Int J Adv Manuf Technol (2013) 68:591–605
DOI 10.1007/s00170-013-4780-9

ORIGINAL ARTICLE

Estimating the effect of process parameters on MRR, TWR and radial overcut of EDMed AISI D2 tool steel by RSM and GRA coupled with PCA

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Received: 16 January 2012 / Accepted: 16 January 2013 / Published online: 13 February 2013

Abstract This paper investigates an optimisation design of the various machining parameters for the electrical discharge machining (EDM) processes on AISI D2 tool steel using a hybrid optimisation method. A new combination of response surface methodology (RSM) and grey relational analysis coupled with principal component analysis (PCA) has been proposed to evaluate and estimate the effect of machining parameters on the responses. The major responses selected for this analysis are material removal rate, tool wear rate and radial overcut or gap, and the corresponding machining parameters considered for this study were pulse current (*Ip*), pulse duration (*Ton*), duty cycle (*Tau*) and discharge voltage (*V*). Thirty experiments were conducted on AISI D2 steel workpiece materials based on a face-centred central composite design. The experimental results obtained were used in grey relational analysis, and the weights of the responses were determined by the PCA and further evaluated using RSM. The results indicate that the grey relational grade (GRG) was significantly affected by the machining parameters considered and some of their interactions. The R^2 value for the GRG model was found to be 0.83, and the optimal parameter setting was determined for the grey relational grades. The analysis of variance results reveal that Tau is the most influencing parameter having 28.57 of percentage contribution followed by Ip, *V* and Ton with 11.52, 5.89 and 5.83 %, respectively. The interaction of the parameters contributes 31.19 % of percentage contribution. These results provide useful information on how to control the machining parameters and thereby responses and ensure high productivity and accuracy of the EDMed component. This method is simple with easy operability, and the results have also been verified by running confirmation tests.

Keywords Electrical discharge machining · Material removal rate · Tool wear rate · Overcut · Response surface methodology · Grey relational analysis · Principal component analysis

1 Introduction

There is a growing trend to use light weight, slim and compact mechanical component in the recent years; hence, there has been an increased interest in the advance materials in modern day industries. These advanced materials having attractive attributes such as high strength, high bending stiffness, good damping capacity, low thermal expansion and better fatigue characteristics, which make them potential materials for modern day industrial application used in mould and die making industries, aerospace component, medical appliance and automotive industries. These industries are facing challenges from such advanced materials, viz. super alloys, ceramics and composites, that are hard and difficult to machine, requiring high precision and surface quality which leads to increase machining cost [1].

For the last six decades, electrical discharge machining (EDM) has been extending inimitable capabilities to machine "difficult to machine" materials with desired shape, size and required dimensional accuracy. It has been impressively applied for machining in the advance industries like automotive, medical, aerospace, consumer electronics and

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optoelectronic industries development. In the past, with the continuing advances of technology, there has been a significant enhancement in EDM technology also, to improve productivity, accuracy and the versatility of the process. The key interest in the active research was to choose the optimal setting of the process parameters in such a way that material removal rate (MRR) and accuracy should increase and, concurrently, overcut or gap, tool wear and surface roughness should reduce. Moreover, a process can be identified better when a model replicates its behavior by its vital parameters. The factors that are significant for the system are to be recognised and different aspects of the process are to be correlated while constructing the model. It is expensive, unpractical or impossible to experiment directly with the process so a good model can be cost-effective to predict the actual process very closely. There are a large number of factors to be considered for the EDM process. Based on experience and literature on EDM research and the working characteristics of the machine, the prime parameters chosen, in the present chapter, are pulse current (*Ip*), pulse duration (*Ton*), duty cycle (*Tau*) (defined as the ratio of pulse on time to the total pulse period) and discharge voltage (V) (Table 1). The motivation as to why these factors have been selected is that these are often used among EDM researchers [2–5] for the aforesaid responses and are found to significantly influence them. Extensive experiments were conducted, and the proposed models used the experimental data on EDMed AISI D2 tool steel and the performances of the developed models are compared.

In the past, significant improvement has been carried out to enhance the productivity, accuracy and versatility of the EDM process. The key issue is to pick the process parameters in such a way that the productivity and accuracy will increase. To identify the factor and their interactions responsible for this issue, various research studies were reported in the literature [6]. Dhar et al. [2] estimated the effect of Ip, Ton and V on MRR, tool wear rate (TWR) and radial overcut or gap (G) on EDM of Al–4Cu–6Si alloy–10 wt% SiC_P composites. Using three factors, three-level full factorial designs, a second-order, non-linear mathematical model has been developed for establishing the relationship among

Table 1 Input variables used in the experiment and their levels

Variable	Unit	Levels		
		1	2	3
Discharge current (Ip)	A	4	7	10
Pulse on time (Ton)	μs	100	200	300
Duty cycle (Tau)	%	80	85	90
Voltage (V)	volt	40	50	60

machining parameters. The MRR, TWR and G increase with increase in Ip and Ton. Salonitis et al. [7] developed a simple thermal-based model to determine the MRR and surface roughness and assert that the increase of Ip, V or Ton results in higher MRR and simultaneously a higher value of surface roughness. On the other hand, on reducing idling time, MRR increases. The model's predictions were compared with experimental results and found to be in good agreement. El-Taweel [8] investigated the correlation of process parameters in EDM of CK45 steel with Al–Cu–Si–TiC composite produced using powder metallurgy technique and evaluated MRR and TWR. It was found that such electrodes are more sensitive to Ip and Ton than conventional electrodes. To achieve maximum MRR and minimum TWR, the process parameters are optimised, and on experimental verification, the results are found to be in good agreement. Pradhan and Biswas [9] investigated the effects of Ip, Ton, Tau and V on various responses using two neuro-fuzzy and one neural network model. Chiang and Chang [10] reported optimisation of the wire EDM process using multiple response analysis based on the grey relational analysis. The Ton and pulse off time, V, wire feed, dielectric flow and cutting radius of workpiece were correlated on responses such as surface removal rate and maximum surface roughness.

Optimal selection of process parameters in EDM is vitally essential since it is an expensive process, and the materials generally used in EDM are difficult to machine and are pretty costly. Consequently, efforts are made to increase a production rate considerably by reducing the machining time. EDM is a highly complex and stochastic process, and it is absolutely difficult to decide optimal parameters for best machining performance, i.e. productivity and accuracy. MRR, TWR and radial over cut are the primary responses that judge the machining performance, but these are contradictory in nature. The higher the MRR, the best, whereas the lower the tool wear and radial over cut, the best. In EDM, it is difficult to find a single optimal combination of process parameters for the performance's parameters, as the process variables influence them differently. Consequently, there is a necessity for a multi-objective optimisation method to succeed at the solutions to this problem. Moreover, in recent years, the customer'sconcern is also oriented towards more than one response, where multi-response experiments and computations on several responses are attained for each setting of a range of control input variables. In design and in designing manufacturing processes, multi-response objectives frequently conflict with each other. To solve this type of multi-optimisation problem, Lin et al. [11] used grey relational analysis based on an orthogonal array and fuzzy-based Taguchi method. Su and Tong [12] and Antony [13] have tried to combine Taguchi method with principal component analysis. Latter on Liao [14] and Wu and Chyu [15] proposed methods on the basis of weighted principal components. Routara et al. [16] performed principal component analysis (PCA) on normalised response variables and compute the multi-response performance index as a weighted summation of PC values. Ribeiro et al. [17] has presented a simultaneous optimisation of correlative multiple response and used score vector of the first principal component acquired from PCA on responses to find optimal conditions by response surface methodology (RSM).

Even though numerous efforts have been made to enhance the productivity, accuracy and versatility of EDM process, association of RSM, grey relational analysis (GRA) and PCA method for obtaining optimal setting on EDM on AISI D2 tool steel has never been attempted. AISI D2 tool steel has abundant growing ranges of applications in manufacturing tools in mould industries. Therefore, the multiple objective optimisation problems have been of increasing interest to the researchers to minimize this complexity. A trouble-free and trustworthy technique based on statistically designed experiments and RSM approach has been adopted with a face-centred central composite design (CCD), as a special case of CCD; later on, it synergies with GRA and PCA methods for maximising MRR and minimising TWR and G to produce intricate precise components. Grey analysis delivers excellent solution to uncertain, multi-input and discrete data problems. As EDM process is analogous in nature, the technique is wonderfully fit in parameter optimisation of such experimental work.

2 Description of the experiments

2.1 Equipment and workpiece material

So as to obtain the data for modelling, a sequence of experiments were accomplished on a CNC Electrical discharge die sinking machine "Electronica Electraplus PS 50ZNC" shown in Fig. 1 , which has the facilities of programming in the Z-vertical axis and manually operated X- and Y-axes. A cylindrical pure copper (99.9 % Cu) was used in this study as a tool electrode with a diameter of 30 mm depicted in Fig. 2, and a commercial grade EDM oil (specificgravity = 0.763, freezingpoint = 94 °C) was used as dielectric fluid. The power supply was linked with the tool electrode (tool: positive polarity, workpiece: negative polarity). A lateral flushing system was used for effective flushing of machining debris from the working gap region with a pressure of 0.4 kgf/cm^2. The workpiece material used was AISI D2 steel plates with the following chemical composition by weight: 1.5 % C, 0.3 % Si, 0.3 % Mn, 1.0 % Mo, 12.0 % Cr, 0.3 % Ni, 0.8 % V and 1.0 % Co, which is widely used in the mould industry.

Fig. 1 Experimental set-up used for experimentation

2.2 Experimental procedure

The workpiece material was initially a circular bar with a diameter of 100 mm and was cut into specimens of thickness 10 mm. The top and bottom faces of the workpiece were ground to make it flat and to have a good quality surface

Fig. 2 Copper electrode and AISI D2 workpiece

finish prior to experimentation. The bottom of the cylindrical electrode was polished by a very fine grade emery sheet prior to every experimental run. Each treatment of the experiment was run for 15 min, and the time was measured with a stopwatch with an accuracy of 0.1 s. The workpiece as well as the tool was detached from the machine, cleaned and dried up, to make it free from the dirt, debris and dielectric. They were weighed, before and after machining, on a precision electronic balance (maximumcapacity = 300 g, precision = 0.001 g). The diameter of the cavity machined on the workpiece was measured by a Tool Maker's microscope (Carl Zeiss, Germany) with an accuracy of 1 μm.

2.3 Measurement of responses

2.3.1 Material removal rate

MRR is calculated by using the volume loss from the workpiece divided by the time of machining. The calculated weight loss is converted to volumetric loss in cubic millimeter per minute as per Eq. 1:

$$\mathrm{MRR} = \frac{\Delta V_{\mathrm{w}}}{T} = \frac{\Delta W_{\mathrm{w}}}{\rho_{\mathrm{w}} g T} \tag{1}$$

where ΔV_{w} is the volume loss from the workpiece, ΔW_{w} is the weight loss from the workpiece, T is the duration of the machining process and $\rho_{\mathrm{w}} = 7{,}700$ kg/m^3 is the density of the workpiece.

2.3.2 Tool wear rate

TWR is expressed as the volumetric loss of tool per unit time, expressed as

$$\mathrm{TWR} = \frac{\Delta V_t}{T} = \frac{\Delta W_t}{\rho_t g T} \tag{2}$$

where ΔV_{t} is the volume loss from the electrode, ΔW_{t} is the weight loss from the electrode, T is the duration of the machining process and $\rho_{\mathrm{t}} = 8{,}960$ kg/m^3 is the density of the electrode.

2.3.3 Radial overcut or gap

G (in micrometer) is expressed as half the difference of the diameter of the hole produced to the tool diameter, that is,

$$G = \frac{(d_{\mathrm{i}} - d_{\mathrm{t}})}{2} \tag{3}$$

where d_{t} is the diameter of the tool and d_{i} is the diameter of the impression or cavity produced by the tool on the workpiece.

3 Analysis method

3.1 Experimental design with RSM

RSM is a collection of mathematical and statistical techniques that are useful for modelling and analysis of problems in which response is influenced by several input variables, and the objective is to find the correlation between the response and the variables investigated [18]. RSM has many advantages, and has effectively been applied to study and optimise the processes. It offers enormous information from a small number of experiments. In addition, it is possible to detect the interaction effect of the independent parameters on the response. The model easily clarifies the effect for binary combination of the independent process parameters. Furthermore, the empirical model that related the response to the independent variables is used to obtain information. It has been widely used in analysing various processes, designing the experiment, building models, evaluating the effects of several factors and searching for optimum conditions to give desirable responses and reduce the number of experiments [19]. In the EDM process, as several machining factors are associated, therefore, RSM can be an appropriate approach to analyse the process. The second-order model is normally used when the response function is not known or non-linear, thus suitable in this study. Based on RSM with a face-centred CCD shown in Table 2, 30 experiments are carried out. The experimental values are analysed, and the mathematical model is then developed that illustrates the relationship between the process variable and response. The following second-order model explains the behavior of the system:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{i=1}^{k} \beta_{ii} X_i^2 + \sum_{i,j=1, i \neq j}^{k} \beta_{ij} X_i X_j + \epsilon \tag{4}$$

where Y is the corresponding response, X_i is the input variables and X_{ii}^2 and $X_i X_j$ are the squares and interaction terms, respectively, of these input variables. The unknown regression coefficients are β_o, β_i, β_{ij} and β_{ii}, and the error in the model is depicted as ϵ.

3.1.1 Data analysis using RSM

Minitab [20] software is used to analyse the process parameters of the response equation, and subsequent analysis of variance (ANOVA) was assessed. Probability (p values) was used to check the significance of the coefficients, which are essential to recognise the pattern of the related interactions between the test variables. The smaller value of the probability reveals a very significant correlation coefficient. The significance of the coefficient was tested by a t test with the

Table 2 CCD and experimental results for four variables in uncoded units

Run order	Ip A	Ton μs	Tau %	V Volt	MRR mm^3/min	TWR mm^3/min	G μm
1	10	300	80	40	21.970	0.060	0.190
2	10	100	80	60	19.360	0.424	0.150
3	10	300	90	60	24.896	0.056	0.230
4	4	100	90	60	6.870	0.089	0.005
5	7	200	85	50	15.370	0.076	0.138
6	7	200	85	50	14.770	0.079	0.132
7	4	100	80	40	5.766	0.075	0.050
8	4	300	80	60	3.320	0.022	0.080
9	10	100	90	40	33.780	0.547	0.160
10	4	300	90	40	5.750	0.000	0.065
11	7	200	85	50	13.760	0.071	0.123
12	4	100	80	60	5.532	0.100	0.038
13	7	200	85	50	13.060	0.067	0.117
14	4	300	90	60	3.130	0.011	0.060
15	4	300	80	40	3.532	0.022	0.090
16	4	100	90	40	8.221	0.045	0.010
17	10	100	80	40	23.480	0.625	0.160
18	10	300	80	60	17.960	0.052	0.170
19	10	300	90	40	31.250	0.033	0.210
20	10	100	90	60	26.662	0.647	0.160
21	10	200	85	50	24.420	0.208	0.181
22	7	200	85	50	14.340	0.073	0.128
23	7	100	85	50	14.768	0.168	0.095
24	7	200	85	40	16.169	0.070	0.142
25	7	200	85	50	13.160	0.068	0.131
26	7	200	85	60	13.078	0.080	0.120
27	4	200	85	50	6.485	0.028	0.080
28	7	200	80	50	12.162	0.083	0.125
29	7	200	90	50	16.234	0.066	0.105
30	7	300	85	50	13.755	0.050	0.130

confidence of 95 %. The excellence of the fit of the model equation was articulated by the coefficient of determination (R^2), and its statistical significance was checked by an F test. Response surface plots were produced, and subsequently, confirmation experiments were accompanied to verify the validity of the statistical experimental strategies.

3.2 Grey relational analysis

GRA is a decision-making technique based on the grey system theory originally developed by Deng [21]. In the grey theory, black represents a system with deficient information, while a white system stands for complete information. However, the grey relation is the relation with incomplete information and is used to characterise the grade of association between two sequences so that the distance of two factors can be measured discretely. When experiments are unclear or if the experimental method cannot be carried out accurately, grey analysis assists to reimburse for the deficiency in statistical regression. Figure 3 shows a flow chart calculation using the grey relational analysis. Grey relational analysis is an effective means of analysing the relationship between sequences with less data and can analyse many factors that can overcome the disadvantages of a statistical method [22].

Besides, it is necessary to know the most significant influential parameters for EDM. EDM is a very complex process involving several branch of engineering such as can be considered as grey systems due to their high complexity and lacking of sufficiently defined or precise information. For such systems, GRA, as one of the most important contents of grey theory, has been applied extensively. The

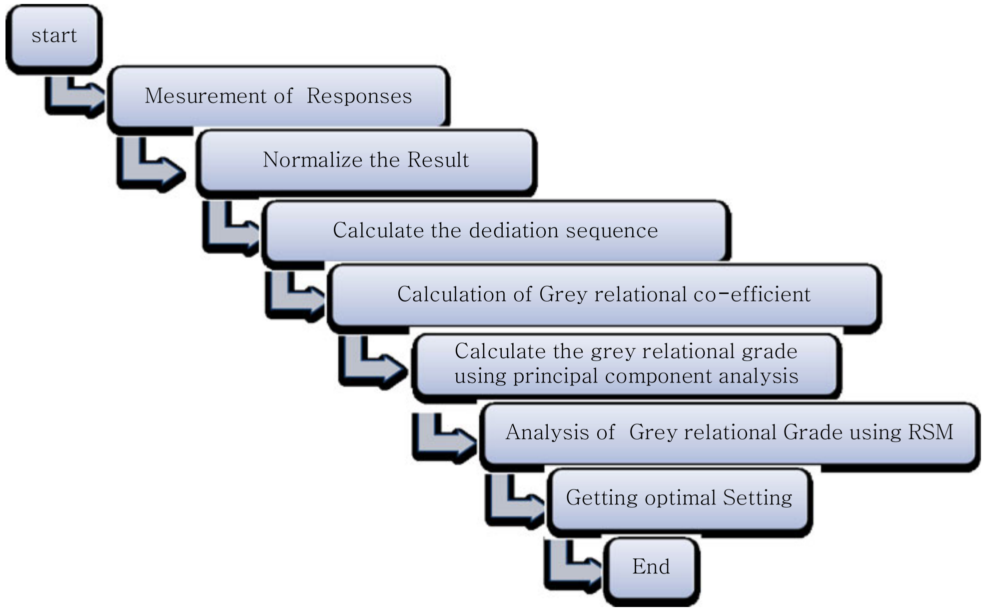

Fig. 3 Flow chart of MRR, TWR and G using grey relational analysis

principle of GRA is to estimate the similarity and degree of the compactness among factors based on the geometric shape of the different sequences [21].

3.2.1 Data preprocessing

Data preprocessing is the method of transferring the original sequence to a comparable sequence, where the original data normalise to a range of 0 and 1. Generally, three different kinds of data normalisations are carried out to render the data, whether the lower is better (LB), the higher is better (HB) or nominal the best (NB). For "the-larger-the-better' characteristics such as productivity or MRR, the original sequence can be HB and should be normalised as [23]

$$X_i^*(k) = \frac{X_i(k) - \min X_i(k)}{\max X_i(k) - \min X_i(k)}. \tag{5}$$

However, if the expectancy is the as small as possible for characteristics such as TWR, G or surface roughness, then the original sequence should be normalised as LB:

$$X_i^*(k) = \frac{\max X_i(k) - X_i(k)}{\max X_i(k) - \min X_i(k)}. \tag{6}$$

Conversely, if a specific target value is to be achieved, then the original sequence will be normalised by the following equation of NB:

$$X_i^*(k) = 1 - \frac{|X_i(k) - X_{ob}(k)|}{\max X_i(k) - X_{ob}(k)} \tag{7}$$

where $i = 1, 2n$; $k = 1, 2, y, p$; $X_i^*(k)$ is the normalised value of the kth element in the ith sequence; $X_{0b}(k)$ is the desired value of the kth quality characteristic; max $X_i^*(k)$ is the largest value of $X_i(k)$; min $X_i^*(k)$ is the smallest value of $X_i(k)$; n is the number of experiments; and p is the number of quality characteristics.

3.2.2 Grey relational coefficient and grey relational grade

After normalising the data, usually grey relational coefficient is calculated to display the relationship between the optimal and actual normalised experimental results. The grey relational coefficient can be expressed as

$$\gamma_i(k) = \gamma(X_0(k) - X_i(k)) = \frac{\Delta\min + \zeta\Delta\max}{\Delta_{0,i}(k) + \zeta\Delta\max}, \tag{8}$$

$$i = 1, ..., n; k = 1, ..., p,$$

where $\Delta_{0,i}(k) = |X_0(k) - X_i(k)|$ is the difference of the absolute value called deviation sequence of the reference sequence $X_0(k)$ and comparability $X_i(k)$. ζ is the distinguishing coefficient or identification coefficient, in which the value range is $0 \le \zeta \le 1$. In general, it is set to 0.5; hence, same is adopted in this study. Deng [21] stated that the value of 0.5 is normally applied. The aim of defining the grey relational coefficient is to express the relational degree between the reference sequence $X_0(k)$ and the comparability sequences $X_i(k)$, where $i = 1, 2, ..., m$ and $k = 1, 2, ..., n$ with $m = 30$ and $n = 3$ in this study. The grey relational grade (GRG) is a weighting sum of the grey relational coefficients and it is defined as

$$\gamma(x_0, x_i) = \sum_{k=1}^{n} \beta_k(x_0, x_i) \tag{9}$$

where β_k represents the weighting value of the kth performance characteristic and $\sum_n^{k=1} \beta_k = 1$. In the present analysis, the weights are computed using principal component analysis discussed in Section 3.3.

The grey relational grade $\gamma(x_0, x_i)$ indicates the level of association between the reference sequence and the comparability sequence. A higher grey relational grade value infers a stronger relational degree between the comparative and referential (ideal) sequence. For illustration, if the two sequences are identically coincidence, then GRG is equal to 1. This grade also specifies the degree of influence that the comparability sequences could employ over the reference sequence. Consequently, if a specific comparability sequence is more vital to the reference sequence than the other comparability sequences, then the GRG for that comparability sequence will be greater than the other. Thus, grey analysis is essentially a measurement of the absolute value of data difference between sequences, and it could be used to measure approximation correlation between sequences.

Table 3 Normalized values for MRR, TWR and *G*

Run order	MRR	TWR	*G*
1	0.615	0.907	0.178
2	0.530	0.345	0.356
3	0.710	0.913	0.000
4	0.122	0.862	1.000
5	0.399	0.883	0.409
6	0.380	0.878	0.436
7	0.086	0.884	0.800
8	0.006	0.966	0.667
9	1.000	0.155	0.311
10	0.085	1.000	0.733
11	0.347	0.891	0.475
12	0.078	0.845	0.853
13	0.324	0.896	0.502
14	0.000	0.983	0.756
15	0.013	0.966	0.622
16	0.166	0.930	0.978
17	0.664	0.034	0.311
18	0.484	0.919	0.267
19	0.917	0.949	0.089
20	0.768	0.000	0.311
21	0.695	0.679	0.217
22	0.366	0.887	0.453
23	0.380	0.741	0.600
24	0.425	0.892	0.391
25	0.327	0.896	0.440
26	0.325	0.876	0.489
27	0.109	0.957	0.667
28	0.295	0.872	0.467
29	0.428	0.898	0.556
30	0.347	0.923	0.444

Table 4 The deviation sequences for MRR, TWR and *G*

Run order	MRR	TWR	*G*
1	0.385	0.093	0.822
2	0.470	0.655	0.644
3	0.290	0.087	1.000
4	0.878	0.138	0.000
5	0.601	0.117	0.591
6	0.620	0.122	0.564
7	0.914	0.116	0.200
8	0.994	0.034	0.333
9	0.000	0.845	0.689
10	0.915	0.000	0.267
11	0.653	0.109	0.525
12	0.922	0.155	0.147
13	0.676	0.104	0.498
14	1.000	0.017	0.244
15	0.987	0.034	0.378
16	0.834	0.070	0.022
17	0.336	0.966	0.689
18	0.516	0.081	0.733
19	0.083	0.051	0.911
20	0.232	1.000	0.689
21	0.305	0.321	0.783
22	0.634	0.113	0.547
23	0.620	0.259	0.400
24	0.575	0.108	0.609
25	0.673	0.104	0.560
26	0.675	0.124	0.511
27	0.891	0.043	0.333
28	0.705	0.128	0.533
29	0.572	0.102	0.444
30	0.653	0.077	0.556

3.3 Principal component analysis

PCA is a mathematical approach that converts a set of observations of probably correlated variables into a set of values of uncorrelated variables. It was invented very early and

Table 5 The Eigenvalues and explained variation for principal components

Principal component	Eigenvalue	Explained variations (%)
First	1.9826	66.1
Second	0.7091	23.6
Third	0.3084	10.3

Table 6 The Eigenvectors for principal components and contribution

Responses	Eigenvectors			
	First principal component	Second principal component	Third principal component	Contribution
MRR	0.643	0.002	0.766	0.413
TWR	−0.541	0.709	0.452	0.293
G	−0.542	−0.705	0.457	0.294

later mostly used as a tool in investigative data analysis and for the formation of predictive models. PCA can be done by eigenvalue decomposition of a data covariance matrix or singular value decomposition of a data matrix. It is used for identifying patterns in data and expressing the data in such a way as to highlight their similarities and differences [24]. The main advantage of PCA is that once the patterns in data have been identified, the data can be compressed, i.e. by reducing the number of dimensions, without much loss of information. The explicit goals of PCA are to:

1. Extract the most significant information from the data,
2. Squeeze the size of the data set by keeping only the significant,
3. Simplify the explanation of the data set, and
4. Analyze the structure of the observations and the variables.

The procedure is described as follows [25]:

1. The original multiple quality characteristic array $X_i(j), i = 1, 2, ..., m; j = 1, 2, ..., n$

$$X = \begin{bmatrix} x_1(1) & x_1(2) & \ldots \ldots & x_1(n) \\ x_2(1) & x_2(2) & \ldots \ldots & x_2(n) \\ : & : & \ldots \ldots & : \\ : & : & \ldots \ldots & : \\ x_m(1) & x_m(2) & \ldots \ldots & x_m(n) \end{bmatrix} \tag{10}$$

   where m is the number of experiment and n is the number of the response. In the present work, x is the grey relational coefficient of each response and $m = 30$ and $n = 3$.
2. Correlation coefficient array

   The correlation coefficient array is evaluated as follows:

$$R_{jl} = \left(\frac{\mathrm{Cov}(x_i(j), x_i(l))}{\sigma x_i(j) \times \sigma x_i(l)}\right), j = 1, 2, ..., m; \quad l = 1, 2, ..., n \tag{11}$$

   where $\mathrm{Cov}(x_i(j), x_i(l)$ is the covariance of sequences $x_i(l)$ and $x_i(l)$, $\sigma_(x_i)(j)$ is the standard deviation of sequence $x_i(j)$ and $\sigma_(x_i)(l)$ is the standard deviation of sequence $x_i(l)$.
3. Determining the eigenvalues and eigenvectors

The eigenvalues and eigenvectors are determined from the correlation coefficient array:

$$(R - \lambda_k I_m)V_{ik} = 0 \tag{12}$$

Table 7 Grey relational coefficient, grey relational grade and rank of the MRR, TWR and G

Run order	Grey relational coefficient MRR	TWR	G	Grey relational Grade	Rank
1	0.565	0.843	0.378	0.572	16
2	0.515	0.433	0.437	0.591	12
3	0.633	0.852	0.333	0.546	26
4	0.363	0.784	1.000	0.594	9
5	0.454	0.810	0.458	0.555	24
6	0.446	0.804	0.470	0.557	23
7	0.354	0.812	0.714	0.565	17
8	0.335	0.936	0.600	0.540	27
9	1.000	0.372	0.421	0.561	20
10	0.353	1.000	0.652	0.549	25
11	0.434	0.820	0.488	0.503	28
12	0.352	0.764	0.773	0.724	1
13	0.425	0.828	0.501	0.574	15
14	0.333	0.967	0.672	0.470	29
15	0.336	0.936	0.570	0.692	2
16	0.375	0.878	0.957	0.580	14
17	0.598	0.341	0.421	0.618	6
18	0.492	0.861	0.405	0.565	18
19	0.858	0.907	0.354	0.596	8
20	0.683	0.333	0.421	0.562	19
21	0.621	0.609	0.390	0.630	5
22	0.441	0.816	0.478	0.645	4
23	0.446	0.659	0.556	0.588	13
24	0.465	0.822	0.451	0.593	10
25	0.426	0.827	0.472	0.558	22
26	0.425	0.802	0.495	0.559	21
27	0.360	0.920	0.600	0.673	3
28	0.415	0.796	0.484	0.609	7
29	0.466	0.830	0.529	0.468	30
30	0.434	0.866	0.474	0.591	11

Table 8 Estimated regression coefficients for GRG (before elimination)

Term	Coef	SE coef	t value	p value
Constant	0.559552	0.006345	88.187	0.001
Block 1	0.000339	0.004719	0.072	0.944*
Block 2	0.003339	0.004719	0.708	0.492*
Ip	−0.023944	0.003963	−6.042	0.001
Ton	0.017000	0.003963	4.290	0.001
Tau	0.037722	0.003963	9.519	0.001
V	−0.017000	0.003963	−4.290	0.001
Ip×Ip	0.016409	0.010553	1.555	0.144*
Ton×Ton	0.000909	0.010553	0.086	0.933*
Tau×Tau	0.013409	0.010553	1.271	0.226*
$V\times V$	0.004909	0.010553	0.465	0.649*
Ip×Ton	0.034375	0.004203	8.178	0.001
Ip×Tau	0.007625	0.004203	1.814	0.093*
Ip×V	−0.016000	0.004203	−3.807	0.002
Ton×Tau	−0.008625	0.004203	−2.052	0.061*
Ton×V	0.001500	0.004203	0.357	0.727*
Tau×V	−0.017500	0.004203	−4.163	0.001
$S = 0.01681$		$R^2 = 95.9$ %		$R^2_{(adj)} = 90.8$ %

t value was obtained from the t test, which indicates the significance of the regression coefficients

* non-significant terms

where λ_x eigenvalue $\sum_{k=1}^{n} \lambda_k = n$, $k = 1, 2, ..., n$, and V_{ik} $[a_{k1}a_{k2}......a_{km}]^T$ is the eigenvectors corresponding to the eigenvalue λ_k.

4. Principal components

The uncorrelated principal component is formulated as

$$Y_{mk} = \sum_{i=1}^{n} x_m(i) \cdot V_{ik} \tag{13}$$

where Y_{m1} is called the first principal component, Y_{m2} is called the second principal component and so on.

The principal components are aligned in descending order with respect to variance, and therefore, the first principal component Y_{m1} accounts for most variance in the data.

4 Results and discussion

The procedure of grey relational analysis coupled with principal analysis to compute the optimal arrangements of the machining parameters for EDM of AISI D2 steel is described step by step as follows:

1. Obtain the experimental data.

Fig. 4 Response graph for grey relational grade

Table 9 The ANOVA table

Source	DF	Seq SS	Adj SS	F value	p value	% contribution
Regression	7	0.0742	0.0742	15.3500	0.0000	83.00
Linear	4	0.0463	0.0463	16.7600	0.0000	51.81
Ip	1	0.0103	0.0103	14.8663	0.0009	11.52
Ton	1	0.0052	0.0052	7.5317	0.0118	5.83
Tau	1	0.0256	0.0256	36.8879	0.0000	28.57
V	1	0.0053	0.0053	7.6094	0.0115	5.89
Interaction	3	0.0279	0.0279	13.4600	0.0000	31.19
Ip × Ton	1	0.0189	0.0189	27.3179	0.0000	21.16
Ip × V	1	0.0041	0.0041	5.8798	0.0240	4.55
Tau × V	1	0.0049	0.0049	7.1276	0.0140	5.52
Residual error	22	0.0152	0.0152			17.00
Total	29	0.0894				

2. Normalize the experimental values.
3. Calculate the equivalent grey relational coefficients.
4. Calculate the grey relational grade using principal component analysis.
5. Accomplish statistical ANOVA.
6. Select the optimal levels of cutting parameters.
7. Run conformation experiments.

4.1 Optimal combination of the process parameters

At the outset, the experiments were conducted as discussed in Section 2 and the experimental results were transformed to the volumetric unit using Eqs. 1, 2 and 3 for the responses such as MRR, TWR and G which are recorded in Table 2. Typically, MRR is considered to be as large as possible and TWR and G are considered to be as small as possible. Therefore, all MRR values were substituted in Eq. 5 and TWR and G were substituted in Eq. 6 to get normalised values which are presented in Table 3. As said by Deng [21], larger values of the normalised results stand for better performance, and the maximum normalised results that are equal to 1 specify the best performance. The results shown in Table 3 were substituted in Eq. 8 to compute grey relational coefficients of the aforementioned responses. The deviation sequences Δ_{0i} (Table 4) in the range are calculated as follows:

$$\begin{aligned}\Delta_{0,1}(k) &= |x_0(1) - x_1(1)| = |1.00 - 0.615| = 0.385\\ \Delta_{0,2}(k) &= |x_0(2) - x_2(2)| = |1.00 - 0.907| = 0.093\\ \Delta_{0,3}(k) &= |x_0(3) - x_3(3)| = |1.00 - 0.178| = 0.822\end{aligned}$$

Therefore, the value of $\Delta_{0,1} = (0.385 \quad 0.093 \quad 0.822)$ and the result of all Δ_{0i} for $i = 1 - 30$ are presented in Table 7. As it is known from Table 3 that

$$\begin{aligned}\Delta_{max} &= \Delta_9(1) = \Delta_{10}(2) = \Delta_4(3) = 1.00\\ \Delta_{min} &= \Delta_{14}(1) = \Delta_{20}(2) = \Delta_3(3) = 0.00\end{aligned}$$

The distinguishing coefficient ζ can be substituted for the grey relational coefficient in Eq. 8. ζ is a distinguishing coefficient to adjust the range of the comparison environment, which was selected as 0.5 in this study [26]. Table 7 lists the grey relational coefficient and grade for each experiment of the FCCD experimental arrangement by applying Eqs. 8 and 9. The table lists the values of each the grey relational grade. Let us say, for the grey relational coefficient, experiment no. 1 can be expressed as

$$\begin{aligned}\gamma_i(1) &= \frac{\Delta_{min} + \zeta\Delta_{max}}{\Delta_{0,i}(1) + \zeta\Delta_{max}} = \frac{0 + 0.5 \times 1}{0.385 + 0.5 \times 1} = 0.565\\ \gamma_i(2) &= \frac{\Delta_{min} + \zeta\Delta_{max}}{\Delta_{0,i}(2) + \zeta\Delta_{max}} = \frac{0 + 0.5 \times 1}{0.093 + 0.5 \times 1} = 0.843\\ \gamma_i(3) &= \frac{\Delta_{min} + \zeta\Delta_{max}}{\Delta_{0,i}(3) + \zeta\Delta_{max}} = \frac{0 + 0.5 \times 1}{0.822 + 0.5 \times 1} = 0.378\end{aligned}$$

The weightage for each response is not same as it reflects its comparative significance in the grey relational analysis which is decided in this study by principal component analysis. The components of the array for multiple response are listed in Table 5, each of which indicates the grey relational coefficient of each response. These data were used

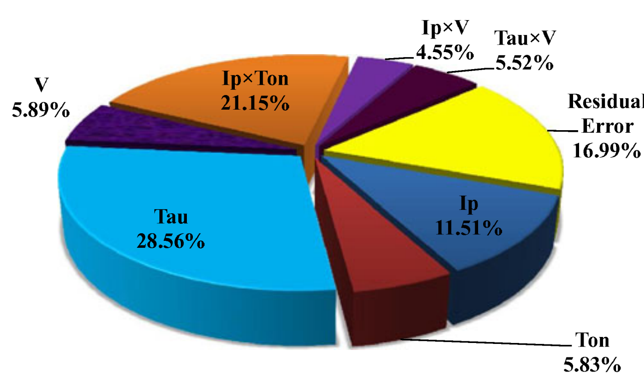

Fig. 5 Percentage contributions of factors on the grey relational grade

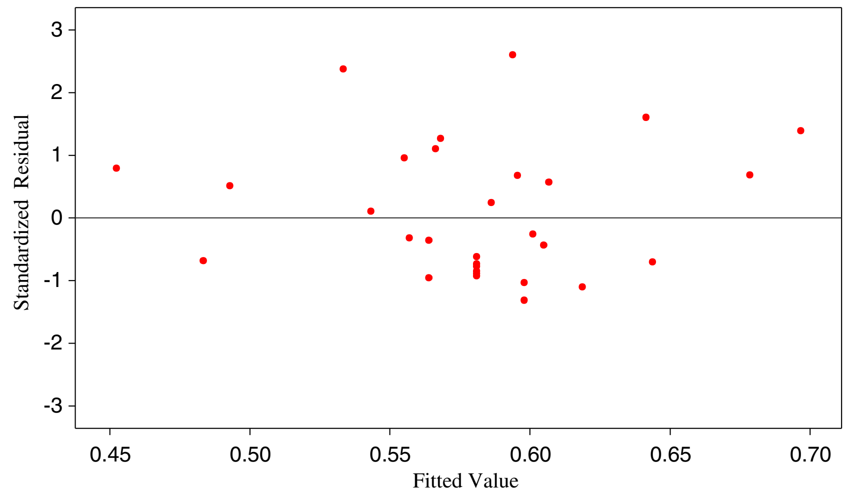

Fig. 6 Residual plots of quadratic model

to assess the correlation coefficient and to decide the corresponding eigenvalues from Eq. 12. The eigenvalues are presented in Table 5. The eigenvector corresponding to each eigenvalue is listed in Table 6. The square of the eigenvalue signifies the contribution of the corresponding response to the principal component. The contribution of MRR, TWR and G is 0.413, 0.293 and 0.294, respectively, and is shown in Table 6. Furthermore, the variance contribution for the first principal component characterising the three responses is as high as 66.1 %. Therefore, for this analysis, the squares of its corresponding eigenvectors were selected as the weighting values of the related response, and coefficients β_1, β_2 and β_3 in Eq. 8 were thus set as 0.413, 0.293 and 0.294, respectively. Based on Eq. 8 and data listed in Table 7, the grey relational grades were calculated as

$$\gamma(x_0, x_i) = 0.565 \times 0.413 + 0.843 \times 0.293 + 0.378 \times 0.294 = 0.636$$

$$\gamma(x_0, x_i) = 0.515 \times 0.413 + 0.433 \times 0.293 + 0.437 \times 0.294 = 0.468.$$

Also, the values were intended for each factor at same level and summarised in Table 7. Therefore, the optimisation design was accomplished relating to a single grey relational grade instead of complex multi-response characteristic. The GRG of each combination is then ranked as per value, and it is found that a set of optimal machining parameters based on the highest GRG value of 0.724 peak value is obtained with Ip = 10, Ton = 300 μs, Tau = 90 % and V = 40 V.

4.2 Statistical analysis of GRG

Statistical analysis was carried out on the GRG data obtained, through face-centred CCD using statistical software Minitab [20]. The experimental conditions with the observations are given in Table 2 and GRG in Table 7. The regression coefficient values, standard deviations, T values and probability (p) values are given in Table 8. Regression analysis is performed to find out the relationship between the input factors and the response GRG. To test

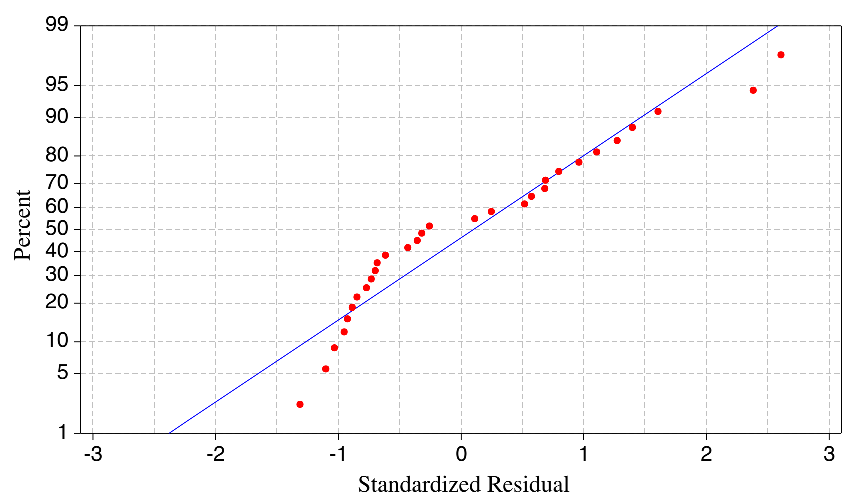

Fig. 7 Normal probability plot

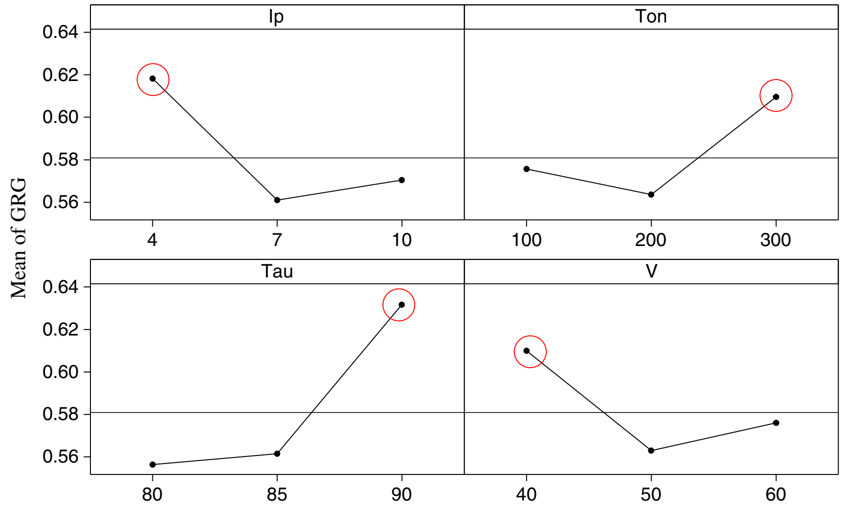

Fig. 8 Influence of process parameters on multiple performances

the adequacy of the model, ANOVA is used for testing the null hypothesis (H_0) of the experimental data with a confidence level of 95 %. The p value for the F statistic is expressing the probability of observing a value of F at least as large if H_0 is true, and with this, the treatments have no effect. If the p value ≤ 0.05, it is concluded that H_α is true and the treatments have a statistically significant effect. Responses obtained from the experiments are compared with the predicted value calculated from the model. Table 8 is an analysis of variance summary that depicts the terms in the model, corresponding coefficients (coef), t statistic and p value to decide whether to reject or fail to reject the null hypothesis. The terms marked with an asterisk in the table are exceeding the α value. Thus, these terms are eliminated for further analysis. The blocking does not have any significant effect on the response GRG (Fig. 4), which reveals that the uncontrollable factors of the experiment conducted were held constant. The backward elimination process discards the insignificant terms (p value greater than 0.05) to adjust the fitted quadratic model. The model, with the rest of the terms, is eliminated after the ANOVA analysis. This way, the simplified truncated model has the highest value of R^2 which is 0.83, indicating a high significance of the model.

ANOVA is a statistical tool employed to understand the experimental results, and it is extensively used to establish the performance of a cluster of parameters under analysis. In this study, ANOVA is effectively applied to inspect the EDM parameter that significantly influences the GRG. The ANOVA table is depicted in Table 9 with various parameters considered for the analysis and their contribution. The percentage contributions for each term influencing grey relational grade are also presented in Fig. 5. It can be seen that Tau is the most significant process parameter due to its highest percentage contribution of 28.56 % followed by pulse current with 11.51 %, V and Ton with 5.89 and 5.83 %, respectively, which total to 51.81 % contribution of linear terms. However, the percentage contribution of interaction terms is 31.1 % with 21.16, 4.55 and 5.52 % for Ip × Ton, Ip × V, and Tau × V, respectively . The estimated standard deviation of the error in the models is $S = 0.0263$

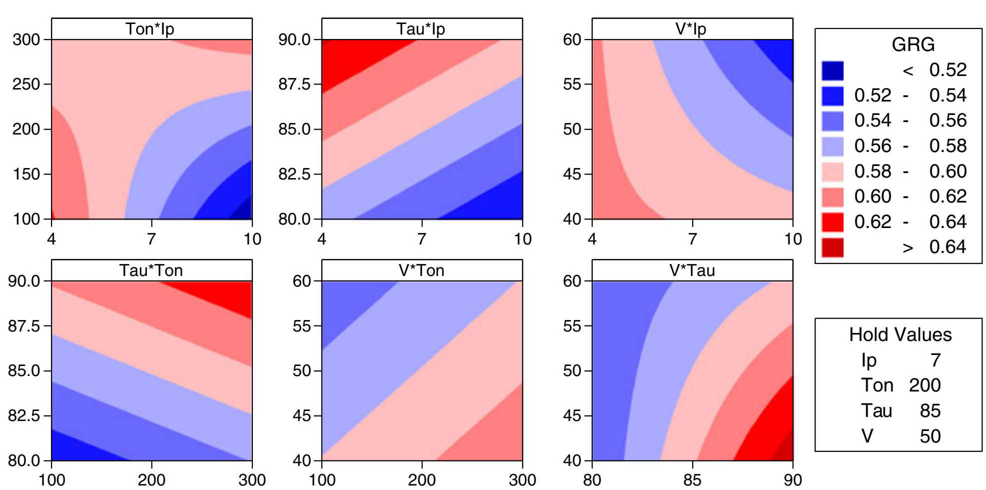

Fig. 9 Two-dimensional contour plots for GRG: effect of **a** pulse current and pulse duration, **b** pulse current and discharge voltage and **c** duty cycle and discharge voltage. * - Other combinations being insignificant

for GRG. It can also be seen that the residual error has a 17 % contribution. Based on the above discussion, the optimal operational conditions established by grey analysis approach are as follows: a pulse current 10 A, pulse duration 300 μs, duty cycle 90 % and discharge voltage 40 V. Therefore, experiment 19 shown in Table 2 fits the optimal process conditions.

The effect of each machining parameter on the GRG at different levels can be independent. The mean of the GRG for each level of the EDM process parameters is presented in Table 7. Furthermore, the total mean of the GRG for all the 30 experiments is computed and listed in Table 7. Fundamentally, the larger the GRG, the better is the multiple performance characteristics. Conversely, the relative importance among the EDM process parameters for the multiple performance characteristics still needed to be investigated so that the optimum combination of the EDM process parameter levels can be decided more correctly. Figure 5 shows the percentage contribution of factors and their interactions on the grey relational grade and Fig. 5 displays the EDM parameter levels on the GRG. From the figure of the GRG for various levels (Table 7), the significance of each parameter can be visually understood. In Fig. 6, a random distribution was noticed for the residual plots for the models, indicating that the residual distribution follows normal and independent patterns. This suggests the adequacy of the quadratic models for evaluating the GRG. The normal probability plot is shown in Fig. 7, and it can be concluded that the points lie close to the straight line, indicating that the data follow a normal distribution, except one outlier. Wherever there is a large slope in the figure, it could be inferred that the parameter has a significant influence on the EDM process. In this study, it can be visually understood and found that Tau and Ip have a significant effect. Figure 8 depicts the plots of the main effects on GRG, and those can be used to graphically assess the effects of the factors on the response. It indicates that Tau and Ip have a significant effect on GRG, which is supported by the results presented in Table 8; however, Tau is the most influencing machining parameter.

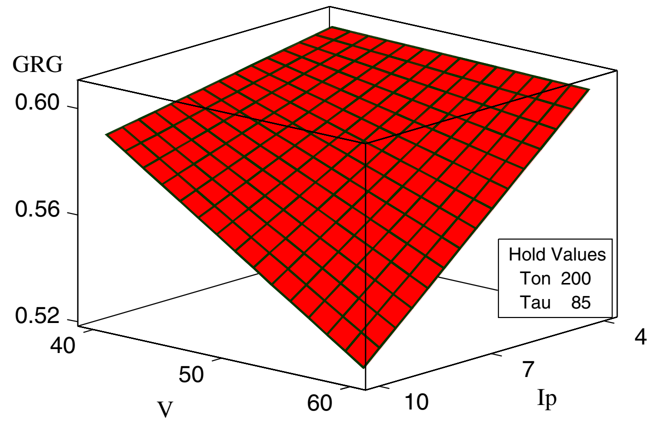

Fig. 11 Response surface plot representing the effect of Ip and V on GRG

With GRG as the response, the contour plots of the model keeping two variables at their mean levels and varying the other two within the experimental ranges are, separately, shown in Fig. 9. The shapes of the contour plots may be curvature with circular, elliptical or saddle implying whether the interactions between the variables are significant or not. The contour plot in Fig. 9a shows that the interactive effects of Ip and Ton on GRG were significant. Similar conclusion was shown by previous researches [3, 9, 27]. Similar counter plots were also observed in Fig. 9b, c showing the interactive effects of Ip and V as well as Tau and V, respectively, on GRG indicating the significance of the said factors. These results are also supported by the p values in Table 8. The other combinations are also presented in the figure but they

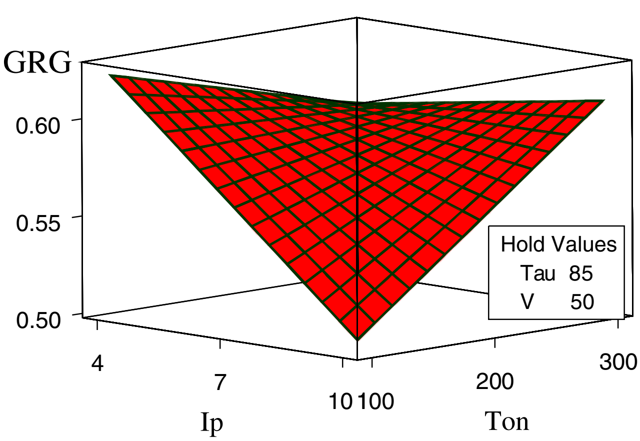

Fig. 10 Response surface plot representing the effect of Ip and Ton on GRG

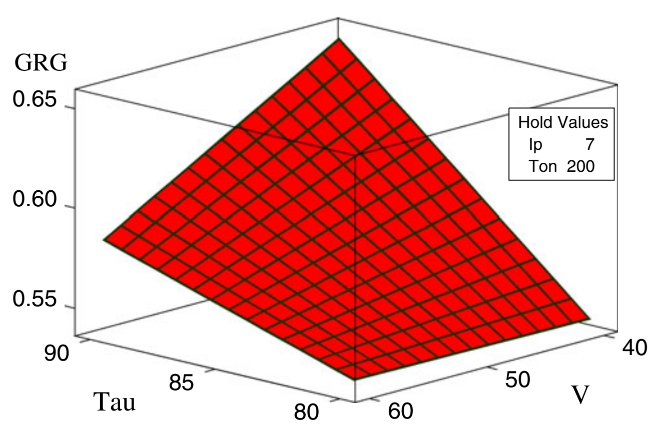

Fig. 12 Response surface plot representing the effect of Tau and V on GRG

Table 10 Conformation results for MRR, TWR and G model

Best combination				MRR	TWR	G
Ip_3	Ton_3	Tau_3	V_1	mm^2/min	mm^2/min	μm
10	300	90	40	32.551	0.036	0.215

do not have a significant contribution on GRG. The surface plot of the said significant factors are also exhibited in Figs. 10, 11 and 12, respectively, for the interactive effect of Ip × Ton, Ip × V and $Tau \times V$, respectively, keeping the other factors at their mean level.

4.3 Confirmation experiments

Once the optimal level of the cutting parameters is identified, which is acquired from the analysis, it is customary to validate the responses. The confirmation experiments are performed to facilitate the verification of the EDM die sinking at the obtained feasible optimal input parametric setting (Ip = 10 A, Ton = 300 μs, Tau = 90 % and $V = 40$ V) for the MRR, TWR and G. The results of the confirmation runs for the responses are listed in Table 10.

5 Conclusion

This investigation proposes a hybrid, integrated approach of grey relational analysis coupled with principal component analysis for the optimisation of the machining parameters of EDM process. Grey relational analysis transforms optimisation of the multiple responses into optimisation of a single response problem, the grey relational grade. Factors having a significant effect are retrieved using ANOVA, and by regulating these process parameters, optimisation of responses was carried out. It has also been found that Tau is the most significant process parameter due to its highest percentage contribution of 28.56 % followed by Ip with 11.51 %, V and Ton with 5.89 and 5.83 %, respectively. However, the percentage contribution of interaction terms such as Ip × Ton, Tau × V and Ip × V is 21.51, 5.52 and 4.55 %, respectively. The optimal operational conditions established by grey analysis approach are as follows: pulse current 10 A, pulse duration 300 μs, duty cycle 90 % and discharge voltage 40 V. This study will help in identifying the significant factors which are efficiently regulated to decrease error, time consumption and cost and to increase quality and productivity. This study may provide the experimenter and practitioners an effective guideline to select optimum parameter settings for achieving the desired MRR, TWR and G during EDM die sinking of AISI D2 tool steel. This method can also be applied for the optimisation of the processing parameters in other manufacturing processes, to promote manufacturing efficiency.

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