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ORIGINAL ARTICLE

Single-machine due date assignment problem with deteriorating jobs and resource-dependent processing times

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Abstract We study a single-machine earliness–tardiness scheduling problem with due date assignment, in which the processing time of a job is a function of its starting time and its resource allocation. We analyze the problem with two different processing time functions and three different due date assignment methods. The goal is to minimize an integrated objective function, which includes earliness, tardiness, due date assignment, and total resource consumption costs. For each combination of due date assignment method and processing time function, we provide a polynomial-time algorithm to find the optimal job sequence, due date values, and resource allocations.

Keywords Scheduling · Single machine · Resource allocation · Due date assignment · Deteriorating jobs

1 Introduction

Scheduling with deteriorating jobs is an important variant of the classical scheduling problems. An extensive survey of different scheduling models and problems involving jobs with deteriorating jobs can be found in [1]. More recent papers which have considered scheduling jobs with deteriorating jobs include [2–20].

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Scheduling problems with controllable processing times have also been extensively studied over the last 30 years. An extensive survey of different scheduling models and problems involving jobs with controllable processing times can be found in [21]. More recent papers which have considered scheduling jobs with controllable processing times include [22–31].

However, to the best of our knowledge, apart from the recent paper of [11] and [19], scheduling problems with deterioration effects and resource-dependent processing times have not been investigated. [11] considered resource allocation problem of scheduling with deteriorating jobs. The objective function is to minimize a cost function containing makespan, total completion (waiting) time, total absolute differences in completion (waiting) times, and total resource cost. They presented polynomial time algorithms for linear resource consumption function [19] considered single-machine scheduling with convex resource-dependent processing times and deteriorating jobs. The objective function is to minimize a cost function containing makespan, total completion (waiting) time, total absolute differences in completion (waiting) times, and total compression cost (i.e., total resource consumption costs). They solved the problem by presenting polynomial time algorithms. In this paper, we study single-machine scheduling problem with deteriorating jobs and resource-dependent processing times in the context of the due date assignment problem, which deals with job scheduling on machines in a just-in-time production environment ([32] and [21]). We show that the due date assignment single-machine scheduling problem with deteriorating jobs and resource-dependent processing times remains polynomially solvable.

The rest of this paper is organized as follows: In Section 2, we will give a formal description of the model under study. In Section 3, we present some relevant preliminary results for the reduced problems with fixed processing times. In Sections 4 and 5, we consider the due date assignment problem for linear resource consumption function and convex resource consumption function, respectively. In Section 6, conclusions are presented.

2 Problem formulation

The focus of this paper is to study the deteriorating jobs and resource allocation simultaneously. The model is described as follows: There are given a single machine and a set $J = \{J_1, J_2, \ldots, J_n\}$ of n independent and nonpreemptive jobs immediately available for processing. The machine can handle one job at a time and job preemption is not allowed. Let p_j be the actual processing time of job J_j. As in [11] and [19], we consider the following models:

A linear resource consumption function:

$$p_j = a_j + bt - \theta_j u_j, \tag{1}$$

where $a_j \geq 0$ is the normal (basic) processing time of the job J_j, $b \geq 0$ is the common deterioration rate for all the jobs, $t \geq 0$ is its start time, $\theta_j \geq 0$, and u_j is the amount of a nonrenewable resource allocated to job J_j, with $0 \leq u_j \leq m_j \leq \frac{a_j}{\theta_j}$ and m_j is the upper bound on the amount of resource that can be allocated to job J_j.

A convex resource consumption function can be formulated with properties very different from those of the linear function:

$$p_j = \left(\frac{a_j}{u_j}\right)^k + bt, u_j > 0, \tag{2}$$

where k is a positive constant.

Let $d_j (\geq 0)$ represent the due date of job J_j and is a decision variable. For a given schedule, π, $C_j = C_j(\pi)$ represents the completion time of job J_j, $E_j = \max\{0, d_j - C_j\}$ is the earliness value of job J_j, and $T_j = \max\{0, C_j - d_j\}$ is the tardiness value of job J_j, $j = 1, 2, \ldots, n$. Further, let α, β, and γ be the per time unit penalties for earliness, tardiness, and due date, respectively. The general objective is to determine the optimal due dates $\mathbf{d}^* = (d_1^*, d_2^*, \ldots, d_n^*)$, resource allocations $\mathbf{u}^* = (u_1^*, u_2^*, \ldots, u_n^*)$, and a schedule π which minimizes

$$f(\mathbf{d}, \mathbf{u}, \pi) = \sum_{j=1}^{n} (\alpha E_j + \beta T_j + \gamma d_j + G_j u_j). \tag{3}$$

where G_j is the per time unit cost associated with the resource allocation u_j of job J_j.

In this paper, we study our problem with the three most frequent due date assignment methods ([32] and [21]):

- The common due date assignment method (usually referred to as CON) [33] is a well-known due date assignment method, under which all jobs are assigned the same due date, i.e., $d_j = d$ for $j = 1, 2, \ldots, n$, where $d \geq 0$ is a decision variable.
- The slack due date assignment method (usually referred to as SLK) [34] is a well-known due date assignment method, under which the jobs are given an equal flow allowance according to the following equation, $d_j = p_j + q$ for $j = 1, 2, \ldots, n$, where $q \geq 0$ is a decision variable.
- The unrestricted due date assignment method (usually referred to as DIF) [35] is a well-known due date assignment method, under which each job can be assigned a different due date with no restrictions.

3 Summary of preliminary results for related problems with fixed processing times

In this section, we present some earlier results obtained by [33, 34] and [35] for the CON, SLK, and DIF due date assignment methods, respectively.

Lemma 3.1 *For each of the three due date assignment methods, there exists an optimal schedule π^* without any machine idle time between the starting time of the first job and the completion time of the last job. Furthermore, the first job in the schedule starts at time 0.*

Lemma 3.2 *For the CON due date assignment method (see [33]), there exists an optimal schedule with the property that d coincides with the completion times of the Kth, where*

$$K = \min\{\max\{\lceil n(\beta - \gamma)/(\alpha + \beta)\rceil, 0\}, n\}. \tag{4}$$

For the SLK due date assignment method, there exists an optimal schedule with the property that q coincides with the completion times of the $K - 1$th, where K is given by Eq. 4.

For the DIF due date assignment method, there exists an optimal schedule with the property that $d_j = 0$ if $\gamma \geq \beta$, otherwise $d_j = C_j$.

Lemma 3.3 *For each of the three due date assignment methods, the optimal total cost can be written as*

$$f(\mathbf{d},\pi)=\sum_{j=1}^{n}\omega_j p_{[j]}, \tag{5}$$

where

$\omega_j=\min\{n\gamma+(j-1)\alpha,(n+1-j)\beta\}$ *for the CON method;*

$\omega_j=\min\{n\gamma+j\alpha,(n-j)\beta\}$ *for the SLK method;*

$\omega_j=\min\{\beta,\gamma\}(n-j+1)$ *for the DIF method.*

4 Solution with a linear resource consumption function

In this section, we will show that with a linear resource consumption function, the optimal schedule for the CON, SLK, and DIF due date assignment methods can be obtained in $O(n^3)$ time.

From [11], the actual processing time of job $J_{[j]}$ can be expressed as follows:

$$p_{[j]}=a_{[j]}-\theta_{[j]}u_{[j]}+b\sum_{l=1}^{j-1}(1+b)^{j-1-l}(a_{[l]}-\theta_{[l]}u_{[l]}). \tag{6}$$

From Eqs. 3, 5, and 6, we obtain the following objective function for all three due date assignment methods.

$$\begin{aligned}
f(\mathbf{d},\mathbf{u},\pi) &= \sum_{j=1}^{n}(\alpha E_j+\beta T_j+\gamma d_j+G_j u_j)=\sum_{j=1}^{n}\omega_j p_{[j]}+\sum_{j=1}^{n}G_{[j]}u_{[j]}\\
&=\sum_{j=1}^{n}\omega_j\left(a_{[j]}-\theta_{[j]}u_{[j]}+b\sum_{l=1}^{j-1}(1+b)^{j-1-l}(a_{[l]}-\theta_{[l]}u_{[l]})\right)+\sum_{j=1}^{n}G_{[j]}u_{[j]}\\
&=\left(\omega_1+b\omega_2+b(1+b)\omega_3+\ldots+b(1+b)^{n-2}\omega_n\right)(a_{[1]}-\theta_{[1]}u_{[1]})\\
&\quad+\left(\omega_2+b\omega_3+b(1+b)\omega_4+\ldots+b(1+b)^{n-3}\omega_n\right)(a_{[2]}-\theta_{[2]}u_{[2]})\\
&\quad+\left(\omega_3+b\omega_4+b(1+b)\omega_5+\ldots+b(1+b)^{n-4}\omega_n\right)(a_{[3]}-\theta_{[3]}u_{[3]})\\
&\quad+\ldots+(\omega_{n-1}+b\omega_n)(a_{[n-1]}-\theta_{[n-1]}u_{[n-1]})+\omega_n(a_{[n]}-\theta_{[n]}u_{[n]})+\sum_{j=1}^{n}G_{[j]}u_{[j]}\\
&=\sum_{j=1}^{n}\Omega_j a_{[j]}+\sum_{j=1}^{n}(G_{[j]}-\theta_{[j]}\Omega_j)u_{[j]},
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
\Omega_1&=\omega_1+b\omega_2+b(1+b)\omega_3+\ldots+b(1+b)^{n-2}\omega_n\\
\Omega_2&=\omega_2+b\omega_3+b(1+b)\omega_4+\ldots+b(1+b)^{n-3}\omega_n\\
\Omega_3&=\omega_3+b\omega_4+b(1+b)\omega_5+\ldots+b(1+b)^{n-4}\omega_n\\
&\ldots\\
\Omega_{n-1}&=\omega_{n-1}+b\omega_n\\
\Omega_n&=\omega_n.
\end{aligned}$$

From Eq. 7, for any sequence, the optimal resource allocation of a job in a position with a negative $G_{[j]}-\theta_{[j]}\Omega_j$ should be its upper bound on the amount of resource $m_{[j]}$, and the optimal resource allocation of a job in a position with a positive $G_{[j]}-\theta_{[j]}\Omega_j$ should be 0. If $G_{[j]}-\theta_{[j]}\Omega_j=0$, then the optimal resource allocation of the job in this position may be any value between 0 and $m_{[j]}$. Hence, we have:

Lemma 4.1 *Given a sequence, for all three due date assignment methods, the optimal resource allocation can be determined by*

$$u^*_{[j]}=\begin{cases} m_{[j]}, & \text{if}\quad G_{[j]}-\theta_{[j]}\Omega_j<0,\\ u_{[j]}\in[0,m_{[j]}], & \text{if}\quad G_{[j]}-\theta_{[j]}\Omega_j=0,\\ 0, & \text{if}\quad G_{[j]}-\theta_{[j]}\Omega_j>0,\end{cases} \tag{8}$$

where $u^*_{[j]}$, $1\le j\le n$, represents the optimal resource allocation of the job in position j.

Lemma 4.2 *For all three due date assignment methods, the optimal sequence can be determined by solving an assignment problem.*

Proof For $1 \le i, j \le n$, let

$$\lambda_{ij} = \begin{cases} \Omega_j a_i, & \text{if} \quad G_i - \theta_i \Omega_j \ge 0, \\ \Omega_j a_i + (G_i - \theta_i \Omega_j) m_i, & \text{if} \quad G_i - \theta_i \Omega_j < 0. \end{cases} \tag{9}$$

Furthermore, let x_{ij} be a 0/1 variable such that $x_{ij} = 1$ if job J_i is scheduled in position j, and $x_{ij} = 0$, otherwise. The optimal matching of jobs to positions requires a solution for the following assignment problem:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{ij} x_{ij} \tag{10}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n,$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n,$$

$$x_{ij} = 0 \text{ or } 1, \quad i, j = 1, 2, \ldots, n.$$

□

The results of our analysis are summarized in the following optimization algorithm that solves our problem with a linear resource consumption function for all three due date assignment methods:

Algorithm 4.1

Step 1	Calculate K by Eq. 4 (apply this step only for the CON and the SLK due date assignment methods).
Step 2	Calculate the λ_{ij} for each of the three due date assignment methods by using Eqs. 5, 7, and 9.
Step 3	Solve the assignment problem (10) to determine the optimal job sequence.
Step 4	Calculate the optimal resources by using Eq. 8.
Step 5	Calculate the optimal processing times by using Eq. 1.
Step 6	For each of the three due date assignment methods, assign the due dates according to Lemma 3.2.

Theorem 4.1 *Algorithm 4.1 solves the scheduling problem for each of the three due date assignment methods and linear resource consumption function in* $O(n^3)$ *time.*

Proof The correctness of the algorithm follows from Lemmas 4.1 and 4.2. Steps 1 and 2 can be performed in linear time; step 3 takes $O(n^3)$ time (using the well-known Hungarian method); steps 4, 5, and 6 can be performed in linear time. Thus, the overall computational complexity of the algorithm is indeed $O(n^3)$. □

5 Solution with a convex resource consumption function

From [19], the actual processing time of job $J_{[j]}$ can be expressed as follows:

$$p_{[j]} = \left(\frac{a_{i[j]}}{u_{i[j]}}\right)^k + b\left(\sum_{l=1}^{j-1}(1+b)^{j-1-l}\left(\frac{a_{i[l]}}{u_{i[l]}}\right)^k\right).$$

For the three due date assignment methods, we have

$$f(\mathbf{d}, \mathbf{u}, \pi) = \sum_{j=1}^{n} \Omega_j \left(\left(\frac{a_{i[j]}}{u_{i[j]}}\right)^k + b\left(\sum_{l=1}^{j-1}(1+b)^{j-1-l}\left(\frac{a_{i[l]}}{u_{i[l]}}\right)^k\right)\right) + \sum_{j=1}^{n} G_{[j]} u_{[j]}, \tag{11}$$

where Ω_j is given by Eq. 7.

In the following lemma, we determine the optimal resource allocation for the three due date assignment methods, denoted by $u^*(\pi)$, as a function of the job sequence.

Lemma 5.1 *For the three due date assignment methods, the optimal resource allocation as a function of the job sequence,* $u^*(\pi)$*, is*

$$u^*_{[j]} = \left(\frac{k\Omega_j}{G_{[j]}}\right)^{\frac{1}{k+1}} \times \left(a_{[j]}\right)^{\frac{k}{k+1}}. \tag{12}$$

Proof By taking the derivative of the objective given by Eq. 11 with respect to $u_{[j]}$, $j = 1, 2, \ldots, n$, equating it to 0 and solving it for $u_{[j]}$, we obtain Eq. 12. Since each of the objectives is a convex function, Eq. 12 provides necessary and sufficient conditions for optimality. □

By substituting Eq. 12 into Eq. 11, we obtain a new unified expression for the cost function for the objective function under an optimal resource allocation and as a function of the job sequence:

$$f(\mathbf{d}, \pi, u^*(\pi)) = \left(k^{\frac{-k}{k+1}} + k^{\frac{1}{k+1}}\right) \times \sum_{j=1}^{n} \theta_{[j]} \phi_j. \tag{13}$$

where

$$\theta_{[j]} = \left(G_{[j]} a_{[j]}\right)^{\frac{k}{k+1}} \tag{14}$$

and

$$\phi_j = \left(\Omega_j\right)^{\frac{1}{k+1}}. \tag{15}$$

In order to find the job sequence for all three due date assignment methods, we have to optimally match the positional penalties ϕ_j with the job-dependent costs θ_j. The optimal matching is as follows:

Lemma 5.2 *For all three due date assignment methods and convex resource consumption function, the optimal job sequence π^* can be obtained in the following way: assign the job with the smallest ϕ_j value to the job with the largest θ_j value, the second smallest ϕ_j value to the job with the second largest θ_j value, and so on.*

Proof See [36]. □

The results of our analysis are summarized in the following optimization algorithm that solves the problem for all three due date assignment methods and convex resource consumption function:

Algorithm 5.1

Step 1 Calculate K by Eq. 4 (apply this step only for the CON and the SLK due date assignment methods).
Step 2 For each due date assignment method, calculate ϕ_j and θ_j for $j = 1, 2, \ldots, n$ by Eqs. (14–15).
Step 3 Sequence the jobs according to Lemma 5.2, and denote the resulting optimal sequence by $\pi^* = [J_{[1]}, J_{[2]}, \ldots, J_{[n]}]$.
Step 4 Calculate the optimal resources allocation $u^*_{[j]}(\pi^*)$ by using Eq. 12.
Step 5 Calculate the optimal processing times by using Eq. 2.
Step 6 For each of the three due date assignment methods, assign the due dates according to Lemma 3.2.

Theorem 5.1 *Algorithm 5.1 solves the scheduling problem for each of the three due date assignment methods and convex resource consumption function in $O(n \log n)$ time.*

Proof The correctness of the algorithm follows from Lemmas 5.1 and 5.2. Steps 1, 2, 4, 5, and 6 can be performed in linear time and step 3 requires $O(n \log n)$ time. Thus, the overall computational complexity of the algorithm is $O(n \log n)$. □

6 Conclusions

In this paper, we have considered the resource allocation scheduling with deteriorating jobs. We studied three different due date assignment methods: CON, SLK, and DIF. For each of the three due date assignment methods, we presented a polynomial-time optimization algorithm to minimize a cost function containing earliness, tardiness, due date assignment, and total resource consumption costs for linear and convex resource consumption functions.

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