Int J Adv Manuf Technol
DOI 10.1007/s00170-012-3897-6

ORIGINAL ARTICLE

Developing a location–inventory model under fuzzy environment

Hassan Shavandi · Bita Bozorgi

Received: 17 August 2011 / Accepted: 3 January 2012
© Springer-Verlag London Limited 2012

Abstract Nowadays, location of distribution centers integrated with inventory or transportation decision play an important role in optimizing supply chain management. Location–inventory models analyze the location and inventory policies in distribution network, simultaneously. Developing location–inventory models under fuzzy environment can enrich the model, and this is our approach in this article. We consider the demand as a fuzzy variable and formulate the problem using credibility theory in order to locate distribution centers (DCs) as well as determining inventory levels in DCs. The derived model belongs to nonlinear mixed integer programming problems, and we presented a genetic algorithm to solve it. Numerical results show that the performance of the proposed algorithm is reasonable.

Keywords Location · Inventory management · Distribution system · Fuzzy logic · Credibility theory · Genetic algorithm

1 Introduction

The new situation in market and changes in technology have made many production companies tend to create a network of close and well-organized communications called supply chain, to more easily supply products to consumers. An important issue in the supply chain is distribution network. Since, decreasing transportation costs and expediting services, are two important factors in competition among companies, effective design of this network is of particular importance. Location policies are among the long-term or mid-term ones, whereas policies for inventory are considered short-term or mid-term policies in product distribution networks; however, the impact of location policies on inventory costs are vividly observable; since inventory costs change consequently, as a result of changing the allocated demand to service providers. Thus, location–inventory models have been offered which simultaneously minimize both the location and inventory costs. More than and before anything, decision makers are faced with uncertain parameters which are of non-deterministic nature and change in the real environment, and also are considered non-deterministic in the problem because of lack of information about these parameters from real world. For instance, if a decision maker wants to schedule their annual plan for next year, obtaining precise value of certain parameters would definitely not be easy (e.g., the demand of customer).

In this paper, we study the location–inventory problem considering fuzzy demand and construct a new model for this problem with solution method to reach a more practical location–inventory model.

In the rest of the paper, Section 2 includes the literature review of related article, and in Section 3, we develop the model of the problem. Solution method and computational results are presented in Section 4, and finally, we conclude the paper in Section 5.

2 Literature review

In recent years, the problem of facility location has received more attention and is formulated as several models such as

H. Shavandi (✉)
Department of Industrial Engineering,
Sharif University of Technology,
Tehran, Iran
e-mail: Shavandi@sharif.edu

B. Bozorgi
Department of Industrial Engineering, Alghadir University,
Tabriz, Iran
e-mail: Bozorgi_bita@yahoo.com

Published online: 28 January 2012

continuous location models, network location models, and mixed integer programming models. Continuous location models have two special properties. One being that the solution space is continuous, according to which one can locate facilities on any point in the plane. The other property is that, distances can be measured by an appropriate metric. For more details, refer to [1, 2]. In network location models, distances are considered as the shortest path in a graph. Nodes represent demand points and potential sites to be allocated to facilities are mapped to a subset of nodes and points on edges. A good review of network location models can be found in [3, 4]. Having a set of potential locations for facilities, many location problems can be formulated by mixed integer programming models [5, 6]. Location–inventory models may be categorized according to inventory control policy capacity of distribution centers (DCs), production capacity of the supplier, type of retailers' demand (deterministic vs. non-deterministic), type of products (single or multiple), number of allocated distribution centers (specified or not), and so on. In the past decades, many people have introduced fuzzy theory to the location problem. Bhattacharya et al. considered the problem where DCs were located under several fuzzy criteria and presented a fuzzy objective programming method to solve the problem [7, 8]. Canos et al. added a set of fuzzy constraints to the p-median problem which introduced a decision-making strategy in which very low costs are obtained by leaving part of the demand uncovered [9]. Also, Chen and Wei, Darzentas, and Rao and Saraswati studied various facility location problems utilizing fuzzy logic methods [10–12]. However, all the parameters in these problems are deterministic, and fuzzy theory is only used to effectively solve the classic programming. Tezeng and Chen have offered a location model with a multi-objective fuzzy approach [13]. This model aids the location of fire stations in an international airport. Because of the combinatorial complexity of this model, a genetic algorithm has been offered and compared with the counting method.

Kuo et al. offered a decision-making support system combining fuzzy set theory and hierarchical analytic procedure in location of a new shop [14]. Chen has offered a new method for multi-criterion decision making to solve the location of distribution centers in a fuzzy environment [15, 16].

Chu proposed a fuzzy TOPSIS for the selection of plant location [17]. Ertugrul and Karakasoglu used fuzzy AHP and fuzzy TOPSIS for the selection of facility location [18]. They considered results of the proposed methods.

With the best of our knowledge, there is no research in literature about the fuzzy location–inventory problem. Therefore, in this article, we develop a new model for fuzzy location–inventory problem using credibility theory.

While possibility theory allows for analysis of a fuzzy variable and mapping it to real space, it lacks the property of duality which in theory and practice is of particular importance. Liu [19] offered credibility theory to investigate fuzzy variables and solve the problem of non-self duality of the credibility measure and further developed it. Liu introduces the foundations of chance theory and credibility theory, while he also investigates many practical problems as well [20].

Zhou and Liu question the location–allocation problem with fuzzy demand and introduce three theoretical models—expected minimization, α-cost minimization, and credibility measure maximization [21]. We discuss in this paper an intelligent hybrid algorithm which solves such models. We also exploit simulation and genetic algorithm to effectively solve the problem.

2.1 Basic definition

Let Θ be a non-empty set and P its power set. An element of P is called an *event*. We define a particular metric to measure the credibility of an event denoted $\mathrm{Cr}\{A\}$, where A is an event. Thus, Cr is a real-valued function over the power set of Θ.

Cr is a credibility measure if and only if it satisfies the following conditions [20]:

$$\mathrm{Cr}\{\Theta\} = 1 \tag{1}$$

$$\mathrm{Cr}\{A\} \leq \mathrm{Cr}\{B\} \quad \text{if } A \subset B \tag{2}$$

$$\mathrm{Cr}\{A\} + \mathrm{Cr}\{A^c\} = 1 \tag{3}$$

$$\mathrm{Cr}\{\cup_i A_i\} = \mathrm{Sup}_i \mathrm{Cr}\{A_i\} \quad \forall \{A_i\} \quad \text{with} \quad \mathrm{Sup}_i \mathrm{Cr}\{A_i\} < 0.5 \tag{4}$$

Self-duality is a fundamental property which distinguishes credibility theory from its predecessors and is defined as follows:

$$\mathrm{Cr}\{A\} + \mathrm{Cr}\{A^c\} = 1 \tag{5}$$

Remark 1 Let μ be a nonnegative function over Θ (e.g., the set of real), such that,

$$\mathop{\mathrm{Sup}}_{X \in \Theta} \mu(X) = 1 \tag{6}$$

Then, the following will establish a credibility measure over Θ,

$$\mathrm{Cr}\{\xi \in B\} = \frac{1}{2}\left(\mathop{\mathrm{Sup}}_{X \in B} \mu(X) + 1 - \mathop{\mathrm{Sup}}_{X \in B^c} \mu(X)\right) \tag{7}$$

Theorem 1 *Let Θ be a nonempty set, P its power set, and Cr a credibility measure, then* [20]

$$\forall A \in P \quad 0 \leq \mathrm{Cr}\{A\} \leq 1 \quad \wedge \quad \mathrm{Cr}\{\phi\} = 0 \tag{8}$$

2.1.1 Critical values

In order to rank fuzzy variables, we may use either of the following [20]:

- α-Optimistic value of ξ: Let ξ be a fuzzy variable and α range over [0,1], then the α-optimistic value of ξ is defined as follows:

$$\xi_{\mathrm{Sup}}(\alpha) = \mathrm{Sup}\{r|\mathrm{Cr}\{\xi \geq r\} \geq \alpha\} \tag{9}$$

- α-Pessimistic value of ξ: Let ξ be a fuzzy variable and α range over [0,1], then the α-pessimistic value of ξ is defined as follows:

$$\xi_{\inf}(\alpha) = \mathrm{Inf}\{r|\mathrm{Cr}\{\xi \leq r\} \geq \alpha\} \tag{10}$$

Theorem 2 [20]: *Let* (a, b, c, d) *be a trapezoidal variable. Then α-pessimistic critical value is defined as follows:*

$$\xi_{\inf}(\alpha) = \begin{cases} 2\alpha(b-a)+a & \text{if } \alpha \leq 0.5 \\ 2(1-\alpha)(c-d)+d & \text{if } \alpha > 0.5 \end{cases} \tag{11}$$

Theorem 3 [20]: *Let ξ be a fuzzy number with continuous membership function. Then,* $\mathrm{Cr}\{\xi \leq r\} \geq \alpha$ *if and only if* $r \geq \xi_{\inf}(\alpha)$, that is:

$$\mathrm{Cr}\{\xi \leq r\} \geq \alpha \Leftrightarrow r \geq \xi_{\inf}(\alpha) \tag{12}$$

Remark 2 [20]: Let $\alpha_1 \leq \alpha_2$ be two parameters, and z_1, z_2 the corresponding values for the fuzzy location–inventory model. Then, we have $z_1 \leq z_2$.

3 Problem formulation

We consider a distribution system where a main supplier delivers its products to merchants. In order to enhance the quality of service, we assume some of the merchants to be distribution centers and the others to be retailers. The main supplier delivers the products to distribution centers and distribution centers deliver the products to assigned retailers. In this distribution system, transportation cost comprises the transportation cost from the main supplier to the distribution centers and from distribution centers to retailers. In this problem, the number of distribution centers, their locations, allocation of retailers to DCs, and the inventory level to be stored in each DC are decision variables. The objective function of the problem is to minimize the total cost of locations, inventory, and transportation costs.

3.1 The assumptions

The assumptions considered to formulate the problem are as follows:

1. Distribution centers receive the cost of transportation of products to the retailers from the main supplier.
2. The location of the main supplier and retailers are given.
3. Planning period is so short, and hence, the total need product is supplied at the beginning of period and the cost of ordering is neglected.
4. Retailers can only be allocated to points which are selected as distribution centers.

3.2 Developing the model considering deterministic environment

In this sub-section, we are going to develop the model of the problem considering the deterministic demand of products in retail. In the following, we define the parameter and decision variable which are common to develop the model under fuzzy environment.

3.2.1 Notation

j	Distribution index
k	Retailer index
I	Set of retailers
D_k	Mean of annual demand at retailer k, $\forall\ k \in I$
F_j	Fixed cost of constructing a distribution center at node j, $\forall\ j \in I$
h_j	Annual cost of storing one unit of products in distribution center j, $\forall\ j \in I$
a_j	Transportation cost of one unit of products from the main supplier to distribution center j $\forall\ j \in I$
c_{jk}	Transportation cost of one unit of products from distribution center j to retailer k $\forall\ k \in I$
X_j	equals 1, if retailer j is selected as a distribution center, and 0, otherwise.
Y_{jk}	equals 1, if retailer k is supplied by distribution center j, and 0, otherwise.

3.2.2 The model

The objective function of the model can be written as:

$$\min \sum_j \left[F_j X_j + \sum_k c_{jk} D_k Y_{jk} + \left[a_j \sum_k D_k Y_{jk} + \frac{h_j \sum_k D_k Y_{jk}}{2} \right] \right] \tag{13}$$

The terms of objective function are described as follows:

$F_j X_j$	Fixed cost of constructing distribution center
$\sum_k c_{jk} D_k Y_{jk}$	Transportation cost of products from DC j to retailer
$a_j \sum_k D_k Y_{jk}$	Transportation cost from the main supplier to DC j
$\frac{h_j \sum_k D_k Y_{jk}}{2}$	Inventory holding cost at DC j

The mathematical of problem in deterministic situation is written below:

$$\min \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k Y_{jk} + \sum_j \left[a_j \sum_k D_k Y_{jk} + \frac{h_j \sum_k D_k Y_{jk}}{2} \right] \tag{14}$$

Subject to:

$$\sum_j Y_{jk} = 1 \quad \forall k \in I \tag{15}$$

$$Y_{jk} - X_j \leq 0 \quad \forall j, k \in I \tag{16}$$

$$Y_{jk} \in \{0, 1\} \quad \forall j, k \in I \tag{17}$$

$$X_j \in \{0, 1\} \quad \forall j \in I \tag{18}$$

The first constraint 15 asserts that each retailer is allocated to just one DC and constraint 16 asserts that retailers may only be allocated to the node j if it is DC.

Constraints 17 and 18 assert the 0–1 type of decision variable.

3.3 Formulating the location–inventory model under fuzzy environment

Since critical values are used to rank fuzzy variables and in this model our goal is to minimize the objective function, we will use α-pessimistic critical value. Since we assume demand to be a trapezoidal fuzzy number in this problem, the objective function will also be fuzzy which means we should assume the total corresponding costs to be fuzzy as well.

In this problem, we want to minimize the α-critical total value rather than the objective function.

Our model will take the form:

$$\min \bar{f} \tag{19}$$

Subject to:

$$\mathrm{Cr}\left\{ \sum_j F_j X_j + \sum_j \sum_k c_{jk} \widetilde{D}_k Y_{jk} + \sum_j \left[a_j \sum_k \widetilde{D}_k Y_{jk} + \frac{h_j \sum_k \widetilde{D}_k Y_{jk}}{2} \right] \leq \bar{f} \right\} \geq \alpha \tag{20}$$

$$\sum_j Y_{jk} = 1 \quad \forall k \in I \tag{21}$$

$$Y_{jk} - X_j \leq 0 \quad \forall j, k \in I \tag{22}$$

$$Y_{jk} \in \{0, 1\} \quad \forall j, k \in I \tag{23}$$

$$X_j \in \{0, 1\} \quad \forall j \in I \tag{24}$$

Now, by applying theorems 2 and 3, the model can be transformed to a solvable model. According to theorem 3, we have:

$$\mathrm{Cr}\left\{ \sum_j F_j X_j + \sum_j \sum_k c_{jk} \widetilde{D}_k Y_{jk} + \sum_j \left[a_j \sum_k \widetilde{D}_k Y_{jk} + \frac{h_j \sum_k \widetilde{D}_k Y_{jk}}{2} \right] \leq \bar{f} \right\} \geq \alpha \Leftrightarrow \bar{f} \geq \xi_{\inf}(\alpha) \tag{25}$$

Then,

$$\overline{f} \geq \xi_{\inf}(\alpha) \tag{26}$$

Now, since $\widetilde{D}_k = (D_k^1, D_k^2, D_k^3, D_k^4)$ is assumed to be a trapezoidal fuzzy number, from theorem 2, the objective function of the fuzzy model will be as follows, and our objective function will also be a trapezoidal number.

$$\min \overline{f} = \begin{cases} 2\alpha(r_2 - r_1) + r_1 & \text{if } \alpha \leq 0.5 \\ 2(1-\alpha)(r_3 - r_4) + r_4 & \text{if } \alpha > 0.5 \end{cases} \tag{27}$$

Subject to:

$$r_2 = \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k^1 Y_{jk} + \sum_j \left[a_j \sum_k D_k^1 Y_{jk} + \frac{h_j \sum_k D_k^1 Y_{jk}}{2} \right] \tag{28}$$

$$r_2 = \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k^2 Y_{jk} + \sum_j \left[a_j \sum_k D_k^2 Y_{jk} + \frac{h_j \sum_k D_k^2 Y_{jk}}{2} \right] \tag{29}$$

$$r_3 = \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k^3 Y_{jk} + \sum_j \left[a_j \sum_k D_k^3 Y_{jk} + \frac{h_j \sum_k D_k^3 Y_{jk}}{2} \right] \tag{30}$$

$$r_4 = \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k^4 Y_{jk} + \sum_j \left[a_j \sum_k D_k^4 Y_{jk} + \frac{h_j \sum D_k^4 Y_{jk}}{2} \right] \tag{31}$$

Thus, the fuzzy location–inventory model will be of general form,

$$\min \overline{f} = \begin{cases} 2\alpha(r_2 - r_1) + r_1 & \text{if } \alpha \leq 0.5 \\ 2(1-\alpha)(r_3 - r_4) + r_4 & \text{if } \alpha > 0.5 \end{cases} \tag{32}$$

Subject to:

$$r_1 = \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k^1 Y_{jk} + \sum_j \left[a_j \sum_k D_k^1 Y_{jk} + \frac{h_j \sum_k D_k^1 Y_{jk}}{2} \right] \tag{33}$$

$$r_2 = \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k^2 Y_{jk} + \sum_j \left[a_j \sum_k D_k^2 Y_{jk} + \frac{h_j \sum_k D_k^2 Y_{jk}}{2} \right] \tag{34}$$

$$r_3 = \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k^3 Y_{jk} + \sum_j \left[a_j \sum_k D_k^3 Y_{jk} + \frac{h_j \sum_k D_k^3 Y_{jk}}{2} \right] \tag{35}$$

$$r_4 = \sum_j F_j X_j + \sum_j \sum_k c_{jk} D_k^4 Y_{jk} + \sum_j \left[a_j \sum_k D_k^4 Y_{jk} + \frac{h_j \sum D_k^4 Y_{jk}}{2} \right] \tag{36}$$

$$\sum_j Y_{jk} = 1 \quad \forall k \in I \tag{37}$$

$$Y_{jk} - X_j \leq 0 \quad \forall j, k \in I \tag{38}$$

$$Y_{jk} \in \{0, 1\} \quad \forall j, k \in I \tag{39}$$

$$X_j \in \{0, 1\} \quad \forall j \in I \tag{40}$$

4 Solution method and computational results

4.1 Solution method

In this section, a genetic algorithm (GA) is presented to solve the model. In the following, the elements of GA are described.

4.1.1 Chromosome

We use a vector for chromosomes, in which a gene with value 0, the node and a gene with value other than 0 means that DC covers the mentioned node. Consider the following example.

Assume we have seven shopping centers. For example, consider the following:

Chromosome	3	6	0	3	6	0	3

The above chromosome illustrates that distribution centers are located in nodes 3 and 6. Retailers 1, 4, and 7 are supplied by distribution center 3, and retailers 2 and 5 are supplied by distribution center 6.

4.1.2 Initial population

To start, a *popsize* by *n* matrix is created in which each element is initially 1. Each chromosome in the population is produced as follows:

Table 1 Analysis of sensitivity to cost of constructing parameter

Size of problem	Repeat count	Cost of constructing	Mean objective	Best solution
7	5	17, 27, 13, 15, 18, 29, 27	18,146.108	18,044.490
7	5	19, 32, 18, 20, 25, 33, 38	19,020.746	18,065.490
7	5	21, 34, 20, 22, 30, 35, 39	19,176.882	18,078.490
7	5	29, 36, 39, 41, 33, 39, 25	19,606.243	18,184.420

Assume we have seven facilities of which centers 3 and 4 are randomly designated to serve as DC. In what follows, we set entries 3 and 4 to be 0. Thus,

1	1	0	0	1	1	1

Now starting from the beginning of the chromosome, we randomly allocate the 1 s in the chromosome to one of the selected distribution centers.

4	3	0	0	4	4	3

The potential problem in this step is that there is a distribution center which does not serve any retailer, which arises from the random nature of distribution center allocation. As an example, consider:

4	3	0	0	4	0	3

A modifier procedure of chromosome is written to solve this potential problem which is explained later.

4.1.3 Crossover operator

The crossover operator being used in this model is called single-point crossover. The problems of this step would be that the produced children may be infeasible. As an example, consider the following two chromosomes as selected parents:

7	7	0	3	3	7	0

3	3	0	0	3	4	3

If the cutting point is point 5, the produced children are as follows:

7	7	0	3	3	4	3

3	3	0	0	3	7	0

As, it is seems the produced children are not feasible; therefore to correct them, we apply the modifier function. Suppose the modifier procedure is applied to above chromosomes and the modified chromosome are as follows:

3	3	0	3	3	3	3

3	3	0	3	3	7	0

4.1.4 Modifier procedure

The modifier procedure is applied to solve two problems in generated chromosome.

1. When a retailer is allocated to a DC which itself is a retailer. As an example:

3	3	0	3	3	**4**	3

In which retailer 6 is allocated to retailer 4. The modifier procedure is to allocate a retailer to a valid DC randomly.

3	3	0	3	3	**3**	3

2. Maybe a distribution center does not provide any retailer. As an example:

3	3	0	**0**	3	7	0

Table 2 Analysis of sensitivity to cost of transportation parameter from distribution center to retailer

Size of problem	Repeat count	Cost of constructing	Mean objective	Best solution
7	5	0.56, 0.67, 0.43, 0.65, 0.74, 0.59, 0.47	18,146.108	18,044.490
7	5	0.66, 0.77, 0.55, 0.74, 0.81, 0.62, 0.53	20,924.471	18,080.872
7	5	0.71, 0.81, 0.63, 0.78, 0.88, 0.71, 0.66	21,212.321	18,095.548
7	5	0.88, 0.90, 0.74, 0.82, 0.95, 0.86, 0.81	22,254.548	18,113.378

Table 3 Analysis of sensitivity to cost of storing parameter

Size of problem	Repeat count	Cost of storing	Mean objective	Best solution
7	5	17, 27, 13, 15, 18, 29, 27	18,146.108	18,044.490
7	5	19, 32, 18, 20, 25, 33, 38	24,609.973	23,212.778
7	5	21, 34, 20, 22, 30, 35, 39	30,875.978	30,561.540
7	5	29, 36, 39, 41, 33, 39, 25	35,566.934	35,399.778

In this example, center 4 is a DC, but supplies no retailer. To correct this error, we remove distribution center 4 and allocate it as a retailer to another DC randomly.

3	3	0	**3**	3	7	0

4.1.5 Mutation operator

This operator is implemented by selecting two retailers and substituting their corresponding distribution centers. This is illustrated in the following example:

Initial chromosome

3	3	0	0	**3**	**4**	3

New chromosome

3	3	0	0	**4**	**3**	3

4.2 A numerical example

Assume a network with n=7 nodes. The construction costs of DC at each node are as:

Centers (j)	1	2	3	4	5	6	7
F_j	17	27	13	15	18	29	27

Transportation costs C_{jk} for any pair of nodes are as follows:

$$c_{jk} = \begin{bmatrix} 0.0 & 0.84 & 0.34 & 0.65 & 0.44 & 0.65 & 0.54 \\ 0.46 & 0.0 & 0.45 & 0.67 & 0.43 & 0.67 & 0.23 \\ 0.53 & 0.34 & 0.0 & 0.46 & 0.87 & 0.34 & 0.75 \\ 0.56 & 0.22 & 0.67 & 0.0 & 0.63 & 0.46 & 0.85 \\ 0.43 & 0.24 & 0.23 & 0.25 & 0.0 & 0.63 & 0.34 \\ 0.53 & 0.27 & 0.54 & 0.24 & 0.67 & 0.0 & 0.85 \\ 0.89 & 0.78 & 0.89 & 0.89 & 0.87 & 0.21 & 0.0 \end{bmatrix}$$

Now, considering the demand of customers to be trapezoidal fuzzy numbers:

$$\widetilde{D}_1 = [50 \quad 55 \quad 60 \quad 65], \widetilde{D}_2 = [94 \quad 98 \quad 102 \quad 105], \widetilde{D}_3 = [150 \quad 155 \quad 160 \quad 165]$$
$$\widetilde{D}_4 = [122 \quad 126 \quad 128 \quad 130], \widetilde{D}_5 = [64 \quad 65 \quad 68 \quad 70], \widetilde{D}_6 = [55 \quad 57 \quad 60 \quad 66]$$
$$\widetilde{D}_7 = [74 \quad 75 \quad 78 \quad 80]$$

Table 4 Analysis of sensitivity to the demand parameter

Size of problem	Repeat count	Demand	Mean objective	Best solution
7	5	[94 98 102 105], [50 55 60 65], [122 126 128 130] [64 65 68 70], [74, 75, 78, 80], [150 155 160 165] [55 57 60 66]	18,146.108	18,044.490
7	5	[110 115 120 125], [62 68 72 76], [84 85 88 90] [132 136 138 140], [160 165 170 175], [65 67 70 76] [74 75 78 80]	20,773.418	19,882.288
7	5	[120 125 130 135], [72 78 82 86], [142 146 148 150] [170 175 180 185], [75 77 80 86], [84 85 88 90] [94 95 98 100]	23,623.168	22,430.088

Table 5 Sensitivity analysis with respect to parameter α

Size of problem	Parameter α	Objective function
7	0.1	12,654.112
7	0.2	12,793.524
7	0.3	12,923.936
7	0.4	13,072.348
7	0.5	13,211.760
7	0.6	14,150.874
7	0.7	14,290.368
7	0.8	17,690.926
7	0.9	17,859.888
7	1	18,022.850

Transportation costs from the main supplier to the distribution centers are:

Centers (j)	1	2	3	4	5	6	7
a_j	0.56	0.67	0.43	0.65	0.98	0.59	0.47

Holding costs for each unit of products for 1 year are indicated in the following table:

Centers (j)	1	2	3	4	5	6	7
h_j	120	180	175	100	290	140	125

Assume α=0.6 and best solution equals 18,044.49

4.3 Sensitivity analysis

4.3.1 Sensitivity analysis to cost of construction

Should the cost of transportation from DCs to retailers increase, the objective function increases as well. Furthermore, increases in the cost of the establishment of distribution centers, transportation cost from the main supplier to the distribution centers, and inventory costs would result in increases in the objective function, with the objective function being most sensitive to increases in inventory cost. The following table shows the analysis of sensitivity to the parameters of costs of establishment, with all input parameters being constant while the establishment costs vary (Table 1).

The following tables show the analysis of sensitivity to other parameters (Tables 2 and 3).

4.3.2 Analysis of sensitivity to demand parameter

By increasing the demand, the objective function increases rapidly. The following table (Table 4) shows the analysis of sensitivity to the demand parameter, with all parameters being constant but the demand parameter.

4.3.3 Sensitivity of solutions to α

It is concluded from the location–inventory model that the value of the objective function depends on parameter α as well. Here, we consider different values for α (Table 5). The objective function is increasing in α which is in conformance with the following remark (2).

4.4 Performance analysis of proposed GA

In this step, we evaluate the performance of proposed GA by comparing the results of optimal method (explicit numeration) and the results of GA. To perform the comparison, we will solve the problem using both methods under same conditions. The running time and the solution from the two methods are recorded, and percentage of discrepancy

Table 6 A comparison of the results obtained from GA against optimal method

Size of problem	GA			Optimal method			Error %
	Run-time (s)	Number of distribution centers	Objective	Run-time (s)	Number of distribution centers	Objective	
5	1	2	14,150.874	1	2	14,150.874	0
6	1	3	14,523.606	2.76	3	14,523.606	0
7	1	3	18,044.490	47.05	3	18,044.490	0
8	1	4	17,523.204	1,569.625	4	17,523.204	0
9	1	4	21,299.688	17,857.93	4	21,299.688	0
10	1	5	24,686.306	56,016.02	5	24,698.100	0.04
11	2	5	28,250.732	101,700.42	5	28,250.732	0
12	2	5	31,248.202	142,128.38	5	31,248.048	0
13	2	6	34,218.202	188,028.22	6	34,218.202	0.02
14	2	6	38,039.322	271,404.35	6	38,039.322	0
15	2	6	40,000.176	328,680.25	6	40,000.176	0

Table 7 Performance analysis of proposed GA for medium and large size problem

Size of problem	Repeat count	Run-time (s)	Number of distribution centers	Mean objective	Best solution
20	5	6	8	52,843.948	49,697.418
30	5	7	11	89,926.599	85,421.242
35	5	8	12	106,607.222	95,676.912
40	5	9	13	123,636.838	113,150.576
45	5	10	15	147,873.335	137,997.488
50	5	12	17	163,708.592	155,735.482
55	5	13	18	187,661.364	177,075.588
60	5	14	22	207,862.834	192,598.090
65	5	15	21	227,905.698	210,636.078
70	5	16	20	246,867.962	239,245.020
75	5	17	24	270,140.900	255,263.500
80	5	18	25	287,243.114	277,329.356
90	5	20	29	323,526.537	303,989.110
100	5	25	33	355,598.934	341,076.418

between the two solutions (error) is calculated using the following formula:

$$\text{Error} = \frac{\text{solution from the optimal method} - \text{solution from the genetic method}}{\text{solution from the optimal method}} \times 100\%$$

As indicated in Table 6, error equals 0 in most cases and is negligible in two cases. Furthermore, it takes about 4 days to obtain the optimal solution for n=15. Thus increasing the number of centers; the calculation time of the optimal solution rises and it is not possible to solve the problems with larger sizes.

4.4.1 Performance analysis of GA for medium and large size

Here, we evaluate the performance of proposed GA in large problems (Table 7). We solved many problems with size 20 to 100 nodes and results are tabulated in Table 6. The convergence graph is presented in Fig. 1.

5 Conclusion

Considering the location decisions and inventory decisions simultaneously play a main role in optimizing the cost of supply chain. Since the supply chain environment is uncertain in real world, therefore, developing the location–inventory model in fuzzy environment is valuable. This is the first attempt on developing the location–inventory model in fuzzy environment. We developed the model and a genetic algorithm to solve it. The performance of proposed genetic algorithm is reasonable based on numerical results. Our model determines the number of distribution centers, location of distribution centers, allocation of retailers to DCs, and inventory levels in DCs. This research can be extended by considering multi products with interdependent relationships and stochastic demand for products. Considering

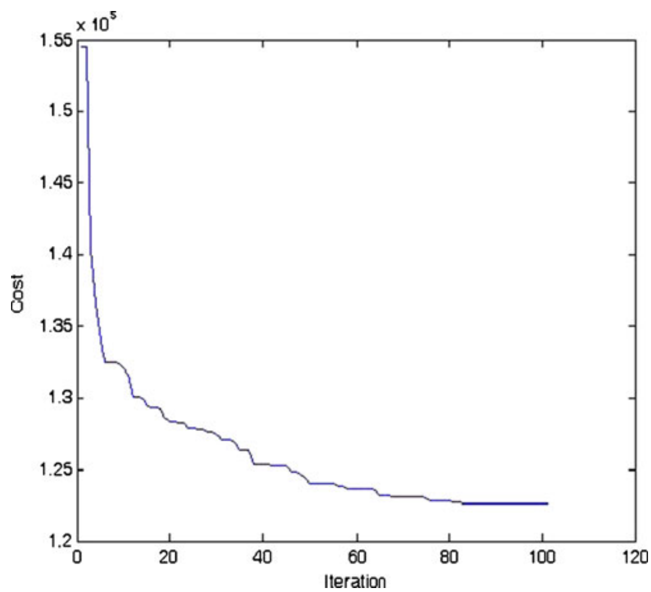

Fig. 1 GA convergence graph

customers behavior on choosing DCs and amount of demand is also another valuable extent of this research.

References

1. Brimberg J, Hansen P, Mladenovic N (2000) Improvements and comparison of heuristics for solving the multisource Weber problem. Oper Res 48:252–263
2. Hansen P, Mladenovic N, Taillard E (1998) Heuristic solution of the multisource Weber problem as a p-median problem. Oper Res 22:55–62
3. Beasley JE (1993) Lagrangean heuristics for location problems. Oper Res 65:383–399
4. Boffey B, Yates D, Galvao RDG (2003) An algorithm to locate perinatal facility in the municipality of Rio de Janeiro. Oper Res 54:21–31
5. Chardaire P, Lutton J, Sutter A (1999) Upper and lower bounds for the two-level simple plant location problem. Ann Oper Res 86:117–140
6. Harkness J, Revell C (2003) Facility location with increasing production costs. Eur J Oper Res 145:1–13
7. Bhattacharya U, Rao JR, Tiwari RN (1992) Fuzzy multi-criteria facility location problem. Fuzzy Sets Syst 51:277–287
8. Bhattacharya U, Rao JR, Tiwari RN (1993) Bi-criteria multi facility location problem in fuzzy environment. Fuzzy Sets Syst 56:145–163
9. Canos MJ, Ivorra C, Liern V (1999) An exact algorithm for fuzzy p-median problem. Eur J Oper Res 116:80–86
10. Chen CB, Wei CC (1998) An efficient fuzzy MADM method for selecting facility location. J Eng Valuat Cost Anal 2:19–32
11. Darzentas J (1987) A discrete location model with fuzzy accessibility measures. Fuzzy Sets Syst 23:149–154
12. Rao JR, Sarawait K (1988) Facility location problem on network under multiple criteria- fuzzy set theoretic approach. Int J Syst Sci 12:2555–2559
13. Tzeng GH, Chen YW (1999) Optimal location of airport fire stations: a fuzzy multi-objective programming and revised genetic algorithm approach. Transport Plann Tech 23:229–241
14. Kuo RJ, Chi SC, Kao SS (1999) A decision support system for locating convenience store through fuzzy AHP. Comput Ind Eng 37:323–326
15. Chen SM (2001) Fuzzy group decision-making for evaluating the rate of aggregative risk in software development. Fuzzy Sets Syst 118:75–88
16. Chen CT (2001) A fuzzy approach to select the location of the distribution center. Fuzzy Sets Syst 118:65–73
17. Chu TC (2002) Selecting plant location via a fuzzy TOPSIS approach. Int J Adv Manuf Technol 20:859–864
18. Ertugrul I, Karakasoglu N (2008) Comparison of fuzzy AHP and fuzzy TOPSIS methods for facility location selection. Int J Adv Manuf Technol 39:783–795
19. Liu B, Liu YK (2002) Expected value of fuzzy variable and expected value models. IEEE Trans Fuzzy Syst 10:445–450
20. Liu B (2009) Theory and practice of uncertain programming, 3rd edn. UTLAB
21. Zhou J, Liu B (2007) Modeling capacitated location-allocation problem with fuzzy demands. Comput Ind Eng 53:454–468