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ORIGINAL ARTICLE

Machining tests to identify kinematic errors of machine tool table rotation axis based on sensitive directions

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Abstract Machining tests based on homogeneous transformation matrix approach are relatively complex and time resuming in the mathematical modeling and the implementation of measurement and calculation. A new method is proposed to identify the kinematic errors of the rotation axis of machine tool table by machining tests based on sensitive directions. In order to identify the kinematic errors in the sensitive direction conveniently, a simple mathematical model of the kinematic errors is developed by optimizing the coordinate system settings based on basic kinematic transformation, and the sensitive direction vector was adopted to identify the kinematic errors from the machining errors of the finished workpiece. Experimental results demonstrate that the proposed method can reduce the complexity and time resuming substantially.

Keywords Machine tool · Rotation axis · Machining tests · Kinematic errors

1 Introduction

Five-axis machine tools can provide 5 degrees of freedom between the workpiece and the tool due to their structural configurations. They can reduce the number of setups for machining a part and improve the machining efficiency because of the flexible orientation of the workpiece to the tool. Consequently, five-axis machine tools have been widely applied in the industry. The structural configurations of five-axis machine tools can be divided into three types by the different layout of two rotary axes: a universal spindle head with two controlled axes, a swivel head with a controlled axis and a rotary table, and a tilting rotary table with two controlled axes [1]. Where, the machine tools with a tilting rotary table are also called cradle-type machine tools. They are the most commonly used small- and medium-sized machine tools.

With the rapid popularization of five-axis machine tools, they are more and more applied in the high-precision machining nowadays. The improvement of their motion accuracies is a crucial demand for the users. So the kinematic errors of the machine tools must be estimated effectively. International Organization for Standardization (ISO) developed the relevant evaluation specifications to identify kinematic errors of universal spindle head type five-axis machine tools [2]. For the machine tools with a tilting rotary table, there is currently no ISO standard to identify their kinematic errors. However, many instruments have been proposed to identify the kinematic errors of the machines tool with a tilting rotary table in recent years, such as telescoping double ball bar [3–5], R-test [6], 3D probe-ball [7, 8], capball sensor [9], and so on.

In 2010, Ibaraki et al. proposed the machining tests to identify the kinematic errors of five-axis machine tools [10]. The machining tests can more accurately reflect the kinematic errors of the machine tools than the non-cutting method and be more intuitively understood to evaluate the accuracy of the machine tools for the users. However, the measurement and computation of their method is relatively

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complex. It is mainly manifested in the following two aspects: firstly, homogeneous transformation matrix (HTM) was adopted to establish the kinematic model in their work. The calculation of HTM is complex because it relies on heavy symbolic manipulation of the matrix multiplication [11]. And the physical meaning of the mathematical based on HTM is difficult to understand. So it is difficult to identify the kinematic errors in the sensitive direction from this kinematic model. Secondly, in order to identify the kinematic errors, not only the difference between the lengths in a Cartesian axis of the finished planes should be measured and calculated but also the other differences of the finished workpiece, such as the difference of the angles of the intersection line between the vertical plane and the horizontal plane around a Cartesian axis, should be measured and calculated. It makes the process of the measurement and calculation relatively complex and time consuming.

In this paper, a simple method is developed and implemented for modeling the kinematic errors of five-axis machine tool rotary table by optimizing the coordinate system settings. It reduces the computations substantially and makes the kinematic model be understandable. It is convenient to identify the kinematic errors in the sensitive direction from this kinematic model. Then, sensitive direction vector is adopted in the machining tests. It reduces the complexity of the process of the measurement substantially because only the differences between the lengths of the finished planes in the sensitive direction need to be considered in the machining tests.

2 A simple mathematical model of kinematic errors

A cradle-type five-axis machine tool is studied in this paper. The structural configuration can be seen in Fig. 1.

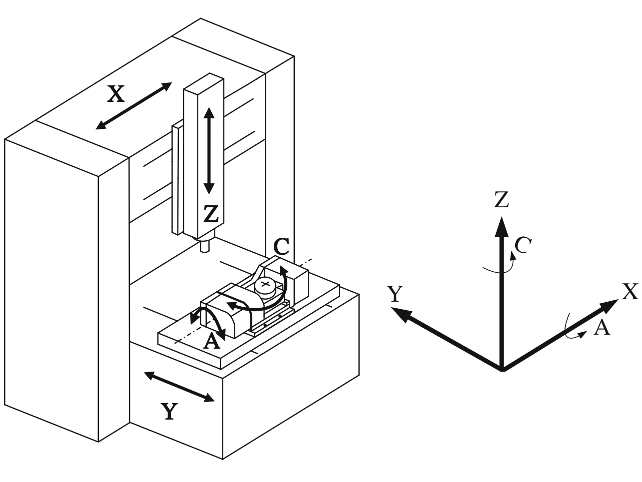

Fig. 1 Structural configurations of cradle-type five-axis machine tool

2.1 Kinematic errors to be identified

There are three linear displacement errors ${}^{C}\delta_x(\gamma)$, ${}^{C}\delta_y(\gamma)$, and ${}^{C}\delta_z(\gamma)$ and three angular errors ${}^{C}\xi_x(\gamma)$, ${}^{C}\xi_y(\gamma)$, and ${}^{C}\xi_z(\gamma)$ in C-axis. Where, ${}^{C}\delta_x(\gamma)$, ${}^{C}\delta_y(\gamma)$, and ${}^{C}\delta_z(\gamma)$ are the linear shifts in X direction, Y direction, and Z direction with respect to A-axis, respectively; ${}^{C}\xi_x(\gamma)$, ${}^{C}\xi_y(\gamma)$, and ${}^{C}\xi_z(\gamma)$ are the angular errors of the center line of C-axis about X direction, Y direction, and Z direction with respect to A-axis, respectively.

The main kinematic errors which have greater impact on the position accuracy of the workpiece are: linear displacement errors ${}^{C}\delta_x(\gamma)$ and ${}^{C}\delta_y(\gamma)$; angular errors ${}^{C}\xi_x(\gamma)$ and ${}^{C}\xi_y(\gamma)$. The other kinematic errors of the workpiece can be ignored in the process of machining. Because ${}^{c}\delta_z(\gamma)$ can be eliminated by tool setting and workpiece localization, and ${}^{C}\xi_z(\gamma)$ is in the direction of the output of C-axis rotation. Therefore, only four kinematic errors of C-axis need to be identified.

2.2 Machine tool table coordinate system settings

According to the structure of the machine tool table, the coordinate systems of C-axis were reasonably set up with a simple method in this paper, as can be seen in Fig. 2.

1. Establish the reference coordinate system $\{F\}$ in the centerline intersection point of A-axis and C-axis (point O) when C-axis remains stationary.
2. Establish the coordinate system $\{C\}$ (nominal coordinate system of C-axis) when the nominal displacement of C-axis is γ. There is a rotation transformation for coordinate system $\{C\}$ to coordinate system $\{F\}$. The rotation operator is represented as ${}^{F}_{C}R$.
3. When the actual displacement of C-axis is γ, the position of C-axis changes from coordinate system $\{C\}$ to coordinate system $\{C'\}$ because of the kinematic errors. The intersection of coordinate system $\{C'\}$ and coordinate system $\{A\}$ is point O'. At this moment, establish the coordinate system $\{C'\}$ (actual coordinate system of C-axis) on point O'.

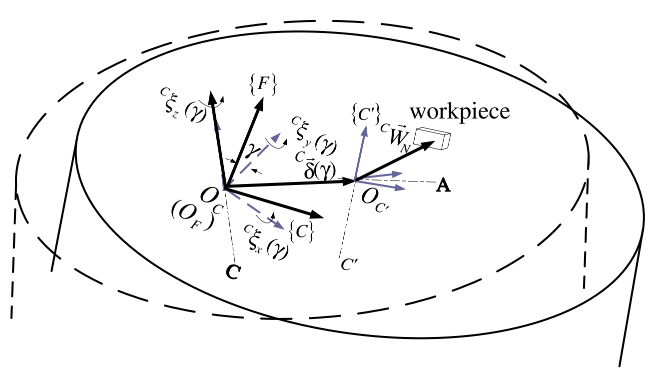

Fig. 2 Kinematic error modeling of C-axis

2.3 The modeling process of the kinematic errors

The modeling process of the kinematic errors is described as follows:

1. Description of the workpiece position

Suppose that the initial position of the workpiece is in the $+X$ direction and has a distance of L from the center of the rotary table. It can be represented as

$$^{C'}\overrightarrow{W}_N = \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

2. The differential transformation

There is a differential transformation from coordinate system $\{C'\}$ to coordinate system $\{C\}$. The differential transformation is completed by the differential movement transformation and the differential rotation transformation, as can be seen in Eq. (2).

$$^{C}\overrightarrow{W} = {}^{C}\overrightarrow{\delta}(\gamma) + {}^{C}_{C'}R\,{}^{C'}\overrightarrow{W}_N \tag{2}$$

Where, $^{C}\overrightarrow{\delta}(\gamma)$ is the differential motion vector of coordinate system $\{C'\}$ to coordinate system $\{C\}$ when C-axis turned the angle of γ around the Z-axis. $^{C}\overrightarrow{\delta}(\gamma)$ is represented as

$$^{C}\overrightarrow{\delta}(\gamma) = \begin{bmatrix} ^{C}\delta_x(\gamma) \\ ^{C}\delta_y(\gamma) \\ ^{C}\delta_z(\gamma) \end{bmatrix} \tag{3}$$

As $^{C}\overrightarrow{\delta}_z(\gamma)$ can be ignored, we can obtain

$$^{C}\overrightarrow{\delta}(\gamma) = \begin{bmatrix} ^{C}\overrightarrow{\delta}_x(\gamma) \\ ^{C}\overrightarrow{\delta}_y(\gamma) \\ 0 \end{bmatrix} \tag{4}$$

Based on the assumptions of small angle [12], the operator of the differential rotation transformation from coordinate system $\{C'\}$ to coordinate system $\{C\}$ can be represented as

$$^{C}_{C'}R = \begin{bmatrix} 1 & -\xi_z(\gamma) & \xi_y(\gamma) \\ \xi_z(\gamma) & 1 & -\xi_x(\gamma) \\ -\xi_y(\gamma) & \xi_x(\gamma) & 1 \end{bmatrix} \tag{5}$$

As $\xi_z(\gamma)$ can be ignored, we can obtain

$$^{C}_{C'}R = \begin{bmatrix} 1 & 0 & ^{C}\xi_y(\gamma) \\ 0 & 1 & -^{C}\xi_x(\gamma) \\ -^{C}\xi_y(\gamma) & ^{C}\xi_x(\gamma) & 1 \end{bmatrix} \tag{6}$$

3. Rotation transformation

The rotation transformation operator is represented as

$$^{F}_{C}R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \tag{7}$$

3 Machining tests description

According to the mathematical modeling of the kinematic errors, the design of the machining tests is described, as can be seen in Fig. 3.

Install the workpiece on the table when Y-axis and A-axis remain stationary and the angel of A-axis is 0°. Two

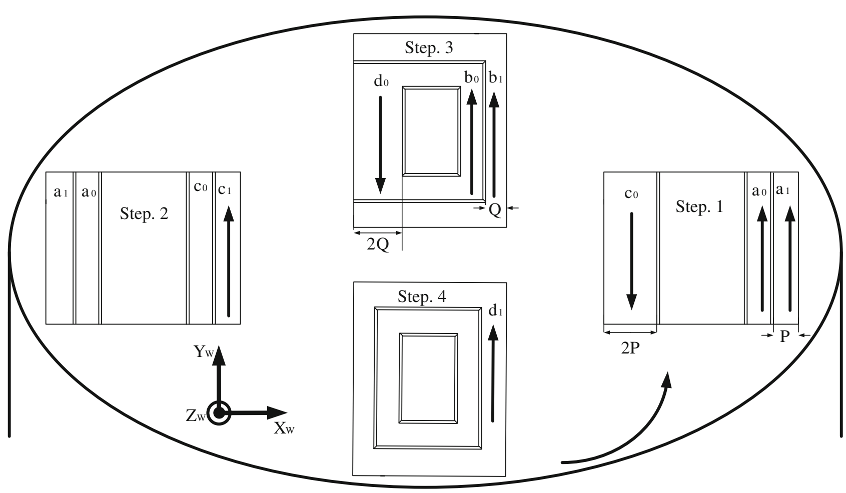

Fig. 3 Machining tests of five-axis machine tool C-axis

machining patterns were developed as follows. They can be finished in single setup of the workpiece.

Machining pattern 1:

Machining pattern 1 can be seen from step 1 and Step.2 in Fig. 3.

The initial position of the workpiece is in the $+X$ direction and has a distance of L from the center of the rotary table. In the machining tests of C-axis, A-axis remains stationary. To simply the calculation, we take the point (0,0,H) as the center point of the reference coordinate system $\{F\}$.

Every machining pattern contains two processes as follows:

1. Machining procedure of the measure datum

Cut the first layer of the workpiece to be the measurement datum plane, the performance is as follows:

Perform a side cutting along the $+Y$ direction on the right side of the workpiece, and perform a side cutting along the $-Y$ direction on the left side of the workpiece. The two planes are both $2P$ of width and h of depth. The intersection lines are a_0 and c_0.

2. Machining procedure of the planes to be measured

After cutting the measure datum, turn the tool back to the starting point to eliminate the impact of the kinematic errors of the displacement axis on the planes to be measured. When the rotation angel of C-axis is 0°, perform a side cutting on the right side of the workpiece along $+Y$ direction. When the rotation angel of C-axis is +180°, perform a side cutting on the right side of the workpiece along $+Y$ direction, too. Thus, the intersection lines a_1 and c_1 were generated.

Where, the intersection line of the vertical plane a_v and the horizontal plane a_h is a_1, and the intersection line of the vertical plane c_v and the horizontal plane c_h is c_1. The horizontal distance between the vertical plane a_v and the vertical plane c_v is P, and the vertical distance between the horizontal plane a_h and the horizontal plane c_h is h_{ca}. The intersection line c_0 is generated when the angel of C-axis is 0°, and the intersection line c_1 is generated when the angel of C-axis is +180°. So, the actual cutting direction of c_0 and c is the same and the impact of the kinematic errors of linear axis Y-axis on the plane can cancel each other out.

Machining pattern 2:

Machining pattern 2 can be seen from step 3 and step 4 in Fig. 3. The initial position of the workpiece is in $+Y$ direction and has a distance of L from the center of the rotary table. The machining procedure is similar with machining pattern 1. Similarly, four intersection lines b_0, d_0, b_1, and d_1 were generated. Where, the horizontal distance between the vertical plane b_v and the vertical plane d_v is Q, and the vertical distance between the horizontal plane b_h and the horizontal plane d_h is h_{bd}.

4 Identification of the kinematic errors

From machining test 1, we can obtain the mathematical representation of the workpiece based on the analysis in Section 2

$$
\begin{aligned}
&{}^{F}\overrightarrow{W} = {}^{F}_{C}R\left[{}^{C}\overrightarrow{\delta} + {}^{C}_{C'}R^{C'}\overrightarrow{W}_N\right] \\
&= \begin{bmatrix} \cos\gamma \times (L + {}^{C}\delta_x) - \sin\gamma \times {}^{C}\delta_y \\ \sin\gamma \times (L + {}^{C}\delta_x) + \cos\gamma \times {}^{C}\delta_y \\ -L \times {}^{C}\xi_y \end{bmatrix}
\end{aligned} \tag{8}
$$

Here, the concept of error-sensitive direction vector [13] is adopted to identify individual kinematic error. The position vector of the workpiece in $+X$ error-sensitive direction can be represented as

$$
P = {}^{F}\overrightarrow{W} \cdot [1 \quad 0 \quad 0] \tag{9}
$$

The error-sensitive direction vector r_1=[1 0 0] represents that the geometry is in $+X$ direction. The difference of the direction vector from P_{a1} to P_{c1} in $+X$ direction is

$$
\begin{aligned}
&P_{c1} - P_{a1} \\
&= {}^{F}\overrightarrow{W}_1(\gamma = 180^0) \cdot r_1 - {}^{F}\overrightarrow{W}(\gamma = 0^0) \cdot r_1
\end{aligned} \tag{10}
$$

Where:

$$
\begin{aligned}
&{}^{F}\overrightarrow{W}_1(\gamma = 180^\circ) \\
&= \begin{bmatrix} \cos 180^\circ \times (L + {}^{C}\delta_x) - \sin 180^\circ \times {}^{C}\delta_y \\ \sin 180^\circ \times (L + {}^{C}\delta_x) + \cos 180^\circ \times {}^{C}\delta_y \\ -L \times {}^{C}\xi_y \end{bmatrix}; \\
&{}^{F}\overrightarrow{W}_1(\gamma = 0^\circ) \\
&= \begin{bmatrix} \cos 0^\circ \times L - \sin 0^\circ \times {}^{C}\delta_y \\ \sin 0^\circ \times L + \cos 0^\circ \times {}^{C}\delta_y \\ 0 \end{bmatrix}
\end{aligned}
$$

Through calculation, we can obtain

$$
P_{c1} - P_{a1} = -2L - {}^{C}\delta_x \tag{11}
$$

When the intersection line a and intersection line c are finished, the tool moved the distance of $-2L$. Consequently, the difference of the width of horizontal plane a_h and the width of horizontal plane c_h is

$$
\Delta P = -{}^{C}\delta_x \tag{12}
$$

We can know that r_1=[1 0 0] is the sensitive direction of the error vector ${}^{C}\delta_x(\gamma)$. Meanwhile, it is the direction in which the error of the workpiece ΔP exists.

Similarly, the position vector in $-Z$ sensitive direction can be expressed as

$$h={}^{F}\overrightarrow{W}\cdot\begin{bmatrix}0 & 0 & -1\end{bmatrix} \tag{13}$$

Thus, r_3=[0 0 −1] is the sensitive direction of the error vector.

$$\begin{aligned}\Delta h_{ac} &= h_{cv}-h_{av}\\ &={}^{F}\overrightarrow{W}_1(\gamma=180^\circ)\cdot r_3-{}^{F}\overrightarrow{W}(\gamma=0^\circ)\cdot r_3\\ &=L\times{}^{C}\xi_y(\gamma)\end{aligned} \tag{14}$$

Where, Δh_{ac} represents the difference between the height of the horizontal plane a_h and the height of the horizontal plane c_h.

From machining pattern 2, we can obtain the mathematical expression of the workpiece error.

$$\begin{aligned}{}^{F}\overrightarrow{W} &={}^{F}_{C}R\left[{}^{C}\overrightarrow{\delta}(\gamma)+{}^{C}_{C'}R^{C}\overrightarrow{W}_N\right]\\ &=\begin{bmatrix}{}^{C}\delta_x(\gamma)\\ \cos\gamma\times\left({}^{C}\delta_y(\gamma)+L\right)-\sin\gamma\times{}^{C}\xi_x(\gamma)\times L\\ \sin\gamma\times\left({}^{C}\delta_y(\gamma)+L\right)+\cos\gamma\times{}^{C}\xi_x(\gamma)\times L\end{bmatrix}\end{aligned} \tag{15}$$

The position vector of the workpiece in $+Y$ error-sensitive direction can be represented as

$$Q={}^{F}\overrightarrow{W}\cdot\begin{bmatrix}0 & 1 & 0\end{bmatrix} \tag{16}$$

The difference of the direction vector from P_{b1} to P_{d1} in $+X$ direction is

$$\begin{aligned}Q_{d1}-Q_{b1} &=\cos 180^\circ\times\left({}^{C}\delta_y(\gamma)+L\right)-\cos 0^\circ\times L\\ &=-2L-{}^{C}\overrightarrow{\delta}_Y(\gamma)\end{aligned} \tag{17}$$

When the intersection lines b_1 and d_1 were finished, the tool moved the distance of $-2L$. Consequently, the difference of the width of horizontal plane b_h and the width of horizontal plane d_h is

$$\Delta Q=-{}^{C}\overrightarrow{\delta}_Y(\gamma) \tag{18}$$

Thus, the position vector of the workpiece in $-Z$ error-sensitive direction can be represented as

$$h={}^{F}\overrightarrow{W}\cdot\begin{bmatrix}0 & 0 & -1\end{bmatrix} \tag{19}$$

The position vector difference of the workpiece geometry from h_a to h_c in $-Z$ direction is

$$\begin{aligned}\Delta h_{bd} &= h_{dv}-h_{bv}\\ &=cos180^\circ\times{}^{C}\xi_X(\gamma)\times L\\ &=-L\times{}^{C}\xi_X(\gamma)\end{aligned} \tag{20}$$

Where, Δh_{bd} represents the difference between the height of the horizontal plane b_h and the height of the horizontal plane d_h. Thus, we can identify four C-axis kinematic errors from the machining error of the finished workpiece.

Fig. 4 Machining tests of five-axis machine tool C-axis

5 Experimental research

Mikron UCP600 machining center is studied for the machining tests, as can be seen from Fig. 4. The finished workpiece is shown in Fig. 5. CMM inspection is performed on the middle line of the vertical and horizontal planes. Because the sensitive direction is in the Cartesian axes, and the workpiece is located in the center point of the rotary table, so the middle line of the plane is just in the sensitive direction. There were n sampling points in every middle line.

The sampling points were fitted into a line by least-square method. The distance in the sensitive direction between two parallel fitted lines is the machining length. The difference between the nominal value and the actual value of the machining length is the machining error, which was obtained through calculation.

For machining pattern 1, we can obtain the results from the sampled pints:

$$\Delta P_{ac}=-{}^{C}\overrightarrow{\delta}_X=-0.0128\,mm \tag{21}$$

$$\Delta h_{ac}={}^{C}\overrightarrow{\xi}_Y\times L=-0.0134\,mm \tag{22}$$

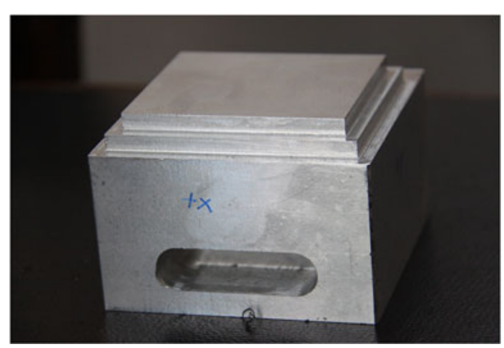

Fig. 5 Finished workpiece

For machining pattern 2, we can obtain the results from the sampled pints:

$$\Delta Q_{bd} = -{}^{C}\overrightarrow{\delta}_{Y} = -0.0190\ mm \tag{23}$$

$$\Delta h_{bd} = -{}^{C}\overrightarrow{\xi}_{X}(\gamma) \times L = -0.0133\ mm \tag{24}$$

We can obtain the results by calculation from Eqs. (21) and (23):

$$\begin{aligned} {}^{C}\overrightarrow{\delta}_{X} &= 0.0128\ mm \\ {}^{C}\overrightarrow{\delta}_{Y} &= 0.0190\ mm \end{aligned}$$

The distance from the center of the finished plane to the center of the table is obtained from the measurement:

$$L = 137.4\ mm$$

Consequently, we can obtain the results by calculation from Eqs. (22) and (24):

$$\begin{aligned} {}^{C}\overrightarrow{\xi}_{X} &= 9.6696 \times 10^{-5} rad = 0.0055^{\circ} \\ {}^{C}\overrightarrow{\xi}_{Y} &= -9.7314 \times 10^{-5} rad = -0.0056^{\circ} \end{aligned}$$

6 Discussions

The advantages of the proposed machining tests based on sensitive direction over the machining tests based on HTM method [11] are discussed in detail, as follows.

1. The description and calculation of the workpiece position vector is based on basic kinematic transformation. The workpiece position in reference coordinate system can be denoted as ${}^{F}\overrightarrow{W} \in R^{3}$. The transformation can be denoted as ${}^{F}_{C}R \in R^{3\times 3}$ and ${}^{C}_{C'}R \in R^{3\times 3}$. This mathematical expression is understandable, and it is convenient to identify the kinematic errors in the Cartesian axes from this mathematical model. And the calculation of the modeling process is simpler than HTM method.
2. The sensitive direction vectors, which are denoted as $\overrightarrow{r}_{1} = [1 \quad 0 \quad 0]$ and r_3=[0 0 −1], are adopted to separate the kinematic errors from the finished workpiece. Only the difference of the lengths in the middle line of the machining plane needs to be measured. The other differences, such as the difference of the angles of the intersection line between the vertical plane and the horizontal plane around a Cartesian axis, do not need to be considered in this method. So, the complexity of the calculation and the time resuming of the machining tests are reduced substantially.

7 Conclusion

In this work, the machining tests based on sensitive directions are proposed to assess the kinematic errors of the rotary axis (*C*-axis) of five-axis machine tool table. Two linear displacement errors and two angular errors of *C*-axis can be identified from the machining differences of the finished workpiece in the sensitive direction conveniently. Experimental results demonstrate that the proposed method can reduce the complexity and time resuming substantially compared with the machining tests based on homogeneous transformation matrix approach. Furthermore, the machining tests based on sensitive directions will be implemented to assess the other errors of five-axis machine tools in our future research.

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References

1. Tsutsumi M, Saito A (2003) Identification and compensation of systematic deviations particular to 5-axis machining centers. Int J Mach Tools Manuf 43(8):771–780
2. ISO 10791 series (1998) Test conditions for machining centres. International Organization for Standardization, Geneva
3. Bryan JB (1982) A simple method for testing measuring machines and machine tools, Part I: principle and applications. Precis Eng 4 (2):61–69
4. Tsutsumi M, Saito A (2004) Identification of angular and positional deviations inherent to 5-axis machining centers with a tilting-rotary table by simultaneous four-axis control movements. Int J Mach Tools Manuf 44(12–13):1333–1342
5. Dawei Z, Peng S, Yanling T, Qiongyi H (2008) A DBB-based alignment error measurement method for rotary axis of 5-axis CNC machine tool. China Mech Eng 19(22):2737–2741
6. Weikert S (2004) *R*-test, a new device for accuracy measurements on five axis machine tools. CIRP Ann Manuf Technol 53(1):429–432
7. Lei WT, Hsu YY (2002) Accuracy test of five-axis CNC machine tool with 3D probe-ball. Part I: design and modelling. Int J Mach Tools Manuf 42(10):1153–1162
8. Lei WT, Hsu YY (2002) Accuracy test of five-axis CNC machine tool with 3D probe-ball. Part II: errors estimation. Int J Mach Tools Manuf 42(10):1163–1170
9. Zargarbashi SHH, Mayer JRR (2009) Single setup estimation of a five-axis machine tool eight link errors by programmed end point constraint and on the fly measurement with capball sensor. Int J Mach Tools Manuf 49(10):759–766
10. Ibarakia S, Sawada M, Matsubara A, Matsushita T (2010) Machining tests to identify kinematic errors on five-axis machine tools. Precis Eng 34:387–398
11. Lin Y, Shen Y (2003) Modelling of five-axis machine tool metrology models using the matrix summation approach. Int J Adv Manuf Technol 21:243–248
12. Youlun X (1993) Robotics. China Machine, Beijing
13. Zargarbashi SHH, Mayer JRR (2006) Assessment of machine tool trunnion axis kinematic error, using magnetic double ball bar. Int J Mach Tools Manuf 46(14):1823–1834