor's winding electrical resistance are shown in Fig. 15 for a linear degradation and Fig. 16 for a nonlinear degradation.

Also, the change in the system's dynamic can be observed through its time response (Fig. 17).

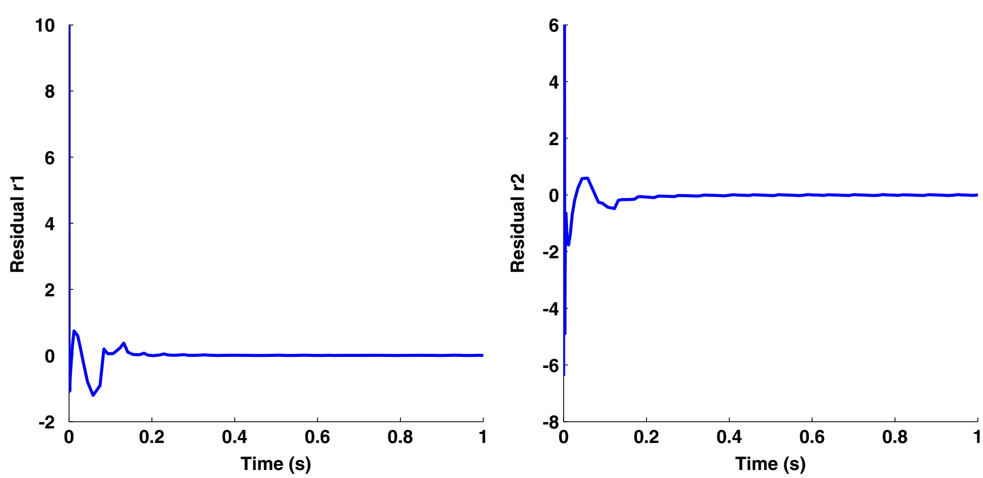
The remaining useful life of the mechatronic system can then be calculated according to defined performance criteria (related to the system's precision, time response, stability, etc.). The performance criteria chosen for the RUL calculation can be prioritized: one can imagine that the stability of the system should be more important than its time response.

In this application, the system remains stable despite the degradation (this can be verified by calculating its eigenvalues for different values of the resistance R_1). For this reason, the criterion taken into account to calculate the RUL is the final value of the system which can be obtained from its transfer function. This function, called $H(p)$, can be derived from the state space model given in Eq. 12:

$$H(p) = \frac{Y(p)}{E(p)} = C \cdot (p \cdot I - A)^{-1} \cdot B + D \tag{22}$$

where E is the input of the system, Y is its output (expressed in the Laplace domain), p is the Laplace variable, and A, B, C, and D are the matrices of the state space model. The steady state of the system for a step input $E(t)$ can then be

Fig. 14 Time response of the residuals in the absence of failures



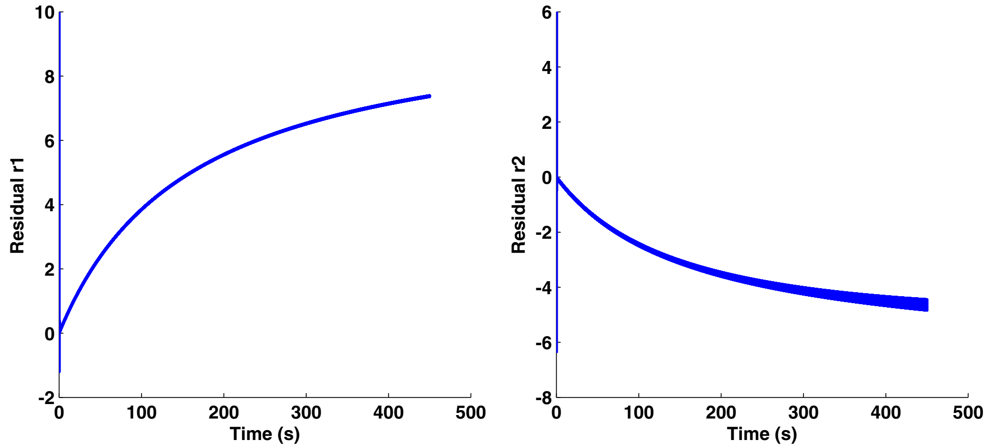


Fig. 15 Time response of the residuals in the presence of a linear degradation

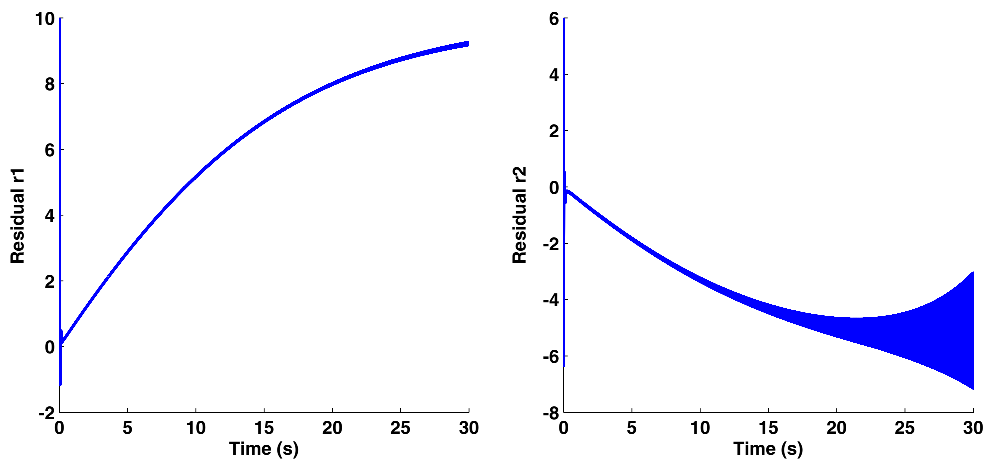


Fig. 16 Time response of the residuals in the presence of a nonlinear degradation

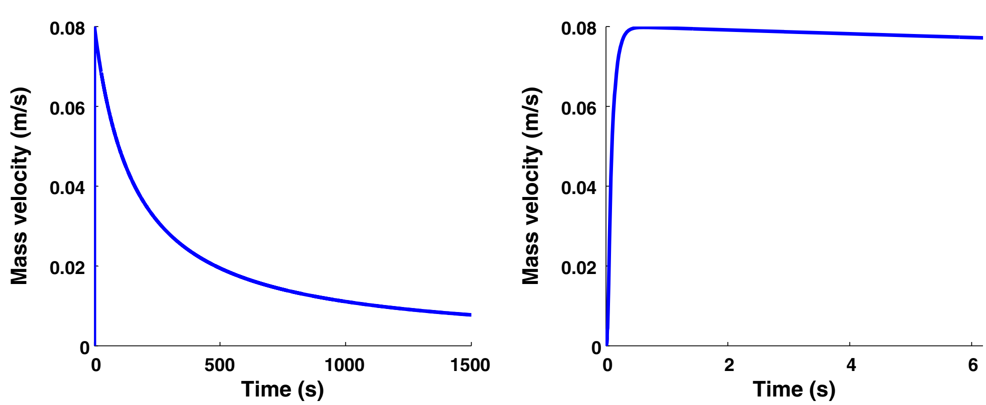


Fig. 17 Time response of the system under a linear degradation (*left figure*) and zoom on the first seconds (*right figure*)

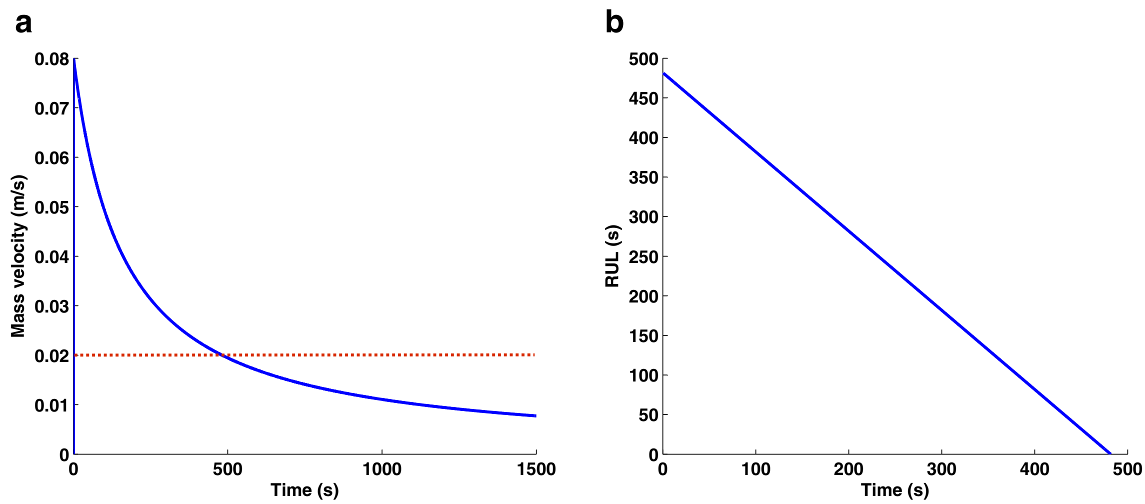


Fig. 18 Fault threshold value equal to 0.02 m/s (**a**) and estimated RUL (**b**) for a linear degradation

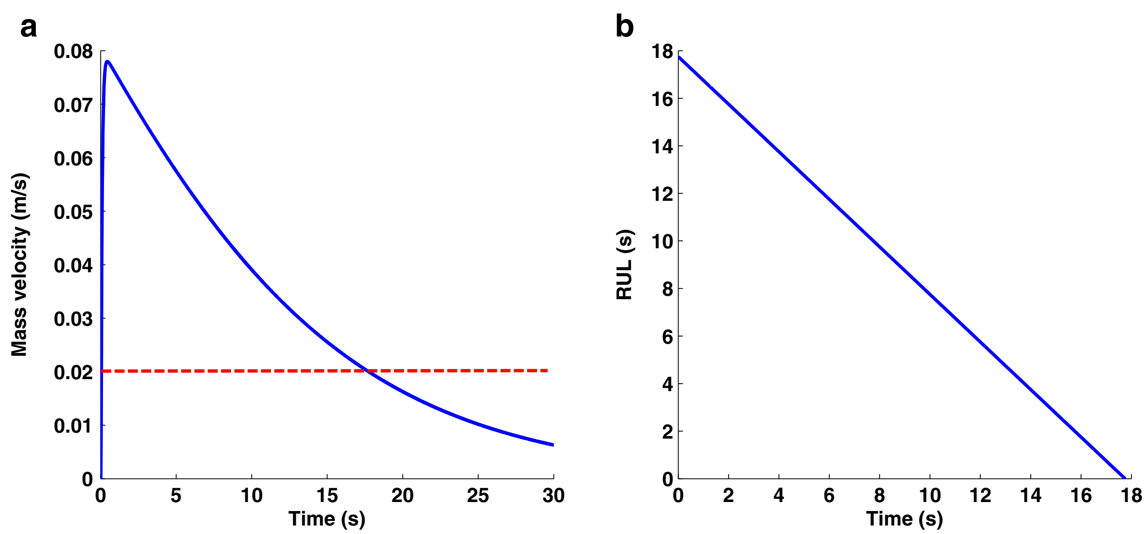


Fig. 19 Fault threshold value equal to 0.02 m/s (**a**) and estimated RUL (**b**) for a nonlinear degradation

calculated by using the final value theorem on the transfer function $H(p)$:

$$y(t) = \lim_{p \to 0} p \cdot H(p) \cdot E(p)$$
$$= \lim_{p \to 0} \left[C \cdot (p \cdot I - A)^{-1} \cdot B + D \right] = -C \cdot A^{-1} \cdot B \tag{23}$$

The result of Eq. 23 shows that the steady state of the system depends on the physical parameters present in the matrices A, B, and C (the matrix D being equal to zero). Initially and in the absence of the degradation, the steady state value is equal to 0.08 m/s. However, in the presence of the degradation, the final value changes and decreases to reach a value which is less than 0.01 m/s. If we set a limit under which the final value is critical (corresponding to failure threshold), the RUL can then be calculated according to the formula given in Eq. 8. Figures 18 and 19 show the estimated RUL for both linear and nonlinear degradation models and for a steady state threshold equal to 0.02 m/s.

5 Conclusion

A hybrid fault prognostic method applied to mechatronic systems is proposed in this paper. The method is a system-oriented approach, which can be applied on a wide range of multi-physical systems. It relies on two main phases. The first phase concerns the construction of the system's behavior and degradation models and also the definition of the thresholds needed in the calculation of the RUL. The second phase deals with the assessment of the system's health state, the prediction of its future one, and the estimation of its RUL. The degradation models are obtained by using the residuals, and the whole behavior model (including the degradations) is used to do simulations, predictions, and RUL calculation. The RUL is calculated according to a final value of the system considered as its acceptable performance.

The advantage of the method is its deterministic aspect, as the dynamic model is obtained through physical modeling leading to precise RUL. Furthermore, different degradation models (linear and nonlinear) are simulated. The method is applied on a mechatronic system, and simulation results are obtained. However, the implementation of the proposed method on data acquired from an experimental platform would allow verifying and validating its effectiveness.

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