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ORIGINAL ARTICLE

Optimization of machining characteristics in electric discharge machining of 6061Al/Al_2O_3p/20P composites by grey relational analysis

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Abstract Present investigation applied the designs of experiments and grey relational analysis (GRA) approach to optimise parameters for electrical discharge machining process of 6061Al/Al_2O_3p/20P aluminium metal matrix composites. Planning of experiments was based on an $L18$ (2^1×3^5) orthogonal array to determine an optimal setting. The process parameters included one noise factor, aspect ratio having two levels and five control factors, viz. pulse current, pulse ON time, duty cycle, gap voltage and tool electrode lift time with three levels each. The material removal rate, tool wear rate and surface roughness were selected as the evaluation criteria, in this study. Optimal combination of process parameters is determined by the grey relational grade (GRG) obtained through GRA for multiple performance characteristics. Analysis of variance for the GRG is also implemented. It is shown that through GRA, the optimization of the multiple performance characteristics can be greatly simplified.

Keywords EDM · AMMCs · DoE · GRA · Optimization · Multiple response evaluation

1 Introduction

Modern industry promotes the use of alternative advanced materials (*composites*, *super alloys and ceramics*) for establishing design and manufacturing. These industrial applications demand materials with a specific set of properties, which has led to the development of composite materials consisting of two or more physically and/or chemically distinct phases [1]. The continuous phase is referred to as the *matrix*, whereas the discontinuous phase is called the *reinforcement*. These advanced materials have superior properties than those depicted by any of its individual components. Matrixes consisting of a metallic base of a ductile metal (e.g. *Al*, *Ti* or *Mg*) and reinforced with ceramic particles (e.g. Al_2O_3, *SiC or* graphite) are known as metal matrix composites (MMCs).

The major innovations have been mainly oriented towards the development of MMCs in the materials world in the past two decades [2]. Lindroos and Talvitie overviewed that recent research and development activity as well as applications has been concentrated on aluminium and its alloy-based MMCs, called as aluminium matrix composites (AMCs) or aluminium metal matrix composites (AMMCs) [3]. AMMCs are high-potential materials for many manufacturing sectors including automobiles, aerospace, electrical, military, sports and engineering components, owing to their better technological properties [4]. AMMCs have been the most exploited material for its low density and the ease of fabrication [5]. They offer a range of property enhancements over conventional engineering materials (*monolithic*) due to their higher strength-to-weight ratio, high bending stiffness, improved high-temperature properties, better wear resistance, corrosion resistance, good damping characteristics and lower thermal expansion [6, 7].

In AMMCs, the reinforcement mixed into the aluminium matrix significantly increases the elastic modulus, wear resistance, strength and fatigue resistance. Further, the addition of reinforcement also reduces the coefficient of thermal expansion of the matrix material. These types of property changes are not generally possible through conventional alloying methods. This fact drives the research towards their advanced industrial applications.

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AMMCs are generally produced by mechanical mixing of particulate reinforcement into the molten aluminium alloy base. They can also be tailored according to some special applications by varying the volume fraction of reinforcing constituents [8]. Some of the other processing techniques to produce AMMCs are preform infiltration, powder metallurgy and spray forming [9].

The density of AMMCs is approximately one third that of steel [10]; offers superior wear resistance [11]; and often competes with other advanced materials, owing to their attractive physical and mechanical properties. Moreover, a fine balance of factors such as cost, damage tolerance (*toughness, ductility and flaw sensitivity*), isotropy (*properties such as strength, stiffness, etc. are the same in all directions*), thermal characteristics, reproducibility, environmental resistance, forming, machining and joining further adds to its increasing popularity [12]. The exceptional properties of MMCs have been documented and reviewed extensively in the past by the researchers [2, 12–15]. Stefanescu reviewed various manufacturing techniques to produce low-cost AMMCs with low volume fraction of particulate, so as to achieve a good wetting between the particle and the matrix [16].

However, full potential of the AMMCs is hindered by the problem of poor machinability and extensive tool wear from conventional machining methods such as turning, milling, drilling, etc. owing to their non-homogeneity, anisotropy, hardness, low ductility, toughness and intrinsic brittleness and also because of the presence of hard abrasive reinforcements [17]. Weinert et al. reported that low material removal, excessive tool wear, poor surface finish and high manufacturing costs are generally associated with machining of such advanced materials [18]. Engineering components of AMMCs produced by casting processes are often having near net shape, but they do require machining to achieve the desired dimensional accuracy and surface finish, especially if close tolerances in complex geometries are required.

Thus, non-conventional machining methods like electric discharge machining (EDM), a thermal process, is being successfully employed for easy machining of AMMCs [19–24]. EDM or spark erosion removes the material through a series of repetitive electrical sparks, when a tool electrode discharges current to the work material separated by a very small distance through a dielectric medium. The thermal energy is utilised to generate high-temperature plasma that erodes the work material through melting and vaporisation and subsequently flushes off the metal particulates (*debris*) from the surface of the work material through cool dielectric fluid [25]. EDM involves no cutting forces, since the two electrodes are in a no-contact position, which prevents mechanical stresses.

In order to achieve the economic objective of the machining process, optimal process parameters are to be determined by various empirical methods based on statistical analysis and optimization approaches. In this study, experimental investigation of machining characteristics such as material removal rate (MRR), tool wear rate (TWR) and surface roughness (SR) of the stir-casted 6061Al/Al_2O_3p/20P work specimens were carried out with a copper tool electrode, by varying various factors affecting the EDM process. This paper proposes an efficient method for multiple criteria evaluation, to find the significant parameters affecting the machining characteristics with the use of design of experiments and grey relational analysis (GRA).

2 Past work

The comprehensive literature review, presented in this section, focused on theoretical and experimental studies by limiting the researches carried out on the EDM of advanced materials, addressing the various aspects of machining. Literature also identifies that EDM technique has proved its merit in the machining of MMCs for achieving improved MRR, less TWR and better surface finish.

During EDM of AlSi7Mg+20% Al_2O_3 MMCs, it has been found that the process parameters affect MRR and surface roughness [21]. Teti overviewed that the machining of composite materials is difficult to carry out due to their anisotropic and non-homogeneous structure [26]. The high abrasiveness of their reinforcing constituent also affects the machining.

De Silva and Rankine performed an experimental study on Al/SiC work material with EDM, and it was observed that during sparking, the matrix surrounding the reinforcement particles was melted, thus easing out the reinforcement particles from the matrix, resulting in improved machining [27]. Hocheng et al. presented the affects of machining parameters such as electrical current, ON time and the crater size produced by a single spark for the Al–SiC composite [28]. After observing the crater diameter, it was found that the increase in discharge energy and discharge current lead to an increase in the diameter of workpiece craters. Some EDM studies on Al/SiC MMCs concluded that this process could also be used to perform precision machining [29]. Hung et al. investigated the feasibility of applying EDM process for cast aluminium AMMCs reinforced with SiC_p, to predict the effect of process parameters on metal removal rate, re-cast layer and surface finish through statistical models [30]. It was found that the SiC particles shield and protect the aluminium matrix from being vaporised, thus resulting in the reduction of MRR.

Statistical analysis of EDM of 10% and 22% Al–Al_2O_3 MMCs with pure copper electrodes has also been performed [20]. Rotary EDM with a disk-like electrode was used to perform cutting of Al_2O_3/6061Al composites, to study the effect of each electrical or non-electrical EDM parameter (*electrical parameters*, *e.g. polarity*, *peak current*, *pulse duration and power supply voltage and non-electrical parameters*, *e.g. circumferential speed of electrode and reciprocating speed*) on the machining characteristics—MRR, electrode wear rate (EWR), relative EWR and SR. The Taguchi analysis concluded that the electrical group more significantly affects the machining characteristics as compared to the non-electrical group [31]. A reduction in MRR was observed with an increase in Al_2O_{3p} reinforcement, after the machining of Al 6061/Al_2O_3 MMCs with rotating round copper tool electrode [32]. A modified EDM setup with a plate electrode has also been employed to cut SiC–TiB_2 and SiCw/Al [33, 34]. Some experiments on blind-hole drilling of the Al_2O_3/6061Al composite have also been performed, using rotary EDM with cylindrical electrodes, and results were analysed by Taguchi methodology [35]. It has been reported that the rotary motion imparted to the tool electrode led to an improvement in MRR, due to effective flushing conditions, while machining Al/SiC MMCs using rotary EDM. However, the increasing content of SiC led to a decrease in MRR and EWR and an increase in SR [36].

Bialo et al. performed the micro-hole EDM of prepared Al+20%Si+3%Cu+1%Mg alloy matrix and Al_2O_3 ceramic reinforcement with 2-mm-thick copper electrode [37]. Aluminium-based Al_2O_{3p}-reinforced composites have also been wire EDMed [38]. Experimental investigations on the EDM of carbon–carbon composite plates using copper electrodes with negative polarity concluded that the EWR decreases substantially, within the region of experimentation, if the parameters were set at their lowest levels, while an increase in the MRR was noticed, if the parameters were set at their highest levels [39].

Some hybrid processes have also been employed on composites to study their effects. Lin et al. performed EDM with ball burnish machining (BBM) with ZrO_2 balls, providing burnishing immediately after the EDM process of an Al–Zn–Mg alloy, to study the effects on performance characteristics [40]. It was found that the combined EDM with BBM effectively improves the surface roughness and eliminates the micro-pores and cracks caused by EDM. Abrasive powder-mixed EDM (APM-EDM) or abrasive EDM (AEDM) has also been utilised to explore the influences of process parameters on the performance measures such as MRR, TWR, dimensional overcut (DOC) and SR, during machining of 6061Al+20% Al_2O_3p with copper tool electrode [41–43]. The study indicated after analysing the results using Lenth's method that the TWR and SR decrease, whereas MRR increases considerably after the addition of powder in the dielectric fluid. DOC, however, was noticed to increase slightly during AEDM [41]. From the preceding literature review, it can be seen that the EDM process and its variations (hybrid processes) have brought about significant advances in the field of machining AMMCs. An attempt has been made to find the optimal machining conditions for forming a micro-hole through EDM to a minimum diameter and a maximum aspect ratio. The work highlights the application of the Taguchi method used to determine the relations between machining parameters and process characteristics and GRA to determine the optimal machining parameters. By the former, it was concluded that that electrode wear and the entrance and exit clearances had a significant effect on the diameter of the micro-hole when the diameter of the electrode was identical, whereas the latter predicted that the input voltage and the capacitance were found to be the most significant [44].

The Grey–Taguchi's approach has also been utilised for optimization of multi-performance characteristics in electric discharge drilling of hybrid MMCs [45]. Further, the optimization of APM-EDM of AMCs with multiple responses using GRA has been successfully achieved by this approach [46].

In majority of the aforesaid past research work, many classical optimization techniques have been utilised such as statistical approach, RSM, Taguchi, etc. Classical optimization methods for solving multi-objective problem suffer from drawback, since they are unable to satisfy the requests as they demand for the large data sets, which become inconvenient to acquire, at times [47]. Furthermore, the original Taguchi method is capable to optimise a single performance characteristic [48]. In the EDM process, it is difficult to find a single optimal combination of process parameters for the performance characteristics, as the process parameters influence them differently [48–50].

Hence, there is a need for a multi-objective optimization method to arrive at the solutions to this problem. Handling the optimization of multi-performance characteristics is an interesting research field and is resolved by the GRA technique. GRA, an approach totally different from the traditional statistical analysis, provides an efficient solution to the uncertain, multi-input and discrete data problem.

Review of the past work has also indicated that few published work on EDM have utilised GRA as one of the optimization technique for multiple performance characteristic optimization [48–53]. It shows that GRA can effectively be recommended as a method for optimising the complicated inter-relationships among multiple performance characteristics. Research is still needed to bridge the gap between the theoretical research and the practical applications of EDM, through the optimisation of process parameters with multiple quality characteristics using GRA technique. Therefore, this paper adopts the GRA technique, a part of grey system theory [54], which fulfils the crucial mathematical criteria for dealing with a poor, incomplete and uncertain system [55, 56]. Through the grey relational analysis, a grey relational grade (GRG) is obtained to evaluate the multiple performance characteristics. As a result, optimization of the complicated multiple performance characteristics can be converted into the optimization of a single grey relational grade. It arrives at salient relationships in a complex system using relatively small amount of data.

3 Experimentation

3.1 Materials

The work specimens were manufactured using *stir-casting method*, one of the liquid metal processing techniques, whereby the reinforcement particulates are incorporated into the molten metal by continuous stirring of the melt [4]. This process is especially attractive, since it can often be implemented with only minor alterations to existing casting equipment. For the present case, the starting material or matrix material was aluminium alloy AA 6061 (DURALCAN). This alloy has liquidus and solidus temperatures of 650.8°C and 582.8°C, respectively [13]. Pure alumina (Al_2O_3p), prominently used in abrasive, ceramics and refractory industries, has been used as reinforcement embedded in the metal matrix, having 20 μm average particle size.

An open induction furnace and a graphite crucible were used to cast AMMCs of 6061Al/Al_2O_3p/20P. After complete stirring, the mixed slurry was poured into the die and allowed to remain until complete solidification occurred, to produce near net shape casting work specimens in the form of plates of 300×100×10 mm. The solidified casting was then ejected from the die, to be ground to achieve a plane surface.

Selvaduray et al. investigated the microstructure and physical properties of 20% Al_2O_3 reinforced AA 6061 alloys [57]. It was observed that the strength and abrasion resistance improved with the addition of reinforcement particles, whereas the ductility and fracture toughness decreased. Further, the shear strength (in longitudinal direction) showed a distinct improvement, whereas the ultimate tensile strength showed a marginal improvement. The published literature further highlights the properties of 20% Al_2O_3 reinforced AA 6061 alloys, as shown in Table 1[57]. Electrolytic copper electrodes (99.9%) were used as tool electrode material, in the present study.

Table 1 Properties of 20% Al_2O_3 reinforced AA 6061 alloys [57]

Properties	Values
Brinell hardness (500 kg)	114
Yield strength	328 MPa
Ultimate tensile strength	366 MPa
Elastic modulus	74.5 GPa
Percentage elongation	5.3%
Impact strength (Charpy's test)	28.8–35.1 J
Ultimate shear strength (longitudinal and transverse)	232 and 226 MPa, respectively
Shear modulus (longitudinal and transverse)	29.2 and 30.0 GPa, respectively

3.2 Experimental machining parameters and performance characteristics

Based on the literature review, one noise factor having two levels and five control factors having three levels each were chosen to be varied during the experimentation. The factors were assigned specific levels determined on the basis of preliminary experiments. The shape of the tool electrode (*with varied aspect ratio*) has been considered as noise factor (*which cannot be changed during experimentation*) in the present study. The two shapes, one square with aspect ratio of 1.0 (size 40×40 mm) and other rectangular with aspect ratio of 0.6 (size 24×40 mm) have been considered. Further, the control factors considered are pulse current, Ip (in amperes); pulse ON time, T_{ON} (in microseconds); duty cycle, ζ (in percent); gap voltage, Vg (in volts); and tool electrode lift time, T_L (seconds), with three levels each. The process parameters, namely Ip (in amperes); T_{ON} (in microseconds); ζ (in percent); Vg (in volts); and T_L (in seconds) were controlled by the E-ZNC machine itself after setup viz. through machine settings. The factors and levels for the present experiments are shown in Table 2.

The experimental investigations were carried out on an electric discharge machine (E-ZNC, Make: Electronica, Pune, India) powered with a PS-50 generator, max. working current of 50 A. The spark erosion oil (SEO 250, flash point 94°C, viscosity CST at 40°C=2.6, Make: IPOL) was used as dielectric fluid with a flushing pressure of 1.15 kgf/cm^2. The polarity of the tool electrode considered as positive and that of the work material as negative. The multiple performance characteristics considered were MRR, TWR and SR. The response variable of MRR and TWR for each run was calculated on the basis of weight difference before and after machining using a Sartorius LA-1200S precision scale (*max. capacity of 1,200 g and precision accuracy of 0.001 g*). The surface roughness (Ra) of the EDMed surface

Table 2 Factors and their levels

Factor	Experimental Parameters	Symbol (Units)	Level 1	Level 2	Level 3
A	Aspect ratio	AR	0.6	1.0	–
B	Pulse current	Ip (A)	10	15	20
C	Pulse ON time	T_{ON} (μs)	50	100	200
D	Duty cycle	ζ (%)	0.4	0.5	0.7
E	Gap voltage	Vg (V)	40	45	50
F	Tool electrode lift time	T_L (s)	2.0	3.0	5.0

was measured by a Taylor–Hobson surface roughness tester, with an accuracy of 0.1 μm.

3.3 Design of experiments

An appropriate Taguchi`s orthogonal array (OA) design for experimentation is selected by computing the total degrees of freedom. In the present study, there are 11 degrees of freedom owing to one two-level machining parameter and five three-level machining parameters in the EDM process. Therefore, an $L18$ (2^1×3^5) mixed orthogonal array with 6 columns and 18 rows was used to accommodate one two-level noise factor and five three-level control factors. Since six process parameters are used in the experiments (occupying six columns), two columns of the OA are left empty, which does not affect its orthogonality. This partial factorial experimental design provides an efficient and systematic approach of determining an optimal parameter condition. Each factor is assigned to a column and 18 machining parameter combinations are required. Therefore, by using the $L18$ OA, only 18 experiments are needed to study the entire process parameters. The experimental layout for the machining parameters using the $L18$ OA is shown in Table 3.

The entire performance measures data for the 18 experimental runs are shown in Table 4. Thus, in optimization by the GRA, the observed values of MRR, TWR and SR were set to maximum, minimum and minimum, respectively. The GRA technique is presented and discussed in Section 4.

Table 3 Experimental layout using an orthogonal array L18 ($2^1 \times 3^5$) design

No. of run/factor	N	C_1	C_2	C_3	C_4	C_5
1	0	0	0	1	1	0
2	0	1	2	2	1	0
3	0	2	2	1	2	0
4	1	0	0	0	0	0
5	1	1	1	2	0	0
6	1	2	1	0	2	0
7	0	0	1	2	2	1
8	0	1	0	0	2	1
9	0	2	0	2	0	1
10	1	0	1	1	1	1
11	1	1	2	0	1	1
12	1	2	2	1	0	1
13	0	0	2	0	0	2
14	0	1	1	1	0	2
15	0	2	1	0	1	2
16	1	0	2	2	2	2
17	1	1	0	1	2	2
18	1	2	0	2	1	2

4 Grey system theory

Grey system theory (GST) proposed by Deng in 1982 is based on the random uncertainty of small samples [55, 56]. Since its inception, GST gradually developed into an evaluation technique to solve certain problems of system that are complex and multivariate. Such systems are often referred to as 'grey' having uncertain or incomplete information. In system control theory, a system for which the relevant information is completely known is a 'white' system, while a system for which the relevant information is completely unknown is termed a 'black' system. Any system between these limits is a 'grey' system having poor and limited information [58, 59]. Conventional statistical approaches for analysis of such systems may not be acceptable without large data sets and data satisfying certain mathematical criteria. The grey theory, on the contrary, makes use of relatively small data sets and does not demand strict compliance to certain statistical laws, simple or linear relationships among the observables [60]. The main grey methods within grey system theory are (grey) systems and control, grey modeling and GRA [56, 61]. GRA is an alternative for traditional statistical methods, relying on few samples and uncertainty conditions, and can be applied in optimization of multiple quality characteristics. In recent researches, GRA has been employed by many scientists in different areas including medicines and has demonstrated satisfactory results [54, 62–69]. Literature also identifies that some of the problems related to the analysis of multiple quality characteristics are that most statistical approaches are not suitable for characteristics with significant correlation, and taking all quality characteristics with the same weight may cause yield loss since the importance of each quality characteristic may be different [70]. To overcome such problems, GRA approach is taken.

This paper utilises the GRA approach to optimise the process parameters taking into account the correlation between multiple performance characteristics. In the following section, the GRA for determining the optimal machining parameters is reported step by step. Further, the optimal machining parameters with considerations of the multiple performance characteristics are obtained and validated.

4.1 Grey relational analysis

GRA is a normalisation-based evaluation technique requiring a sample of only limited (*and from a statistics point of view generally insufficient data*) size, of discrete sequential (time series) data to enable reliable modeling and estimation of system behaviour [60]. In GRA, it is assumed that the input attributes satisfy three conditions for comparability of the set of series, referred to as scaling (for the order of magnitude), polarisation (*for the attribute type*) and non-dimension (for the measurement scale) [55]. Normalisation

Table 4 Experimental results obtained for material removal rate, tool wear rate and surface roughness

No. of run	MRR (g/min)			TWR (g/min)			SR, Ra (μm)		
	1	2	3	1	2	3	1	2	3
1	0.1135	0.1142	0.1158	0.0059	0.0088	0.0048	6.5	6.0	5.2
2	0.2818	0.2659	0.2638	0.0198	0.0187	0.0200	6.9	6.1	6.8
3	0.3004	0.3234	0.3362	0.0260	0.0232	0.0228	6.3	6.8	7.3
4	0.1482	0.1531	0.1436	0.0068	0.0066	0.0073	5.8	6.4	6.1
5	0.2915	0.2752	0.2823	0.0085	0.0095	0.0087	6.8	6.2	6.5
6	0.2983	0.2911	0.3016	0.0149	0.0153	0.0154	5.9	6.4	7.5
7	0.1322	0.1384	0.1374	0.0073	0.0079	0.0069	6.7	7.2	5.6
8	0.1502	0.1578	0.1570	0.0086	0.0090	0.0085	7.3	5.7	6.2
9	0.2875	0.3054	0.3446	0.0238	0.0243	0.0239	7.5	7.4	6.8
10	0.1637	0.1711	0.1932	0.0094	0.0089	0.0093	6.6	6.9	6.0
11	0.2644	0.2818	0.2968	0.0094	0.0098	0.0099	7.3	7.1	6.9
12	0.3299	0.3484	0.3327	0.0118	0.0123	0.0119	7.1	7.4	7.4
13	0.1701	0.1869	0.1680	0.0168	0.0166	0.0161	6.3	6.6	6.3
14	0.2653	0.2422	0.2635	0.0145	0.0141	0.0140	6.0	6.4	6.5
15	0.2671	0.2839	0.2770	0.0183	0.0179	0.0184	6.4	6.8	6.9
16	0.1964	0.2231	0.2105	0.0132	0.0139	0.0134	6.6	6.5	6.1
17	0.1849	0.1773	0.1928	0.0031	0.0038	0.0033	6.5	6.4	6.9
18	0.3116	0.2841	0.3013	0.0123	0.0130	0.0128	6.6	6.7	7.1

of the input data prior to GRA processing is required, if the above stated three conditions are not satisfied.

Linear normalisation involves data pre-processing in compliance with the three conditions to be achieved. This step is also called 'grey relational generating' [70]. Data pre-processing translates the trend relationship between an 'original sequence' to a 'reference sequence', at a given point in a system. After normalisation, the reference sequence is identified. In the present study, normalisation of the experimental results obtained for material removal rate, tool wear rate and surface roughness was performed, in the range between 0 (*black*) and 1 (*white*). Figure 1 depicts the flowchart showing the experiment and grey relational analysis. In general, for a maximum value type attribute, viz. MRR, the highest value is taken, for a minimum value type attribute, viz. TWR and SR, the lowest value is considered.

The respective formulae to obtain normalised experimental results are as follows:

'Higher-the-better' (HB) value:

$$x_{ij} = \frac{y_{ij} - \min_i y_{ij}}{\max_i y_{ij} - \min_i y_{ij}} \tag{1}$$

'Lower-the-better' (LB) value:

$$x_{ij} = \frac{\max_i y_{ij} - y_{ij}}{\max_i y_{ij} - \min_i y_{ij}} \tag{2}$$

where y_{ij} is the jth performance characteristic in the ith experiment. Furthermore, in Eq. 1 and Eq. 2, $\max_i y_{ij}$ and $\min_i y_{ij}$ are the maximum and minimum value of jth performance characteristic for alternative i, respectively. Equation 1 is used for the HB value, whereas Eq. 2 is used for LB values. The entire results of normalisation (data preprocessing) of experimental results obtained for performance measures by Eq. 1 for MRR and Eq. 2 for TWR and SR are shown in Table 5.

4.2 Grey relational coefficient

Normalisation creates a new matrix of difference vectors. From this matrix, a grey relational coefficient (GRC) is calculated, expressed as:

GRC

$$\xi_{ij} = \frac{\min_i \min_j \left| x_j^0 - x_{ij} \right| + \zeta \max_i \max_j \left| x_j^0 - x_{ij} \right|}{\left| x_j^0 - x_{ij} \right| + \zeta \max_i \max_j \left| x_j^0 - x_{ij} \right|} \tag{3}$$

x_j^0 is the ideal normalised result for the jth performance characteristic. In Table 4, the row labelled with 'ideal' (x_j^0= 1) is the reference (ideal) sequence. The entire results for the GRC are shown in Table 4. GRC (ξ_{ij}) is computed by selecting a proper distinguishing coefficient ζ (in general, ζ=1) by using Eq. 3.

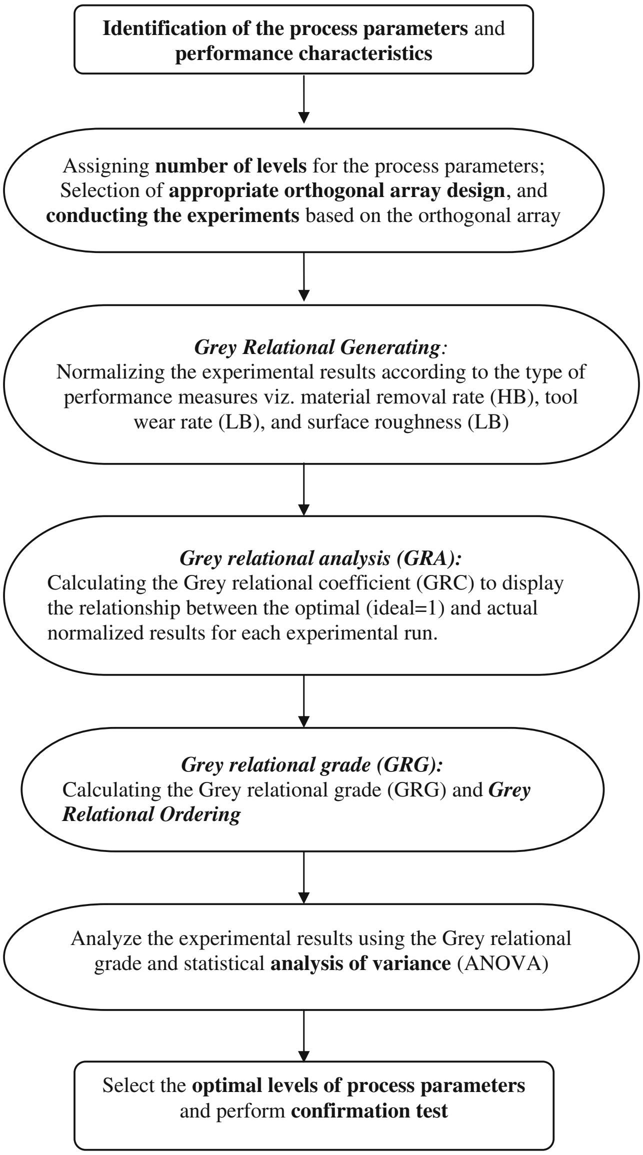

Fig. 1 Flow chart for the experimental analysis

4.3 Grey relational grade

Finally, the GRG is obtained by averaging the grey relational coefficient corresponding to each performance measures.

GRG:

$$r_i = \frac{1}{m} \sum_{j=1}^{m} \xi_{ij} \tag{4}$$

Thus by applying Eq. 4, all grey relational grades can be computed. The grey relational grades of the set of compared series provide a ranking of the alternatives, where a higher value determines a better alternative. By analysis of the grey relational grade, we can understand which factors will crucially affect reference factors. This relationship is held for any distinguishing coefficient. The higher grey relational grade represents that the corresponding result is closer to the ideal normalised value [48]. The grey relational grade obtained for each experimental run and the ranking order of the experiment is shown in Table 5. It is seen that experiment #1 has the best multiple performance characteristics among the 18 runs performed, having highest relational grade. Hence, it is the optimal. It is followed by experiments #4 and #13, being ranked as second and third, respectively.

5 Results and interpretation

In this paper, Matlab 6.5 and MiniTab 14 versions have been successfully utilised to perform GRA and analysis of variance (ANOVA). The effect of each process parameters on the basis of GRG at each level has been shown in Table 6. The orthogonal experimental design separates out the effect of each machining parameter on the grey relational grade at different levels. For example, the mean of grey relational grade for the factor A, viz. aspect ratio, at levels 1 and 2 can be calculated by taking the average of the grey relational grade for the experiment nos. 1–9 and 10–18, respectively (shown in Table 2). Similarly, the mean of the grey relational grade for each level of other machining parameters can also be computed. In addition, the total mean of the grey relational grade for the 18 experiments is also calculated and listed in Table 6. The total mean value of the grey relational grade is 0.65218.

The grey relational grade represents the level of corelation between the reference sequence and the comparability sequence [72]. The greater value of the grey relational grade means that the comparability sequence has a stronger correlation to the reference sequence. Therefore, the optimal level of the machining parameters is the level with the greatest grey relational grade value. The level value marked asterisks (*) indicates that they results in a better EDM performance. Based on the grey relational grade given in Table 6, the optimal machining performance for MRR, TWR and SR was obtained for aspect ratio (level 1), pulse current (level 1), pulse ON time (level 1), duty cycle (level 1), gap voltage (level 3) and tool electrode lift time (level 1) combination. Accordingly, A1B1C1D1E3F1 is the optimal level of EDM parameters in the case of multiple performance characteristics because higher GRG values yield better quality.

The difference between the maximum and the minimum value of the grey relational grade is also calculated and tabulated in Table 6 and ranked accordingly. As listed in Table 6, the difference between the maximum and the minimum value of the average grey relational grade of the EDM machining parameters is as follows: 0.0761 for aspect ratio, 0.0851 for pulse current, 0.0434 for pulse ON time, 0.0191

Table 5 Normalisation (data pre-processing) of the experimental results for each performance measures

No. of run	MRR			TWR			SR		
Ideal	1	1	1	1	1	1	1	1	1
1	1.00000	1.00000	1.00000	0.50626	0.54345	0.49256	0.68610	0.83607	1.00000
2	0.53644	0.58149	0.58183	0.76874	0.76715	0.82620	0.58175	0.79275	0.56403
3	0.51030	0.50188	0.48302	1.00000	0.94373	0.94399	0.75369	0.58175	0.49640
4	0.84878	0.84420	0.88105	0.51771	0.51037	0.52760	1.00000	0.68610	0.69697
5	0.52248	0.56695	0.55292	0.54080	0.55489	0.54950	0.60474	0.75369	0.61424
6	0.51312	0.54370	0.52568	0.64995	0.67213	0.68565	0.93865	0.68610	0.47368
7	0.91240	0.89701	0.90506	0.52429	0.52941	0.52167	0.62963	0.50495	0.83806
8	0.84144	0.82860	0.83328	0.54223	0.54667	0.54626	0.50495	1.00000	0.67427
9	0.52815	0.52435	0.47368	0.90355	1.00000	1.00000	0.47368	0.47368	0.56403
10	0.79507	0.78743	0.72681	0.55388	0.54505	0.55944	0.65665	0.56044	0.72125
11	0.56344	0.55706	0.53220	0.55388	0.55994	0.56976	0.50495	0.52218	0.54907
12	0.47368	0.47368	0.48702	0.59207	0.60591	0.60707	0.54064	0.47368	0.48478
13	0.77482	0.74354	0.79777	0.69138	0.70554	0.70387	0.75369	0.62963	0.65300
14	0.56198	0.62217	0.58232	0.64186	0.64398	0.65190	0.88439	0.68610	0.61424
15	0.55908	0.55398	0.56091	0.72801	0.74245	0.77121	0.71831	0.58175	0.54907
16	0.70143	0.65935	0.68498	0.61688	0.63951	0.63843	0.65665	0.65665	0.69697
17	0.73174	0.76961	0.72784	0.47368	0.47368	0.47368	0.68610	0.68610	0.54907
18	0.49575	0.55369	0.52608	0.60070	0.62017	0.62551	0.65665	0.60474	0.52141

for duty cycle, 0.0224 for gap voltage and 0.0493 for tool electrode lift time. The most significant factor affecting performance characteristics is determined by comparing these values. The most effective controllable factor was the maximum of these values. As per Table 6, the maximum value among the controllable factors is for pulse current, viz. 0.0851. This higher value indicates that the pulse current has the strongest effect on the multi-performance characteristics among the other machining parameters. Furthermore, the significance of role that every process parameter plays over the multi-performance characteristics can be predicted by examining these values.

Table 6 Grey relational grade for each experimental run

Run No.	Grey relational grade	Order
1	0.78494	1
2	0.66671	7
3	0.69053	6
4	0.72364	2
5	0.58447	15
6	0.63207	13
7	0.69583	5
8	0.70197	4
9	0.66013	9
10	0.65623	10
11	0.54583	17
12	0.52650	18
13	0.71703	3
14	0.65433	11
15	0.64053	12
16	0.66121	8
17	0.61906	14
18	0.57830	16

The order of importance of the machining parameters to the multi-performance characteristics in the EDM process, in

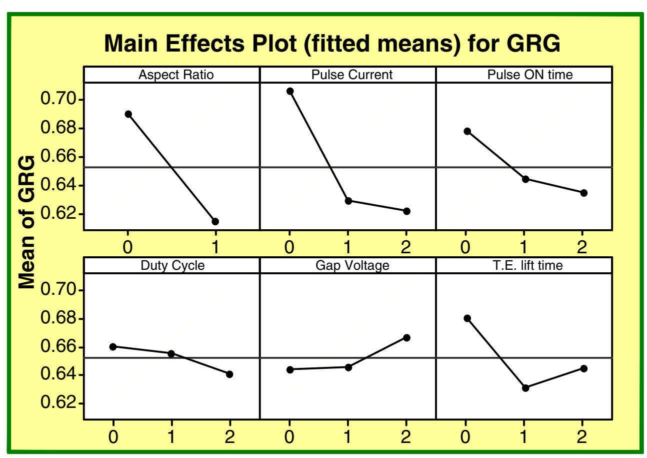

Fig. 2 Effects plot for grey relational grade

Table 7 Response table for grey relational grade

Factor	Machining parameter	Average grey relational grade			Max.–Min.	Rank
		Level 1	Level 2	Level 3		
A	Aspect ratio	0.6902[a]	0.6141	–	0.0761	2
B	Pulse current	0.7065[a]	0.6287	0.6213	0.0851	1
C	Pulse ON time	0.6780[a]	0.6439	0.6346	0.0434	4
D	Duty cycle	0.6602[a]	0.6553	0.6411	0.0191	6
E	Gap voltage	0.6444	0.6454	0.6668[a]	0.0224	5
F	Tool electrode lift time	0.6804[a]	0.6311	0.6451	0.0493	3

Total mean value of the grey relational grade=0.65218

[a]Results in a better EDM performance

sequence can be ranked as: factor B (pulse current), A (aspect ratio), F (tool electrode lift time), C (pulse ON time), E (gap voltage) and D (duty cycle). This indicates that the EDM performance was strongly affected by the pulse current.

Figure 2 shows the main effect plot (response graph) based on grey relational grade where the dash line indicates the value of the total mean of the grey relational grade (viz. 0.65218). Basically, the larger the grey relational grade, the better is the multi-performance characteristics, since it is closer to the ideal value, viz. 1. Accordingly, A1B1C1D1E3F1 is the optimal level of EDM parameters in the case of multiple performance characteristics. The greater values in Fig. 2 depict the high MRR, low TWR and surface roughness. However, the relative importance among the process parameters for the multiple performance characteristics still needs to be known, to determine the optimal combinations of the parametric levels. Thus, ANOVA is performed.

5.1 Analysis of variance

The main purpose of the ANOVA is the application of a statistical method to identify the effect of individual factors. Results from ANOVA can determine very clearly the impact of each factor on the process results [73].

In the present study, ANOVA investigates which EDM process parameters significantly affect the performance measures. This is revealed by separating the total variability of the grey relational grades, which is measured by the sum of squared deviations from the total mean of the grey relational grade, into contributions by each process parameter and error. In addition F test [74] and p value (probability) have also been determined. Table 7 shows the results of ANOVA for multi-performance characteristics. Since there are four p values less than 0.05, these factors have a statistically significant effect on grey relational grade at 95.0% confidence level.

The results of ANOVA (Table 7) indicate that noise factor–aspect ratio and pulse current are the most significant process parameters affecting the multiple performance measures. The other factors having significant effects are tool electrode lift time and pulse ON time, respectively. Furthermore, the other parameters are not significant at 95% confidence level.

Figure 3 depicts the graphical ANOVA plot showing the effects of each factor scaled so that they can be compared to the variability of the residuals. For each factor, the deviations of the adjusted level means from the estimated grand mean are displayed. Any factor that shows considerably larger variability than the residuals is likely to be an important factor. It also indicates that aspect ratio and pulse current are the most significant process parameters affecting the multiple performance measures.

5.2 Confirmation tests

Once the optimal level of the process parameters is identified, the final step is to predict and validate the improvement of the performance measures using the optimal level. The purpose of the confirmation experiment is to verify the conclusions drawn during the analysis phase. The estimated γ_m using the optimal levels of the process parameters can be computed by using the following formula:

$$\hat{\gamma} = \gamma_m + \sum_{i=1}^{n} (\hat{\gamma}_i - \gamma_m) \quad (5)$$

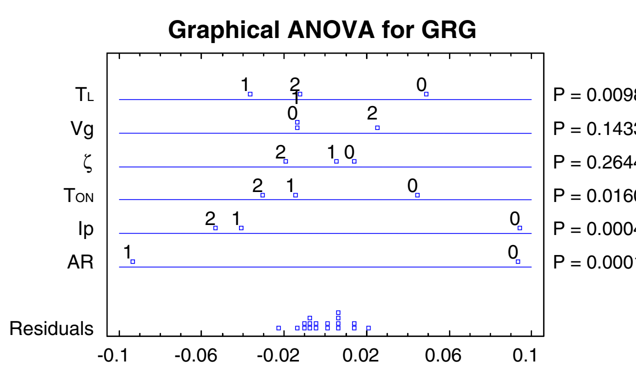

Fig. 3 Graphical ANOVA plot

Table 8 Results of analysis of variance for multi-performance characteristics

Factor	Machining parameter	Degree of freedom	Sum of square	Mean square	*F* value	Probability value
A	Aspect ratio	1	0.0260445	0.0260445	74.11[a]	0.0001
B	Pulse current	2	0.0266962	0.0133481	37.98[a]	0.0004
C	Pulse ON time	2	0.0062594	0.0031297	8.91[a]	0.0160
D	Duty cycle	2	0.0011764	0.0005882	1.67	0.2644
E	Gap voltage	2	0.0019204	0.0009602	2.73	0.1433
F	Tool electrode lift time	2	0.0077495	0.0038748	11.03[a]	0.0098
	Residual error	2	0.0021085	0.0003514		
	Total	17	0.0719550			

[a]Significant at 95% C.I. level

where γ_m is the total mean of the grey relational grade, $\hat{\gamma}_i$ is the mean of the grey relational grade at the optimal level and n is the number of the process parameters that significantly affects the performance characteristics.

From Eq. 5, the estimated grey relational grades using the optimal EDM parameters are computed. Table 8 shows the results of the confirmation tests using the optimal levels of EDM parameters. As noted from Table 8, the MRR is increased from 0.2216 to 0.293 g/min, when the tool wear and surface roughness are minimised from 0.0128 to 0.0076 g/min and from 6.678 to 4.512 μm, respectively. Also an improvement of 0.2450 is noted in grey relational grade, after validation.

6 Conclusions

In the present study, the GRA approach, based on the orthogonal experimental design table, has been proposed as a way of studying the optimization of EDM factors. The GRA approach easily converts the optimization of the multiple performance characteristics into the GRG, thus simplifying the complicated analysis of multiple performance characteristics. The effectiveness of this approach has been later verified by confirmation experiments and analysis of variance. The optimal EDM parameters were determined for the multi-performance characteristics for maximum MRR and minimum TWR and SR.

From the response table (Table 9) of the average grey relational grade, the largest value of grey relational grade for the EDM parameters was found. It was found that the pulse current has the strongest effect among the other process parameters used to study the multi-performance characteristics. The order of importance of the process parameters to the multi-performance characteristics is pulse current, aspect ratio, tool electrode lift time, pulse ON time, gap voltage and duty cycle. Experimental results have shown clearly that the material removal rate, tool wear rate and surface roughness in the EDM process can be improved effectively through the proposed approach. This study indicated that GRA approach could be applied successfully to other operations in which performance measures are determined by many process parameters at multiple quality requests [75].

To conclude, as per the findings, GRA, an advanced statistical method of multi-factorial analysis, embodies rich philosophical thought of the unity of opposites, such as continuity and discontinuity, quality and quantity, statics and dynamics, etc. Empirical research on high-tech industries and systems are often constrained, since traditional statistical methods require large sets of data. On the other hand, grey system theory is designed to work with system where the available information is insufficient to characterise the system.

Table 9 Results of performance measures for initial and optimal process parameters

	Initial machining parameters	Optimal machining parameters	
		Predicted	Experimental
Combination level	A2B1C1D1E1F1	A1B1C1D1E3F1	A1B1C1D1E3F1
MRR (g/min)	0.2216	–	0.293
TWR (g/min)	0.0128	–	0.0079
SR, Ra (μm)	6.678	–	4.712
Grey relational grade	0.6529	0.7687	0.8979

Improvement of the grey relational grade=0.2450

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