ORIGINAL ARTICLE

Integration of fuzzy logic with response surface methodology for thrust force and surface roughness modeling of drilling on titanium alloy

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Abstract In recent years, a lot of extensive research work has been carried out in drilling operations for achieving better hole quality. Drilling operation is one of the machining processes, and it widely used in aeronautical and automotive industries for assembling the parts. The surface roughness is one of the significant factors in drilling operation because the poor surface finish will affect the material condition during the assembly. The spindle speed and feed rate are the important factors to affect the surface finish. In addition, the detailed analysis of the thrust force is also to be investigated for characterizing the cutting process. However, for examining the machining characteristics more trial runs are required, and it increases the time and cost of the experiment. In this paper, the integration of fuzzy logic (FL) with response surface methodology (RSM) has been introduced to reduce the cost and the time consumption for investigation. The low, middle, and upper levels of spindle speed with low and upper levels of feed rate combinations were examined on cutting force and surface finish through the experimental setup with the systematic manner. The FL model for thrust force and surface finish were obtained from the collected experimental data. The FL model has developed another two combinations of data without experimentation through universal partitioning. The results show that the predicted FL values are within the range of experimental value. Therefore, the FL model values were selected for further investigation with RSM. The result of FL-RSM

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N. Baskar e-mail: baskarnaresh@yahoo.co.in model values are also within the range of experimental value. The proposed FL-RSM model and FL model are validated with experimental results. Finally, the validated results show that hybrid FL-RSM produces the effective output than the FL model.

Keywords Drilling operation · Fuzzy logic · Response surface methodology · Thrust force · Surface roughness

1 Introduction

Nowadays, a lot of research work was carried out for reducing the experimental work. In the previous research, many of the researchers used the trial-and-error-based experimental analysis. It increases the experimental time and associated costs. In current days, some researchers used the design of experiment concepts for developing experimental plan through factorial design. These techniques were used to reduce the cost and time of experiment. However, this work tries further reduction on the experimental time and cost by using fuzzy logic concepts. And further data were developed from fuzzy logic modal investigated with analysis of variance (ANOVA) and regression analysis. The developed regression model's performances were evaluated from experimental data. Arghavani et al. [17] proposed that the fuzzy logic approach would reduce experimental runs, test points, time, and ultimately reduce associated costs. Tarng et al. [18] proposed that fuzzy logic is a novel and efficient approach for quality optimization of manufacturing systems with a consideration of multiple performance characteristics. Karthikeyan et al. [16] found that the fuzzy logic model is effective for modeling the drilling characteristics.

Most of the products needed drilled holes for assembly of parts. So the drilling operation was selected for the

investigation of the proposed technique. The manufacturing industries would like to produce the component with high quality and with minimum effort required for latest materials. The machining is a significant and precise process operation of major importance to manufacturing industry [1]. Sharif et al. [3] illustrated that drilling process accounts for 40-60 % of the total material removal processes and that it is an essential technique in aerospace industries. Kurt et al. [2] proposed modern metal cutting methods to improve in manufacturing industry, including drilling, electron-beam machining, ultrasonic machining, electrolytic machining, and abrasive jet machining; conventional drilling still remained one of the most common machining processes because of economical reasons and simplicity. In addition, Strenkowski et al. [4] delivered that drilling problems can result in costly production waste because many drilling operations are usually among the final steps in fabricating a part. Nowadays, the aeronautical industries used the titanium alloys for its inherent properties. Kao et al. [5] analyzed and proposed that the Ti-6Al-4V alloy was an important material in modern industry. Its exceptional properties such as high strength-weight ratio, high temperature stability, and outstanding corrosion resistance make it widely used in the aero space, automobile, chemical, and biomedical fields [1]. The study of parameter effects on such material is very important to reduce the drilling problems.

Modeling of drilling process is also one of the important task for achieving better surface finish and minimum thrust force. These are the responses considered in this work. Drilling process mainly depends on the cutting conditions like spindle speed and feed rate. The decision concerning for the selection of these cutting parameters have a significant influence on the extent of production, production costs, and production quality [6]. Surface roughness and thrust force



Fig. 1 Schematic diagram for experimental setup

models are needed to monitor the process to obtain machining accuracy and process efficiency [7]. The thrust force produced during drilling contains important information related to the quality of the hole [8]. The objective of most drilling experiments was to reduce a number of experiments with design of experiment concepts [9-12]. Much theoretical and experimental work was carried out to develop the mathematical relationship between independent and dependent variables. In this sense, the mechanistic and numerical models were developed for predict the response values. Langella et al. [13] developed a mechanistic model for thrust force and the developed model was suitable for predicting the thrust force. Redouane Zitoune and Francis Collombet [14] proposed a numerical model to calculate the thrust force. The numerical model has been advantageous to predict the plastic deformation and drill geometry influences on drilling operation. ANOVA and RSM have become the statistical tool for analyzing the effect of process parameters to improve the quality of the products and to reduce the effort required to produce the products from existing systems in manufacturing processes. Noorulhag et al. [15] studied the multi-response such as surface roughness, cutting force, and torque on drilling operation by using



Fig. 2 Flow chart for fuzzy logic-RSM procedure

statistical techniques. The optimum levels of parameters have been identified and significant contribution of parameters is determined by ANOVA. Recently, the focus has been on artificial intelligence tools such as fuzzy logic, neural network, and RSM for modeling [16]. Arghavani et al. [17] proposed that the fuzzy logic approach would reduce experimental runs, test points, time, and ultimately reduce associated costs. Tarng et al. [18] proposed fuzzy logic is a novel and efficient approach for quality optimization of manufacturing systems with a consideration of multiple performance characteristics. In this work, the efficiencies of the modeling techniques are increased by analyzing the thrust force and surface roughness in drilling operation.

In this work, the experimental procedure was followed by the full factorial design. The fuzzy logic model also developed and the fuzzy output analyzed with response surface methodology. The fuzzy logic model and combination of fuzzy logic—RSM is verified with experimental data. The rest of the paper is organized as follows: Section 2 explains proposed methodology, the experimental procedure,

Fig. 3 a Spindle speed input membership function. b Thrust force and surface roughness output membership functions

measurement of response, and modeling concepts; comparison of modeling techniques handled in results and conclusion Section 3; conclusion and scope for future research are proposed in Section 4.

2 Methodology

The proposed work methodology starts with experimental approach and the experimental data are analyzed with fuzzy logic concepts and RSM. Finally, the modeling data are validated with experimental data. The proposed methodology of this work as follows:

2.1 Experimental setup and cutting conditions

The drilling experiments were conducted in a radial drilling machine. The tools were used 4-mm TiAlN-coated carbide twist drill. Ti–6Al–4V was used for this study. The sample work piece of diameter 43×10 mm thickness was used. The



experimental design follows three levels of spindle speed (120, 180, and 260 rpm) and two levels of feed rate (0.02–0.05 mm/rev). These cutting conditions were selected based on tool manufacturer recommendation and machine tool capacity. The combinations between these two factors were created, and each experiment observed two replicate values. In total, 12 experiments were conducted. The work piece was clamped in the special fixture to measure the force generated during drilling. Figure 1 shows the schematic of drilling experimental setup.

2.2 Thrust force and surface roughness measurement

The thrust force mainly depends on feed, drill bit, and the work piece thickness [20]. The thrust force generated was measured by using a digital dynamometer recorder for all 12 holes. The digital drill tool dynamometer is used to measure the thrust force. The various factors that affect the surface roughness are vibrations, material of the work piece, type of machining, rigidity of the system (consisting of machine tool, work holding devices, material of tool, and work piece), and cutting conditions. Surface roughness plays an important role in product quality and manufacturing process planning [21]. The surface roughness values of the machined hole surface were measured in order to analyze the surface finish quality. The surface roughness of the job was measured by using the surface roughness tester of Mitutoyo make and SJP 210P.

2.3 Fuzzy logic and regression modeling for surface roughness and thrust force prediction

Vinay Sharma et al. [19] integrated the fuzzy rule based reasoning into Taguchi loss function. The best combination of a parameter found from fuzzy-Taguchi integrated approach for water jet technology was proposed. Similarly, this work assumes to combine the fuzzy logic and RSM to reduce the experimental run and cost for drilling operation. The operations of fuzzy-RSM model are shown in Fig. 2. The experimental outputs were used to formulate the structure of input and output membership functions. This modeling technique was carried out to compute the various inputs and output combinations. The fuzzy logic model is built based on experimental results. The output data generated from fuzzy logic model is analyzed with RSM. The mathematical model is developed from fuzzy output data. Finally, the results of fuzzy model and fuzzy-RSM model are compared.

Generally, the fuzzy logic procedure involves evaluation of fuzzy rule, universe partitioning, making relationship between input and output membership function and rules combination. The input, output functions and membership functions are shown in Fig. 3a and b.

Table 1 Fuzzy expression								
Input (spindle speed)	Output (thrust force and surface roughness)							
Low	Low							
Medium	Medium							
High	High							

2.3.1 Evaluation of fuzzy rule

El bardie [22] developed the stages of a fuzzy logic model for metal cutting, this work followed for drilling operation thrust force and surface roughness modeling. Fuzzy logic provides foundations for approximate reasoning with imprecise fuzzy propositions using fuzzy set theory as a main rule. Fuzzy logic could be used for modeling these potential threats more effectively. Fuzzy logic rules are developed in linguistic terms that address the relationship between the inputs and the outputs from a system. The data collected from experiments are ordered in a logic control system. In this experiment, the input parameter considered is spindle speed and the output responses are thrust force and surface roughness. The input machining parameter is divided into three membership function called low, medium, and high. The output also divided into three sets like low, medium, and high. The modeling of the thrust force and surface roughness followed the triangular type as shown in Fig. 3a, b from the available membership functions like Gaussian, trapezoidal, etc. Table 1 shows the fuzzy expression of input and outputs.

The fuzzy rules note down in the form of if-then rules and connect the input membership function to output membership function. The fuzzy reasoning concept for one input and four outputs are as follows.

For thrust force

- Rule 1 If spindle speed is low, then thrust force is high.
- Rule 2 If spindle speed is medium, then thrust force is medium.
- Rule 3 If spindle speed is high, then thrust force is low.

For entry surface roughness

Rule 1 If spindle speed is low, then entry surface roughness is high.

 Table 2 Universe of input membership (spindle speed)

Fuzzy terms	Discrete universe of input membership (spindle speed)						
	0	1	2	3	4		
Low	1	0.5	0	0	0		
Medium	0	0.5	1	0	0		
High	0	0	0	0.5	1		

 Table 3 Universe of output membership (thrust force)

Fuzzy terms	Discrete universe of output membership (thrust force						
	0	1	2	3	4		
Low	1	0.5	0	0	0		
Medium	0	0.5	1	0	0		
High	0	0	0	0.5	1		

Rule 2 If spindle speed is medium, then entry surface roughness is medium.

If spindle speed is high, then entry surface rough-Rule 3 ness is low.

For middle surface roughness

- Rule 1 If spindle speed is low, then middle surface roughness is high.
- Rule 2 If spindle speed is medium, then middle surface roughness is medium.
- Rule 3 If spindle speed is high, then middle surface roughness is low.

For exit surface roughness

- Rule 1 If spindle speed is low, then exit surface roughness is high.
- Rule 2 If spindle speed is medium, then exit surface roughness is medium.
- Rule 3 If spindle speed is high, then exit surface roughness is low.

These rules are framed based on experimental investigation.

2.3.2 Universe partitioning

The fuzzy rules are derived from Section 2.3.1, the additional method is to partition the universe of the input and output using membership function plots. The input membership function is partitioned regarding minimum and maximum values that allowed controlling the system (spindle speed_{min}-spindle speed_{max}). Based on this, the input membership range is 0-4. If the value is more than spindle speed_{max} then the range is assumed to be infinity and the

Table 4 Relationship between high input to low output

		Unive	Universe of output (thrust force)					
		0	1	2	3	4		
Universe <i>R</i> 1=spindle speed	0	0	0	0	0	0		
I I	1	0	0	0	0	0		
	2	0	0	0	0	0		
	3	0.5	0.5	0	0	0		
	4	1	0.5	0	0	0		

 Table 5 Relationship between medium input to medium output

		Uni	verse of a	output (tl	nrust fo	orce)	
		0	1	2	3	4	
Universe R1=spindle speed	0	0	0	0	0	0	
	1	0	0.5	0.5	0	0	
	2	0	0.5	1	0	0	
	3	0	0	0	0	0	
	4	0	0	0	0	0	

input membership is almost zero when spindle speed is minimum value.

The value allocated for spindle speed is as follows:

- 0 Minimum spindle speed
- 4 Maximum spindle speed

Similarly, the output membership partitioning based on the range of thrust force. There are 12 experimental data for thrust force is carried out for the work material of Ti-6Al-4V.

The value assigned for thrust force as follows:

- 0 Minimum thrust force
- 4 Maximum thrust force

The Tables 2 and 3 illustrate the universe of input membership and universe of output membership based on membership function plot.

2.3.3 Fuzzy relation

Fuzzy relation builds the relation between input membership function to output membership function. The relationship established based on the "High" spindle speed and the "Low" thrust force with Cartesian product. The Cartesian product represents all feasible combinations of rows and columns from the Tables 2 and 3.

Cartesian product contains $m \times n$ rows. Where

- Number of rows in the Table 2 т
- Number of rows in the Table 3 n

Table 6 Relationship between low input to high output

		Universe of output (thrust force)					
		0	1	2	3	4	
Universe R1=spindle speed	0	0	0	0	0.5	1	
	1	0	0	0	0.5	0.5	
	2	0	0	0	0	0	
	3	0	0	0	0	0	
	4	0	0	0	0	0	

Table 7 Combination of rule1 and rule 2 for thrust force

		Universe of output (thrust force)				
		0	1	2	3	4
Universe of spindle	0	0	0	0	0	0
speed $R1 + R2 =$	1	0	0.5	0.5	0	0
	2	0	0.5	1	0	0
	3	0.5	0.5	0	0	0
	4	1	0.5	0	0	0

For rule 1—if spindle speed is low, then thrust force is high.

From the universe of input function Table 2 the high fuzzy set is defined as

High = 0/0 + 0/1 + 0/2 + 0.5/3 + 1/4

And from the universe of output function Table 3 the low fuzzy set is defined as

Low = 1/0 + 0.5/1 + 0/2 + 0/3 + 0/4

The link among high input function and low output function will be shown in Table 4. Similarly, medium input to medium output and low input to high output link are developed and tabulated in Tables 5 and 6.

2.3.4 Rules combination

The "OR" function is one of the operators, which are used to represent the maximum of the membership values of the two different relations as follows. If spindle speed is high, then thrust force is low "OR" if spindle speed is medium, then thrust force is medium. This can be expressed in Eq. 1

$$\mu \,\mathrm{R1} + \mu \,\mathrm{R2} = \max \,\{\mu \,\mathrm{R1}, \ \mu \,\mathrm{R2}\} \tag{1}$$

The combination of rule 1 and rule 2 is expressed in Table 7 $% \left(1-\frac{1}{2}\right) =0$

Table 8 Combination of rule1, rule 2, and rule 3 for thrust force

		Universe of output (thrust force)					
		0	1	2	3	4	
Universe of spindle	0	0	0	0	0.5	1	
speed $R1 + R2 =$	1	0	0.5	0.5	0.5	0.5	
	2	0	0.5	1	0	0	
	3	0.5	0.5	0	0	0	
	4	1	0.5	0	0	0	

ble 9 Averaged val- of thrust force for cific spindle speed	Spindle speed universal partitioning	Averaged thrust force
verse	0	3.66
	1	2.5
	2	1.66
	3	0.5
	4	0.33

Thus the fuzzy algorithm as follows:

Tab ue o spe

> If spindle speed = high THEN thrust force = low OR If spindle speed = medium THEN thrust force = medium OR If spindle speed = low THEN thrust force = high.

> This algorithm can be represented in the relation R which has a membership function as shown Eq. 2.

$$\mu R = \max \{ \mu R1, \ \mu R2, \ \mu R3 \}$$
(2)

The combination of rule 1, rule 2, and rule 3 is expressed in Table 8

Combining this relation with any value the spindle speed lies in its universe (0-4) results required average thrust force output. The average spindle speed can be calculated from the Eq. 3.

Average value =
$$\frac{\sum \text{Thrust force } \times \ \mu(s)}{\sum \mu(s)}$$
 (3)

The averaged thrust force value is shown in Table 9

The following equations, Eqs. 4 and 5, are used for calculate the range factor for fuzzy modeling.

Thrust force range factor

$$= \frac{\text{Thrust force}_{(\text{max})} - \text{Thrust force}_{(\text{min})}}{\text{Averaged thrust force}(0)}$$
(4)

Surface roughness range factor

$$= \frac{\text{Surface roughness}_{(\text{max})} - \text{Surface roughness}_{(\text{min})}}{\text{Averaged surface roughness}(0)}$$
(5)

	Table 10	Output for	thrust	force	from	fuzzy	logic	model
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Spindle speed	Universe partitioning	Averaged thrust force	Averaged thrust force×range factor	(Averaged thrust force×range factor)+minimum thrust force
120	0	3.66	35.25	98.25
155	1	2.5	24.08	87.08
190	2	1.66	15.99	78.99
225	3	0.5	4.82	67.82
260	4	0.33	3.18	66.18



Fig. 4 a Thrust force interaction effect and b response surface plot

For example, feed rate (0.02 mm/rev and spindle speed 120 rpm):

Thrust force range factor (RF) =
$$\frac{98 - 63}{3.66} = 9.63$$

Thrust force value from fuzzy logic model tabulated in Table 10.

Entry surface roughness range factor (RF) = $\frac{0.76-0.5}{3.66} = 0.071$ Middle surface roughness range factor (RF) = $\frac{1.52-0.36}{3.66} = 0.317$ Exit surface roughness range factor (RF) = $\frac{1.75-0.94}{3.66} = 0.221$

The surface roughness values are tabulated in Tables 13 and 14.

2.3.5 Application of fuzzy model into RSM

The developed fuzzy model values are input for response surface methodology and data analysis carried out with design expert software. The historical data format is used for statistical analysis. The response surface plots are presented in Figs. 4, 5, 6 and 7. These plots show the effects of spindle speed and feed rate on thrust force and surface roughness. Figure 4a, b shows the interaction plots for thrust force, plot represents that the mid level of spindle speed while high level feed rate gives minimum thrust force. Figures 5, 6 and 7a, b show the interaction plots for entry, middle, and exit thrust force. These plots are illustrated that intermediate level of spindle speed with high level of feed rate produces lower surface roughness.

2.3.6 Analysis of variance

0.03

0.02 120.00

ANOVA is the statistical method used to calculate the size of the difference between data set. The main elements of ANOVA table are source of variance, sum of squares, degrees of freedom, mean square, F ratio, and the probability associated with the F ratio. The source of variance deals with independent variables that are called factors (spindle speed and feed rate). Sum of squares (SS) SSspindle speed, SSfeed rate, and SStotal denotes the sum of squares of spindle speed, feed rate, and total variance. The degrees of freedom are equal to the number of levels for each factor minus 1. Mean square as a variance static, it is calculated by the sum of squares of each factor divided by the corresponding degrees of freedom. F ratio is defined as the



Fig. 5 a Entry surface roughness interaction effect and b response surface plot

225.00

A: SPINDLE SPEED

90.00

155 00



Fig. 6 a Middle surface roughness interaction effect and b response surface plot

ratio between mean square value of each factor with mean square value of residual. Table 1 shows the ANOVA table for experimental data of thrust force, surface roughness as dependent variables, and spindle speed and feed rate as independent variables.

Table 11 shows the model summary of thrust force and surface roughness. The table shows that the cubic model is best for predict the thrust force and surface roughness value. The significant parameters are identified by using ANOVA table. Table 12 shows the ANOVA results of thrust force and surface roughness response surface cubic model for titanium alloy drilling.

Thrust force data analysis From the model F value of Table 12, 5.35 imply that the model is significant for thrust force. There is only a 0.12 % chance that a "model F value" could be large due to the noise. The values of "prob>F" less than 0.0500 indicate that model terms are significant. In this case, A2 are significant model terms. Values greater than 0.1000 indicate that the model terms are not significant. The "lack of fit F value" of 35.40 implies that the lack of fit is significant. There is only a 0.01 % chance that a "lack of fit

F value" could occur be large due to the noise. The "pred R squared" of 0.4210 is in reasonable agreement with the "adj R squared" of 0.4655. "Adeq precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. This analysis ratio of 4.570 indicates an adequate signal. Therefore, the model can be used to navigate the design space.

Entry surface roughness data analysis The model F value of Table 12, 5.23 implies that the model is significant. There is only a 0.14 % chance that a "model F value" could be large due to the noise. The values of "prob>F" less than 0.0500 indicate that model terms are significant. In this case, A2 are significant model terms. The values greater than 0.1000 indicate that the model terms are not significant. The "lack of fit F value" of 15.27 implies that the lack of fit is significant. There is only a 0.01 % chance that a "lack of fit F value" could be large due to the noise. The "pred R squared" of 0.3894 is in reasonable agreement with the "adj R squared" of 0.4583. "Adeq precision" measures the signal to noise ratio. A ratio greater than 4 is desirable. This analysis ratio of 5.082 indicates an adequate signal. This model can be used to navigate the design space.



Fig. 7 a Exit surface roughness interaction effect and b response surface plot

Table 11Model summary ofresponses

Model	SD	<i>R</i> 2	Adj. <i>R</i> 2	SS	LF	Ade. Pr.	
Thrust force	;						
Linear	13.21	0.24	0.18	1,457.74	97.93	4.95	Suggested
2FI	13.34	0.25	0.16	1,543.04	12.63	4.25	
Cubic	39.41	0.57	0.46	49,886.67	32,659.05	4.57	Suggested
Quadratic	13.59	0.25	0.13	1,549.22	6.45	3.73	Aliased
Entry surfac	e roughnes	s					
Linear	0.30	0.29	0.24	1.00	0.066	6.629	Suggested
2FI	0.30	0.31	0.23	1.04	0.022	5.69	
Cubic	0.63	0.57	0.46	12.35	7.12	5.08	Suggested
Quadratic	0.31	0.31	0.19	1.05	0.0093	5.001	Aliased
Middle surfa	ace roughne	ess					
Linear	0.17	0.42	0.38	0.55	0.29	8.87	Suggested
2FI	0.14	0.63	0.59	0.81	0.026	12.02	
Cubic	0.59	0.61	0.52	13.14	7.87	5.36	Suggested
Quadratic	0.14	0.63	0.58	0.82	0.016	10.66	Aliased
Exit surface	roughness						
Linear	0.26	0.27	0.22	0.68	0.12	6.21	Suggested
2FI	0.26	0.31	0.24	0.78	0.016	5.43	
Cubic	0.63	0.58	0.47	12.71	7.69	5.05	Suggested
Quadratic	0.26	0.27	0.22	0.68	0.12	6.21	Aliased

Middle surface roughness data analysis The model *F* value of Table 12, 6.32 implies that the model is significant. There is only a 0.04 % chance that a "model *F* value" could be large due to the noise. Values of "prob>*F*" less than 0.0500 indicate that model terms are significant. In this case, A2 are significant model terms. Values greater than 0.1000 indicate that the model terms are not significant. The "lack of fit *F* value" of 86.92 implies that the lack of fit is significant. There is only a 0.01 % chance that a "lack of fit *F* value" could be large due to the noise. The "pred *R* squared" of 0.4842 is in reasonable agreement with the "adj *R* squared" of 5.355 indicates an adequate signal. This model can be used to navigate the design space.

Exit surface roughness data analysis The model *F* value of Table 12, 5.41 implies that the model is significant. There is only a 0.12 % chance that a "model *F* value" could be large due to the noise. Values of "prob>*F*" less than 0.0500 indicate that model terms are significant. In this case, A2 are significant model terms. Values greater than 0.1000 indicate that the model terms are not significant. The "lack of fit *F* value" of 22.74 implies that the lack of fit is significant. There is only a 0.01 % chance that a "lack of fit *F* value" could be large due to the noise. The "pred *R* squared" of 0.4156 is in reasonable agreement with the "adj *R* squared" of 0.4689. "Adeq precision" measures the signal

to noise ratio. A ratio greater than 4 is desirable. This analysis ratio of 5.045 indicates an adequate signal. This model can be used to navigate the design space.

Figure 8a–d shows that the thrust force, entry surface roughness, and exit surface roughness cubic models are produced; the values nearer to experimental values than factorial and linear models. But the linear model gives the nearest value of middle surface roughness experimental value.

2.3.7 Regression analysis

The relationship between dependent and independent variable requires a statement of statistical model. This work contains more than one independent variable, so that it needed a regression model. As per interaction effect, plots are shown in Figs. 4, 5, 6 and 7. The thrust force and surface roughness plot do not fall on straight line, so that these models are used polynomial regression models. Equations 6, 7, 8 and 9 are the empirical relationship between independent and dependent variables. Here, N and f are known as spindle speed and feed rate, respectively.

Thrust force =
$$1.43968 \times N - 688.49362 \times f + 0.53175$$

 $\times N \times f - 8.58407E - 003 \times N^2 + 4.24296E$
 $-003 \times N^2 \times f + 1.50397E - 005 \times N^3$
(6)

 Table 12
 Analysis of variable (ANOVA) for all responses

$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	Source	ce Sum of squares		Mean square	F value	P value prob. $>F$	
Model49,886.6768,314.455,350,0012Spindle speed (N)38.73138.730.0250.8758Feed rate (r)45,3171453.170.0290.5940N×f85.30185.300.0550.8167N2 48,339.99148,339.9931.13<0.001	Thrust force ANOVA ta	able					
Spindle speed (N)38.73138.730.0250.8758Fed rut (F)453.171453.170.290.5940N=f85.30185.300.00550.8167N248,339.99148,339.9931.13<0.001	Model	49,886.67	6	8,314.45	5.35	0.0012	
Feed rate (F)453.171453.170.290.5940 $N \times f$ 85.30185.300.0550.8167 $N 2$ 48,339.99148,339.9931.13 ~ 0.0001 $N 2 \times f$ 2.2312.231.439E-0030.9701 $N 3$ 1.4011.409.024E-0040.9763Residual37.271.442.41.552.98 ~ 0.0014 Total87.158.1130 ~ 0.0014 ~ 0.0014 S/N ratio4.57S.D39.41Mean < 51.96 Entry surface roughness ANOVA table ~ 0.044 0.0890.230.6388Feed rate (F)0.1710.0890.230.63188 $N \times f$ 0.04410.04440.1110.7402 $N 2 \wedge f$ 1.296E-00313.682E-0039.352E-0030.9537 $N 3$ 3.662E-00313.682E-0039.352E-0030.9238Residual9.452.40.39 ~ 1.5037 0.0004Middle surface roughness ANOVA table ~ 0.663 Mean0.78Middle surface roughness ANOVA table ~ 0.297 0.0300.0300.030N 21.2180.0580.200.6611Feed rate (F)0.03010.0300.0880.7695N 4f0.270.7770.3904 ~ 1.232 3.552 ~ 0.0001 N2 $\wedge f$ 0.23510.0580.0200.8877N32.667E-00310.0580.0	Spindle speed (N)	38.73	1	38.73	0.025	0.8758	
N×f85.30185.300.0550.8167N2 × f2.231.439F-030.9701N3 1.4011.409.024F-0040.9763Residual37,271.442.41.409.024F-0040.9763Residual37,271.442.41.409.024F-0040.9763Straito4.57SD39.41Mean51.96Straito4.57SD39.41Mean51.96Entry surface roughness ANOVA table9.02430.63880.035Spindle speed (N)0.08910.0890.230.6388Fed rate (F)0.1710.170.430.5168N×f0.0440.110.74020.932E-0030.9328Ns/f1.296E-00313.293E-0030.9328Residual9.45240.399.352E-0030.9238Residual9.45240.399.352E-0030.9238Yir taio5.082SD0.63Mean0.78Middle surface roughness ANOVA table10.0300.0880.7695Nx fi0.2710.270.770.3044Yir taio5.335SD0.0200.8877N21.23211.2323.552-0.0001N×f0.0580.0200.88770.330.90200.8877N32.667E-00317.058E-0030.0200.8877N21.23211.2323.542-0.001	Feed rate (<i>F</i>)	453.17	1	453.17	0.29	0.5940	
$N2$ 48,339.99148,339.9931.13<0.0001 $N2 \times f$ 2.2312.231.439E-0030.9701 $N3$ 1.4011.409.024E-0040.9763Residual37,271.44241.552.981Total87.158.1130301SiN ratio4.57SDSD39.41MeanEnty surface roughness ANOVA table10.0690.230.6388Feed rate (f)0.1710.170.430.5168 $N \times f$ 0.04410.0440.110.7402 $N2 \times f$ 1.296E-00311.296E-0033.292E-0030.9537 $N3$ 3.682E-00313.682E-0039.352E-0030.9238Residual9.45240.390.9238 $N3$ 3.682E-00310.6680.200.6611 Na 5.082SD0.63Mean0.78Middle13.140.74020.270.770.3904Syn ratio5.082SD0.63Mean0.78Middle13.140.4680.200.6611Syn ratio5.082SD0.63Mean0.78Middle13.140.730.30940.9304Na2.667E-00310.2600.9308Syn ratio5.335SD0.59Mean0.82Syn ratio2.335SD0.59Mean0.20Na2.667E-00310	$N \times f$	85.30	1	85.30	0.055	0.8167	
N2×f 2.23 1 2.23 1.439E-003 0.9701 N3 1.40 1 1.40 9.024E-004 0.9763 Residual 37,271.44 24 1,552.98	N2	48,339.99	1	48,339.99	31.13	< 0.0001	
N31.4011.409.024E-0040.9763Residual37,271.44241,552.98Total87,158.1130S// tatio4.57SD39.41MeanModel12.3562.065.230.0014Spindle speed (N)0.08910.0890.230.6388Feed rate (P)0.1710.170.430.5168N×f0.04410.0440.110.7402N211.3111.3128.73<0.0001	$N2 \times f$	2.23	1	2.23	1.439E-003	0.9701	
Residual $37,271.44$ 24 $1,552.98$ Total $87,158,11$ 30 SN ratio 4.57 SD 39.41 Mean 51.96 Entry surface roughness ANOVA table1 0.089 0.23 0.0014 Spindle speed (N) 0.089 1 0.089 0.23 0.6318 N×f 0.044 1 0.044 0.11 0.762 N2 <	N3	1.40	1	1.40	9.024E-004	0.9763	
Total $87, 158, 11$ 30 S/N ratio 4.57 SD 39.41 Mean 51.96 Entry surface roughness ANOVA table 6 2.06 5.23 0.0014 Spindle speed (N) 0.089 1 0.089 0.23 0.6388 Fed rate (P) 0.17 1 0.17 0.43 0.5168 $N \simeq fr0.04410.0440.110.7402N^211.3111.3128.73<0.0001N^2 \checkmark f1.296E-00313.682E-0030.9547N^33.682E-00313.682E-0030.9238Residual9.45240.3620.0228Residual21.803010.63MeanNratio5.082SD0.63Mean0.78Middle surface roughness ANOVA table10.0680.200.6611Feed rate (P)0.03010.0300.0880.7095N < f0.2710.270.770.3904N < f2.32240.059Mean0.82N < f2.467E-00312.667E-0030.93360.9308N < f2.467E-0031.267E-0037.690E-0030.9308N < f0.059Mean0.820.9308N < f0.130.130.330.5716N < f0.130.130.13$	Residual	37,271.44	24	1,552.98			
S/N ratio 4.57 SD 39.41 Mean 51.96 Entry surface roughness ANOVA table	Total	87,158.11	30	,			
Intry surface roughness ANOVA tableModel12.3562.065.230.0014Spindle speed (N)0.08910.0890.230.6388Feed rate (F)0.1710.170.430.5168N×f0.04410.0440.110.7402N211.31111.3128.73<0.0001N2×f1.296E-00312.862E-0030.923E-0030.9547N33.682E-00313.682E-0039.352E-0030.9238Residual9.45240.39Total21.8030S/N ratio5.082SD0.63Mean0.78Middle surface roughness ANOVA tableModel13.1462.196.320.0004Spindle speed (N)0.06810.0680.200.6611Feed rate (F)0.03010.0300.0880.7695N×f0.270.770.3904N2 f7.058E-00312.667E-0037.690E-0030.9038Residual8.32240.35N32.667E-00312.667E-0037.690E-0030.9308Residual8.32240.35Model13.1462.196.32N×f0.0570.59Mean0.82 <th< td=""><td>S/N ratio</td><td>4.57</td><td>SD</td><td>39.41</td><td>Mean</td><td>51.96</td></th<>	S/N ratio	4.57	SD	39.41	Mean	51.96	
Model12.3562.065.230.0014Spindle speed (N)0.08910.0890.230.6388Fed rate (P)0.1710.170.430.5168 $N eq f$ 0.04410.0440.110.7402 N^2 11.31111.3128.73<0.001	Entry surface roughnes	s ANOVA table					
Spindle speed (N) 0.089 1 0.089 0.23 0.6388 Feed rate (P) 0.17 1 0.17 0.43 0.5168 $N \times f$ 0.044 1 0.044 0.11 0.7402 $N2$ 11.31 1 11.31 28.73 <0.0001 $N2 / f$ $1.296E-003$ $3.293E-003$ 0.9547 $N3$ $3.682E-003$ 1 $3.682E-003$ $9.352E-003$ 0.9238 Residual 9.45 24 0.39 $Total21.8030$	Model	12.35	6	2.06	5.23	0.0014	
Fed rate (F)0.1710.170.430.5168 $N \times f$ 0.04410.0440.110.7402 $N2$ 11.31111.3128.73<0.001	Spindle speed (N)	0.089	1	0.089	0.23	0.6388	
$N \neq f$ 0.04410.0440.110.7402N211.31111.3128.73<0.0001	Feed rate (<i>F</i>)	0.17	1	0.17	0.43	0.5168	
N211.3111.3128.73<0.001 $N2 \times f$ 1.296E-00311.296E-0033.293E-0030.9547 $N3$ 3.682E-00313.682E-0039.352E-0030.9238Residual9.45240.39	$N \times f$	0.044	1	0.044	0.11	0.7402	
$N2 \times f$ 1.296E-00311.296E-0033.293E-0030.9547N33.682E-00313.682E-0039.352E-0030.9238Residual9.45240.39	N2	11.31	1	11.31	28.73	< 0.0001	
N3.682E-00313.682E-0039.352E-0030.9238Residual9.45240.39Total21.8030S/N ratio5.082SD0.63Mean0.78Middle surface roughness ANOVA table00.6680.200.6611Feed rate (F)0.03010.0680.200.6611Feed rate (F)0.03010.0300.0880.7695N×f0.2710.270.770.3904N2×f7.058E-00317.658E-0030.0200.8877N32.667E-00312.667E-0037.690E-0030.9308Residual8.32240.3511.267E-0030.682S/N ratio5.335SD0.59Mean0.82Exit surface roughness ANOVA table10.0480.120.7282Feed rate (F)0.1310.0330.5716N/ ratio5.335SD0.59Mean0.82Exit surface roughness ANOVA table10.0480.120.7282Feed rate (F)0.1310.0330.5716N×f0.1010.100.260.6176N211.9230.48<0.0001	$N2 \times f$	1.296E-003	1	1.296E-003	3.293E-003	0.9547	
Residual9.45240.39Total21.8030 S/N ratio5.082SD0.63Mean0.78Middle surface roughness ANOV- table $$	N3	3.682E-003	1	3.682E-003	9.352E-003	0.9238	
Total21.8030 S/N ratio5.082SD0.63Mean0.78Middle surface roughness ANOVA table0.004Model13.1462.196.320.004Spindle speed (N)0.06810.0680.200.6611Feed rate (F)0.03010.0300.0880.7695 $N arr f$ 0.2710.270.770.3904 $N2 arr f$ 7.058E-00317.058E-0030.0200.8877 $N3 arr f$ 2.667E-00312.667E-0030.93080.9308Residual8.32240.350.0200.827Total21.4630S/N ratio5.335SD0.59Mean0.82Exit surface roughness ANOVA tableModel12.7162.125.410.0012Spindle speed (N)0.04810.0480.120.7282Spindle speed (N)0.1310.130.330.5716 $N ar f$ 0.1010.100.260.6176 $N ar f$ 0.1010.100.260.6176 $N ar f$ 0.19211.923.048<0.0001	Residual	9.45	24	0.39			
S/N ratio5.082SD0.63Mean0.78Middle surface roughness ANOVA tableModel13.1462.196.320.0004Spindle speed (N)0.06810.0680.200.6611Feed rate (F)0.03010.0300.0880.7695 $N \times f$ 0.2710.270.770.3904 $N2 \times f$ 7.058E-00317.058E-0030.0200.8877 $N3$ 2.667E-00312.667E-0037.690E-0030.9308Residual8.32240.35St/N ratio5.335SD0.59Mean0.82Exit surface roughness ANOVA tableModel12.7162.125.410.0012Spindle speed (N)0.04810.0480.120.7282Feed rate (F)0.1310.130.330.5716 $N \times f$ 0.1010.100.260.6176 $N \times f$ 0.1011.9230.48<0.001	Total	21.80	30				
Middle surface roughness ANOVA tableModel13.1462.196.320.0004Spindle speed (N)0.06810.0680.200.6611Feed rate (F)0.03010.0300.0880.7695 $N \times f$ 0.2710.270.770.3904 $N2$ 12.32112.3235.52<0.001	S/N ratio	5.082	SD	0.63	Mean	0.78	
Model13.1462.196.320.004Spindle speed (N)0.06810.0680.200.6611Feed rate (F)0.03010.0300.0880.7695 $N \times f$ 0.2710.270.770.3904 $N2$ 12.32112.3235.52<0.0001	Middle surface roughne	ess ANOVA table					
Spindle speed (N) 0.068 1 0.068 0.20 0.6611 Feed rate (F) 0.030 1 0.030 0.088 0.7695 $N imes f$ 0.27 1 0.27 0.77 0.3904 $N2$ 12.32 1 12.32 35.52 <0.001 $N2 imes f$ $7.058E-003$ 1 $7.058E-003$ 0.020 0.8877 $N3$ $2.667E-003$ 1 $2.667E-003$ $7.690E-003$ 0.9308 Residual 8.32 24 0.35 $Total$ 21.46 S/N ratio 5.335 SD 0.59 Mean 0.82 Exit surface roughness ANOVA table V V V 0.048 0.12 0.7282 Feed rate (F) 0.13 1 0.13 0.33 0.5716 $N imes f$ 0.10 1 0.10 0.26 0.6176 $N2 imes f$ $2.858E-003$ 1 $2.858E-003$ $7.309E-003$ 0.9326 $N3$ $1.707E-003$ 1 $1.707E-003$ $4.364E-003$ 0.9479 Residual 9.39 24 0.39 24 0.39	Model	13.14	6	2.19	6.32	0.0004	
Feed rate (F) 0.03010.0300.0880.7695 $N imes f$ 0.2710.270.770.3904 $N2$ 12.32112.3235.52<0.001	Spindle speed (N)	0.068	1	0.068	0.20	0.6611	
$N \times f$ 0.27 1 0.27 0.77 0.3904 $N2$ 12.32 1 12.32 35.52 <0.001 $N2 \times f$ $7.058E-003$ 1 $7.058E-003$ 0.020 0.8877 $N3$ $2.667E-003$ 1 $2.667E-003$ $7.690E-003$ 0.9308 Residual 8.32 24 0.35 $7.690E-003$ 0.9308 Total 21.46 30 $$	Feed rate (<i>F</i>)	0.030	1	0.030	0.088	0.7695	
N12.32112.3235.52<0.001 $N2 \times f$ 7.058E-00317.058E-0030.0200.8877 $N3$ 2.667E-00312.667E-0037.690E-0030.9308Residual8.32240.350.350.9308Total21.4630	N×f	0.27	1	0.27	0.77	0.3904	
$N2 \times f$ 7.058E-00317.058E-0030.0200.8877 $N3$ 2.667E-00312.667E-0037.690E-0030.9308Residual8.32240.35 V V Total21.4630 V V V S/N ratio5.335SD0.59Mean0.82Exit surface roughness ANOVA table V V V V Model12.7162.125.410.0012Spindle speed (N)0.04810.0480.120.7282Feed rate (F)0.1310.130.330.5716 $N2 \times f$ 2.858E-00312.858E-0037.309E-0030.9326 $N3$ 1.707E-00311.707E-0034.364E-0030.9479Residual9.39240.39 V V V Total22.0930 V V V V	N2	12.32	1	12.32	35.52	< 0.0001	
N3 $2.667E-003$ 1 $2.667E-003$ $7.690E-003$ 0.9308 Residual 8.32 24 0.35 Total 21.46 30S/N ratio 5.335 SD 0.59 Mean 0.82 Exit surface roughness ANOVA tableModel 12.71 6 2.12 5.41 0.0012 Spindle speed (N) 0.048 1 0.048 0.12 0.7282 Feed rate (F) 0.13 1 0.13 0.33 0.5716 N $\times f$ 0.10 1 0.10 0.26 0.6176 N2 11.92 1 11.92 30.48 <0.0001 N2 $\times f$ $2.858E-003$ 1 $2.858E-003$ $7.309E-003$ 0.9326 N3 $1.707E-003$ 1 $1.707E-003$ $4.364E-003$ 0.9479 Residual 9.39 24 0.39 2.09 30	$N2 \times f$	7.058E-003	1	7.058E-003	0.020	0.8877	
Residual 8.32 24 0.35 Total 21.46 30 S/N ratio 5.335 SD 0.59 Mean 0.82 Exit surface roughness ANOVA tableModel 12.71 6 2.12 5.41 0.0012 Spindle speed (N) 0.048 1 0.048 0.12 0.7282 Feed rate (F) 0.13 1 0.13 0.33 0.5716 $N \times f$ 0.10 1 0.10 0.26 0.6176 $N2$ 11.92 1 11.92 30.48 <0.0001 $N2 \times f$ $2.858E-003$ 1 $2.858E-003$ $7.309E-003$ 0.9326 $N3$ $1.707E-003$ 1 $1.707E-003$ $4.364E-003$ 0.9479 Residual 9.39 24 0.39 21.99 0.92	N3	2.667E-003	1	2.667E-003	7.690E-003	0.9308	
Total 21.46 30 S/N ratio 5.335 SD 0.59 Mean 0.82 Exit surface roughness ANOVA tableNodel 12.71 6 2.12 5.41 0.0012 Model 12.71 6 2.12 5.41 0.0012 Spindle speed (N) 0.048 1 0.048 0.12 0.7282 Feed rate (F) 0.13 1 0.13 0.33 0.5716 $N \times f$ 0.10 1 0.10 0.26 0.6176 $N2 \times f$ $2.858E-003$ 1 1.92 30.48 <0.0001 $N2 \times f$ $2.858E-003$ 1 $1.707E-003$ $4.364E-003$ 0.9326 N3 $1.707E-003$ 1 0.39 24 0.39 Total 22.09 30 30 30 30 30	Residual	8.32	24	0.35			
S/N ratio5.335SD0.59Mean0.82Exit surface roughness ANOVA tableModel12.7162.125.410.0012Spindle speed (N)0.04810.0480.120.7282Feed rate (F)0.1310.130.330.5716 $N \times f$ 0.1010.100.260.6176 $N2 \times f$ 2.858E-00312.858E-0037.309E-0030.9326 $N3$ 1.707E-00311.707E-0034.364E-0030.9479Total22.093030303030	Total	21.46	30				
Exit surface roughness ANOVA tableModel12.7162.125.410.0012Spindle speed (N)0.04810.0480.120.7282Feed rate (F)0.1310.130.330.5716 $N \times f$ 0.1010.100.260.6176 $N2$ 11.92111.9230.48<0.001	S/N ratio	5.335	SD	0.59	Mean	0.82	
Model12.7162.125.410.0012Spindle speed (N)0.04810.0480.120.7282Feed rate (F)0.1310.130.330.5716 $N \times f$ 0.1010.100.260.6176N211.92111.9230.48<0.0001	Exit surface roughness	ANOVA table					
Spindle speed (N) 0.048 1 0.048 0.12 0.7282 Feed rate (F) 0.13 1 0.13 0.33 0.5716 $N \times f$ 0.10 1 0.10 0.26 0.6176 $N2$ 11.92 1 11.92 30.48 <0.0001 $N2 \times f$ $2.858E-003$ 1 $2.858E-003$ $7.309E-003$ 0.9326 $N3$ $1.707E-003$ 1 $1.707E-003$ $4.364E-003$ 0.9479 Residual 9.39 24 0.39 $7.309E-003$ 0.9479	Model	12.71	6	2.12	5.41	0.0012	
Feed rate (F) 0.1310.130.330.5716 $N \times f$ 0.1010.100.260.6176 $N2$ 11.92111.9230.48<0.0001	Spindle speed (N)	0.048	1	0.048	0.12	0.7282	
$N \times f$ 0.1010.100.260.6176 $N2$ 11.92111.9230.48<0.0001	Feed rate (<i>F</i>)	0.13	1	0.13	0.33	0.5716	
N2 11.92 1 11.92 30.48 <0.0001 N2×f 2.858E-003 1 2.858E-003 7.309E-003 0.9326 N3 1.707E-003 1 1.707E-003 4.364E-003 0.9479 Residual 9.39 24 0.39 1 1.7012 Total 22.09 30 30 30 30	$N \times f$	0.10	1	0.10	0.26	0.6176	
N2×f 2.858E-003 1 2.858E-003 7.309E-003 0.9326 N3 1.707E-003 1 1.707E-003 4.364E-003 0.9479 Residual 9.39 24 0.39 1 1.7012 1 1.7012 Total 22.09 30 30 30 30 30 30 30	N2	11.92	1	11.92	30.48	< 0.0001	
N3 1.707E-003 1 1.707E-003 4.364E-003 0.9479 Residual 9.39 24 0.39 Total 22.09 30	$N2 \times f$	2.858E-003	1	2.858E-003	7.309E-003	0.9326	
Residual 9.39 24 0.39 Total 22.09 30 30	N3	1.707E-003	1	1.707E-003	4.364E-003	0.9479	
Total 22.09 30	Residual	9.39	24	0.39			
	Total	22.09	30				
<i>S/N</i> ratio 5.045 SD 0.63 Mean 0.81	S/N ratio	5.045	SD	0.63	Mean	0.81	



Fig. 8 Mathematical model output comparison for experimental, 2FI, linear, and cubic models (a) thrust force (b) entry surface roughness (c) middle surface roughness, and (d) exit surface roughness

Entry surface roughness = $0.029981 \times N - 17.34654 \times f$ + $0.045072 \times N \times f - 1.96946E - 004 \times N^{2}$ + $1.18127E - 005 \times N^{2} \times f + 3.59811E$ - $007 \times N^{3}$ (7)

Middle surface roughness =
$$0.034294 \times N - 33.40541$$

 $\times f + 0.18514 \times N \times f - 2.37711E - 004$
 $\times N^2 - 1.62448E - 005 \times N^2 \times f + 4.39074E$ (8)
 $-007 \times N^3$

Exit surface roughness =
$$0.030284 \times N - 23.03896 \times f$$

+ $0.089006 \times N \times f - 1.98962E - 004 \times N^{2}$
- $3.67020E - 005 \times N^{2} \times f + 3.61953E$
- $007 \times N^{3}$ (9)

3 Results and discussion

The fuzzy logic and fuzzy–RSM predictive models were compared on the basis of their prediction. The models were validated with 12 data sets of full factorial design used for the model development. The predicted values of surface roughness and thrust force were compared with the corresponding experimental values and the percentage of error tabulated in Tables 13 and 14. Based on experimental and theoretical investigation, the following discussions are made. In this section, Tables 13 and 14 show the comparison of experimental results with fuzzy model and fuzzy–RSM model results.

Figures 9, 10, 11 and 12 show the validation of fuzzy and fuzzy–RSM model with experimental results for thrust force, surface roughness. The average deviation between

Spindle speed	Feed rate (mm/rev)	Experimental results				Fuzzy model results				% of deviation			
(rpm)		Thrust force (kgf)	Surface roughness (Micrometer)			Thrust force (kgf)	Surface roughness (micrometer)			Thrust force (kgf)	Surface roughness (µm)		
			Entry	Middle	Exit		Entry	Middle	Exit		Entry	Middle	Exit
120	0.02	98	0.50	1.52	1.75	98.3	0.76	1.52	1.75	0.3	34.2	0	0
180	0.02	49	0.93	0.94	0.58	78.9	0.62	0.89	1.31	37.9	-50.0	-5.6	55.7
260	0.02	44	1.84	1.20	1.18	66.2	0.52	0.47	1.02	33.5	-253.9	-155.3	-15.7
120	0.05	54	0.59	0.70	0.65	54.0	0.63	0.72	0.79	0	6.4	2.8	17.7
180	0.05	50	0.63	0.72	0.79	52.4	0.61	0.71	0.75	4.5	-3.3	-1.4	-5.3
260	0.05	38	0.36	0.75	0.64	51.3	0.59	0.7	0.67	25.9	38.9	-7.1	4.5
120	0.02	63	0.76	0.36	0.94	53.0	0.93	0.94	0.58	-15.9	18.3	61.7	-62.1
180	0.02	53	0.76	0.57	0.55	50.8	0.84	0.74	0.56	-4.1	9.5	22.9	1.8
260	0.02	45	0.74	0.84	0.61	49.4	0.78	0.6	0.55	8.8	5.1	-40.0	-10.9
120	0.05	51	0.61	0.72	0.64	50.0	1.34	0.89	0.85	-1.9	54.5	19.1	24.7
180	0.05	46	1.34	0.89	0.85	47.8	0.95	0.8	0.74	3.8	-41.1	-11.3	-14.9
260	0.05	35	0.44	0.74	0.62	46.4	0.69	0.74	0.67	24.5	36.2	0	7.5
						Average % o	% of deviation			9.77	-12.08	-9.52	0.252

Table 13 Comparison between experimental, fuzzy logic and fuzzy-RSM value for thrust force

experimental results and fuzzy model results are 9.77 %, -12.08 %, -9.52 % and 0.25 % and for fuzzy—RSM model deviations are 0.57, 3.62, -4.53 and 3.9 % for thrust force, entry surface roughness, middle surface roughness, and exit surface roughness, respectively. The fuzzy–RSM mathematical model produces lesser error than fuzzy model. The difference between the predicted values of fuzzy–RSM mathematical model and experimental value is very small. Thus the equations can be used to predict the thrust force

and surface roughness value for drilling of titanium alloy for any combinations of drilling parameters within the range of experiments.

4 Conclusions

Main objective of this work is to develop the empirical model using fuzzy logic and combination of fuzzy logic with RSM.

Table 14 Comparison between experimental, fuzzy logic and fuzzy-RSM value for thrust force

Spindle speed	Feed rate	Experimental results				fuzzy -RSM model results				% of deviation			
(rpm)	(mm/rev)	Thrust force (kgf)	Surface roughness (Micrometer)		Thrust force (kgf)	Surface roughness (micrometer)			Thrust force (kgf)	Surface roughness (micrometer)			
			Entry	Middle	Exit		Entry	Middle	Exit		Entry	Middle	Exit
120	0.02	98	0.50	1.52	1.75	63.9	1.14	1.18	1.14	-53.4	56.1	-28.8	-53.5
180	0.02	49	0.93	0.94	0.58	58.1	0.87	0.86	0.91	15.7	-6.9	-9.3	36.3
260	0.02	44	1.84	1.20	1.18	53.1	0.67	0.64	0.74	17.1	-1.8	-87.5	-59.5
120	0.05	54	0.59	0.70	0.65	46.9	0.77	0.77	0.75	-14.9	23.4	9.1	13.3
180	0.05	50	0.63	0.72	0.79	45.1	0.59	0.74	0.68	-10.9	-6.8	2.7	-16.2
260	0.05	38	0.36	0.75	0.64	45.2	0.51	0.75	0.67	15.9	29.4	0	4.5
120	0.02	63	0.76	0.36	0.94	63.9	1.14	1.18	1.14	1.4	33.3	69.5	17.5
180	0.02	53	0.76	0.57	0.55	58.1	0.87	0.86	0.91	8.8	12.6	33.7	39.6
260	0.02	45	0.74	0.84	0.61	53.1	0.69	0.64	0.74	15.3	-7.3	-31.3	17.6
120	0.05	51	0.61	0.72	0.64	46.9	0.77	0.77	0.75	-8.6	20.8	6.5	65.0
180	0.05	46	1.34	0.89	0.85	45.1	0.60	0.74	0.68	-2.0	-123.3	-20.3	-25.0
260	0.05	35	0.44	0.74	0.62	45.2	0.51	0.75	0.67	22.6	13.7	1.3	8.1
						Average % o	Average % of deviation			0.57	3.62	-4.53	3.97



Fig. 9 Comparison plot for experimental, fuzzy logic, and fuzzy-RSM value of thrust force

Fuzzy logic with RSM method presents an effective methodology for modeling than usual fuzzy logic modeling. The following points are originated for the analysis of the drilling parameters from the proposed methodology

- Fuzzy model generally needs rule evaluation based on experimental or expert's knowledge. RSM modeling needs minimum experimental data with experimentation.
- The fuzzy models were developed based on fuzzy rules with spindle speed as an input and thrust force and surface roughness were responses.
- The collected fuzzy model data was used as an input for RSM-based thrust force and surface roughness models. The developed cubic model can be used for predict the thrust force and surface roughness values within the levels of experimental data.



Fig. 10 Comparison plot for experimental, fuzzy logic, and fuzzy-RSM value of entry surface roughness



Fig. 11 Comparison plot for experimental, fuzzy logic, and fuzzy-RSM value of middle surface roughness

- The fuzzy logic model is very useful for generating data based on universal partitioning. The developed fuzzy logic and fuzzy with RSM models were validated with experimental data.
- The fuzzy–RSM model has smaller deviation from experimental data than fuzzy logic model. This confirms that the developed model can be used to predict the thrust force and surface roughness value in effective manner.
- The fuzzy–RSM model can reduce the experimental cost and time for drilling operation. This proposed methodology can be used to predict the response values of other processes of cost and time consuming experiments like laser machining, water jet machining, and any other traditional and non-traditional machining processes.



Fig. 12 Comparison plot for experimental, fuzzy logic, and fuzzy-RSM value of exit surface roughness

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