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ORIGINAL ARTICLE

Classification and modeling for in-plant milk-run distribution systems

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Abstract Material handling is one of the most important issues that should be taken into account for eliminating waste and reducing the cost. It is one of the seven wastes defined in the concept of lean manufacturing. In this study, for a lean material handling system under the lean manufacturing conditions such as pull-based and repetitive manufacturing, a system consisting of periodically moving vehicles in certain routes is taken into consideration. This system is also called milk-run distribution system. Application of milk-run distribution systems in plants standardizes the material handling system and eliminates the waste. Although there are huge numbers of studies related with inbound and outbound logistics, there are few studies especially related with milk-run applications in the manufacturing area. Within this study, based on the observations in real manufacturing environment and limited literature, the milk-run distribution problem in the plants is categorized and explained. For one of the main categories, modeling is performed. The objective of the developed models is to minimize the number of vehicles and the distance traversed. A numerical example inspired by real applications is presented for showing the applicability of the developed models.

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1 Introduction

In the competitive environment of the manufacturing, the elimination of activities that do not add value to the products gets much more important than it was before. One of the most important wastes is the material handling waste within the seven wastes of the lean manufacturing [1]. If a good material handling system design is accomplished, it is claimed that the cost will decrease between 10% and 30% [2]. In a typical industry, material handling composes the 25% of the workers, 55% of the factory area, and 87% of the production time [3].

Material handling is regarded as an integral part of the facility layout [4]. Each affects the other and cannot be regarded individually. The aims of the material handling systems are as follows [5]:

- To increase the material flow efficiency by providing the required materials where and when needed,
- To decrease the material handling costs,
- To increase the usage rate of the material handling vehicles,
- To improve the safety and working conditions,
- To ease manufacturing process, and
- To increase productivity.

One of the most important subjects to design a good material handling system is to select the right material handling equipment. At this point, there are mainly three approaches that can be used [6]. These are deterministic approach, probabilistic approach, and knowledge-based approach. Within probabilistic approach, simulation and queuing theory-based approaches are also regarded.

The logistics applications in lean manufacturing environments can be called as lean logistics. Lean logistics is defined as the logistics part of the lean manufacturing and consists of three groups as the in-bound logistics (from supplier to factory), in-plant logistics (logistics in the factory), and the outbound logistics (from factory to the customer) [7]. Within this study, the scope is in-plant lean logistics.

There can be a lot of applications for the elimination of waste in the lean logistics. Such application areas can be the design of the storages, material handling equipment selection, the design of flows, etc. The one which is focused in this study is the design of the material flow from storages to cells, from cells to cells, and from cells to assembly areas and storages. For this aim, under the lean manufacturing conditions where pull system is applied and there is smooth manufacturing, a transportation vehicle called milk-run train periodically moving on a predetermined route is regarded.

Periodically moving automatic or manual systems provide a just-in-time working environment for the manufacturing systems [8]. The importance of periodically moving handling vehicles on a predefined path in lean manufacturing environments is also stressed in different studies [7]. Besides the milk-run applications for the inbound and outbound logistics, there are the milk-run applications for the plant logistics. A milk-run train shown as in Fig. 1 is used in such applications.

By using the milk-run train in the lean manufacturing areas, as it is seen in the studies of Akillioglu et al. and Domingo et al. [9, 10], material handling system is standardized and becomes easy to operate and control in the real-world applications.

For the design of a milk-run train system aiming to minimize the work in process (WIP) and transportation costs, proper routes, and time periods for each route are investigated. The milk-run problem in the plants is categorized and each presented category is explained. For the category "determined time assignment problem," models are developed. Presented models are applied and compared with a numerical example.

The main contribution of this study to the existing literature is that it presents a classification scheme for in-plant milk-run distribution systems for the first time and provides models for a specific new category presented in the classification scheme.

The remainder of the article is organized as follows. Section 2 includes literature review. In Section 3, in-plant milk-run problem categorization is presented. Models for "Determined Time Periods Assignment Problem" are provided in Section 4. A numerical example and applications of the models are presented in Section 5, and finally, conclusions are included in Section 6 with the reference following.

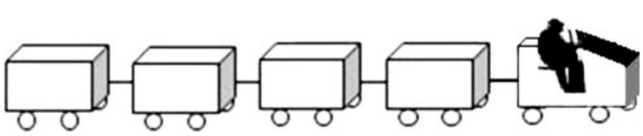


Fig. 1 A milk-run train

2 Literature review

The design of a pull-based material handling system in the plants is related with vehicle routing problems (VRP) and more specifically about milk-run distributions. In some ways, VRP in plants seems similar to the classical VRP outside the plant, since there are both deliveries and pickups and the vehicle is capacitated, this problem looks like capacitated vehicle routing problem with pickups and deliveries. On the other hand, since it deals with determining the quantities to be delivered, it also seems like inventory routing problem but as Vaidyanathan et al. [11] state that it differs from the classical VRP problems by having some special properties such as the amounts required by the stock usage points are the functions of the routes that serve them. That is, the quantities needed by the stock points changes according to the periodic cycle of the moving vehicle on a predefined route. The other differences can be counted as the relationship between the cells, inventory depending on the vehicle cycle time rather than a fixed quantity, physical constraints of the stock areas, multiple product categories, etc. Due to the mentioned differences, it is difficult to model the milk-run problem in manufacturing environment with the existing methods used for inbound and outbound logistics.

One of the latest and extensive literature reviews about VRPs is made by Eksioglu et al. [12]. Totally 1,494 bibliographical entities were reviewed and classified mainly according to the type of study, scenario characteristics, problem physical characteristics, information characteristics, and data characteristics. Regarding their study, it is concluded that there is need for studies about VRPs in manufacturing environments. Since the scope of this article is plant logistics, the literature about the logistics including the vehicle routing problems outside the plant is omitted. There are few studies which are directly related with the design of the milk-run distribution in lean manufacturing environment. Subjects studied under material handling system are mostly simultaneously design of material handling system and facility layout, minimizing the number of material handling equipments, and selection of the most proper material handling equipment.

Since facility layout and material handling system design subjects affect each other, they are regarded together in most of the studies. A framework of research in the areas of facility layout and discrete material handling system design are presented by Rajagopalan and Heragu [13]. By aiming to observe the trend in the literature in the area of manufacturing, they tried to combine the fields of facility layout design and material flow network design. A genetic algorithm was developed by Hu et al. [14] for the inter-cell layout and material handling system design.

Finding the optimum number of material handling vehicles is another subject studied under material handling system design. A heuristics was developed by Hwang [15] to find the minimum number of transporters for a given fixed facility layout and predetermined material flow paths. Sayarshad [16] also used bees algorithm to determine the optimal number of material handling equipments used on the manufacturing centers. A mathematical programming approach was developed by Beamon and Deshpande [17] to specify the unit load and fleet sizes, based on material transportation requirements and system characteristics.

Decision making methods are generally used to select the most proper material handling equipment. Chakraborty and Banik [18] used analytic hierarchy process for this aim. Onut et al. [19] used fuzziness in their selection models. Kulak [20] developed a model including axiomatic design and fuzziness. Mirhosseyni and Webb [21] presented a two-stage method consisting of fuzzy expert system and genetic algorithm to select the most proper material handling equipment.

The vehicle routing problem in just-in-time systems was studied by Vaidyanathan et al. [11]. They constructed a nonlinear programming model with the aim of minimizing the total trip time of the vehicles. Since the model they constructed was nonlinear, they proposed a heuristics and a lower bound for the comparison of the heuristic.

A mixed integer linear programming model was developed and applied in a real manufacturing environment for designing a milk-run distribution system [9]. A simulation of the model was also made. The improvements in some of the lean metrics such as WIP and transportation costs are shown. According to the application results, the percentage of the improvements in transportation and WIP costs is 66%.

Another milk-run distribution system in a real lean manufacturing environment was made by Domingo et al. [10]. According to the empirical results of their application, dock-to-dock time and cycle times were decreased. Also unnecessary inventories, excessive transportation, and idle times were reduced.

A simulation of material delivery system using a logistics train for the manufacturing lines of an electronic company was made by Costa et al. [22]. The experimental results obtained from the model showed that the periodically moving train had financial advantages.

As a consequence, in literature, there are a lot of publications about VRPs but few of these publications include the design of plant logistics in lean manufacturing environments. Although there are a limited number of publications presenting the advantages of the use of milk-run trains in lean manufacturing environment, there is no classification scheme presented for milk-run distribution problems in lean manufacturing environment. Depending on the authors' observations in different fields of automotive industry (bus plant, motor injector plant, automotive safety systems plant, and automotive, electronic, and electromagnetic parts plant) where lean manufacturing is applied, a detailed classification scheme for milk-run distribution problem is presented. For one main category under the classification scheme, two models are developed and tested on a numerical example. By presenting a classification scheme and models for a specific category, this study differs from the other studies. It also shows that there is more to do about milk-run distribution systems in lean manufacturing environment.

3 In-plant milk-run problem categorization

Similar to the classical milk-run systems outside the plant, also in the milk-run systems inside the plant, the vehicle starts from the storage and visits the assigned cells or stations on its route and then returns again to the starting point periodically. For the application of the milk-run systems in the plants, the manufacturing system must be lean. That is, it must be smooth and repetitive. The milk-run distribution problem in a plant where lean manufacturing is applied can mainly be defined as determining the routes and time periods of routes under the constraints such as space requirements, number of vehicles, and capacity of vehicles, etc. The aim is to minimize the cost consisting of the WIP cost, transportation cost of the vehicles, and the fixed cost of the vehicles. There is a trade-off between WIP cost and transportation costs. If the vehicle makes frequent tours, the WIP cost decreases but on the other side, the transportation cost increases, and oppositely if the vehicle makes rare tours, the transportation costs decrease but the WIP costs increase.

The main problem as stated before is to determine the routes and their time periods. These two critical points make up the classification scheme. In this study, assignment methodology is used for the easiness to solve the problem. So the categorization mainly depends on the assignment type. There are three main categories. The first one is called "general assignment problem," the second one is called "dedicated assignment problem," and the last one is "determined time periods assignment problem." Each main problem category consists of subcategories related with the assignable route numbers to the vehicles and time period types of the vehicles. The deterministic or stochastic types of the parameters are not regarded in the classification scheme. Although deterministic parameters are used in the models, stochastic parameters or any other type of parameters can also be used under the same classification structure.

The first category is the “general assignment problem” which the routes and time periods are not known. This category is divided into two subcategories. These are one routed and multiple routed vehicles. These categories are also divided into two categories according to the time period type of the vehicles such as differently timed routes and equally timed routes. “Dedicated assignment problem” is the second category where the groups of cells or stock points are determined before, that is, the routes are known but the problem is to determine the time periods of the vehicles on the related routes. This category has two sub-categories: these are differently and equally timed routes. The last category is “determined time periods assignment problem”; in this category, time periods are known but the routes are not known. This category is divided as one routed and multiple routed vehicles.

In real manufacturing environment, there are one and multiple routed milk-run trains. For one routed milk-run trains, the vehicle has only one route and travels on that route in specific time periods. This is good from the side of lean management. But as it is observed, for one routed milk-run trains, the idle time of the vehicle can be very high. So as to minimize the idle time and the number of vehicles, multiple routed milk-run trains are used. Multiple routed milk-run trains have more than one route, and in specific time periods, they travel on the related routes. They are not as lean as one routed vehicles, but they provide effective usage of milk-run trains by minimizing idle time and the number of vehicles.

Another point observed in lean manufacturing environments is the time periods of the milk-run trains. The time periods of the vehicles can be equal or different. From the side of leanness, the milk-run trains having equal time periods is better, but in some cases, is not as economical as the differently timed vehicles. Depending on the conditions of the manufacturing environment, either of these cases can be proper.

Determining the probable routes is common in all of the categories. A lot of combinatorial routes can be defined in a physical layout, but since defining all the routes makes the problem harder to solve, some points should be taken into consideration for reducing the number of routes at this stage. These are:

- The routes should be compatible with the physical conditions in the layout and the product flow.
- Routes should not pass by a cell more than twice.
- In case there are related cells, a route passing by such a cell, should pass by the other related cell.

The general scheme of the milk-run problems in plants when assignment methodology is used is shown in Fig. 2. As mentioned before, this classification scheme is obtained as a result of observations in different real-life applications. For each of the categories, modeling can be made. The model developed by Akillioglu et al. [9] can be regarded in the general assignment problem including one routed vehicle with differently timed periods. In this study, all of the models for each of the categories are not given. The scope is restricted and different from the presented models in the literature, modeling for “determined time periods assignment problem” which has a wide usage in real-life applications is made both for one routed and multiple routed vehicles. With the models presented, a systematic approach will be provided for real-life applications. Developed models are tested on a numerical example.

4 Models for “determined time periods assignment problem”

As stated before, in this category, the time periods of the routes are known but the routes and the numbers of vehicles required are not known. According to the manufacturing requirements, in some facilities, the time periods of the vehicles can be a specific value or some probable values. In such a case, the models presented will be useful for finding the routes and the number of vehicles in different specific time period cases. While developing the model, it is benefited from the study of Akillioglu et al. [9]. In the presented mixed integer linear programming models, assumptions for one routed and multiple routed vehicles are same except the number of routes that can be assigned to the vehicle. Since the time period of the milk-run trains are determined, different from the other categories, the WIP cost is not regarded in the objective function. The objective here is to minimize the cost occurring from the number of vehicles and the distance traversed.

The objectives are same for one routed and multiple routed vehicles.

Objective Minimization of the fixed and variable cost of the vehicle.

Assumptions

- Model is designed according to a lean manufacturing environment where there is repetitive and smooth manufacturing.
- Manufacturing system is pull-based.
- The layout is not changed. The material handling system design is made according to the current layout.
- Each stock point corresponds to a material; the demand/supply rate of each stock point is constant.
- The material handling within the cells is not taken into account. The material handling in the input and output stock areas of the cells are taken into consideration.

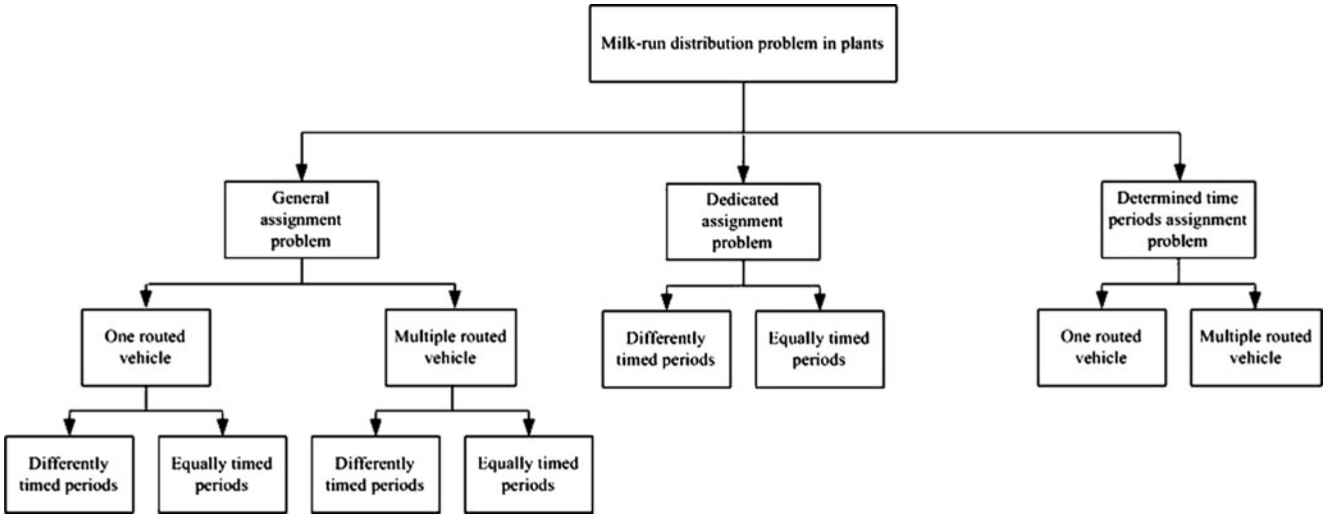


Fig. 2 Classification of the milk-run distribution problem in plants under the assignment approach

- The speed of the vehicle is constant.
- No traffic problems occur during the movements of the vehicles.
- The cost of each tour of a vehicle is constant.
- No breakdown of the vehicles occurs during the tours.
- All the vehicles in the model are identical.

4.1 Determined time periods assignment problem for one routed vehicles

As explained before, in this subcategory, a vehicle has only one route. The objective is to minimize the number of vehicles and the distance traversed within specific time periods under some constraints such as the capacity of the vehicle, determined time periods, maximum number of vehicles, related stations, and so on.

Objective Minimization of the fixed and variable costs of the vehicle.

$$\min z = \sum_r (\text{route}_\text{r} \times \text{vehicle} \cos t) + \sum_r (\text{route}_\text{r} \times \text{C}_\text{r} \times (24 \times 60)/t_{\text{fixed}}) \quad (1)$$

In the objective, there are two parts. In the first part, the capital recovery cost of one vehicle is multiplied by the number of vehicles used. In the second part, total traveling cost of the vehicles in a day are obtained by multiplying traveling cost of one tour by the number of total tours made in a day.

The subscripts, decision variables, sets, parameters, and constraints are shown below.

Subscripts

i, j: Stock points (Each stock point corresponds to a material)
a, b: Stock areas (Stock points in a specific place make up a stock area)
r: Route or vehicle
t: Time period
s_r, ss_r: The assignment order of stock points that can be allocated to route “r”

Decision variables

route_r: Route “r” is chosen or not chosen (1 or 0)
X_{ir}: Stock point “i” is assigned or not assigned to the route “r” (1 or 0)
D_{ir}: One cycle demand quantity of the stock point “i” that is assigned to the route “r”
L_{ir}: One cycle loading-unloading time of the stock point “i” that is assigned to the route “r”
Y_{ar}: The 0 or 1 binary variable determining the route of the stock area “a”
vehcycle_r: The total cycle time of the route/vehicle “r” (Including fixed trip time of the vehicle and total loading–unloading time)

Sets

A: The set of stock areas in the material storage and assembly station (a,b)
B: The set of stock points related with each other (pulling or pushing) (i,j)
C: The stock areas in the same cell (a,b)
N_r: The set of stock points that can be assigned to the route “r”
N_a: The set of stock points in the stock area “a”

Parameters

VT_r: The fixed trip time of the route/vehicle “r” (only the trip time not including loading-unloading times)

seq_{sri}: The 0 or 1 integer value showing the assignment orders "s" of the possible assignable stock points "i" to the route "r"
C_r: The transportation cost of one cycle of the route "r"
$sign_i$: The "−1" or "1" values depending on the stock point "i"s type ("supply" or "demand")
vol(a): The volume of the stock area "a"
vehiclecost: The daily fixed cost (capital recovery) of the vehicle/route "r".
maxvehicle: Maximum number of vehicles that can be used in the system
t_{fixed}: The determined time period
d_i: The output and input ratio (quantity/time) of the stock point "i".
l_i: The loading–unloading time of the stock point "i".
cap: The capacity of the vehicle
v_i: The volume of the stock point "i"
M: A big number

Constraints

$$\sum_r X_{ir} = 1 \quad \forall i \tag{2}$$

$$X_{ir} = X_{jr} \quad \forall (i,j) \in B, \forall r \tag{3}$$

$$\sum_{i \notin Nr} X_{ir} = 0 \quad \forall r \tag{4}$$

$$\sum_r Y_{ar} = 1 \quad \forall a,\ \text{a} \notin \text{A} \tag{5}$$

$$Y_{ar} = Y_{br} \quad \forall (a,b) \in C, \forall_r \tag{6}$$

$$M \cdot X_{ir} \geq D_{ir} \quad \forall i, \forall r \tag{7}$$

$$M \cdot X_{ir} \geq L_{ir} \quad \forall i, \forall r \tag{8}$$

$$L_{ir} = D_{ir} \cdot l_i \quad \forall i, \forall r \tag{9}$$

$$D_{ir} = t_{\text{fixed}} \cdot d_i \cdot X_{ir} \quad \forall i, \forall r \tag{10}$$

$$M \cdot Y_{ar} \geq \sum_i X_{ir}\ i \in N_a, \forall r, \forall a, a \notin A \tag{11}$$

$$\text{vehcycle}_r = \sum_j L_{jr} + \text{route}_r \cdot VT_r\, \forall r \tag{12}$$

$$\text{vehcycle}_r \leq t_{\text{fixed}} \quad \forall r \tag{13}$$

$$\sum_r \text{route}_r \leq \text{max vehicle} \tag{14}$$

$$M \cdot \text{route}_r \geq \sum_i X_{ir} \quad \forall r \tag{15}$$

$$M \cdot \text{route}_r \geq D_{ir} \quad \forall i, \forall r \tag{16}$$

$$M \cdot \text{route}_r \geq L_{ir} \quad \forall i, \forall r \tag{17}$$

$$\sum_{ss_r=1}^{s_r} \sum_i D_{ir} \cdot v_i \cdot \text{seq}_{s_r i} \cdot \text{sign}_i \leq \text{cap}\, \forall r, \forall s_r \tag{18}$$

$$\begin{matrix} X_{ir}, Y_{ar}, \text{route}_r \in \{0,1\} \\ \text{others} \geq 0 \end{matrix} \ \forall i, \forall a, \forall r, \forall s_r \tag{19}$$

Description of the model constraints are as follows:

(2) Each stock point "i" is assigned to one route.
(3) The related stock points are assigned to same "r" route.
(4) A stock point "i" is not assigned to "r" route, if it is not on the way of the route "r".
(5) Each stock area except the ones in material storage and assembly station is assigned to one "r" route.
(6) The related stock areas are assigned to same "r" route.
(7) The demand of stock point "i" at "r" route becomes zero, if it is not assigned to that route.
(8) The loading-unloading time of stock point "i" at "r" route becomes zero, if it is not assigned to that route.
(9) The total loading-unloading time of a stock point "i" at "r" route is determined.
(10) The total demand of a stock point "i" at "r" route is determined.
(11) All the stock points in the stock areas except the ones in the material storage and assembly lines are assigned to the same "r" route.
(12) The cycle time of vehicle (route) "r" is determined.
(13) The cycle time of a route is restricted with a fixed determined time.
(14) The total number of selected routes (vehicles) is restricted with an upper limit.
(15) A stock point "i" is not assigned to an unselected "r" route.
(16) The demand of a stock point "i" becomes zero at an unselected "r" route.
(17) The loading–unloading time of a stock point "i" becomes zero at an unselected "r" route.

(18) The capacity of the vehicle is controlled at each assignment of a stock point to a route.
(19) Decision variables are defined as either 0–1 binary integer variable or real numbers.

The capacity of the vehicle is controlled at each assignment of a material. Because of this, the materials (stock points) are classified as demand or supply stock points. The sign of "supply stock points" are "1." This means that the vehicle is loaded. On the other hand, the sign of "demand stock points" are "−1." This means that the vehicle is unloaded.

4.2 Determined time periods assignment problem for multiple routed vehicles

Different from the one routed vehicle in determined time periods, in this category, a vehicle can have more than one route. Although it is easier to manage the milk-run trains with one routed vehicles, it is possible to utilize the milk-run trains more efficiently with the multiple routed vehicles group than the one routed vehicles.

Objective Minimization of the fixed and variable costs of the vehicle.

$$\min z = \sum_v (\text{vehicle}_v \times \text{vehiclecos } t) + \sum_v \sum_r (\text{vehicleroute}_{vr} \times C_r \times (24 \times 60)/t_{\text{fixed}}) \quad (20)$$

The subscripts, decision variables, sets, parameters, and constraints in the multiple routed vehicles model are shown below. The identical ones described before at the problem with the one routed vehicles are not given.

Subscripts

v: Vehicle

Decision variables

X_{ivr}: Stock point "i" is assigned or not assigned to the "v" vehicle's "r" route (1 or 0)
D_{ivr}: One cycle demand quantity of the stock point "i" at "v" vehicle's "r" route
L_{ivr}: One cycle loading-unloading time of the stock point "i" at "v" vehicle's "r" route
Y_{avr}: The stock area "a" is assigned or not assigned to "v" vehicle's "r" route. (1 or 0)
vehicleroutecycle$_{vr}$: The total cycle time of the "v" vehicle's "r" route (Including fixed trip time of the vehicle and total loading-unloading time)
vehicle$_v$: Vehicle "v" is chosen or not chosen (1 or 0)
vehicleroute$_{vr}$: Route "r" is assigned or not assigned to vehicle "v" (1 or 0)

Constraints

$$\sum_v \sum_r X_{ivr} = 1 \quad \forall i \quad (21)$$

$$X_{ivr} = X_{jvr} \quad \forall (i,j) \in B, \forall v, \forall r \quad (22)$$

$$\sum_{i \notin Nr} X_{ivr} = 0 \quad \forall v, \forall r \quad (23)$$

$$\sum_v \sum_r Y_{avr} = 1 \quad \forall a,\ \text{a} \notin \text{A} \quad (24)$$

$$Y_{avr} = Y_{bvr} \quad \forall (a,b) \in C, \forall_v, \forall_r \quad (25)$$

$$M \cdot X_{ivr} \geq D_{ivr}\ \forall i, \forall v, \forall r \quad (26)$$

$$M \cdot X_{ivr} \geq L_{ivr} \quad \forall i, \forall v, \forall r \quad (27)$$

$$L_{ivr} = D_{ivr} \cdot l_i \quad \forall i, \forall v, \forall r \quad (28)$$

$$D_{ivr} = t_{\text{fixed}} \cdot d_i \cdot X_{ivr}\ \forall i, \forall v, \forall r \quad (29)$$

$$MY_{avr} \geq \sum_i X_{ivr}\ \ i \in N_a, \forall v, \forall r, \forall a, a \notin A \quad (30)$$

$$\text{vehicleroutecycle}_{vr} = \sum_i L_{ivr} + \text{vehicleroute}_{vr}.VT_r \quad \forall i, \forall v, \forall r \quad (31)$$

$$\sum_r (\text{vehicleroutecycle}_{vr}) \leq t_{\text{fixed}} \forall v \quad (32)$$

$$M \cdot \text{vehicleroute}_{vr} \geq \sum_i X_{ivr} \quad \forall v, \forall r \quad (33)$$

$$M \cdot \text{vehicleroute}_{vr} \geq D_{ivr} \quad \forall i, \forall v, \forall r \quad (34)$$

$$M \cdot \text{vehicleroute}_{vr} \geq L_{ivr} \quad \forall i, \forall v, \forall r \quad (35)$$

$$M \cdot \text{vehicle}_v \geq \text{vehicleroute}_{vr} \forall v, \forall r \quad (36)$$

$$\sum_{v} \text{vehicle}_v \leq \max \text{vehicle} \tag{37}$$

$$\sum_{ss_r=1}^{s_r} \sum_{i} D_{ivr} \cdot v_i \cdot \text{seq}_{s_r i} \cdot \text{sign}_i \leq \text{cap}\, \forall v, \forall r, \forall s_r \tag{38}$$

$$\begin{aligned} &X_{ivr}, Y_{avr}, \text{vehicle}_v, \text{vehicleroute}_{vr} \in \{0, 1\} \\ &\text{others} \geq 0 \\ &\forall i, \forall a, \forall v, \forall s_r, \forall r \end{aligned} \tag{39}$$

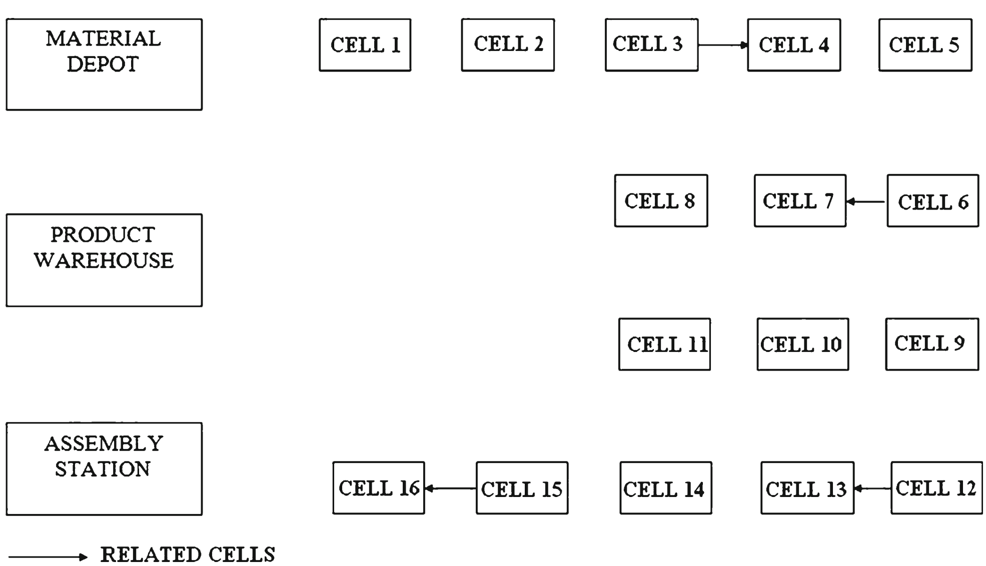
Description of the model constraints are as follows:

(21) Each stock point "i" is assigned to one "v" vehicle's one "r" route.
(22) The related stock points are assigned to same "v" vehicle's same "r" route.
(23) A stock point "i" is not assigned to a "v" vehicle's "r" route, if it is not on the way of route "r".
(24) Each stock area except the ones in material storage and assembly station is assigned to one "v" vehicle's one "r" route.
(25) The related stock areas are assigned to same "v" vehicle's same "r" route.
(26) The demand of stock point "i" at "v" vehicle's "r" route becomes zero, if it is not assigned to that route.
(27) The loading–unloading time of stock point "i" at "v" vehicle's "r" route becomes zero, if it is not assigned to that route.
(28) The total loading–unloading time of a stock point "i" at a "v" vehicle's "r" route is determined.
(29) The total demand of a stock point "i" at a "v" vehicle's "r" route is determined.
(30) Each stock area except the ones in material storage and assembly station is assigned to one "v" vehicle's one "r" route.
(31) The cycle time of "v" vehicle's "r" route is calculated.
(32) The cycle time of a vehicle "v" is restricted with the fixed determined time.
(33) A stock point "i" is not assigned to an unselected "v" vehicle's "r" route.
(34) The demand of a stock point "i" becomes zero at an unselected "v" vehicle's "r" route.
(35) The loading–unloading time of a stock point "i" becomes zero at an unselected "v" vehicle's "r" route.
(36) No route assignment is made to an unselected "v" vehicle.
(37) The total number of selected vehicles is restricted with an upper limit.
(38) The capacity of the vehicle is controlled at each assignment of a stock point to a vehicle.
(39) Decision variables are defined as either 0–1 binary integer variable or real numbers.

5 A numerical example and application of the models

For the application of the solution approaches, a numerical example is constructed in parallel to the real-life manufacturing system as shown in Fig. 3. There are material storage, 16 cells, assembly station and a product warehouse. There are totally 98 stock points in the system and there are 36 stock areas consisting of the stock points. There are two stock areas in each cell which are called input and output areas. The arrows between the cells mean that there is a material flow between the cells.

Fig. 3 The layout of the numerical example



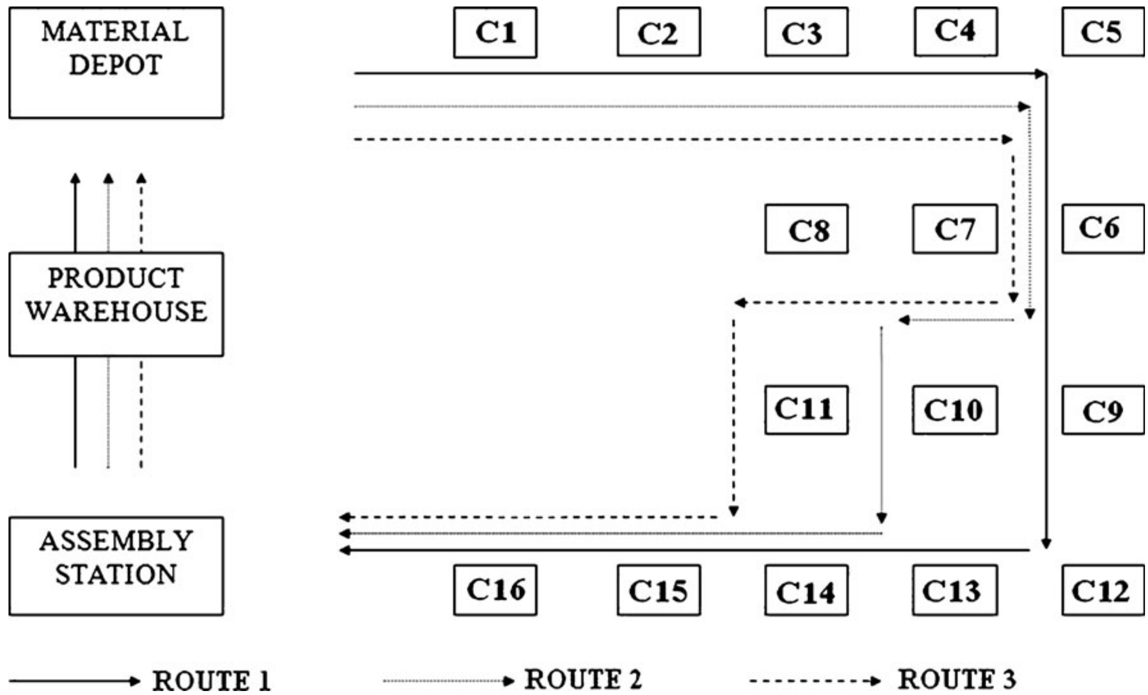


Fig. 4 Route 1, route 2, and route 3

The first step in the design of the system is constructing the routes according to the principles stated before such as layout, flow of the products, and physical conditions, etc. Although there could be more routes, for the easiness of the problem, nine probable routes are constructed as shown in Figs. 4, 5, and 6.

The values of the parameters can change according to the conditions of the application area. In this numerical example, some of the parameters are determined small for the easiness of operations. The demand rates of each stock point are defined as cases per minute and the capacity of the vehicles and stock areas are also defined as cases.

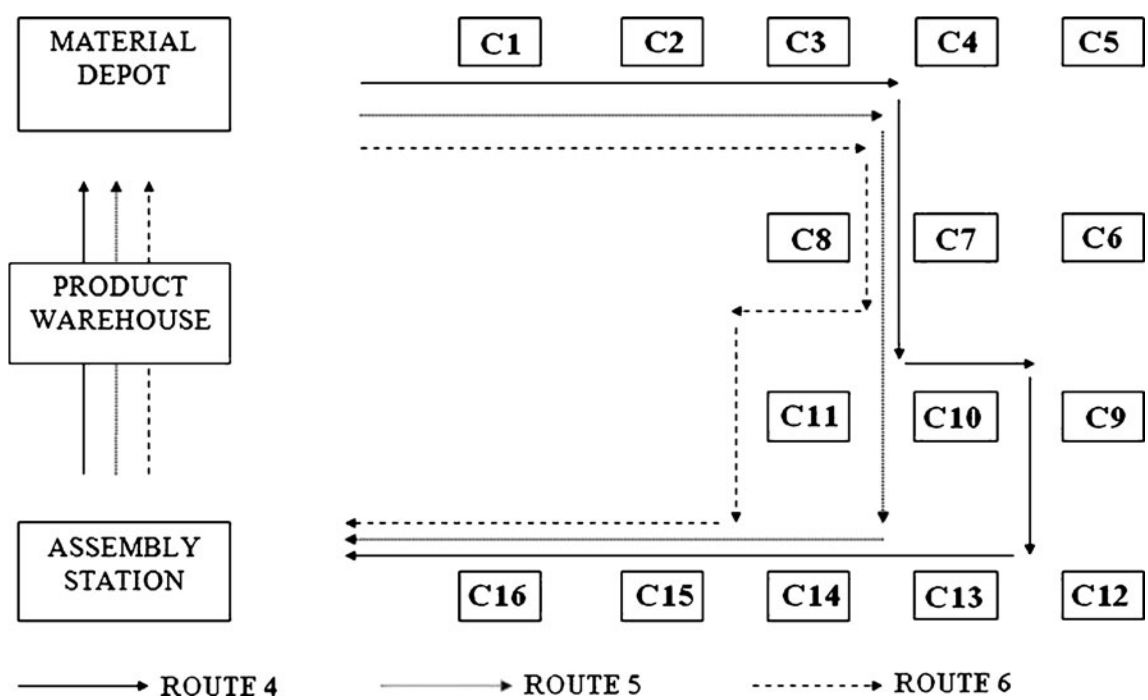


Fig. 5 Route 4, route 5, and route 6

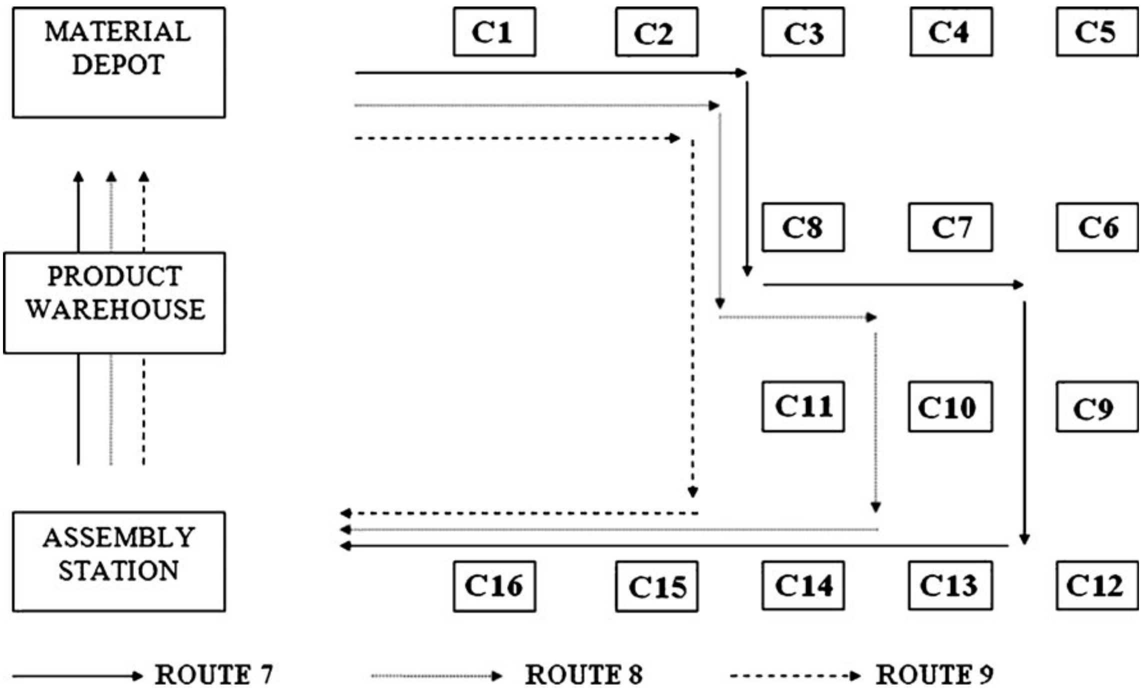


Fig. 6 Route 7, route 8, and route 9

Parameters

The related cells: Cell 3–Cell 4, Cell 6–Cell 7, Cell 12–Cell 13, and Cell 15–Cell 16

The time for the vehicles to traverse the routes: The average velocity of the vehicles is 60 m/min. So the time to traverse each route is: route 1, 4.7 min; route 2, 4.7 min; route 3, 4.7 min; route 4, 4.7 min; route 5,

Table 1 The supply or demand quantity of each stock point per time

Stock point (sign)	Case/ min	Stock point (sign)	Case/ min	Stock point (sign)	Case/ min	Stock point (sign)	Case/ min	Stock point (sign)	Case/ min
1 (1)	0.1	21 (1)	0.05	41 (−1)	0.05	61 (1)	0.05	81 (−1)	0.05
2 (1)	0.1	22 (1)	0.05	42 (1)	0.05	62 (−1)	0.05	82 (−1)	0.05
3 (1)	0.05	23 (1)	0.05	43 (1)	0.05	63 (−1)	0.05	83 (−1)	0.10
4 (1)	0.05	24 (1)	0.05	44 (−1)	0.10	64 (1)	0.05	84 (−1)	0.10
5 (1)	0.05	25 (1)	0.10	45 (−1)	0.10	65 (−1)	0.05	85 (−1)	0.10
6 (1)	0.05	26 (−1)	0.1	46 (1)	0.10	66 (1)	0.05	86 (−1)	0.10
7 (1)	0.05	27 (−1)	0.1	47 (−1)	0.10	67 (−1)	0.05	87 (−1)	0.05
8 (1)	0.05	28 (−1)	0.05	48 (−1)	0.10	68 (−1)	0.05	88 (−1)	0.05
9 (1)	0.05	29 (1)	0.05	49 (1)	0.10	69 (1)	0.05	89 (−1)	0.05
10 (1)	0.10	30 (1)	0.05	50 (−1)	0.10	70 (−1)	0.05	90 (−1)	0.05
11 (1)	0.10	31 (−1)	0.05	51 (−1)	0.10	71 (1)	0.05	91 (−1)	0.05
12 (1)	0.10	32 (−1)	0.05	52 (1)	0.10	72 (1)	0.05	92 (−1)	0.05
13 (1)	0.10	33 (1)	0.05	53 (1)	0.10	73 (−1)	0.05	93 (−1)	0.10
14 (1)	0.10	34 (1)	0.05	54 (−1)	0.10	74 (−1)	0.10	94 (−1)	0.10
15 (1)	0.10	35 (−1)	0.05	55 (−1)	0.05	75 (1)	0.05	95 (−1)	0.10
16 (1)	0.05	36 (1)	0.05	56 (1)	0.10	76 (−1)	0.05	96 (−1)	0.10
17 (1)	0.05	37 (−1)	0.05	57 (−1)	0.05	77 (−1)	0.05	97 (−1)	0.10
18 (1)	0.05	38 (−1)	0.05	58 (1)	0.05	78 (−1)	0.05	98 (−1)	0.10
19 (1)	0.05	39 (1)	0.05	59 (−1)	0.05	79 (−1)	0.05		
20 (1)	0.05	40 (−1)	0.05	60 (−1)	0.05	80 (−1)	0.05		

4.2 min; route 6, 4.2 min; route 7, 4.7 min; route 8, 4.2 min; route 9, 3.7 min.
The average cost of the 1-cycle tour of each route: The cost of the transportation mainly depends on the energy type used by the material handling equipment and the material that is carried. So it has a wide range of price. Cost per kilometer is regarded as $0.25 per km for all the vehicles and the cost of 1 cycle of each route is: route 1, $0.0705; route 2, $0.0705; route 3, $0.0705; route 4, $0.0705; route 5, $0.063; route 6, $0.063; route 7, $0.0705; route 8, $0.063; route 9, $0.055
The number of time intervals: Each time interval is 2 min and the total number of time intervals is 120.
The cycle time: The determined cycle time is 30 min for all the vehicles.
The quantity of safety stock: One cycle demand is the safety stock for each stock point.
The costs related with the milk-run train: Regarding the depreciation and the driver cost of the vehicle, the daily cost is assumed to be $60 for one milk-run train.
The loading–unloading time of the material cases: It is assumed to be 0.3 min for one case. But for the materials in the material storage, taking into account of the searching time, it is assumed to be 0.45 min for one case.
The capacity of the train: The train has four wagons and each wagon can carry 16 cases.
The demand or supply quantity of stock points per time: According to the stock point type (demand or supply), the quantity per time and the sign showing the type of the stock point (supply=1 or demand=−1) are shown in Table 1.

Table 2 The results for one routed vehicles

The chosen routes	Tour time (min)	The stock points assigned to the route
r1	23.15	sp6, sp7, sp8, sp9, sp10, sp11, sp12, sp35, sp36, sp37, sp38, sp39, sp40, sp41, sp42, sp43, sp44, sp45, sp46, sp47, sp48, sp49, sp80, sp81, sp82, sp83.
r4	24.95	sp4, sp5, sp15, sp16, sp17, sp20, sp21, sp22, sp23, sp31, sp32, sp33, sp34, sp54, sp55, sp56, sp57, sp58, sp62, sp63, sp64, sp65, sp66, sp67, sp68, sp69, sp78, sp79, sp86, sp87, sp89, sp90, sp94, sp97.
r5	12.30	sp13, sp14, sp50, sp51, sp52, sp53, sp84, sp85.
r9	23.95	sp1, sp2, sp3, sp18, sp19, sp24, sp25, sp26, sp27, sp28, sp29, sp30, sp59, sp60, sp61, sp70, sp71, sp72, sp73, sp74, sp75, sp76, sp77, sp88, sp91, sp92, sp93, sp95, sp96, sp98.

Table 3 The results for multiple routed vehicles

The chosen routes	Tour time (min)	The stock points assigned to the route
v1	r5 (12.92)	sp6, sp7, sp17, sp18, sp19, sp22, sp23, sp24, sp25, sp35, sp36, sp37, sp38, sp39, sp57, sp58, sp59, sp60, sp61, sp67, sp68, sp69, sp70, sp71, sp72, sp73, sp74, sp75, sp80, sp87, sp88, sp90, sp91, sp92, sp94, sp97.
	r9 (16.26)	sp1, sp2, sp3, sp4, sp5, sp13, sp14, sp26, sp27, sp28, sp29, sp30, sp31, sp32, sp33, sp34, sp50, sp51, sp52, sp53, sp76, sp77, sp78, sp79, sp84, sp85, sp93, sp95, sp96, sp98.
v2	r1 (23.56)	sp8, sp9, sp10, sp11, sp12, sp15, sp16, sp20, sp21, sp40, sp41, sp42, sp43, sp44, sp45, sp46, sp47, sp48, sp49, sp54, sp55, sp56, sp62, sp63, sp64, sp65, sp66, sp81, sp82, sp83, sp86, sp89.

5.1 Solution of the models

Both of the models are coded in the GAMS optimization program. Under the given parameters, the models are solved and the results in Tables 2 and 3 are obtained.

For one routed vehicles, optimum solution can be found in a short time. There are four milk-run trains in the system. They have different tour times. Since the cycle time is determined before and it is 30 min, each vehicle has different idle times.

However, in the multiple routed vehicles, optimum solution could not be found in a 24-h time and the optimization program is interrupted and the feasible solution is given. This situation also shows that there is need for developing heuristics for multiple routed vehicles. There are two milk-run trains in the system; one having two routes and the other having one route. In the multiple routed model, the vehicles also have idle times but not as much as the one routed vehicles.

When one routed and multiple routed vehicles are compared in terms of idle time, number of vehicles and cost, it is seen that multiple routed vehicles are more advantageous than the one routed ones as shown in Table 4. But as

Table 4 The comparison of one routed and multiple routed vehicles

Performance indicators	One routed vehicle	Multiple routed vehicle
Average idle time ratio of vehicle (%)	29.71	12.10
Number of vehicles	4	2
Cost ($)	244.15	123.14

mentioned before, it is easier to manage the milk-run trains in the one routed vehicles than the multiple routed vehicles. According to the manufacturing environment and managerial skills, the most proper alternative can change.

6 Conclusion

Within this study, a pull-based material handling system problem is analyzed. The problem can also be called as milk-run problem, because vehicle starts from a storage area and visits the cells on a predefined path and again returns to the storage area periodically. The problem is classified into categories on the basis of assignment methodology regarding the time periods and number of assignable routes to vehicles. Each category is introduced and explained. Mixed integer linear programming models are presented for a specific category where the cycle time of the milk-run trains are known but the routes and the number of vehicles are not known. To show the applicability of the presented models, a numerical example is constructed. Obtained results are analyzed and compared in terms of performance indicators such as idle time, number of milk-run trains in the system and cost. As a result, although it is more difficult to manage the milk-run trains, it is more advantageous to use them in the multiple routed vehicles method than one routed vehicles method.

Although the literature about vehicle routing problem is extensive, there is not much more about plant logistics especially in lean manufacturing systems. By presenting a detailed classification scheme about plant logistics and developing models, it is aimed to fill this research gap. During the study, it is highly benefited from the real applications especially in the automotive sector where lean manufacturing is applied. The next step of this study can be to improve the classification scheme and develop models. When the problem gets bigger, especially for the multiple routed vehicles, it becomes difficult to find the optimum solution. This shows that there is need to develop heuristics for such kinds of problems. Moreover, the parameters of the papers by [23, 24] and [25] may be modified in the context of present work to incorporate multiple-item situations in the real environment.

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