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ORIGINAL ARTICLE

An adaptive feedrate scheduling method of dual NURBS curve interpolator for precision five-axis CNC machining

Yuwen Sun · Yurong Bao · Kaixuan Kang · Dongming Guo

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Abstract Non-uniform rational b-spline (NURBS) tool path is becoming more and more important due to the increasing requirement for machining geometrically complex parts. However, NURBS interpolators, particularly related to five-axis machining, are quite limited and still keep challenging. In this paper, an adaptive feedrate scheduling method of dual NURBS curve interpolator with geometric and kinematic constraints is proposed for precision five-axis machining. A surface expressed by dual NURBS curves, which can continuously and accurately describe cutter tip position and cutter axis orientation, is first used to define five-axis tool path. For the given machine configuration, the calculation formulas of angular feedrate and geometric error aroused by interpolation are given, and then, the adaptive feedrate along the tool path is scheduled with confined nonlinear geometric error and angular feedrate. Combined with the analytical relations of feed acceleration with respect to the arc length parameter and feedrate, the feed profiles of linear and angular feed acceleration sensitive regions are readjusted with corresponding formulas and bi-directional scan algorithm, respectively. Simulations are performed to validate the feasibility of the proposed feed scheduling method of dual NURBS curve interpolator. It shows that the proposed method is able to ensure the geometric accuracy and good machining performances in five-axis machining especially in flank machining.

Keywords Interpolator · NURBS · Feed rate · Five-axis machining · Tool path

Y. Sun (✉) · Y. Bao · K. Kang · D. Guo
Key Laboratory for Precision and Non-Traditional Machining Technology of the Ministry of Education, Dalian University of Technology, Dalian 116024, China
e-mail: xiands@dlut.edu.cn

1 Introduction

Five-axis tool paths, respectively, include cutter tip and orientation data. Compared with discrete data format, the five-axis tool path that is defined in terms of non-uniform rational b-spline (NURBS) has the advantage of improved path smoothness and reduced size of machining codes. However, geometrically smooth tool path does not mean that it must generate a smooth cutter movement. Different from three-axis machining, five-axis machining involves simultaneous and coupled translation and rotation movements, the coupling effects between two types of movements lead to that the feed interpolation with constrained geometric error and kinematic characteristics has become a challenging issue. It gets more and more significant due to the increasing requirements of high-speed and high-accuracy machining [1–3].

After a long-term evolution process of parametric interpolator, particularly NURBS interpolator, in NC machining, some methods [4–15] with constraints have been proposed respecting different application backgrounds for three-axis machining. For example, NURBS interpolator for short linear segments [16], NURBS curve interpolator with acceleration/deceleration (Acc/Dec) control [17–19], dynamics-based NURBS interpolator [20–22], NURBS interpolator with minimal feed fluctuation and continuous feed rate [23, 24], NURBS surface interpolator [25, 26], and NURBS curve interpolator based on control of speed and precision [27–29]. Generally, the interpolator algorithms used in three-axis machining cannot be directly translated to five-axis machining owning to the two additional rotation axes. Currently, limited methods are available for five-axis machining. For example, aiming at constant feed interpolator for five-axis machining, Fleisig and Spence [30] presented a reduced angular acceleration interpolation algorithm for off-line interpolation of a set of discrete cutter location points consisting of a position vector, representing the tool tip, and an orientation unit vector, representing the tool axis. In the method, three splines, namely position, orientation, and reparameterization, are computed. Among them, the position and orientation splines are both near arc length parameterized splines. Also, the coordinated motion is accomplished with an orientation reparameterization spline. To some extent, the ability of controlling angular acceleration is limited since adaptive feedrate is not used in the interpolator. Xu. et al. [31] proposed a novel angular interpolation method for bi-parameter curves. Fatan and Feng [32] presented a method to access the geometry-based errors for interpolated tool paths in five-axis surface machining. Their work focused on the error due to tool path discretization done by the CAM software. They pointed out that besides the local surface geometry, the configuration of the kinematic chain of the CNC machine has been found to be the primary factor controlling the resulting value and type of the geometry-based errors. Implementations with a typical complex free-form surface validated that the conventional chordal deviation method was not reliable and could underestimate significantly the geometry-based errors in five-axis machining. Li et al. [33] presented a NURBS pre-interpolator with three function options for a CNC system so that the NURBS interpolator can be thoroughly applied for five-axis machining. The first function is called the NURBS converter function, which is used to convert a series of linear/circular segments exactly into a NURBS curve. The second function is the NURBS smoother function, by which, a series of linear segments are fitted to a NURBS curve. The third option provides two kinds of NURBS G code definition, by which, the NURBS trajectory with five axes can be represented directly. Upon using the three options of the NURBS pre-interpolator, a unified NURBS curve can be obtained for further interpolation.

In five-axis machining, the feed in machining codes is very different from that of cutter tip due to the nonlinear coupling relations between the rotation axes and translation axes of the machine. Even constant feed of machine control center is also possible to generate significant feed fluctuations of tool center point (TCP). Therefore, with a view to ensure the machining accuracy and quality, an effective interpolator with rotation tool center point (RTCP) function, which has the ability of accurate control of cutter tip, has become a research focus. To achieve this goal, the data format of dual NURBS tool path [34, 35] has been developed recently. Different from others, RTCP interpolator usually defines the five-axis tool path in part coordinate system. Then, inverse kinematic transformation is carried out in the NC unit, and thus, the TCP coordinates are transformed to the actual position of each machine axis.

This paper presents a rapid off-line feedrate scheduling method of NURBS interpolator for five-axis NC machining in order to simultaneously consider the factors that affect machining accuracy such as geometric and kinematic constraints. The main features in the proposed method are (1) geometric error limit—the nonlinear geometric error generated in five-axis machining especially with a long cutter is usually larger than the chord error; (2) maximum angular feed limit—it is helpful for reducing the rotation angle of cutter axis at regular time interval and the inconsistency of cutting feed; (3) kinematic constraints of cutter axis—the change rate of cutter axis rotation speed is restricted to avoid the excess fluctuations of angular feed; (4) kinematic constraints of cutter tip: the linear feed Acc/Dec is limited for maintaining feed steadiness of cutter tip. Using the off-line scheduled feed spline, the complicated calculation of adaptive feedrate profile in real time is avoided while maintaining the desired geometric precision and kinematics characters. In the real-time process, it only needs to perform inverse transformation according to the two splines of tool path and feed profile for generating interpolation point.

2 Dual NURBS curve interpolator

2.1 Dual NURBS curve

As one of the most popular industry standards, NURBS are widely used in tool path planning due to its exact analytic representation and the ability of local adjustment. Since the cutting location data consists of a position vector and an orientation unit vector, as shown in Fig. 1, a continuous spatial trajectory segment of a given cutter in five-axis

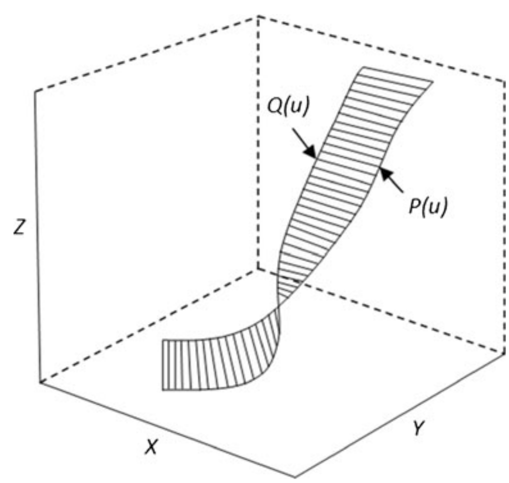

Fig. 1 Tool path surface defined by a ruled surface

machining is essentially described by dual NURBS curves with the following expressions in the part coordinate system.

$$S(u,v) = (1-v)P(u) + vQ(u) \quad v \in [0,1], \quad u \in [0,1] \tag{1}$$

where v is the parameter of the tool path surface, $P(u)$ is the trajectory of the cutter tip point in part coordinate system, and $Q(u)$ is the trajectory of a fixed point except the cutter tip point on the cutter axis. To make the two trajectories keep pace with each other, the two curves are given the same parameter u. $\boldsymbol{P}(u)$ and $\boldsymbol{Q}(u)$ are two NURBS curves given as follows:

$$\begin{aligned} P(u) &= \sum_{i=0}^{n} N_{i,k}(u)\omega_i p_i \Big/ \sum_{i=0}^{n} N_{i,k}(u)\omega_i \\ Q(u) &= \sum_{i=0}^{n} N_{i,k}(u)\omega_i q_i \Big/ \sum_{i=0}^{n} N_{i,k}(u)\omega_i \; u \in [0,1] \end{aligned} \tag{2}$$

where $\boldsymbol{p}_i$ and $\boldsymbol{q}_i$ represent the control points, ω_i is the weight factor, u is the normalized arc length parameter of the spline curve $\boldsymbol{P}(u)$. $n+1$ is the number of the control points, and $N_{i,k}(u)$ is the b-spline basis function with the following recursive formulas:

$$N_{i,0}(0) = \begin{cases} 1 & u_i \le u \le u_{i+1} \\ 0 & otherwise \end{cases}$$

$$N_{i,k}(u) = \frac{u - u_i}{u_{i+k} - u_i} N_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} N_{i+1,k-1}(u)$$

where $[u_0,\cdots,u_{n+k+2}]$ represents the knot vectors.

The unit vector $\boldsymbol{H}(u)=(H_x(u), H_y(u), H_z(u))$ of cutter axis orientation at parameter u is calculated as

$$H(u) = (Q(u) - P(u))/\|Q(u) - P(u)\| \tag{3}$$

2.2 Principle of NURBS interpolation

NURBS curve interpolator involves the computation of the next sampling period parameter according to the curve information and the feedrate of current sampling period. By using the Taylor series expansion of $u(t)$ at $t=t_i$, the second-order interpolation algorithm for parameter u with respect to time t is expressed as

$$u_{i+1} = u_i + T\frac{du}{dt}\bigg|_{t=t_i} + \frac{1}{2}T^2\frac{d^2u}{dt^2}\bigg|_{t=t_i} + HOT \tag{4}$$

where T is sampling period and HOT represents the high-order truncation error.

The instantaneous feed rate $V(u_i)$ along the cutter tip path $\boldsymbol{P}(u)$ is written as

$$V(u_i) = \left\|\frac{dP(u)}{dt}\right\|_{u=u_t} = \left\|\frac{dP(u)}{du}\right\|_{u=u_i} \frac{du}{dt}\bigg|_{t=t_i} \tag{5}$$

Then Eq. (4) can be further given as

$$u_{i+1} = u_i + \frac{V(u_i)T}{\left\|\frac{dP(u)}{du}\right\|_{u=u_i}} + \frac{T^2}{2}\frac{d^2u}{dt^2}\bigg|_{u=u_i} \tag{6}$$

where

$$\frac{d^2u}{dt^2} = \frac{d\left(V\Big/\left\|\frac{dP(u)}{du}\right\|\right)}{dt} = \frac{\frac{dV}{dt}\left\|\frac{dP(u)}{du}\right\| - Vd\left\|\frac{dP(u)}{du}\right\|/dt}{\left\|\frac{dP(u)}{du}\right\|^2}$$

Furthermore, if the path parameter u is normalized arc length parameterization, then $\|d\boldsymbol{P}(u)/du\|=\kappa$ where κ is the length of the curve. Thus, Eq. (6) can be simplified as

$$u_{i+1} = u_i + \frac{V(u_i)}{\kappa}T + \frac{a(u_i)}{2\kappa}T^2 \tag{7}$$

where a is acceleration. From Eq. (7), it can be seen that the position of next sampling point in interpolator has relations with the feedrate and geometric properties of the cutter tip path. However, once the path parameter is given as normalized arc length parameter, the position of next sampling point has only relations with the kinematic parameters of cutter tip.

3 Proposed interpolator method

Compared to the three-axis machining, the cutter movement is more complex and difficult to control in five-axis machining owning to the orientation change of cutter axis. The structure of the proposed feedrate scheduling method of five-axis NURBS interpolator is shown in Fig. 2, which mainly includes four function modules: maximum angular feedrate calculation module, geometry-based error calculation module, adaptive feedrate scheduling module, and Acc/Dec control module. The first module is mainly used to confine the excess rotation of cutter axis and thus prevent the dramatic change of cutting parameters such as cutting load. Otherwise, excessive orientation fluctuations will cause the deterioration of machining surface quality. Geometry error calculation module is used to ensure the machining accuracy. Nonlinear error rather than chordal error is considered in case that the geometric error in five-axis interpolator is underestimated. The third module, adaptive feedrate scheduling module, will schedule the feedrate to confine chord error and angular feedrate within the prescribed value. The angular/linear feed Acc/Dec control

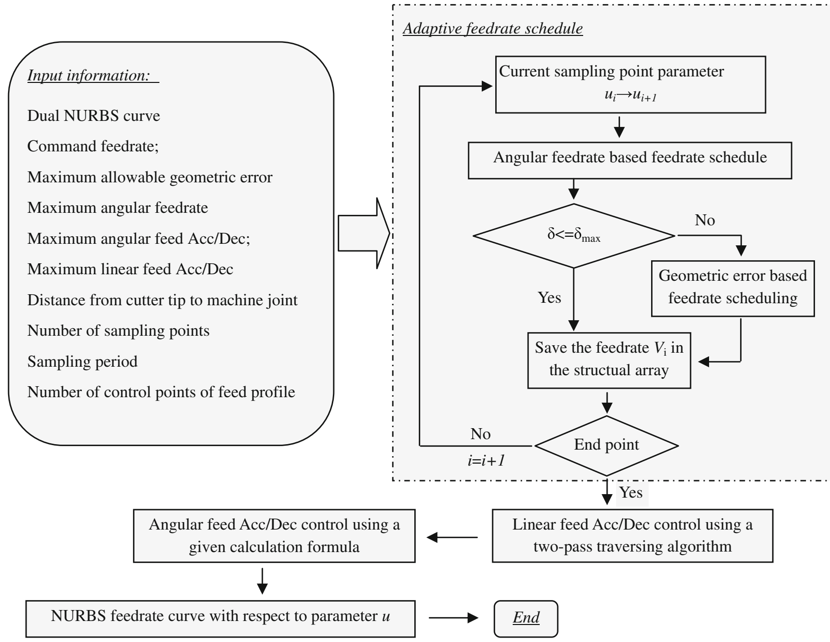

Fig. 2 Proposed feedrate scheduling method of dual NURBS curve interpolator for five-axis machining

module is to detect the feed sensitive region in interpolator and readjust feed profile using a derived formula and bi-directional scan scheme. Excessive Acc/Dec may probably lead to the significant change of cutting load. As a result, it will cause the unsteadiness of machining process. The maximum Acc/Dec is limited within a preset value when the angular or linear feed Acc/Dec exceeds the desired range. It is also helpful to reduce the shock to the machine and cutter caused by feed Acc/Dec and improve the machining quality. After the feedrate scheduling, the corresponding parameter value of the sampling point can be calculated with Taylor series expansion formula.

3.1 Adaptive feedrate scheduling

3.1.1 Feedrate determined by geometric error

In five-axis machining, only considering chord error will underestimate the geometry error aroused by parametric interpolation. At each sampling period, the resulting nonlinear error is related to the configuration and size of a specific five-axis machine. For a cutter location point $W(\boldsymbol{P}, \boldsymbol{H})$ in part coordinate system, where $\boldsymbol{P}=(P_x,P_y,P_z)$ is the cutter position data and $\boldsymbol{H}=(H_x,H_y,H_z)$ is the unit orientation vector, to make the cutter tip move to the exact position with a desired cutter orientation, the corresponding machine control data M $(\boldsymbol{G}, \boldsymbol{D})$ needs to be defined with five parameters, namely the two rotation angles $\boldsymbol{G}=(a, b)$ and three translation coordinates $\boldsymbol{D}=(x, y, z)$. The transformation relations between the machine control data and cutter location data are derived by establishing a kinematic model according to the concrete machine configuration. Without losing generality, a five-axis with dual rotary head machine configuration is used here as an example [36]. Let the direction of each coordinate axis of part coordinate system be aligned with that of machine coordinate system, and at initial status, the origin of the part coordinate system be coincided with the cutter tip. For the spindle-tilting machine configuration, the inverse kinematic transformations are given as

$$\begin{aligned} \left[H_x\ H_y\ H_z\ 0\right]^T &= T(E)B(b)B(a)[0\ 0\ 1\ 0]^T \\ \left[P_x\ P_y\ P_z\ 1\right]^T &= T(E)B(b)B(a)[0\ 0-L\ 1]^T \end{aligned} \tag{8}$$

where $\boldsymbol{T}$ is translation matrix, $\boldsymbol{E}$ is the coordinate of the origin of machining coordinate system in the part coordinate system while the machine is in the original status, $\boldsymbol{B}$ is unit rotation matrix, $\boldsymbol{r}_1=[0,\ 0,-L]^T$, and L is the distance from the machine pivot to the cutter tip center.

Then, for a given cutter location $\left(P_x{}^i, P_y^i, P_z{}^i, H_x{}^i, H_y{}^i, H_z{}^i\right)$, the corresponding rotation angle of machine control data about the primary and second rotation axes are obtained as

$$a_i = \arcsin\left(-H_y^i\right) \quad a_i \in \left[-\frac{\pi}{2}\ \frac{\pi}{2}\right]$$

and

$$\begin{cases} b_i = \arctan 2\left(H_x^i, H_z^i\right) & H_z^i \neq 0 \\ b_i = \pi/2 & H_z^i = 0, H_x^i \succ 0 \\ b_i = -\pi/2 & H_z^i = 0, H_x^i \prec 0 \\ b_i = 0 & H_z^i = 0, H_x^i = 0 \end{cases}$$

The relations between the cutter location coordinates and the translation coordinates of machine pivot can be given as

$$\begin{cases} x_i = P_x^i + L\cos a \sin b \\ y_i = P_y^i - L\sin a \\ z_i = P_z^i - L + L\cos a \cos b \end{cases} \tag{9}$$

Thus, using the inverse kinematic transformation, for two given interpolation points, their corresponding machine control coordinates can be calculated. Then, the machine joint will move linearly from the start to the end as the same time the cutter orientation also changes linearly. In this case, in the interpolator period, nonlinear geometric error will be aroused due to the simultaneous translation and rotation movements of the cutter, so it is not enough to only take the chord error as an evaluation criterion of geometric error like in three-axis machining. As illustrated in Fig. 3, assuming P_i and P_{i+1} are the initial and final cutter tip points at some period. Their corresponding machine control coordinates ($D_i(x_i, y_i, z_i)$, $\boldsymbol{G}_i(a_i, b_i)$), and ($D_{i+1}$, $\boldsymbol{G}_{i+1}$) can be obtained, respectively. Then, the intermediate control coordinates between the two points are given using the following formulas

$$\begin{aligned} D(\lambda) &= (1-\lambda)D_i + \lambda D_{i+1} \\ G(\lambda) &= (1-\lambda)G_i + \lambda G_{i+1} \end{aligned} \tag{10}$$

where λ is the interpolation parameter and $0 \le \lambda \le 1$.

To calculate the nonlinear geometric error in a sampling period, some intermediate control coordinates between two adjacent interpolation points are first obtained using the above formula. Then, the real cutter tip point $P_i\left(P_x{}^i, P_y{}^i, P_z{}^i\right)$ accompanied by the intermediate machine control coordinates can be calculated with Eq. (9) in part coordinate system. After finding the nominal points of cutter tip path $\overline{P}_i\left(\overline{P}_x^i, \overline{P}_y^i, \overline{P}_z^i\right)$

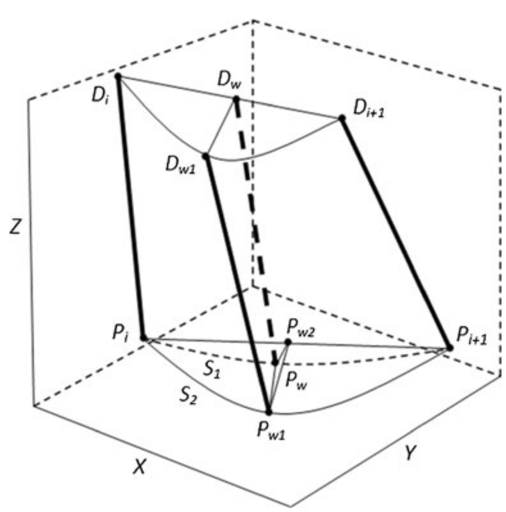

Fig. 3 Chord error and geometric error in curve interpolation

corresponding to these real ones, the geometric error of each sampling point is computed, and the largest error of the sampling points can be viewed as the maximum geometric error at current interpolation period, that is, $\varepsilon = \max\left(\left\|P_1 - \overline{P}_i\right\|, \cdots, \left\|P_i - \overline{P}_i\right\|, \cdots, \left\|P_m - \overline{P}_m\right\|\right)$. Empirically, the maximum error point is close to the middle position of the interpolation curve segment, so the maximum geometric error can be simply given as the geometric error generated at the position of middle point in some cases.

3.1.2 Angular feed

The angular feed of cutter axis is related to the cutter tip path $\boldsymbol{P}(u)$, path parameter u, and cutter orientation vector $\boldsymbol{H}(u)$. It can be derived by means of kinematic analysis. At each sampling period in interpolation process, it is necessary to ensure that the angular feed is within the allowable range. The magnitude of instantaneous angular velocity $\omega(u)$ can be described as the modulus of the first derivative of the unit cutter orientation vector $\boldsymbol{H}(u)$ with respect to time parameter t, that is,

$$\omega(u_i) = \left\|\frac{dH(u)}{dt}\right\|_{u=u_i} = \left\|\frac{dH(u)}{du}\right\|_{u=u_i} \left.\frac{du}{dt}\right|_{t=t_i} \tag{11}$$

From Eqs. (5) and (11), the relation between the feedrate $V(u)$ and angular feedrate $\omega(u)$ is derived as follows:

$$\omega(u_i) = \left\|\frac{dH(u)}{du}\right\|_{u=u_i} \frac{V(u_i)}{\left\|\frac{dP(u)}{du}\right\|_{u=u_i}} \tag{12}$$

If the maximum angular feedrate is $\omega_{\max}$, and then, the feedrate $V(u_i)$ of current sampling point is given as

$$\begin{cases} V(u_i) = V_c & \text{if } \omega_i \le \omega_{\max} \\ V(u_i) = \left\|\frac{dP(u)}{du}\right\|_{u=u_i} \frac{\omega_{\max}}{\left\|\frac{dH(u)}{du}\right\|_{u=u_i}} & \text{otherwise} \end{cases} \tag{13}$$

where V_c is the maximum allowable feedrate. Similarly, when the path parameter u is normalized arc length parameterization, $\|d\mathbf{P}(\mathbf{u})/du\| = \kappa$ can be utilized to further simplify the above formula.

3.1.3 Adaptive feedrate scheduling algorithm

According to the current interpolation point, the next candidate interpolation point position is first determined based on the maximum angular feed criterion, and then, the exact position is found with constraint of nonlinear geometric error using a bisection method. The detailed algorithm of determining the parametric coordinate of next interpolation point is given as follows:

Input: maximum allowable geometric error $\delta_{\max}$, maximum angular feedrate $\omega_{\max}$, maximum allowable linear feedrate V_c, the number of sampling points n, and dual NURBS tool path.

(1) Calculate sampling parameter $(u_i)_{i=1}{}^n$ of the tool path position using the formula $u_i=(i-1)/(n-1)$, and store them in the given structural array.
(2) For each given parameter position u_i, calculate its attached feasible feedrate V_i^f according to the maximum angular feedrate $\omega_{\max}$ and formula (13). Then, store the obtained feedrate V_i^f in the structural array.
(3) For each given parameter position u_i, calculate its attached feasible feedrate V_i^g using the geometry-based criterion. The concrete details are as follows:

   ① Select reference feedrate V_i^f as the initial feedrate, and then, the parameter increment δu_i of next interpolation point corresponding to the current position u_i can be calculated using the Taylor formula.
   ② For the given two successive interpolation positions u_i and $u_i+\delta u_i$, calculate the maximum nonlinear geometric error δ_i.
   ③ If $\delta_i \le \delta_{\max}$, it means that the feedrate V_i^f also satisfies the requirement of geometric error and keeps the feed rate unchanged.
   ④ Else if $\delta > \delta_{\max}$, determine the new feasible parameter increment δu using a bisection method until $|\delta_{\max}-\delta|<\xi$. ξ can be given as 0.0001 mm in the interpolator. Then, for the given interpolator period T, calculate the desired feedrate using the formula $V_i^g=\kappa(u_{i+1}-u_i)/T$. Let $V_i^f = V_i^g$ and update the structural array.

(4) If $i=n$, exit the adaptive feedrate scheduling procedure.

3.2 Linear and angular feed Acc/Dec control

The limits of maximum geometric error and angular feed have been considered in the above adaptive interpolator algorithm. However, the interpolator might lead to the rate of increasing/reducing linear feed and angular feed beyond machining process requirements especially in the regions with sensitive points such as high curvature point and angular feed sensitive point with dramatic change of cutter posture. Essentially, the linear and angular accelerations are influenced by each other. Here, a sequent linear and angular Acc/Dec control strategy is proposed for simplifying the adjustment process of feed Acc/Dec control

3.2.1 Linear Acc/Dec control

In linear Acc/Dec control, it is essential to estimate the position of deceleration start point and ensure the distance requirement of reducing feedrate with maximum allowable deceleration when approaching the sensitive point. Curve interpolation is the process of finding a sequence of cutter location points with a time interval of sampling cycle between any two successive points. For two successive points $\mathbf{P}_i(u_i)$ and $\mathbf{P}_{i+1}(u_{i+1})$, their corresponding linear feed and angular feedrate are $V(u_i)$, $\omega(u_i)$ and $V(u_{i+1})$,$\omega(u_{i+1})$, respectively. If assuming the cutter feeds forward from position u_i to u_{i+1} with a maximum acceleration $a_{\max}$, then we have

$$\begin{cases} V_{i+1} = V_i + a_{\max} T \\ V_i T + \frac{1}{2} a_{\max} T^2 = \kappa(u_{i+1} - u_i) \end{cases} \tag{14}$$

Furthermore, arrange the above formula, and we can obtain the following expression:

$$\left(V_{i+1}^2 - V_i^2\right)/(u_{i+1} - u_i) = 2a_{\max}\kappa = const \tag{15}$$

The method of linear feed Acc/Dec control is shown in Fig. 4. The vertical coordinate of the figure is half of the square of linear feedrate and the horizontal coordinate is the arc length of cutter tip path. In the figure, the curve segment DE is feed deceleration sensitive region. From kinematic analysis, we know that the $v^2/2-s$ curve is a linear segment when the cutter tip point moves from point E to C with a constant maximum acceleration. This gives us a simple and intuitive linear feed Acc/Dec control method. For example, after linear feed deceleration control, the initial feed profile of the deceleration sensitive region DE is modified as CE. The process of treating the sensitive regions of Acc/Dec mainly includes the detection of Acc/Dec sensitive point and the profile modification of corresponding feed sensitive region. Before giving the detailed algorithm, the following necessary elements are defined first.

- Acceleration sensitive point: for a point P_i, if $V_i<V_{i+1}$,$a_i<a_{\max}$ and $|a_{i+1}|>a_{\max}$, then the point P_i such as point F in Fig. 4 is an acceleration sensitive point. The path parameter u_i associated with point P_i is acceleration sensitive position.
- Deceleration sensitive point: for a point P_i, if $V_{i-1}>V_i$, $|a_{i-1}|>a_{\max}$ and $a_i>-a_{\max}$, then the point P_i such as point E in Fig. 4 is a deceleration sensitive point. Similarly, its corresponding parameter position is deceleration sensitive position.

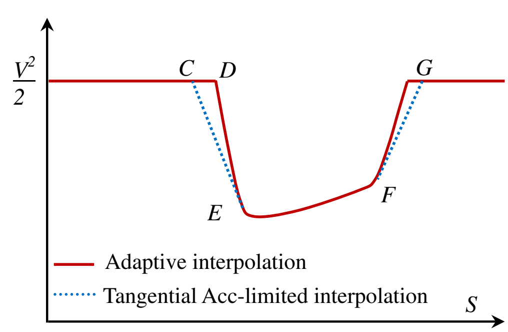

Fig. 4 Tangential acceleration control at feed sensitive region

From Fig. 4, we can see that acceleration sensitive point F is also the start point of this acceleration sensitive region. Thus, we can check and modify the feedrate in the structural array from left to right. However, for the deceleration sensitive region, it is difficult to find the start point C on the feed profile of deceleration sensitive region since only the deceleration sensitive point E keeps unchanged after feed profile modification. This issue is usually solved by a look-ahead strategy. For off-line operation, an alternative way is inversely tracing the structural array from right to left. In this case, the deceleration sensitive point becomes an acceleration sensitive point. Then, the feedrate associated with each parameter position u_i can be checked one by one. Based on this idea, a bi-directional scan algorithm of feed Acc/Dec control is given and summarized as follows:

Input: the number of sampling point n, parameter interval Δu maximum allowable linear feed Acc/Dec $a_{\max}$, and the structural array including the information of sampling points.

(1) Traverse the structural array from left to right. For the ith array element, judge whether the parameter position u_i associated with this element is an acceleration sensitive position. The acceleration corresponding to the current parameter position is calculated as

$$|a_{i+1}| = \left|V_{i+1}^2 - V_{\mathrm{i}}^2\right| / (2\kappa \Delta u)$$

(2) If parameter position u_{i+1} is an acceleration sensitive point and $i<n-1$, update the feedrate V_{i+1}in the $i+1$th element using the following formula:

$$V_{i+1} = \sqrt{V_i^2 + 2\kappa a_{\max} \Delta u}$$

(3) If $i<n-1$, let $i=i+1$, and go to step 1.

(4) Traverse the structural array from right to left. Using the same strategy to check and modify the deceleration sensitive region of the given feed data until the whole array is traversed.

(5) Exit the module of linear feed Acc/Dec control.

From the bi-directional scan algorithm of feedrate modification, it can be seen that Acc/Dec control is easily conducted and can be checked one by one without using a look-ahead strategy.

3.2.2 Angular feed Acc/Dec control

In five-axis machining, the cutter axis orientation $\boldsymbol{H}(u)$ is continuously changing when the cutter tip feeding along the tool path $\boldsymbol{P}(u)$. From Eq. (11), the change rate of the magnitude of angular velocity with respect to time t can be given as

$$\begin{aligned} \frac{d\omega(u)}{dt} &= \frac{d\left\|\dot{H}(u)\right\|}{du}\left(\frac{du}{dt}\right)^2 + \left\|\dot{H}(u)\right\|\frac{d^2u}{dt^2}, \dot{H}(u) \\ &= \frac{dH(u)}{du} \end{aligned} \tag{16}$$

According to the relations between the normalized path parameter u and the total path length, we have

$$\frac{du}{dt} = \frac{d(s/\kappa)}{dt} = \frac{V(u)}{\kappa},\ \frac{d^2u}{dt} = \frac{d(V(u)/\kappa)}{dt} = \frac{a(u)}{\kappa} \tag{17}$$

Then, if $\omega(u)\neq 0$ the magnitude of angular feed acceleration $A(u)$ is given as

$$\begin{aligned} |A(u)| &= \left|\frac{d\omega(u)}{dt}\right| \\ &= \left|\frac{1}{\kappa^2}\frac{d\left\|\dot{H}(u)\right\|}{du}V(u)^2 + \frac{\left\|\dot{H}(u)\right\|}{\kappa}a(u)\right| \end{aligned} \tag{18}$$

It shows that angular feed acceleration has a linear relation with the square of linear feedrate $V(u)^2$ and linear feed acceleration $a(u)$. For a point on the cutter tip path $\boldsymbol{P}(u)$, the feasible region of linear feedrate $V(u)$ and linear feed acceleration $a(u)$ of this point can be determined by the following inequality:

$$-A_{\max} \le \frac{1}{\kappa^2}\frac{d\left\|\dot{H}(u)\right\|}{du}V(u)^2 + \frac{\left\|\dot{H}(u)\right\|}{\kappa}a(u) \le A_{\max} \tag{19}$$

If the above inequality is specified as constraint condition to obtain the feed profile of the whole tool path using an optimization strategy, it is time-consuming and even possibly unable to give exact solutions when simultaneously considering various constraints such as geometric error, angular feed, and feed acceleration. Hence, it is desirable to find an effective and easy-to-realize method of controlling angular feed acceleration. Here, a proportional adjustment method of feedrate is proposed to control the angular feed acceleration within the preset value. For the parameter position u_i and u_{i+1} with an infinitesimal parameter interval, we have

$$\rho_o = \left\|\frac{dH(u)}{du}\right\|,\ \ \rho_p = \left\|\frac{dP(u)}{du}\right\| \tag{20}$$

Then Eq. (12) is given as

$$\omega(u_i) = \frac{\rho_o^i}{\rho_p^i} V(u_i), \quad \omega(u_{i+1}) = \frac{\rho_o^{i+1}}{\rho_p^{i+1}} V(u_{i+1}) \tag{21}$$

Further

$$A = \frac{(\omega(u_{i+1}) - \omega(u_i))}{\Delta t} = \frac{\rho_o^{i+1} V_{i+1} \big/ \rho_p^{i+1} - \rho_o^i V_i \big/ \rho_p^i}{\Delta t} \tag{22}$$

The time from parameter position u_i to u_{i+1} with a constant acceleration can be given as

$$\Delta t = \frac{V_{i+1} - V_i}{a} = \frac{2(V_{i+1} - V_i)\kappa(u_{i+1} - u_i)}{V_{i+1}^2 - V_i^2} = \frac{2\kappa(u_{i+1} - u_i)}{V_{i+1} + V_i} \tag{23}$$

If the feedrates associated with the two points are changed by the same coefficient τ, then the moving time is

$$\Delta t* = \frac{2\kappa(u_{i+1} - u_i)}{\tau(V_{i+1} + V_i)} = \frac{\Delta t}{\tau} \tag{24}$$

Correspondingly, the angular acceleration becomes

$$A^* = \frac{\tau(\rho_o^{i+1} V_{i+1} \big/ \rho_p^{i+1} - \rho_o^i V_i \big/ \rho_p^i)}{\Delta t / \tau} = \tau^2 A \tag{25}$$

From the above equation, one can see that, If $A > A_{\max}$, the angular acceleration can be reduced to the maximum value $A_{\max}$ by reducing the feedrate with a coefficient $\tau(\tau<1)$. Thus, we have

$$A_{\max} = \frac{\tau(\rho_o^{i+1} V_{i+1} \big/ \rho_p^{i+1} - \rho_o^i V_i \big/ \rho_p^i)}{\Delta t / \tau} = \tau^2 A \tag{26}$$

It shows that if the angular acceleration A is beyond the preset value $A_{\max}$, we can control the angular acceleration with the following approximation formula:

$$V_i^* = V_i \tau, \quad \tau = \sqrt{\mathrm{A_{max}/A_i}} \tag{27}$$

It also verified by Eq. (15). For example, if the feedrate of each point along the cutter tip path is reduced by multiplying a coefficient τ, when the cutter tip is assumed to feed between the two points with a constant acceleration, the linear feed acceleration is given as

$$a^*(u) = \tau^2 \left(V_{i+1}^2 - V_i^2\right) / (2\kappa(u_{i+1} - u_i)) = \tau^2 a(u) \tag{28}$$

As shown in Fig. 5, for an angular acceleration sensitive segment FG where the angular accelerations of two regions AB and DC are beyond the preset values, the two regions can be readjusted, respectively, with the formula (27), and then, an operation of linear Acc/Dec control is conducted on the modified feed profile. On the other hand, in order to reduce the feed fluctuations and the complexity of adjustment process, the whole feed profile of this sensitive region can also be adjusted with the formula (27) according to the real maximum angular acceleration. Thus, the angular accelerations of all points in this region must, thus, be within the preset values. This case only occurs in the valley region such as curve segment FG whose feedrate is less than command feedrate except that of two end points. Another case of possible feedrate adjustment is that the feedrate of angular acceleration sensitive region is constant and equal to command feedrate, and thus the sensitive region whose attached angular feed Acc/Dec are all beyond the allowable maximum values at sampling positions needs to be adjusted. The concrete process of angular feed Acc/Dec control is given as follows:

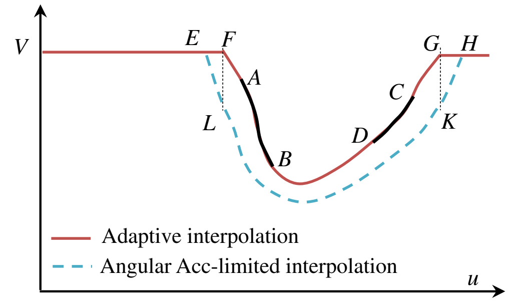

Fig. 5 Angular feed acceleration control at feedrate sensitive region

Input: the number of sampling point n, parameter interval Δu, maximum allowable angular feed Acc/Dec $A_{\max}$, and the structural array including the feedrate information of sampling points.

(1) Let $J=0$ and traverse the structural array from left to right. For the whole array, find all the angular acceleration sensitive regions and recorder the number J of sensitive regions as well as the start and end positions of each sensitive region.
(2) If $J>0$, let $j=1$; else, go to step 5.
(3) For the jth angular feed acceleration sensitive region, determine the parameter position u^j with a maximum magnitude of angular feed Acc/Dec, after computing the angular feed acceleration $A(u^j)$ the corresponding coefficient τ^j can be obtained, and then update the corresponding feedrate of this sensitive region with the formula (27).
(4) If $j<J$, let $j=j+1$, and go to step 3. Else, for the updated structural array, a bi-directional scan operation is conducted on the whole tool path to further control the linear feed Acc/Dec.
(5) Fit the NURBS feed curve to the feed data saved in the array with respect to the path parameter, and then, exit the model of angular feed Acc/Dec control.

From the above algorithm, it can be seen that feed Acc/Dec control is an iterative process of angular feed acceleration and linear feed acceleration. Using this way, the angular and linear feed accelerations can be easily confined within the preset range.

4 Illustrated examples

In the section, the illustrated example is given to evaluate the feasibility and applicability of the developed five-axis NURBS interpolator algorithm. A good desirable performance of the interpolator method has been obtained. The proposed NURBS interpolator algorithm for five-axis machining was validated on a given dual NURBS tool path, as shown in Fig. 6. The two curves of constructing the cutter axis surface both have 42 control points, and their weights are set to one. The cutter tip curve is parameterized by normalized arc length. From Fig. 6, one can see that the cutter axis orientation and path curvature both have dramatic change at some sensitive regions. It is quite possible that geometric error, angular feedrate or Acc/Dec sensitive points will be generated in these regions. All parameters of the five-axis NURBS interpolator for numerical simulations are listed in Table 1 unless stated otherwise. The comparison results of the chord error and complete geometric error are given first in Fig. 7a. Simulation results of feedrate, geometric error, angular feed, angular feed Acc/Dec, and linear feed Acc/Dec are shown in Fig. 7b–f. The comparisons are made among the constant feedrate interpolator, adaptive interpolator, and Acc/Dec limited adaptive interpolator.

From Fig. 7a, it can be seen that the nonlinear geometric error aroused by the interpolator in five-axis machining is dominant compared with the chord error. The chord error is only about 1 μm, while the maximum geometric error is about 10 μm. Meanwhile, the geometric error increases with the increase of distance from the machine joint point to the cutter tip point. The longer the cutter shank is, the larger the geometric error is. It is highly desirable to use a short cutter during five-axis machining.

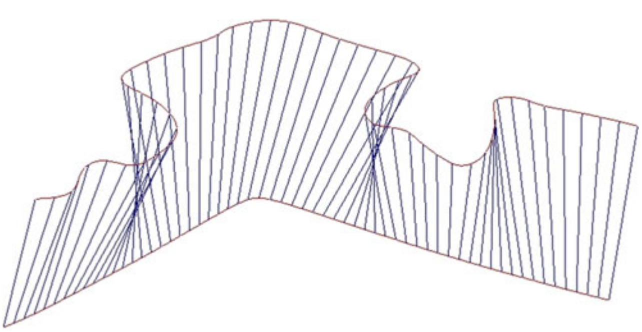

Fig. 6 The dual NURBS cutter axis surface

Table 1 Five-axis NURBS interpolator parameters

Parameter	Value
Sampling time	0.002 s
Geometric error	0.003 mm
Maximum angular feedrate	4 rad/s
Maximum feedrate	100 mm/s
Maximum linear feed acceleration	500 mm/s^2
Maximum angular feed acceleration	50 rad/s^2

The feed profiles using three different interpolator methods are illustrated in Fig. 7b. Compared with the only adaptive interpolator (ADI), the feedrate of the linear feed Acc/Dec limited adaptive interpolator (L-ADI) has a modification. The reason is that although ADI can ensure the geometric error and angular feedrate are within preset ranges, it cannot avoid the abrupt change of the feedrate, which probably leads to the linear feed Acc/Dec exceeding the allowable range. Furthermore, the feedrate profile of the angular and linear feed Acc/Dec limited adaptive interpolator (AL-ADI) is made another modification near the first valley of the whole feedrate curve for the purpose of ensuring the angular feed Acc/Dec also within the preset value. Using AL-ADI, the feedrate is kept constant during most of the machining time, and the machining quality is ensured at sensitive regions by constraining the geometric error, angular feedrate, and feed Acc/Dec. Similarly, as shown in Fig. 7c, the peak of the angular feedrate profile of ADI is obviously smaller than that of constant feedrate interpolator. The largest error appearing due to sharp cutter axis variations is reduced by slowing down the feedrate in sensitive areas using the proposed algorithm. It proves that the adaptive interpolator is also able to confine angular feedrate within the allowable range.

Figure 7d shows the geometric errors of four interpolator algorithms. It can be seen that the maximum geometric error is significantly reduced and can be controlled within 0.003 mm using the proposed adaptive algorithms, while the maximum geometric error is up to 6.8 μm if constant feedrate interpolator is adopted. Meanwhile, although the adaptive interpolators with or without Acc/Dec limited function, all can keep the geometric error under control, the geometric error generated by Acc/Dec limited adaptive algorithm is still smaller than those of ADI in some narrow parameter intervals owning to the feedrate modification in the sensitive regions.

Figure 7e shows that if only with ADI, the feed acceleration has abrupt jump with a maximum value approaching 1,600 mm/s^2 in the two feedrate sensitive areas. However, if using L-ADI, the maximum value is reduced to 1,000 mm/s^2, and the linear Acc/Dec always changes within the preset range (−1,000 mm/s^2, 1,000 mm/s^2). Thus, the

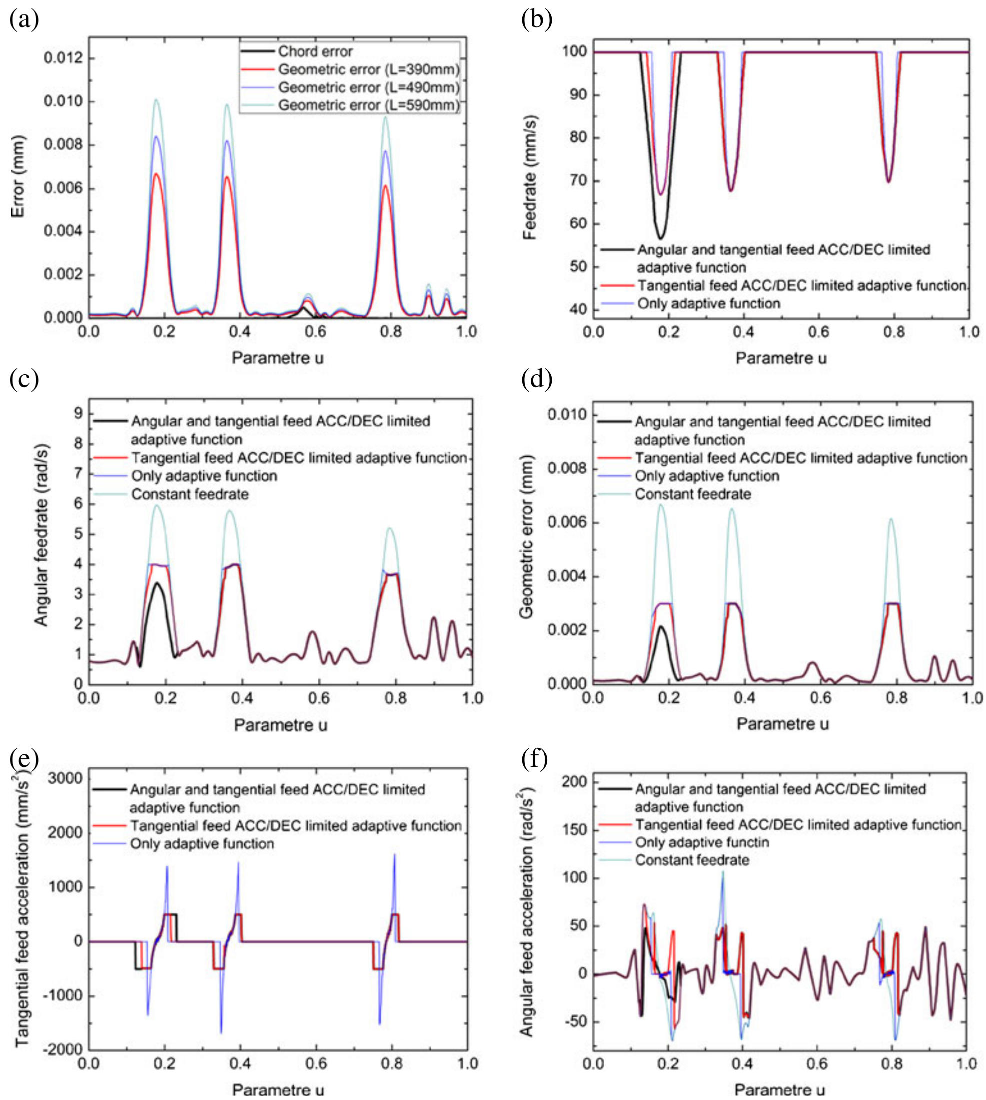

Fig. 7 Feedrate scheduling for a dual NURBS ruled cutter axis surface using different interpolator algorithms: **a** comparison results of chord error and geometric errors; **b** feedrate profile; **c** angular feedrate; **d** geometric error; **e** tangential feed Acc/Dec; **f** angular feed Acc/Dec

cutter shock can be supervised to ensure the quality requirements of the machined parts. Furthermore, as shown in Fig. 7f, the magnitude of angular feed acceleration is 105 and 75 rad/s^2, respectively, when using ADI and L-ADI. Only using AL-ADI, the angular feed Acc/Dec can be confined within the preset range (−50 rad/s^2, 50 rad/s^2). It is certain that AL-ADI has the ability to confine the geometric error, angular feedrate, linear and angular feed Acc/Dec within the allowable range in feedrate sensitive regions while maintaining command feed throughout the rest of the dual NURBS tool path in five-axis machining.

5 Conclusions

In this study, an adaptive feedrate scheduling of dual NURBS curve interpolator, which is able to control the geometric error, angular feedrate, and linear and angular feed Acc/Dec within the preset values, is proposed for five-axis machining, especially for flank machining. The scheduling method mainly concentrates on the assurance of geometric accuracy and kinematic performance of cutter axis. The property of the NURBS interpolator with variable feedrate is that each constraint of the interpolator is independent and one can freely determine its maximum allowable magnitude or relax it according to the real requirements. In general, the interpolator is independent of machine configurations except geometric error module. In the interpolator, the whole feed profile curve is first off-line constructed, and then, the interpolation is real-time performed using a Taylor expansion formula. The feasibility of the proposed five-axis NURBS interpolator method has been validated by illustrated examples. The comparison results show that the geometric error and feed fluctuations can be effectively constrained and the proposed interpolator is helpful for the improvements of surface machining quality, accuracy, and efficiency in open five-axis machine tools.

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