

Collision and interference correction for impeller machining with non-orthogonal four-axis machine tool

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Abstract Aiming at the four-axis machining with inclined rotary working table, an approach for collision and interference correction is proposed in this paper. Based on the principle that the tool axis vector is limited in the corresponding rotation plane of four-axis machine tools, collision and interference can be eliminated by using the rotation of the ellipse which is the intersection of the plane and the cutter surface. And the minimum necessary rotation angle for the cutter to eliminate interference is obtained by solving the quartic polynomial equations with numerical method. Finally, the process techniques involved in impeller machining with non-orthogonal four-axis machine tools are developed thoroughly, and the experiments of free-form centrifugal impeller machining are rendered with details. Experiment results show that the approach is an effective approach to detect collision and avoid interference for four-axis machining and it can be directly implemented into current CAD/CAM software to promote impeller machining in industry.

Keywords Integral 3D impeller · Four-axis machining · Inclined rotary working table · Interference correction · Non-orthogonal machine tool

1 Introduction

The impeller is an important part of major equipment. Thanks to advanced CAD/CAM techniques, impellers with complex features have been widely adopted in many

industries and they are often cut with multi-axis computer numerical control (CNC) milling [1]. In order to get more feasible accessibility, five-axis CNC milling technique is often used to machine impellers, including roughing for high cutting efficiency and finishing for high accuracy [2–4]. Therefore, five-axis machining center are the ideal equipment for impeller machining. However, in some manufacturing enterprises, there are also lots of four-axis machine tools. How to make full use of these four-axis machine tools to produce high value-added impeller is an important issue in these enterprises. Besides, the five-axis machine tool is usually more expensive than the four-axis machine tool, the utilization of four-axis machine tool may reduce the equipment purchase cost, thus reduce the machining cost of impellers especially for small enterprises[5].

While machining free-form surface impellers, the collision and interference between the cutter and the workpiece must be avoided to get qualified final workpiece. During the last few decades, lots of collision detection and interference correction approaches, such as C-space method [6], bounding volume method [7], distance monitoring method, and radial projection method, have been developed [8–11]. Balasubramaniam et al. [12] applied an efficient bounding volumes hierarchy, i.e., a point cloud representation for the workpiece, a constructive solid geometry one for the tool to reduce the interference problem by sampling point inclusion queries. But it tends to lose efficiency and requires a substantial amount of memory. Ilushin [13] proposed an approach that uses space subdivision techniques and ray-tracing algorithms to derive a highly accurate polygon/surface–tool intersection algorithm for global collision detection and avoidance. Tang et al. [14] presented a tool path verification method through computing parallel slices of the workpiece, the reference part and the tool geometry with constant intervals named sweep plane algorithm, which allows general tool geometry and supports collision detection

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between the workpiece and the tool, tool holder, and fixtures, but the precision relies on the distance between the slices and the behavior of the surface.

Since the tool can only reach specified orientations with four-axis machine tool, the accessible region is much smaller than it with five-axis machine tool. Aiming at tool interference in four-axis machining, Suh [15] proposed a handling algorithm based on the cutter location (CL) plane that determines an interference-free tool axis. While machining impeller with four-axis machine tool, Wu [16] proposed a method based on intersection of two conical surfaces to determine interference-free tool axis. Luo [17] proposed to generate representative tool orientation first; then, other tool orientations can be obtained by interpolation algorithm to get smooth tool orientations in four-axis machining. All the above research studies focus on orthogonal machine tools which cannot be employed directly for non-orthogonal four-axis machine tools.

Aiming at the four-axis machining with inclined rotary working table, an approach of collision and interference correction is proposed in this paper. First, four-axis machining of impeller is introduced in Section 2. Collision and interference correction for four-axis machining is proposed in Section 3. Finally, in Section 4, the proposed method and experiments are discussed to demonstrate the effectiveness and advantages of this innovative method.

2 Four-axis milling of impeller

The rotary coordinate of four-axis NC machine tools is usually realized by the working table, which is provided as an accessory of the machine to meet the technical requirement and then reach any motion forms according to the commands. As shown in Fig. 1, when machining a 3D impeller with non-orthogonal four-axis machine tools, the working table is usually settled at an inclined angle to get open space and reduce tool length as well

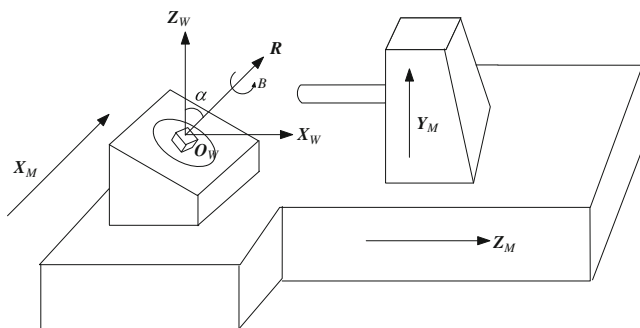


Fig. 1 Four-axis machining with inclined rotary working table

as to avoid the collision and interference; the inclined angle is represented by α . With the inclined angle, the rotary axis of the working table is axis R , and then the rotary of the working table can be treated as the cutter's rotation about axis R .

Different from five-axis milling, tool orientations of this method are constrained within a certain conical surface, the rotation axis of the conical is R and its half apex angle is $(\pi/2 - \alpha)$. Therefore, the main issue of four-axis milling is to find the proper tool orientation on the cone surface while avoiding collision. Let $\{O_W; X_W, Y_W, Z_W\}$ be the workpiece coordinate system (WCS) and $\{O_M; X_M, Y_M, Z_M\}$ be the machine tool coordinate system (MCS). The two coordinate systems satisfy the right-hand rule and their origin points coincide, then the following equations is valid:

$$\begin{cases} X_W = Z_M \\ Y_W = X_M \\ Z_W = Y_M \end{cases}$$

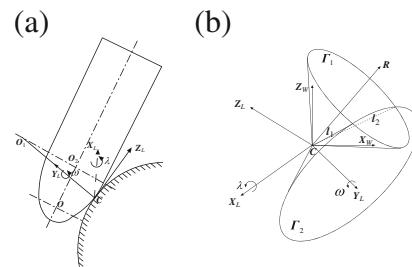
Hence, the rotation matrix about Y_M in the WCS is

$$M_{\text{roty}} = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix} \tag{1}$$

Then, the rotation axis R in the MCS can be expressed as:

$$R = (0 \ 0 \ 1)M_{\text{roty}} = (\sin \alpha \ 0 \ \cos \alpha)$$

As shown in Fig. 2a, at cutter contact point C , $\{O_L; X_L, Y_L, Z_L\}$ is the local coordinate system. X_L is the feed direction, Y_L is the normal direction at point C , and Z_L is the step over direction. The cutters can rotate about X_L and Y_L in the local coordinate system in five-axis milling. But in four-axis milling, there are more constraints. As shown in Fig. 2b, after the tool rotate angle λ about X_L , it always lies on the conical surface F_2 when rotating about Y_L . In addition, while machining with inclined working table, tool orientation also lies on the conical surface F_1 shown in Fig. 2b. Therefore, the final tool orientation is the intersection line l_1 and l_2 between F_1 and F_2 .



Local coordinate system Tool orientation selection in four-axis milling

Fig. 2 Tool orientation selection in four-axis milling. a Local coordinate system. b Tool orientation selection in four-axis milling

3 Collision and interference correction for four-axis machining

3.1 Collision and interference correction

In five-axis machining, the tool can reach any orientation within the working space, so the interference can be corrected by rotating the tool around an arbitrary direction. But in four-axis machining with a non-orthogonal machine tool, as shown in Fig. 3, the tool can only rotate around R axis. Therefore, when collision and interference occur, the correction of the tool is constrained in the conical surface, which causes more difficulties in interference correction than in five-axis machining case.

As shown in Fig. 4, while eliminating interference in four-axis machining with an inclined working table, P is the interference point, Γ is the plane perpendicular to R axis and point P lies on Γ , and E is the intersection profile of plane Γ and the cutter envelope. Actually, the correction of collision is to locate the point P outside the cutter envelope by adjusting the tool axis vector. Within plane Γ , the rotation of cutter around R axis is the same as the rotation of ellipse E around R axis. When ellipse E is rotated to a certain correction angle where point P locates outside the ellipse, this means that point P is located outside the cutter surface, i.e., elimination of interference. Therefore, the next step is to determine the correction angle.

3.2 Correction angle calculation

In the workpiece coordinate system, the equation of plane Γ can be expressed as:

$$R_x x + R_y y + R_z z + D = 0 \tag{2}$$

Where $D = -(R_x P_x + R_y P_y + R_z P_z)$. Let the cutter location at current contact point be $CL = (CL_x, CL_y, CL_z)$, then the equation of axis R' which is parallel to R can be expressed as:

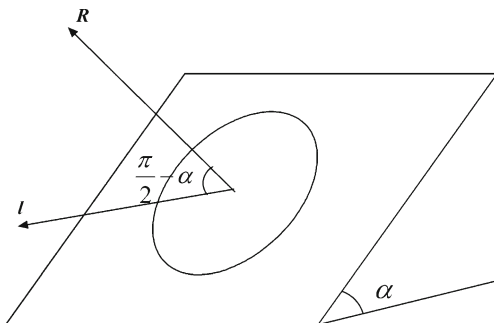


Fig. 3 Tool rotation in four-axis machining with non-orthogonal machine tool

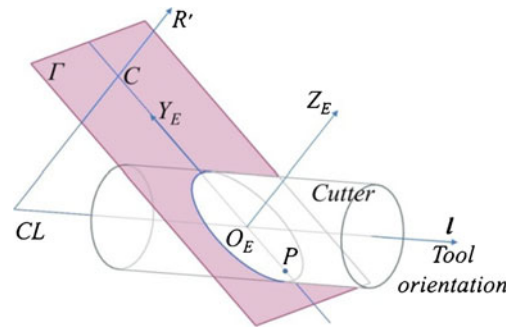


Fig. 4 Interference correction for four-axis with inclined rotary working table

$$\frac{x - CL_x}{R_x} = \frac{y - CL_y}{R_y} = \frac{z - CL_z}{R_z} = k_1 \tag{3}$$

Where k_1 can be obtained by solving Eq. (2) and Eq. (3) simultaneously:

$$k_1 = -\frac{R_x CL_x + R_y CL_y + R_z CL_z + D}{R_x^2 + R_y^2 + R_z^2} \tag{4}$$

Point C is the intersection of R' and Γ , and the location of point C in the workpiece coordinate system is (x_0, y_0, z_0) , determined by substituting k_1 into Eq. (4). Point C , the rotation center of ellipse E around line R' during interference correction, is called as oscillation center of the cutter, as shown in Fig. 4. Let the tool orientation be $l = (l_x, l_y, l_z)$, as tool lines on the line CL , then the equation of CL is:

$$\frac{x - CL_x}{l_x} = \frac{y - CL_y}{l_y} = \frac{z - CL_z}{l_z} = k_2 \tag{5}$$

Based on Eq. (4), the following expression can be obtained:

$$k_2 = \frac{-(D + R_x CL_x + R_y CL_y + R_z CL_z)}{R_x l_x + R_y l_y + R_z l_z} \tag{6}$$

Point $O_E(X_E, Y_E, Z_E)$ is the intersection point of the tool axis and the plane perpendicular to the revolution axis in the workpiece coordinate system; it can be obtained by substituting k_2 into Eq. (5). Point O_E is the center of ellipse E .

The axis of tool cylinder is l , tool radius is r_f , normal vector of plane Γ is R , the angle between l and R is $\theta = \pi/2 - \alpha$, according to knowledge of descriptive geometry, when the tool cylinder is truncated obliquely by the plane Γ , resulted ellipse E has a minor semi-axis which is equal to tool radius r_f , in direction $R \times l$, and a major semi-axis is equal to $R/\cos \theta$, in the direction $R \times (R \times l)$, as shown in Fig. 5.

In Fig. 6, set $X_E = R \times l$, $Y_E = R \times (R \times l)$, $Z_E = R$ and take the center O_E of ellipse E as the origin of coordinate, X_E, Y_E, Z_E as the base vectors to establish local coordinate system, which is called elliptic coordinate system.

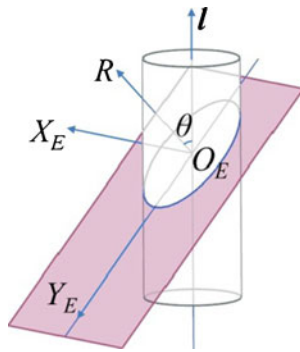


Fig. 5 Ellipse on a cylinder truncated obliquely by a plane correction

Let

$$\begin{aligned}
 a_1 &= O_E(1); & a_2 &= O_E(2); & a_3 &= O_E(3); \\
 a_{11} &= X_E(1); & a_{21} &= X_E(2); & a_{31} &= X_E(3); \\
 a_{12} &= Y_E(1); & a_{22} &= Y_E(2); & a_{32} &= Y_E(3); \\
 a_{13} &= Z_E(1); & a_{23} &= Z_E(2); & a_{33} &= Z_E(3);
 \end{aligned}
 \tag{7}$$

The transformation matrix between ellipse and workpiece coordinate systems is M

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
 \tag{8}$$

Matrix M is an orthogonal matrix, and $M^{-1} = M^T$.

In the workpiece coordinate system, the oscillation center of the tool is $C(X_0, Y_0, Z_0)$, and then point C under the elliptic coordinate system becomes:

$$(x_0, y_0, z_0) = [(X_0, Y_0, Z_0) - (a_1, a_2, a_3)]M
 \tag{9}$$

Obviously, $z_0=0$.

Figure 6 shows the interference during four-axis machining with an inclined working table along R direction in the elliptic coordinate system, i.e., projection of Fig. 5 on plane $X_E O_E Y_E$. As shown in Fig. 6, C is the intersection point of plane Γ vertical to rotary axis and rotary axis R , P is the interference point. In this plane view, the rotation of tool means the rotation of ellipse around point C , and the process of interference correction is to make the point P outside ellipse E by rotating the ellipse.

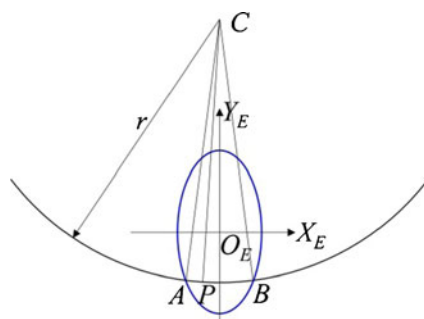


Fig. 6 Plane view of interference during four-axis machining

In order to eliminate interference ellipse E , an approach is proposed here. The circle, whose center is C and radius is $|CP|$, intersects ellipse E at points A and B . In the elliptic coordinate system $X_E O_E Y_E$, the expressions of arc \widehat{AB} and ellipse E are in Eq. (10).

$$\begin{aligned}
 (x - x_0)^2 + (y - y_0)^2 &= r^2 \\
 x^2 + y^2 \cos^2 \alpha &= r_f^2
 \end{aligned}
 \tag{10}$$

Where $r = |CP|$, and r_f is the tool radius. Points A and B in Fig. 6 can be obtained by solving Eq. (10). Solution of this quartic polynomial can be directly obtained by numerical method or command “roots” from MATLAB. $\angle ACP$ or $\angle PCB$ is the minimum necessary rotation angle for the tool to eliminate interference. But in actual integral impeller machining, there is only one valid rotation direction, and the rotation in the opposite direction will lead cutter to pierce the blade. Hence, suitable direction and tool rotation angle must be determined.

3.3 Determination of tool rotation direction

When interference occurs, rotation of tool axis should keep the cutter away from interference area. For impeller machining, when interference occurs between the tool and the machined surface, the tool axis vector has to be rotated toward the direction of outward normal vector of machined surface. And when interference occurs on the adjacent sculptured surface, the tool axis should be rotated against to the direction of outwards normal vector of machined surface.

At the cutter contact point, the normal vector of machined sculptured surface is n , and let β be the required tool rotation angle around rotary axis R to correct interference. There are two cases when determining angle β :

1. If interference occurs on the machined surface and meets the following equation

$$n \cdot AP > 0, n \cdot BP < 0$$

$$\text{then } \beta = \angle ACP; \text{ otherwise, } \beta = \angle PCB.$$

2. If interference occurs on the adjacent surface and satisfies the expression:

$$n \cdot AP > 0, n \cdot BP < 0$$

$$\text{then } \beta = \angle PCB; \text{ otherwise, } \beta = \angle ACP.$$

After the determination of β , the rotation direction of tool axis must be defined to make the interference point outside the tool. Assuming that the rotation angle $\beta = \angle ACP$, then the rotation direction of tool can be determined according to the following rules:

1. If $(CA \times CP) \cdot R > 0$, one can rotate the tool counter clockwise around R angle β ;

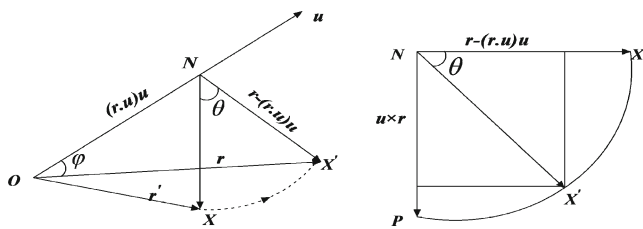


Fig. 7 Rotation of the vector around an arbitrary axis through original point

- If $(CA \times CP) \cdot R < 0$, one can rotate the tool clockwise around R angle β .

In addition, to ensure accuracy and security, a safe angle β_{safe} in correction direction is needed based on the rotary angle β , here set β_{safe} as 2° . So the rotation angle of tool axis vector is:

$$\beta_T = \text{sign}(\beta)(|\beta| + \beta_{safe}) \tag{11}$$

3.4 Calculation of the new tool orientation

Tool rotation around R can be regarded as the rotation of the vector around an arbitrary axis through original point. As shown in Fig. 7a, θ is the angle and the vector r rotates around the line through original point in u direction (agree with right-hand rule, counterclockwise is positive and clockwise is negative). During rotation, vector r in u direction ON does not change, and the component NX arrives at NX' after a rotation angle θ . For $ON = r \cdot u$, the vector NX can be expressed as $r - (r \cdot u)u$. Figure 7b shows view along the u direction and vector $u \times r$ is expressed by NP . As in $|r - (r \cdot u)u| = |r| \sin \phi = |u \times r|$, then $|NX| = |NX'| = |NP|$.

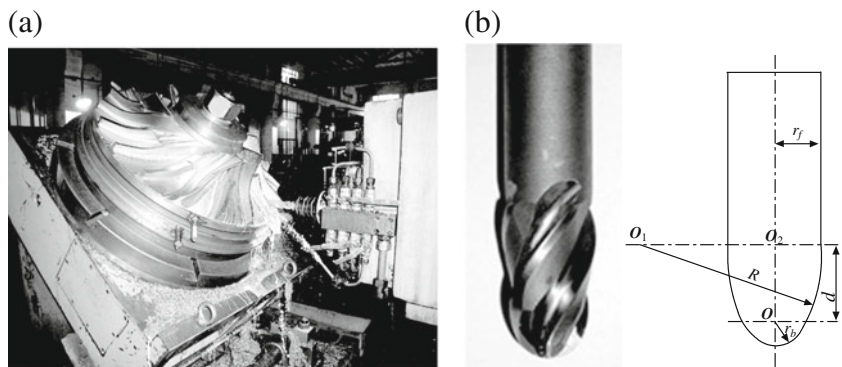
NX' can be decomposed in two directions NX and NP ,

$$NX' = [r - (r \cdot u)u] \cos \theta + (u \times r) \sin \theta \tag{12}$$

The vector after rotation becomes $r' = ON + NX'$, and

$$r' = (r \cdot u)u + [r - (r \cdot u)u] \cos \theta + (u \times r) \sin \theta \tag{13}$$

Fig. 8 Four-axis non-orthogonal machine tool and olive cutter used in the experiments. a Four-axis non-orthogonal machine tool. b Olive cutter



Four-axis non-orthogonal machine tool

Olive cutter

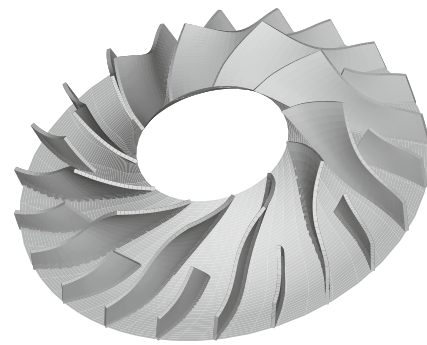


Fig. 9 3D model of the impeller

The position of tool after rotary correction can be obtained by substituting β_T into Eq. (12). Set the tool position after correction in workpiece coordinate system as l_{new} , then

$$l_{new} = (I \cdot R)R + [I - (I \cdot R)R] \cos \beta_T + (R \times I) \sin \beta_T \tag{14}$$

Detection and correction of collision and interference is a reduplicative process. To eliminate the collision between tool and all surfaces, usually one time correction cannot assure the non-collision condition. In addition, the largest interference depth is calculated based on the center of mass which is the intersection between the tool and surfaces, and this may cause deviation from the actual value. So the detection and correction of multi-axis machining is an iterative process. If a tool with bigger radius is chosen, the appropriate tool position to avoid collision may not be found. Therefore, a permitted iterative number is needed. When the collision correction operation exceeds the given number but the collision still exists, tool with smaller radius should be considered.

4 Experiments and discussions

To demonstrate the effectiveness of the presented methods in this paper, machining experiments have been implemented.

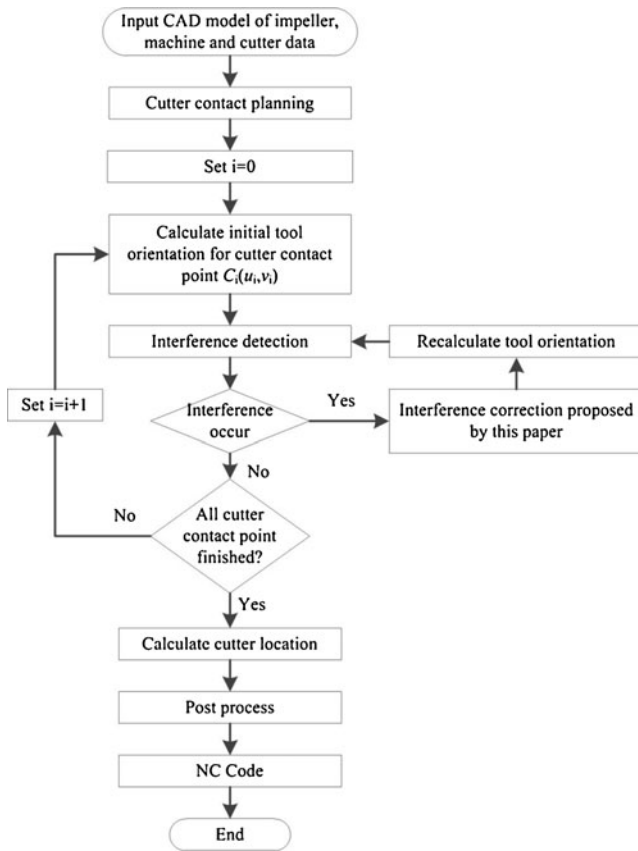


Fig. 10 NC Code generation process for the impeller

4.1 Experimental setup

The machine tool used here is a four-axis non-orthogonal CNC machine tool made by Shanghai Machine Tool Ltd. The machine has translation motion of cutter in *Y* and *Z* directions, and rotation angle *B* and motion in *X* are realized by the rotation and translation of working table. Meanwhile, the inclination angle of working table can be modified by adjusting the wedge-shape filling block. While determining the inclination angle of the working table, collision and interference between the given cutter and impeller can be

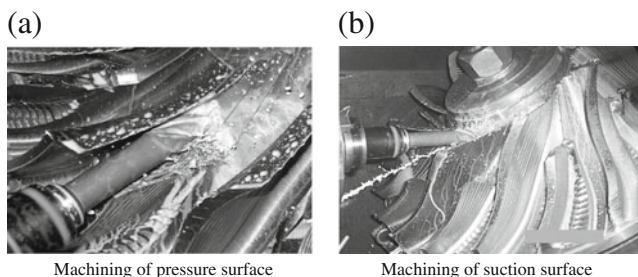


Fig. 11 Clean-up machining process of long blade. a Machining of pressure surface. b Machining of suction surface

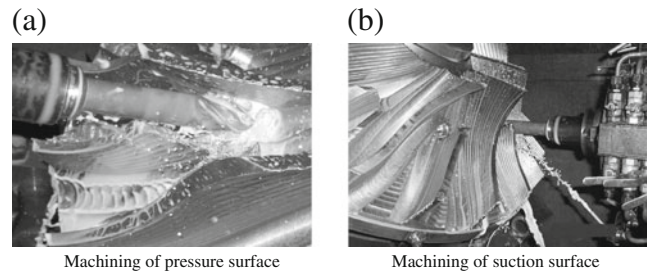


Fig. 12 Clean-up machining process of short blade. a Machining of pressure surface. b Machining of suction surface

analyzed firstly for different inclination angles according to existing approaches [1, 18]. Then, the inclination angle without collision and interference can be chosen as the working table inclination angle. The machine tool and olive cutter used for machining are shown in Fig. 8, the inclination angle α of working table is 40° . As for the olive cutter, the diameter of the ball part is 12.5 mm; it is 16 mm for the cylinder part. As shown in Fig. 8b, the olive cutter is composed of three parts: shank, side edge, and ball tip. The side edge profile is a revolution surface which is tangent to the shank surface, the bus of the revolution surface is an arc with radius of *R* and center *O*₁. The profile of the ball tip is spherical surface and is tangent to the side edge profile and its radius is *r*_b. The side edge is mainly used for blade machining while the ball tip is mainly used for clean-up and hub machining.

4.2 NC code generation

The 3D model of the impeller is shown in Fig. 9, it has 11 long blades and 11 short blades. The thickness of the blade is 8 mm, and the diameter of the impeller is about 870 mm.

NC code generation process for the impeller is shown in Fig. 10. For tool path planning of the impeller, the constant scallop height approach can be used in cutter contact points calculation. The collision detection and correction procedure using method proposed in the paper is as follows. For

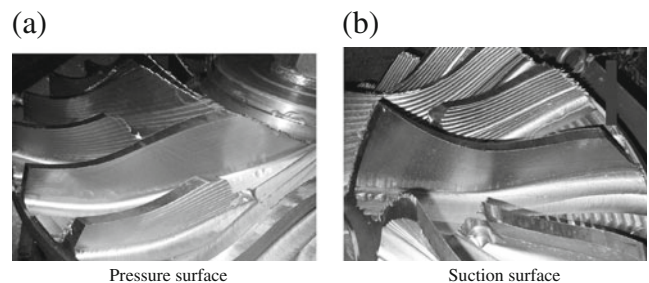


Fig. 13 Finish machining of long blade. a Pressure surface. b suction surface

cutter contact point $C_i(u_i, v_i)$, cutter rotation angle can be calculated according to Eq. (11) when interference occurs. Then, the tool orientation can be calculated by Eq. (14) and the interference check process is carried out again. Furthermore, since the maximum interference depth is calculated by the centroid of the intersection profile between tool and surface, it may lead to a deviation with the actual maximum interference depth. Therefore, the multi-axis machining collision interference checking and correction is an iterative process. If the tool is too large, it may not be able to find a suitable collision-free tool orientation. A permitted iteration number is set in the presented approach; if the collision cannot be eliminated when the iteration number exceeds the permitted iteration number, the tool is too large to machine the surface and a smaller cutter should be employed.

When the collision and interference is checked and corrected for all cutter contact points, cutter location file for impeller machining can be calculated according to the cutter contact points and the tool orientation. NC code can be generated after the post-process and is ready for use in impeller machining.

4.3 Machining results and discussion

In the machining processes, the spindle speed is set to 1,200 rpm and the feed rate is set to 400 mm/min. Figure 11 reveals clean-up machining process of long blade, where interference between tool and workpiece often arises during machining free-form surface impellers with long and short splitter blades.

Figure 12 shows clean-up machining process of a short blade near pressure and suction surfaces of long blade. Interference often emerges between tool and entrance of pressure and suction surfaces on a short blade. The distance between tool and short blade is much larger in machining suction surface. For impeller with short and long blades, the selection of tool radius depends on the minimum distance between pressure surface of long blade and suction surface of short blade.

Figure 13 shows the finished pressure and suction surfaces of long blade, respectively. As seen, the finished blade has excellent surface and evenly well-distributed tool path, and no interference or collision occurs during machining.

5 Conclusions

The most effective machining method to eliminate interference and collision is side milling especially with respect to workpiece like impeller which is of complicated

space shape, narrow flow distortion. Aiming at four-axis machining impeller with non-orthogonal machine tool, this paper proposed an approach to determine the tool axis vector, and then achieved a uniform programming manner according to the establishment of the relationship between the workpiece coordinate and elliptic one. According to the position of interference point in the ellipse, the correction angle and tool orientation are calculated. By timely adjusting the direction and angle value of the tool axis vector, non-interference condition can be obtained, and finally get a clean, smooth, evenly distributed machining surface. Experimental results of finishing operation on pressure and suction planes of long blade verified the accessibility of the method, and it can be applied to other complex parts machining with non-orthogonal four-axis machine tool.

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