Int J Adv Manuf Technol (2011) 57:1245–1258
DOI 10.1007/s00170-011-3681-z

ORIGINAL ARTICLE

Geometric parameter-based optimization of the die profile for the investment casting of aerofoil-shaped turbine blades

Yiwei Dong · Dinghua Zhang · Kun Bu · Yangqing Dou · Wenhu Wang

Received: 17 April 2011 / Accepted: 26 September 2011 / Published online: 13 October 2011

Abstract Dimensional accuracy is one of the most important criteria in the investment casting of turbine blades for aero engine applications. Although investment casting is an ideal route for producing net-shaped components, it necessitates accurate determination of the casting die profile. To avoid extensive modifications on the die cavity shape, this study proposes a simple but efficient inverse design method that adjusts certain geometric parameters to establish the die profile. Parameterized modeling is achieved by identifying the geometric parameters that describe the mean camber line of the blade profile. Meanwhile, blade shrinkages at different positions are effectively remedied through the adjustment of the parameters using the inverse iteration algorithm proposed in this paper. The relative positions among the cross-sections can also be varied to rectify the twist deformation of the blade, thereby guaranteeing the dimensional accuracy of the turbine blades. The applicability of this method is validated using numerical simulation data and experimental results.

Keywords Die design · Inverse iteration algorithm · Geometric feature parameter · Numerical simulation · Investment casting · Deformation compensation

Y. Dong (✉) · D. Zhang · K. Bu · Y. Dou · W. Wang
The Key Laboratory of Contemporary Design and Integrated Manufacturing Technology,
Northwestern Polytechnical University,
P.O. Box 552, 127 Youyi Xilu,
Xi'an 710072 P.R. China
e-mail: dongyiwei618@gmail.com

1 Introduction

Turbine blades and vanes, which consist of rotor blades and stationary vanes, are some of the critical components of an aero engine [1, 2]. The blades are operated under extreme and complex environmental conditions. They contend with air dynamics and the centrifugal forces that cause tensile and bending stresses. In manufacturing these components, highly stringent quality requirements should be satisfied to ensure that they withstand high mechanical loads at temperatures of up to 1050°C for several thousand hours. Therefore, the materials used in making turbine blades must have enough high-temperature tensile strength, endurance strength, and creep strength [3, 4]. Close dimensional tolerances are specified for the components because engine performance significantly depends on the shape and dimensions of these parts.

A Turbine blade has close dimensional and geometrical tolerances and is manufactured by the investment casting process employed in producing high quality and net-shaped complex parts [5]. The size of a turbine blade produced by the investment casting process is smaller than that of the die cavity because of the shrinkages of the wax and solidifying alloy material. The shape of the cast significantly depends on the geometry of the metal die cavity in the investment casting process [6]. A precise die profile, which generally takes into account the various shrinkages involved in the casting process, is therefore important in improving the quality of the net-shaped products. Therefore, to ensure dimensional accuracy, positional accuracy, and surface roughness, the die profile design for turbine blade should consider compensation for the shrinkages brought about by the solidification process [7].

In the die design and manufacturing process, the die profile design is the most important consideration. The cast deformation of the castings depends on the integrated operation of many factors; even for a single casting, different parts have varied deformation conditions [8]. The principle of designing the die profile lie in the compensation for certain deformations at the positions where flaws occur. The establishment of the die profile is currently based on the linear scaling method, which can be classified into the following techniques: uniform scaling, chord length scaling, mean camber line scaling and shrinkage center scaling methods. These methods are simple and neutral, but the shrinkage ratio used to design the profile remains constant. These features bring about disadvantages such as disregard of the flexural-torsional deformation and shrinkage established by the linear scaling method. Huang and Lan [9] employed the reverse compensation method to improve dimensional precision and volume shrinkage in the rapid prototyping process. Sabau [10] and Ferreira and Mateus [11] presented a method that uses different shrinkage rates for various directions. However, such calculations do not consider the influence of casting structures and various constraint conditions. Establishing an appropriate numerical model also necessitates sufficient understanding of such influence and accordant efforts such as experimentation.

With the help of commercially developed solidification simulation software based on finite element analysis, researchers have extensively used large-scale finite element analysis software in the die profile design optimization and shrinkage prediction during the casting process. Bonilla et al. [12] developed a methodology that uses computer-aided heat transfer simulation, and experimentally derived factors for injection parameters to predict wax pattern shrinkages in the investment casting process. Wang et al. [13] discussed a springback compensation method, which involves minimizing the elastic recovery of the part and adjusting the tooling shape. However, no related research on how to compensate for shrinkage to produce a precision investment cast turbine blade has been published. Modukuru et al. [14] proposed a grid displacement reverse stacking method, in which the calculated deformations are reversed and stacked into each node. This process is iterated until the deformed shape exhibits good agreement with the ideal shape. With three-dimensional graphical output, this method can be regarded as a virtual process of modifying the die profile until an accurate numerical simulation result is achieved. Nevertheless, comprehensive simulation demands accurate boundary conditions and a solidification constitutive model; therefore, attempting this route provides no value if focus is trained exclusively on the die profile design.

The present work proposes a simple but efficient inverse design scheme that adjusts certain geometric parameters to establish the die profile. On the basis of the geometric design methodology of turbine blades, parameterized modeling is achieved by identifying geometric parameters that describe the camber line, after which the blade shrinkages at different positions are effectively remedied through the adjustment of the parameters using the inverse iteration algorithm. Simultaneously, considering that the die profile design is focused mainly on the internal shape of the blade body, the goal of the present work is to identify a simple but universal methodology that can determine the die cavity for the investment casting of turbine blades. Moreover, the implementation of the suggested approach is demonstrated through an example.

2 Parametric design of turbine blades

The design of high-efficiency turbine blades is essential for optimum aerodynamic, thermo-economic, and overall performance of turbomachinery-based power plants [15–17]. Turbine blades are three-dimensional objects operating in a complex flow field; this problem is usually reduced to a series of radially "stacked" two-dimensional problems. Blade design is both a science and an art. The blade can be designed by direct or inverse methods. In the direct method, the designer incorporates the geometry of the blade, and the output is the performance of the airfoil. Related research achievements are detailed in Ref [18]. In the inverse method, the designer typically begins from a certain initial blade shape and performance, and incorporates the desired modifications into the performance. The output is a new shape with improved performance, i.e., performance that is as close to the desired performance as permitted by the equations modeling the flow.

With the development of computer-aided design (CAD), Bézier, B-spline and Non-Uniform Rational B-Splines (NURBS) curves provide effective solutions for section curve modeling, which can handle more complicated geometric shapes and respond more quickly than can classical equation of conic sections. Consequently, the modeling technique using these three kinds of curves is extensively applied [19–21]. The direct method for designing turbine blades is to directly build the model from the given blade geometric parameters. Using a kind of analytic curve as a basic curve, this method establishes sufficient geometric constraints according to the aerodynamic requirements, so that the designers can construct the section curve by manipulating the geometric parameters, such as the blade inlet and blade exit angles, stagger angle, maximal thickness, as well as the blade leading edge (LE) and trailing edge (TE) radii.

This study uses geometric parameters as initial data to establish the die profile. Therefore, the identification and extraction of the geometric parameters are necessary.

2.1 Recognition of the parametric features of the turbine blade

Some parametric features describing the two-dimensional blade profile are extracted on the basis of the direct method. With surface feature modeling technology, the complex surface features, such as the turbine blade, can be transformed by directly using the performance parameters. Designers do not need to address large numbers of initial data points or curves; only by means of some parametric operations, such as swap, add, delete, modify, etc., the blade profile can be easily designed. In using parametric modeling technology, defining the parameters should be the first step.

More than 20 kinds of parameters are related to the structure, strength, and manufacturing process of turbine blades; however, this paper focuses mainly on the method of determining the die profile of turbine blades. The blade-design method can be based on specifying the thickness distribution around the mean camber line; thus, the parameters that can be used to represent and reconstruct the mean camber line are discussed. Figure 1 shows the geometric parameters, which can be used to represent a typical turbine cascade, as well as their locations.

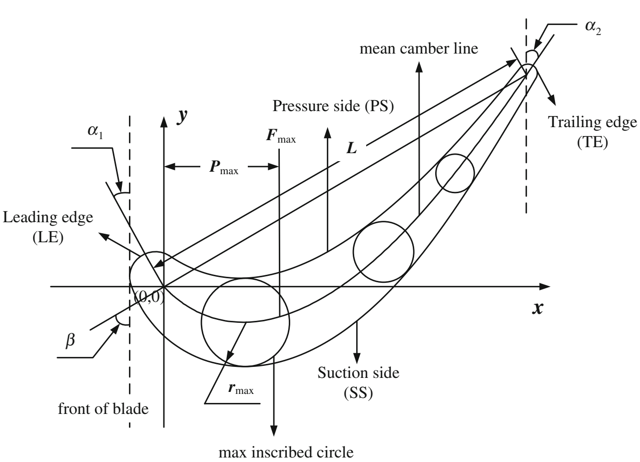


Fig. 1 Sketch of the camber line and thickness parameters

For familiarization with turbine blade terminology, the following terms are explained:

- suction side (SS): the curve on the convex side of a blade where the gas flow pressure is decreased (Fig. 1);
- pressure side (PS): the curve on the concave side of a blade where the gas flow pressure is increased (Fig. 1);
- LE: the edge where the gas flow first hits the blade (Fig. 1);
- TE: the edge where the gas flow leaves the blade (Fig. 1);
- L: the chord length, which represents the distance between the centers of LE and TE;
- F_{max}: the maximum deflection (Fig. 1);
- P_{max}: the position of the maximum deflection (Fig. 1);
- α_1: blade inlet angle, or the angle between the tangent and the camber line at the LE (Fig. 1);
- α_2: blade exit angle, or the angle between the tangent and the camber line at the TE and turbine axial direction (Fig. 1);
- β: angle between the chord line of the airfoil section and turbine axis at a radial view; the deviation of the actual stagger angle against its nominal value is called "twist" (Fig. 1).

2.2 Parametric modeling of the mean camber line

According to the demand of blade section design, the section curve is normally separated into four segments: LE, TE, pressure curve, and suction curve, which constitute the piecewise closed section profile. To ensure that the surface velocity field and stress field of the blade surface are uniform, a high-order polynomial is often used to construct the PS and SS of the blade section. Korakianitis and Pantazopoulos [22] proposed a method for generating the blade shape using fourth-order parametric spline segments; this approach has been proven efficient in specifying the geometry of a turbine cascade.

On the basis of the definition of the mean camber line, its discrete points should have high-order polynomial relationships. To verify the parametric modeling of the mean camber line, two groups of discrete points used to design a turbine blade cascade are chosen, and the mean camber line parameterizations are solved by least squares fitting method using an orthogonal polynomial.

Given a set of input discrete points $\{(x_i, y_i)\}_{i=0}^{n}$, a continuous curve represented by continuous function

$y = \varphi(x)$ can be designed. One can define $\varphi^*(x) \in \phi = \{\varphi(x, c_0, c_1, ..., c_n), c_i \in R\}$, and yield

$$\left\|r(\varphi^*)\right\|_2 = \min_{\varphi\in\phi} \|r(\varphi)\|_2 \tag{1}$$

The fifth-order polynomial can be used to construct the PS and SS curves. In vector space $\{1, x, x^2, x^3, x^4, x^5\}$, we can obtain the normal equation as follows:

$$\mathbf{A^T A C} = \mathbf{A^T y} \tag{2}$$

where $\mathbf{A}$ is the positive definite matrix and $\mathbf{C}$ is the coefficient matrix, and

$$\mathbf{A^T} = \begin{bmatrix} \varphi_0(x_0) & \varphi_0(x_1) & \dots & \varphi_0(x_n) \\ \varphi_1(x_0) & \varphi_0(x_1) & \dots & \varphi_1(x_n) \\ \varphi_2(x_0) & \varphi_1(x_1) & \dots & \varphi_2(x_n) \\ \varphi_3(x_0) & \varphi_2(x_1) & \dots & \varphi_3(x_n) \\ \varphi_4(x_0) & \varphi_3(x_1) & \dots & \varphi_4(x_n) \\ \varphi_5(x_0) & \varphi_4(x_1) & \dots & \varphi_5(x_n) \end{bmatrix}$$

Thereby, the polynomial coefficients can be obtained as $\mathbf{C^T} = \{C_0, C_1, C_2, C_3, C_4, C_5\}$. However, in the polynomial fitting process, the selection of the fitting orders is important. If the polynomial order is inappropriate, the fitting result will be rough, or noise data will be included in the fitting model. Therefore, the optional polynomial order is selected by testing the partial fitting square sum on the basis of the variance analysis of the fitting equation.

For a given set of data $(x_i.y_i)$, $(i = 1, 2, ..., n)$, using the polynomial fitting model as

$$P(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_m x^m (m < n)$$

we can define $K^2 = \sum_{j=1}^{n} [P(x_j) - y_i]^2$.

To solve K^2, the coefficients of the fitting function are incorporated into the equation, written as

$$K^2 = \sum_{j=1}^{n} [P(x_j) - y_i]^2 = \sum_{j=1}^{n} \left[\sum_{i=0}^{m} a_i x_j^i - y_j\right] \tag{3}$$

The appropriate fitting order should take degree of freedom $n - m - 1$ as the mathematical expectation, where n is the original data length and $m + 1$ denotes the number of polynomial terms.

Both fifth- and third-order polynomials are used to fit the same group of discrete points of the mean camber line, and the variance analysis method (ANOVA) is used to study the characterization and effectiveness of the polynomial fitting. The data fitting results of one group of the mean camber line data are derived by ANOVA (Table 1).

Table 1 indicates that the multiple correlation coefficient of the third-order polynomial fitting is 99.99%, the linear relationship is good, and the fitting equation is markedly effective.

Therefore, to generate an efficient solution process for the parametric modeling of the mean camber line, the third-order polynomial is chosen to model the mean camber line.

The mean camber line is essentially a two-dimensional free-form curve. If the length of the mean camber line remains constant, its freedom can be restricted by four parameters: α_1, α_2, L, and P_{max}.

Figure 2 shows the sketch of the parameters that can be used to control the two-dimensional turbine blade cascade. First, the center of the LE is used as the starting point of the mean camber line. The origin of the coordinates is defined as the center of LE after coordinate transformation. Little difference between Figs. 1 and 2 is observed. In Fig. 1, the X direction is defined as the chord direction. In the new coordinate system (Fig. 2), however, the direction perpendicular to the front of the blade direction is used as the X-axis direction. Therefore, the definitions of the geometric parameters are held constant, and only the symbolic forms are modified.

The third-order polynomial model is selected to establish the mean camber line. The cubic curve can be represented as

$$y(x) = ax^3 + bx^2 + cx + d \quad 0 < x \le L \tag{4}$$

Table 1 ANOVA results of data fitting for the mean camber line

Polynomial orders	Item	Degree of freedom	Sum of square	Mean square	F-value
Third order	Regression	3	16911.60984	5637.20328	3.80426E6
	Residual errors	398	0.58976	0.00148	
	Sum	401	16912.19961		
Fifth order	Regression	6	17993.84144	2998.97357	3.81039E7
	Residual errors	396	0.03117	7.87051E-5	
	Sum	402	17993.87261		

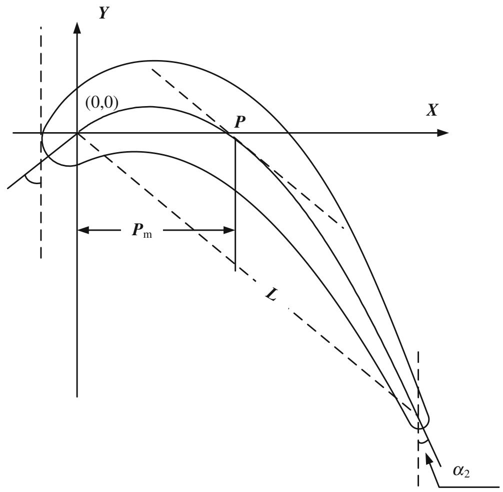


Fig. 2 Sketch of the geometric parameters of a turbine blade cascade

Then, the coefficients of the cubic curve can be established by the following control equation

$$\begin{cases} y(0) = 0 \\ y'(0) = \cot\alpha_1 \\ y'(P_m) = M/m \\ y'(m) = -\cot\alpha_2 \end{cases} \tag{5}$$

where $m = \sqrt{L-M}$, and (m, M) is the coordinate value of the last point of the mean camber line. The coefficients of the cubic curve can be solved by Eq. 5.

$$\begin{cases} b = -\dfrac{(\cot\alpha_1(L-M) - M\sqrt{L-M} - P_m^2\cot\alpha_1 + P_m^2\cot\alpha_2)}{2P_m\sqrt{L-M}(\sqrt{L-M}-P_m)} \\ a = \dfrac{(-P_m\cot\alpha_1 + \cot\alpha_1\sqrt{L-M} - M + P_m\cot\alpha_2)}{3P_m\sqrt{L-M}(\sqrt{L-M}-P_m)} \\ c = \cot\alpha_1 \\ d = 0 \end{cases} \tag{6}$$

The third-order polynomial model of the mean camber line can then be established.

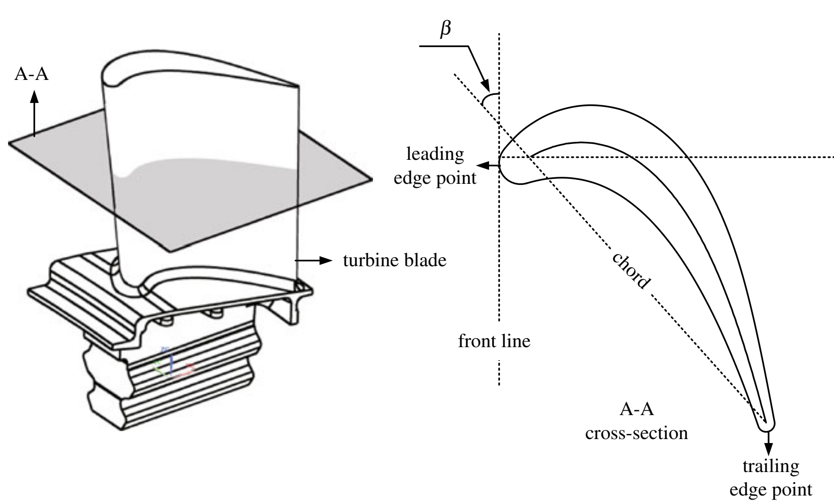
3 Parameter extraction of the blade section curve

3.1 Extraction of β and L

When the direction perpendicular to the front of the blade direction is used as the X-axis direction, the LE point is the point that has the minimum abscissa value, whereas the TE point is the point that has the maximal abscissa value. The chord can be defined as the line connecting the LE and TE points. Then, the stagger angle β is defined as the angle between the chord and front of the blade (Fig. 3).

Determining the chord before the extraction of β is important. The key technology for chord extraction is obtaining the points of tangency of the chord and LE/TE. The two-dimensional profile of the blade can be treated as a set of ordered points without self-intersection; it can also be viewed as a kind of polygon. Perceptually speaking, the SS is the maximal concave edge of the polygon. According to the characteristics of a turbine blade, the end-points of this concave edge can be regarded as the points of tangency, which require resolution. To evaluate the longest span in all the concave edges of the polygon, the distances between each pair of adjacent points of the profile are calculated, and the adjacent points that have the maximum distance are determined as the end-points of the concave edge, that is, the points of tangency of the chord and profile. Thereafter, the stagger angle β can be extracted.

Fig. 3 Sketch of the stagger angle and chord



3.2 Extraction of mean camber line

The mean camber line extraction is based on the "equidistance line" method, The collection of central arced curves can be transformed into the determination of the self-intersection points of the equidistant curves [23]. This approach involves the following procedures:

1. In the beginning, the smoothing section model is established.
2. Then, the accurate equidistant lines of the section are solved at different offset distances.
3. The self-intersection points of each equidistant line are determined using the family of equidistant lines.
4. Finally, all the self-intersection points are arranged and interpolated to generate the smoothing mean camber line.

3.3 Extraction of α_1, α_2, and P_{max}

Blade inlet angle (α_1) is the angle between the tangent and the camber line at the LE; it represents the slope of the blade front. If the numbers of discrete points on mean camber line are sufficient, the angle between the line connecting the first two points of mean camber line and the front of the blade can be regarded as the blade inlet angle (Fig. 4).

The method used to extract the blade exit angle (α_2) is similar to the blade inlet extraction method. Considering the angle between the line connecting the last two points and the front of the blade, the blade exit angle can be established.

If the chord is considered the horizontal axis in the coordinate system, the maximum deflection position (P_{max}) can be regarded as the point that has the maximum distance from the mean camber line to the chord. Thus, a line parallel to the chord can be drawn tangent to the mean camber line, and the point of tangency can be identified as the maximum deflection position.

4 Optimized compensation of geometric parameters

4.1 Optimization method for die profile determination

To efficiently reduce the deformation generated during the solidification process, the traditional technology is used in designing the die profile. This technology involves providing certain appropriate reverse deformation dimensions that have been previously used; these dimensions are obtained through numerical simulation or experimentation on opposing deformation directions. Figure 5 shows the basic principle of the reverse deformation method: Plate P is assumed as the initial shape of the casting, plate Q represents the shape after deformation, and plate R refers to the shape after reverse deformation. This method can also be used in the design of the die profile for investment casting.

Let D be the objective function of the surface profile of a wax pattern for the turbine blade, P be the shape function of the turbine blade before investment casting, and Q be the shape function of the blade after investment casting. The displacement field function, expressed as $\Delta D = Q - D$, is the deformation function. If error ΔD is less than the maximum form error ΔMax, P can be considered the ideal deformed shape. Otherwise, the deformed shape should be re-modified.

However, because of the complicated shape of turbine blades, the accuracy requirement cannot be satisfied using only one-step compensation. In this study, the iteration method is used for reverse deformation, which can enable a more precise approximation of the

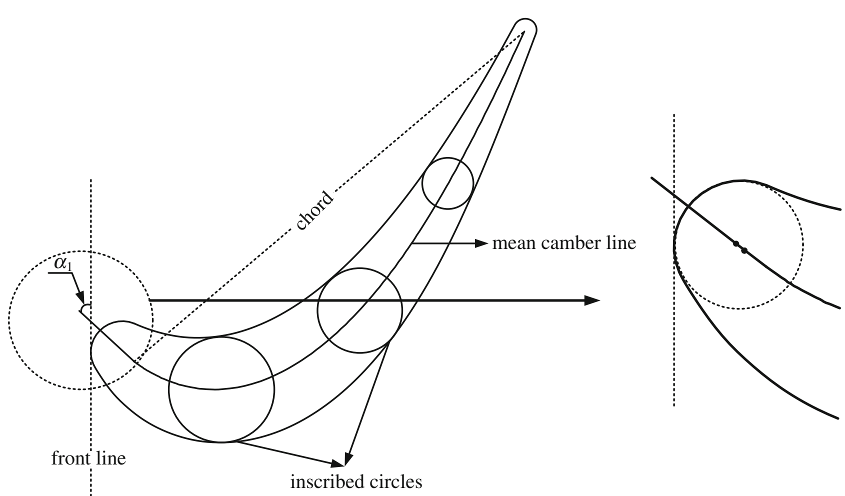


Fig. 4 Sketch of blade inlet angle α_1

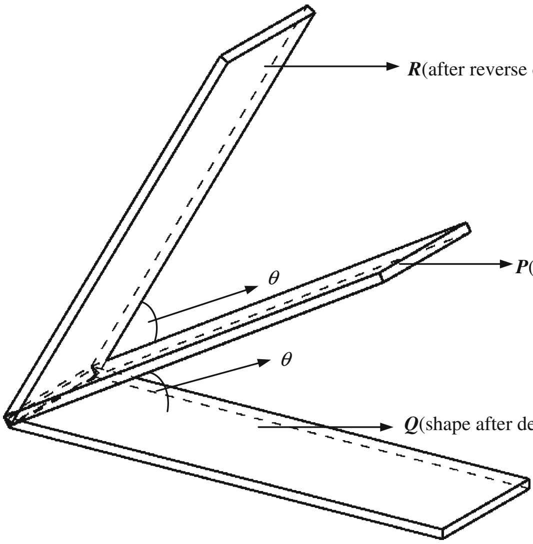


Fig. 5 Basic principle of reverse deformation

target shape. Let the initial shape be denoted as P_i, which is transformed into the deformed shape after i times of inverse operations. It is then denoted as Q_i. The casting process is implemented once on the basis of the shape of the casting, which has been subjected to reverse deformation operation i times. Comparing the casting result with the target shape, the error denoted as ΔD_i can be described as

$$\Delta D_i = Q_i - D \tag{7}$$

If error ΔD_i is less than the maximum form error ΔMax, P_i can be considered the ideal deformed shape. Otherwise, the deformed shape should be re-modified. According to the reverse result from the previous step, reverse compensated error ΔD_i is expressed as

$$P_i = -\Delta D_{i-1} + P_{i-1} \tag{8}$$

Calculating P_{i+1} and Q_{i+1} using the iterative method, casting form error ΔD_{i+1} can be obtained from Eq. 7. If the error satisfies the requirement of form error precision, the program is terminated. The final reverse deformation function can be obtained as $P = P_i$.

Given that the form error is related to the structure of the casting, each form error shows some similarities. Hence, a shape coefficient, denoted as λ, is introduced and the Eq. 8 can be written as

$$P = -(1 + \lambda)\Delta D + P_0 \tag{9}$$

where P_0 represents the initial turbine blade model and ΔD is the displacement field. By introducing shape coefficient λ, the multiple compensations can be converted into a single compensation. Thus, the accuracy requirement can be satisfied using only one-time compensation. The physical significance of Eq. 8 is that for large deformation castings, the displacement increment of the compensation can cause an even larger deformation, also a necessary type of compensation.

However, if the casting is large, ΔD will also be large and displacement increment $\lambda \times \Delta D$ will be oversize. Therefore, the compensation of the displacement increment can again be executed. The equation can be written as

$$\begin{aligned} P &= -\Delta D - \lambda\Delta D - \lambda^2\Delta D - \cdots - \lambda^n\Delta D + P_0 \\ &= -(1 + \lambda + \lambda^2 + \cdots + \lambda^n)\Delta D + P_0 \end{aligned} \tag{10}$$

The value of the shape coefficient must ranges from must 0 to 1; hence, the optimized compensation method can be written as

$$P = -\left(\lim_{n\to+\infty} \sum_{i=1}^{n} \lambda^i\right)\Delta D + P_0 = -\frac{1}{1-\lambda}\Delta D + P_0 \tag{11}$$

The deformation of castings during the solidification process must first be established because of the optimization of the wax pattern die profile for the investment casting of turbine blades. That is, the displacement field is established. Turbine blades are frequently manufactured using unidirectional solidification technology. When a gating system is designed, the feeding system is essential in preventing the contraction of molten metal during the investment casting process. The gating and feeding systems are shown in Fig. 6

According to the unidirectional solidification process, the deformation along the Z direction can be decreased or prevented by the feeding system. Meanwhile, the design of three-dimensional turbine blades

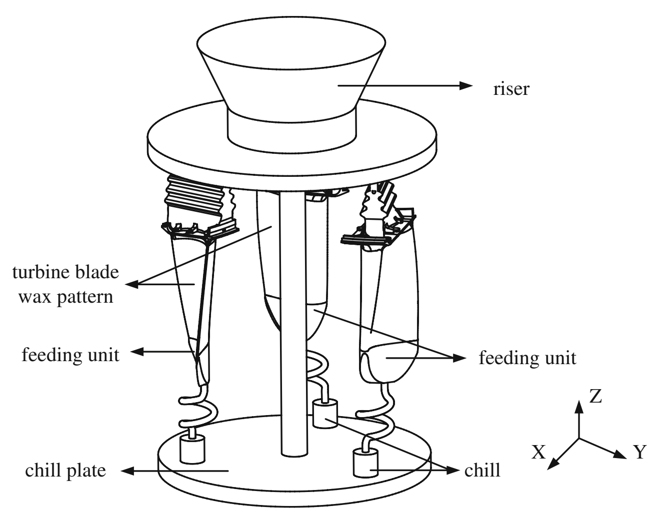


Fig. 6 Schematic of the gating and feeding systems

are commonly generated by "stacking" 2-dimensional designs. Therefore, the analysis of the deformation of turbine blades and the optimization of the die profile are based on the 2-dimensional sections of turbine blades. In this work, the deformation along the Z direction is negligible.

4.2 Determination of optimized geometric parameters

The extracted geometric parameters can be used to represent the deformation, as explained in Section 3. The purpose of the optimization of the geometric parameters is to compensate for each parameter based on the method proposed in Section 4.1. However, these parameters are not independent of one another, and are mutually interacting and restricted. Therefore, first determining the relationship among the geometric parameters is necessary.

Stagger angle β is the parameter that can be used to represent the torsional deformation of the blade; hence, compensating for the torsional deformation means compensating for the deformation of β.

The bending deformation can be denoted by blade inlet angle α_1, blade exit angle α_2, maximum deflection position P_m, and chord length L. The relationship among these parameters can be established according to the geometric relationship and characteristics of the geometric parameters (Fig. 7).

First, the coordinate value of dot A (x_A, y_A) and the last point of mean camber line B (m, M) should be determined. The geometrical meaning of dot A is the intersection point of the tangent lines of the first and last points of the mean camber line. Hence, the establishment of the coordinate value of dot A can be calculated based on the two-point form of a straight-line equation.

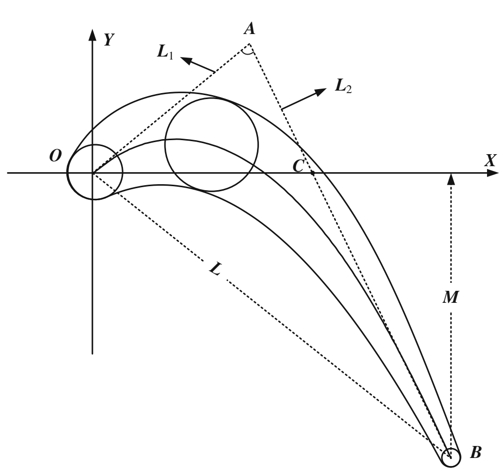


Fig. 7 Sketch of the control parameters of bending deformation

Then, by calculating the coordinate values of dot A in the CAD and measurement models of a certain turbine blade, the optimized coordinate value of dot A can be determined on the basis of the compensation of discrete points:

$$P_{xy}(x, y) = -\left(1 + \frac{y_2 - y_1}{x_2 - x_1}\right) W_{xy}(x, y) + P_0(x_0, y_0) \tag{12}$$

where $P_0(x_0, y_0)$ is the coordinate value of an arbitrary dot of the CAD model, $P_{xy}(x, y)$ denotes the optimized coordinate value of this dot, and $W_{xy}(x, y)$ is the distance between dot A in the CAD and measurement models.

The coordinate value of dot B, which is the last point of the mean camber line, can be solved in the same manner using the distance compensation method (Eq. 12).

By solving a quadratic equation with $\cot\alpha_1$ as the variant, the dependent variables are the coordinate value of the last point of mean camber line (m, M), the X-coordinate value of dot A (A), and angle $\tan\theta$. The relationship between chord length L and $\cot\alpha_1$ can be established as

$$\begin{aligned}\cot\alpha_1 = {} & 1/2TA\big[\sqrt{(L-M)} + MT + \big(\sqrt{(L-M)} \\ & + 2\sqrt{(L-M)}MT + M^2T^2 - 4T^2A^2 - 4TA \\ & + 4T^2\sqrt{(L-M)}A\big)^{(1/2)}\big]\end{aligned} \tag{13}$$

Similarly, both the relationship between $\cot\alpha_2$ and L, and that between P_m and L can be established. The compensation of the parameters used to control the bending deformation can be transformed into the compensation of chord length L.

Shape coefficient λ must first be determined using Eq. 11 to compensate for the bending deformation. Assuming that length of the mean camber line remains unchanged after investment casting, the entire mean camber line can be divided into equal infinite units on the basis of the calculus relationship (Fig. 8). Assuming that the length of the curve unit is sufficiently small, this curve unit can be interpreted as a straight line, which can be formed with a right-angled triangle by the chord length and height of segment H.

Figure 8 shows that the change in chord length is influenced by the variation in segment height. Therefore, shape coefficient λ can be chosen as $\Delta H_{i+1}/\Delta H_i$. ΔH represents the difference between the height of the segments of the measurement and CAD models. ΔH_{i+1} and ΔH_i represent the difference in the intervallic deformation between two iterative calculation steps.

The optimized chord length can be determined using Eq. 11. Meanwhile, the optimized blade inlet angle,

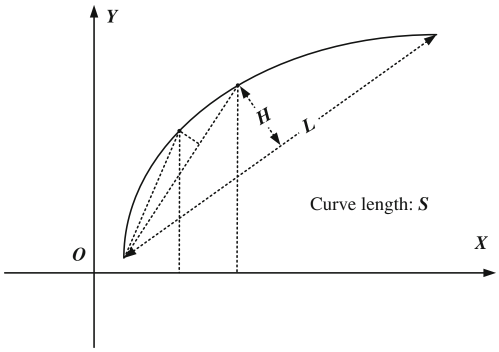


Fig. 8 Sketch of the constant curve length

optimized blade exit angle, and optimized maximum deflection position can be obtained by the relationship among $\cot\alpha_2$, P_m, and L. Consequently, the bending deformation can be compensated for.

For the torsional deformation of the turbine blade, stagger angle β can be used to represent this kind of deformation. Thus, the compensation process can be regarded as the optimization of β. As shown in Fig. 9, the solid line represents the schematic of the blade section, and the dashed line represents the section after torsional deformation. β_1 represents the stagger angle of the CAD model, and β_2 represents the stagger angle of the measurement model.

The torsional deformation can be expressed as $\Delta\beta = \beta_2 - \beta_1$. This equation can be rearranged as follows using Eq. 11:

$$\beta_m = -\left(\lim_{n\to\infty}\sum_{i=1}^{n}\Delta\beta^i\right)\Delta D + \beta_c$$
$$= -\frac{1}{1-\Delta\beta}\Delta D + \beta_c \qquad (14)$$

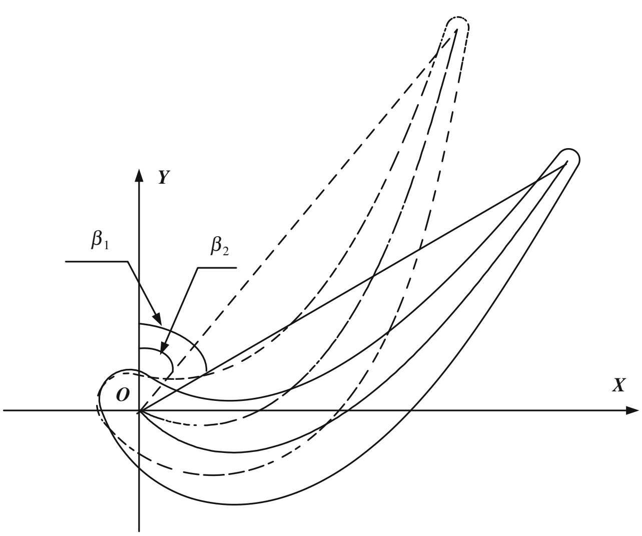


Fig. 9 Torsion of the discrete points of the section line

Fig. 10 CAD model of the turbine blade

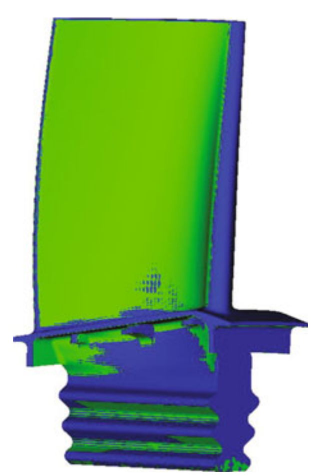
where β_m is the optimized stagger angle, ΔD is equal to $\Delta\beta$, and β_c is the stagger angle of the CAD model.

Therefore, the bending and torsional deformations can be correspondingly compensated for. The optimized die profile for the investment casting of aerofoil-shaped turbine blades can be established using the optimized geometric parameters as bases.

5 Determination of the optimized die profile on the basis of the mean camber line

The optimized mean camber line can be determined by the optimized geometric parameters determined in Section 4 using the parametric model of the mean camber line introduced in Section 2.2 as basis. With the establishment of the optimized mean camber line,

Fig. 11 Accurately aligned result



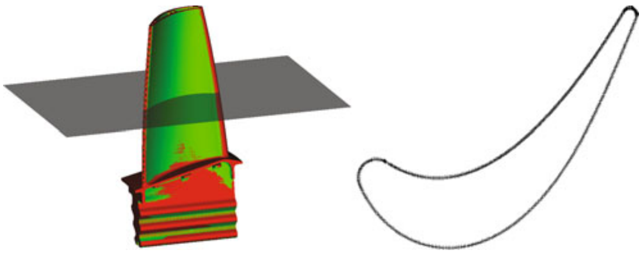


Fig. 12 Cross-section of the turbine blade at a given position ($Z = 50mm$)

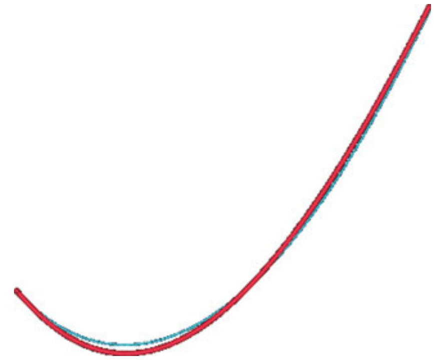


Fig. 13 Schematic of the optimized mean camber line ($Z = 50mm$)

the reconstruction of the die profile is carried out as follows.

1. First, the mean camber line is generated using the optimized parameters established by the compensation method.
2. Subsequently, the thickness distribution rule along the mean camber line is established using the given thickness control parameters.
3. On the basis of the first two steps, the envelope curve of the die profile section can be obtained.

Given that the actual turbine blade model is highly complicated, the complex shape and structure cause uneven heat dissipation during cooling, which in turn, also generates non-linear and non-uniform shrinkage distribution. Necessary simplifications should be made for the thickness distribution on the basis of the experiential parameter formulas of the die profile design, assuming that the shrinkage ratio is constant during the solidification process. Thus, the die profile is determined by the constant shrinkage rate, with an enlargement factor of 1.012.

6 Typical optimization example of a die profile of turbine blades

A certain type of turbine blade (Fig. 10) was used for optimization. An optical photography device, ATOS II, was adopted to obtain a set of real measurement data on a turbine blade, and the measurement data were expressed as an STL file. The accurately aligned result of the CAD and measurement models is shown in Fig. 11.

Table 2 Optimized parameters for mean camber line of the cross-section

Parameter	Theoretical value	Optimized value
α_1	40.769	39.1338
α_2	26.565	28.2000
P_m	(13.117, 6.154)	(13.732, 6.078)
L	32.401	32.8090

As an example, the extraction procedure and result for the case in which the section at the position where $Z = 50mm$ is chosen are shown in Fig. 12. P_m can be obtained on the basis of the optimized parameters of chord length L, blade inlet angle α_1, blade exit angle α_2, and maximum deflection position. The optimized parameters are shown in Table 2.

Using the optimized parameters, the mathematical model of the mean camber line can be established. The mean camber line model at the cross-section of $Z = 50mm$ is shown in Fig. 13. The thick line represents the optimized mean camber line, while the thin line represents the mean camber line of the CAD model.

The cross-section of the die profile can be established via the thickness distribution model (Fig. 14) because the optimized mean camber line model has been obtained. To decrease computational complexity, four sections at different heights ($Z = 28mm$, $Z = 31mm$, $Z = 50mm$, $Z = 63.5mm$) are chosen to reconstruct the die profile. On the basis of the surface reconstruction technology that uses UG NX7™ (Siemens PLM Software, Munich, Germany), the optimized die profile for turbine blade can be reconstructed.

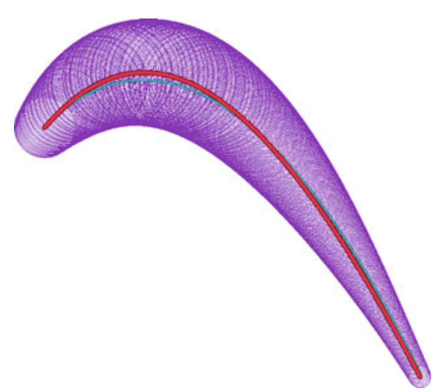


Fig. 14 Optimized cross-section of the die profile

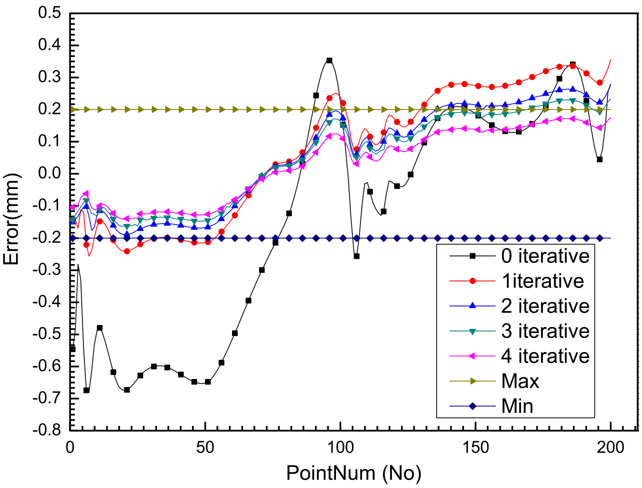


Fig. 15 Two-dimensional deviation of the numerical simulation and CAD models at the same height

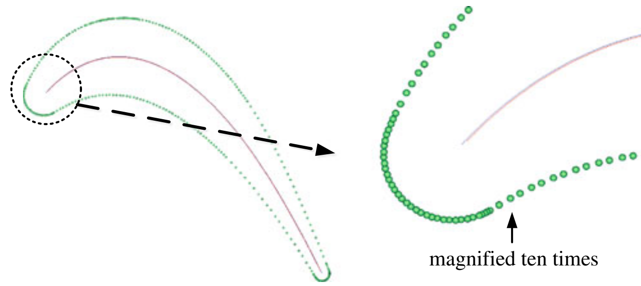


Fig. 16 Comparison of the mean camber lines after four iterations

Although the optimized die profile for the investment casting of the turbine blade is established, the die profile availability obtained by the proposed method still requires further investigation by numerical simulation. ProCAST™ (ESI Group, Paris, France) [24] and the finite element method (FEM) solver were used to obtain the displacement field of the XYZ directions.

To calculate two-dimensional deviation, the sections of the numerical simulation and CAD models, which are located at the same height ($Z = 50mm$), were intercepted. The deviation of two-dimensional cross-section, derived by four times iterative calculations, is shown in Fig. 15, and the calculation results are listed in Table 3.

As shown in Fig. 15, the two-dimensional deviations of the discrete points are all within the specific tolerances ($\pm 0.2mm$). During each iteration, the deviation significantly decreases. Moreover, after four iterations in the numerical simulation model, the two-dimensional mean deviation decreases to 0.11151 mm from the previous 0.35094 mm (Table 2) compared with that derived for the CAD model. Therefore, the optimization of the die profile for turbine blades proposed in this confirms a better effect on the die profile design of turbine blades. The comparison of the mean camber lines of the CAD and numerical simulation models after four iterations is shown in Fig. 16.

The three-dimensional CAD model of the optimized die profile for investment casting of aerofoil-shaped turbine blades can be obtained by reconstructing several optimized cross-sections, as shown in Fig. 17. The CAD model can be used to optimize the investment die (Fig. 18).

On the basis of the optimized result, the optimized investment cast die of the turbine blade was produced, and the experiment for evaluating the optimized result was conducted. Figure 19 shows the image of the optimized investment cast die.

A coordinate measuring machine (CMM; Brown & Sharpe Global Status 121510, Hexagon Metrology, Qingdao, China) was used to measure the airfoil of the casting blades that were produced by the optimized investment cast die (Fig. 19). For the measurement process, a touch-type probe is generally used; this probe carries out point-to-point motions to obtain the three-dimensional coordinates of the airfoil surface, usually one point at a time. In the experiment, four blades were measured. The measurement process is shown in Fig. 20.

The measured data derived by the CMM are the spherical center coordinates of the probe. Three-dimensional offset technology was used for the compensation of the probe, and the measured surface was reconstructed. The iterative closest point method [25]

Table 3 Statistical table of the 2-dimensional deviation

Number of iterations	Point number	Average	Standard deviation	Total deviation	Min deviation	Max deviation
0	200	−0.13526	0.35094	−27.05133	−0.68014	0.35308
1	200	0.07585	0.20723	15.17093	−0.25616	0.35524
2	200	0.05936	0.16152	11.87152	−0.18885	0.27766
3	200	0.05192	0.14087	10.38331	−0.16351	0.23327
4	200	0.03077	0.11151	6.15418	−0.14093	0.17463

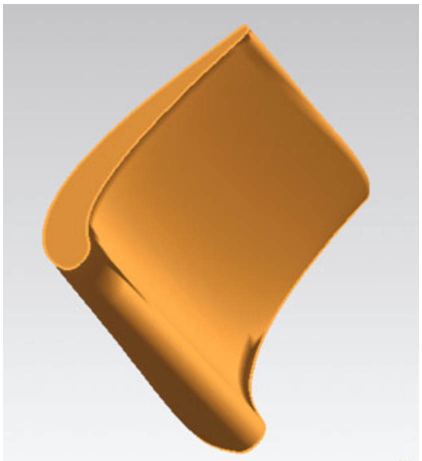


Fig. 17 Image of the optimized die profile of the turbine blade

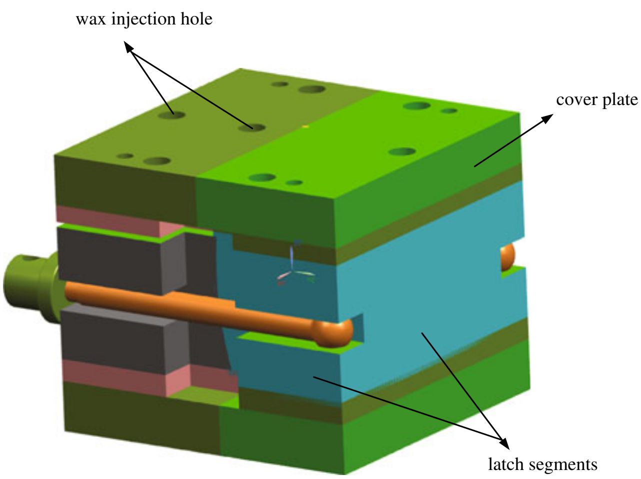


Fig. 18 Cavity optimization result

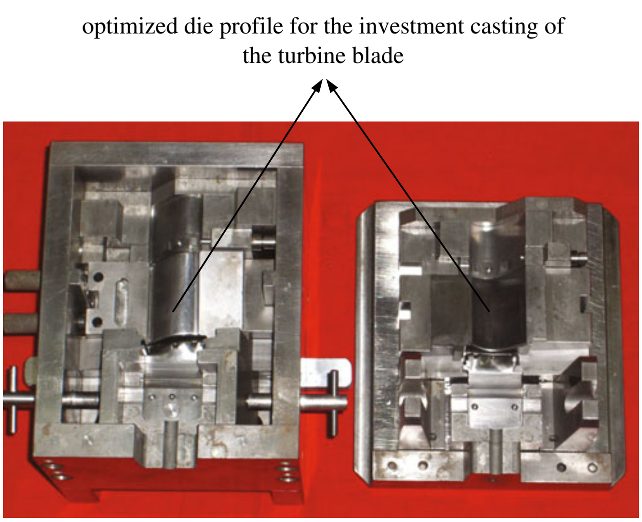


Fig. 19 Photograph of optimized investment cast die

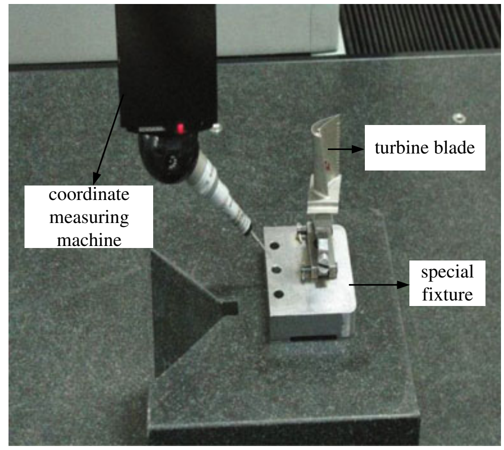


Fig. 20 Measurement of the casting blade

was employed to realize accurate registration of the CAD model and casting blades.

After registration, the next step is displacement calculation. To decrease computational complexity, five cross-sections were chosen for the calculation of the displacement (Fig. 21). The cross-sections were extracted and dispersed into 200 discrete points because of the number of measurement points of the CMM. Then, the distance between the discrete points of the measurement and CAD models were computed and saved for the evaluation of the optimized result.

The blade airfoils before and after optimization were cast under the same conditions. The displacements before and after optimization of the parts are shown

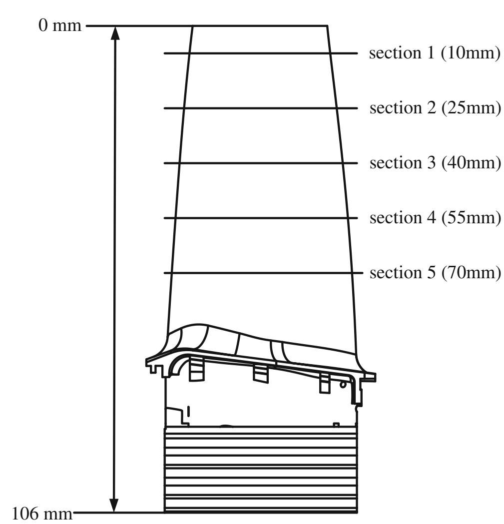


Fig. 21 Measurement of the cross-sections of the turbine blade

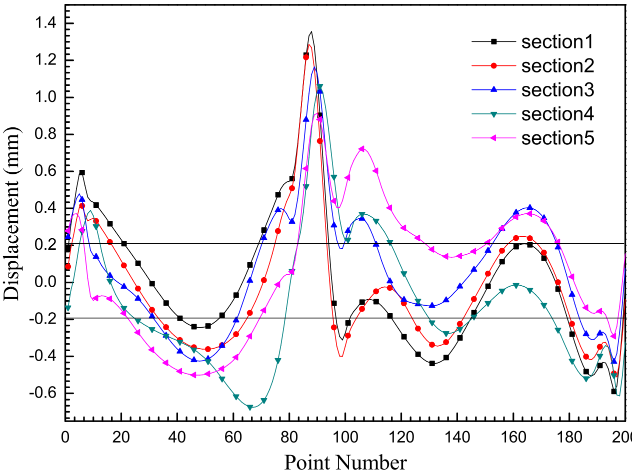


Fig. 22 Displacement before optimization

in Figs. 22 and 23. For convenient analysis, the mean values of the displacement of every discrete point of the four airfoils in each cross-section were calculated. From the comparison of Figs. 22 and 23, we can conclude that the displacement of the turbine blade is non-uniform. The LE and TE have larger shrinkage compared with the other parts; the shrinkage of the pressure component is larger than that of the blade back. Although there is a little difference between the result of simulation and that of the experiment, the displacement distribution tendency is similar. After optimization, the displacement decreases to a low level, the maximum displacement is 0.197*mm*, which occurs at the LE of the Section 1. The goal of die profile optimization is achieved.

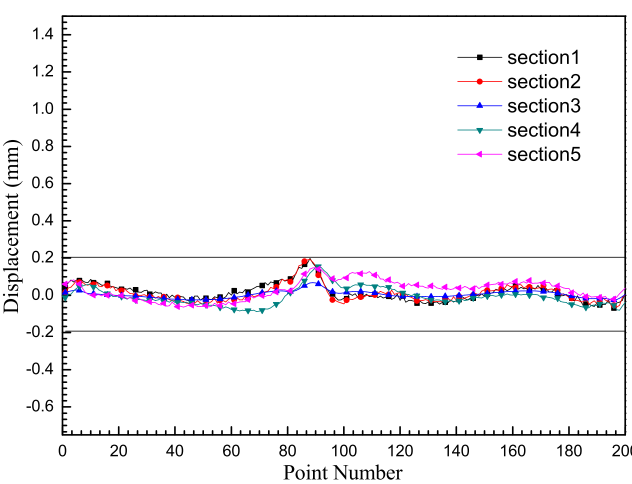


Fig. 23 Displacement after optimization

7 Conclusions

A novel method employed to optimize the die profile of the investment cast die for turbine blades, and its related key technologies, have been proposed in this paper. The conclusions drawn can be summarized as follows.

1. The geometric parameters that describe the mean camber line of the blade profile are identified, analyzed, and adjusted using the inverse iterative method to counteract blade deformations.
2. The optimized wax pattern die profile can be established with the reconstruction of the die profile; turbine blade deformation that occurs during the solidification process can be compensated for.
3. This method therefore has an advantage over the traditional linear scaling method. By investigating the investment casting of a typical blade, substantial reduction in dimensional and shape tolerances is achieved with the proposed die optimization method.

Acknowledgements This research was supported by Aviation Science Program of China with Grant No. 2008ZE53042. The authors would like to appreciate Yangliu Dou, Yingjie Xu and Ruisong Jiang for their generosity, useful discussions and helpful assistance.

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