

Evaluation of routing flexibility of a flexible manufacturing system using simulation modelling and analysis

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Abstract Routing flexibility is a major contributor of the flexibility of a flexible manufacturing system (FMS). The present paper focuses on the evaluation of the routing flexibility of an FMS with the dynamic arrival of part types for processing in the system. A typical FMS configuration is chosen for detailed study and analysis. The system is set at five different levels of routing flexibility. Operations of part types can be processed on alternative machines depending upon the level of routing flexibility present in the system. Two cases have been considered with respect to the processing times of operations on alternative machines. A discrete-event simulation model has been developed to describe the operation of the chosen FMS. The performance of the system under various levels of routing flexibility is analyzed using measures such as mean flow time, mean tardiness, percentage of tardy parts, mean utilisation of machines, mean utilisation of automatic-guided vehicles, and mean queue length at machines. The routing flexibility for producing individual part types has been evaluated in terms of measures such as routing efficiency, routing versatility, routing variety and routing flexibility. The routing flexibility of the system has been evaluated using these measures. The flexibility levels are ranked based on the routing flexibility measure for the system. The ranking

thus obtained has been validated with that derived using fuzzy logic approach.

Keywords Flexible manufacturing system · Routing flexibility · Simulation · Fuzzy logic

1 Introduction

There is an increasing trend towards higher product variety, smaller lot sizes and shorter lead times in the market place. In this environment, manufacturing companies are forced to implement systems that can provide flexibility and efficiency. Emergence of flexible manufacturing systems (FMSs) is an important development in this direction. MacCarthy and Liu [1] state that a flexible manufacturing system is a production system in which groups of numerically controlled or computer numerically controlled machine tools and an automated Material Handling System (MHS) work together under computer control. Goswami et al. [2] state that the objective of FMSs is to achieve the efficiency of transfer lines, while maintaining the flexibility of low volume job shops. Stecke [3] identifies four hierarchical levels in which the decision problems in FMS are partitioned: design, planning, scheduling and control problems. Scheduling of FMSs continues to attract the interest of both the academic and industrial sectors. Chan et al. [4] report that simulation is the most widely used tool for modelling FMSs. Wang and Chatwin [5] observe that discrete-event simulation is an integral business tool giving flexibility and convenience in designing, planning and analysing complex manufacturing systems.

Flexibility is an important feature that distinguishes FMSs from other manufacturing systems. Chan [6] defines flexibility of a manufacturing system as the ability to cope

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with changes in product mix, volume, or timing of its activity in an efficient and effective manner. According to Koste and Malhotra [7], the competitive potential of flexibility at the organisational level is widely recognised by managers, leading many to proclaim flexibility as the next competitive battle. The flexibility of an FMS is dependent upon its components (machines, MHS, etc.), capabilities, interconnections, and the mode of operation and control. Routing flexibility can be regarded as the main contributor to the flexibility of an FMS. It is the ability of a system to provide multiple alternate routes to produce a set of parts economically and efficiently. The greater the number of alternate options available for the production of a part, the more flexible the system is. Although flexibility has drawn attention from researchers and practitioners, measurement of flexibility continues to be a difficult task as described by Beskese et al. [8]. Tidd [9] states that flexibility is a relative attribute, rather than an absolute one. Swamidass and Newell [10] measure flexibility in aggregate and Upton [11] measures it for individual dimensions. Chang et al. [12] state that two attributes of flexibility, namely efficiency and versatility, should be considered in the measurement of flexibility. Chang [13] extends this approach by considering another attribute for measurement, namely routing variety.

The purpose of this research is to evaluate the routing flexibility of an FMS using simulation-based experimentation for a dynamic order arrival environment. In the existing literature, Chang [13] evaluates the routing flexibility of a job shop production system operating in a static environment by considering the attributes such as routing efficiency, routing versatility and routing variety. Yu and Greene [14] present a method for the measurement of routing flexibility for a static multi-stage flow shop using the available routes and the work load of the machines. In both these recent works, no detailed analytical/simulation modelling of the system has been carried out. Furthermore, a realistic evaluation of the system performance requires the consideration of operational control decisions. The present work deals with the measurement of routing flexibility of a typical FMS operating with a dynamic arrival of part types. A discrete-event simulation model has been developed to describe the operation of the FMS under different flexibility levels. Two cases have been considered with respect to the processing times of operations on alternative machines. The operational control decisions such as part launching, part routing and part sequencing are also considered. The attributes of the routing flexibility measures are characterised by the factors such as the number of alternative routes, the efficiency of the routes, the variety of the routes and the uniformity of the routes. Detailed simulation experimentation has been carried out. The simulation results have been subjected to statistical analysis. This paper constitutes a

systematic study of the behaviour of an FMS in relation to its flexibility. Hence, evaluation of the various measures of routing flexibility for an FMS with a dynamic arrival pattern of part types is a significant contribution of this research work. It is specifically intended to rank the flexibility levels of the FMS based on the value of routing flexibility measure. The ranking thus obtained has been validated with that derived using fuzzy logic approach. Fuzzy logic provides a method for synthesizing the attributes such as routing efficiency, routing versatility and routing variety, which are not homogeneous. Thus, this paper makes a novel contribution to the literature.

The remaining sections of the paper are organised as follows. Section 2 provides a review of the relevant literature. The approach adopted in the present study is briefly outlined in section 3. The methodology used for the measurement of routing flexibility is also described in this section. The salient aspects of the development of simulation model are presented in section 4. Section 5 presents the details of experimentation. The evaluation of routing flexibility is illustrated in section 6. The analyses of results are provided in section 7. The validation of routing flexibility measures is described in section 8. Finally, conclusions are presented in section 9.

2 Literature review

The literature on manufacturing flexibility describes several types of flexibility. Browne et al. [15] describe eight types of flexibility as follows: machine flexibility, process flexibility, product flexibility, routing flexibility, volume flexibility, expansion flexibility, operation flexibility and production flexibility. Gupta and Goyal [16] provide a review of the concepts of flexibility. Sethi and Sethi [17] enhance the types of flexibility to include the flexibility types such as material handling flexibility, program flexibility and market flexibility. Vokurka and O'Leary-Kelly [18] describe four additional flexibility dimensions such as automation flexibility, labour flexibility, new design flexibility, and delivery flexibility.

The operational decisions of an FMS can be categorised as pre-release (planning) and post-release (scheduling) decisions. The machine loading problem is an important pre-release decision problem. It is concerned with the allocation of operations of parts to be produced in the ensuing planning period and their associated tools to machines to meet the specified objectives subject to the technological and capacity constraints of the system. Kumar et al. [19] state that the loading problem encompasses various types of flexibility aspects pertaining to part selection and operation assignments along with constraints ranging from simple algebraic to potentially very complex

conditional constraints. Formulations and solution techniques for solving the machine loading problems in an FMS have been presented by several researchers including Berrada and Stecke [20], Mukhopadhyay et al. [21], Nayak and Acharya [22], and Tiwari et al. [23]. Grieco et al. [24] provide a survey of different approaches proposed in the literature to solve the loading problem. They describe the characteristics of the FMS that affect the loading problem.

Vidyarthi and Tiwari [25] use fuzzy logic approach for the operation-machine allocation decisions in an FMS. Each operation-machine allocation is judged by its contribution to the objectives and constraints of the loading problem by framing appropriate membership function. Gamila and Motavalli [26] analyse the problems of part loading, tool loading, part routing, and part scheduling using an integrated planning model. Swarankar and Tiwari [27] have considered the machine loading problem with the bicriterion objectives of minimising system unbalance and maximising throughput. A hybrid algorithm based on tabu search and simulated annealing is employed to solve the problem. Bilkay et al. [28] propose a two-stage approach for the machine loading and scheduling problems. In the first stage, a fuzzy logic-based algorithm for assigning priorities to part types that are to be machined is proposed. In the second stage, an operation-machine allocation and scheduling algorithm is presented. The proposed algorithm can re-generate the schedule in case of a machine breakdown, and therefore can be used as an on-line controller. Prabakaran et al. [29] consider the operation-tool scheduling problem in a flexible manufacturing cell consisting of several machines and a common tool magazine. Two heuristic algorithms namely, priority dispatching rules algorithm and simulated annealing algorithm are proposed to derive optimal solutions.

Kumar et al. [19] propose a methodology known as constraint-based genetic algorithm to handle a complex variety of variables and constraints in a typical FMS loading problem. Three constraint-based genetic operators are introduced which help avoid getting trapped at local minima. Tiwari et al. [23] address the combined job sequencing and machine loading problem using minimization of system unbalance and maximization of throughput as objective functions, while satisfying the constraints related to available machining time and tool slots. They describe two heuristics to deal with the problems. Heuristic I uses predetermined fixed job sequencing rules whereas heuristic II uses genetic algorithm-based approach. Biswas and Mahapatra [30] propose a meta-heuristic approach based on particle swarm optimization to improve the solution quality and reduce the computational effort in solving the machine loading problem in an FMS. Das et al. [31] investigate the issues of machine loading, tool allocation, and part type grouping with the intent of

developing an operation-sequencing technique capable of optimizing operation time, non-productive tool change times, and orientation change times when processing a group's design features. Integer programming models are formulated to group the parts and to address the operation-sequencing problem. Zeballos et al. [32] present an integrated constraint programming model to tackle the problems of tool allocation, machine loading, part routing, and scheduling in an FMS. The formulation, which is able to take into account a variety of constraints found in industrial environments, as well as several objective functions, has been successfully applied to the solution of various case studies of different sizes.

Chung and Chen [33] propose a routing flexibility measure based on the average number of available routes for each part type. Lin and Solberg [34] indicate that flexible processing could reduce mean flow time while increasing system throughput and machine utilisation. Bernardo and Mohamed [35] utilise the inverse of the number of available routes as a term of the routing flexibility measure. Barad [36] investigates the relative impact of versatility as a physical characteristic and operating strategies on FMS performance. Das and Nagendra [37] measure routing flexibility as the sum of the average differences between each route and all other routes. The difference between two routes is expressed as a function of the difference in processing time for each machine. Benjaafar and Ramakrishnan [38] suggest a hierarchical classification of flexibility as being either product or process related. Generally, entropy is used to measure the uncertainty or disorder of a system. The entropy approach has been applied for the measurement of operation flexibility and routing flexibility. Chang et al. [12] state that the entropy approach could be used for depicting the meaning of versatility rather than flexibility. They combine efficiency and versatility as a revised entropy approach for measuring single-machine flexibility.

Garavelli [39] reports on a simulation study conducted to analyse the performance of several flexible manufacturing systems, each of which is characterised by a specific degree of routing flexibility. The researcher finds that instead of complete flexibility, a system with limited flexibility performs better in terms of lead time and work-in process. Mohammed et al. [40] present a study wherein the relationship between the degree of machine flexibility and the level of system performance is analysed. Chan et al. [4] present a review of scheduling studies of FMS, which employ simulation as an analysis tool. Chan [6] reports on a simulation study of the effects of dispatching and routing decisions on the performance of an FMS. ElMekkawy and ElMaraghy [41] use flexible routing in order to avoid system deadlock caused by machine breakdowns and downtimes. Chang [13] proposes a multi-attribute approach

to measure routing flexibility that incorporates routing efficiency, routing versatility and routing variety. Bilge et al. [42] describe product flexibility as a potential flexibility and its utilisation during execution as routing flexibility. They state that the realised level of routing flexibility depends on the technological capabilities and the operational control strategies.

Fuzzy logic is used to deal with problems in which a source of vagueness is involved [43]. Due to the characteristics of impreciseness and vagueness, environmental variables can be expressed as fuzzy variables and used in the scheduling decision. Yu et al. [44] propose a fuzzy inference-based scheduling decision for FMSs with multiple objectives. Chan et al. [45] present a fuzzy approach for operation and routing selection via simulation. The proposed fuzzy approach is compared with conventional selection rules. Chan et al. [46] extend their fuzzy approach for machine loading at multiple decision points in an FMS. Srinoi et al. [47] propose a fuzzy logic approach to select a part for the next operation on a machine. The input variables include machine-allocated processing time, machine priority, due date priority, setup time priority, and the part priority is the output fuzzy variable. Caprihan et al. [48] provide a fuzzy dispatching strategy for due-date scheduling of FMSs with information delays. Information delays, called status review delays, occur when information is reviewed only at fixed intervals of time. A fuzzy dispatching strategy is developed for deployment within FMSs where information delays exist.

Yazgan [49] present a fuzzy analytical network process approach for the selection of dispatching rules. The model contains different performance criteria, details of FMS information, a company's strategic criteria, and different dispatching rules. The fuzzy information is introduced in the evaluation process. Lu and Liu [50] develop a dynamic dispatching strategy for multiple performance measures based on fuzzy inference. Initially, the variables affecting the system performance are identified. Then, the fuzzy membership functions and the fuzzy inference rules are established based on the simulation data. According to the statuses of environment variables, the fuzzy inference is performed to find an appropriate dispatching rule at each decision point to meet the best multiple performance measures.

Sometimes, flexibility parameters cannot be accurately defined as for instance, the versatility of a workstation. Tsourveloudis and Phillis [51] suggest a knowledge-based method that consists of an implementation of fuzzy logic approach to assess manufacturing flexibility. Knowledge is represented via If (fuzzy antecedents) and THEN (fuzzy consequent) rules, which are used to draw conclusions about the value of flexibility. Beskese et al. [8] evaluate flexibility elements in monetary terms using fuzzy

approaches based on mathematical programming and present worth analysis.

The present paper focuses on the measurement of routing flexibility of an FMS using a discrete-event simulation model. The purpose of this research is to evaluate the routing flexibility of an FMS using the measures such as routing efficiency, routing versatility and routing variety for the situation wherein part types to be produced in the system arrive continuously in a random manner.

3 Approach adopted

The objective of this research is to evaluate the routing flexibility of the FMS considered for detailed study and analysis. The first step has been to establish the FMS configuration and data regarding the processing requirements of part types to be produced in the system. Then, operational control procedures have been established in terms of how the orders for part types arrive, how the parts are launched into the system, how these launched parts are routed through various machines and how parts are sequenced for processing on a machine. Operations of part types can be processed on alternative machines depending upon the level of routing flexibility present in the system. A discrete-event simulation model has been developed to describe the operation of the chosen FMS. Mean flow time is used as the performance measure for the evaluation of routing flexibility. The multi-attribute approach proposed by Chang [13] has been appropriately modified for the evaluation of routing flexibility for producing individual part types and the total routing flexibility of the system. The routing flexibility thus obtained is validated using fuzzy logic-based approach. How all these aspects are carried out is described in the following sections.

3.1 Measurement of routing flexibility

The three attributes of routing flexibility are described as follows:

3.1.1 Routing efficiency

Throughput time or flow time of a part can be considered as a comprehensive measure of routing efficiency. Determination of routing efficiency of a route involves comparing the flow time of the route with the minimum flow time in the set of routes. Thus, the efficiency of route i in producing part j denoted as e_{ij} is expressed as

$$e_{ij} = \frac{\text{Min}\{F_{ij}\}}{F_{ij}} \quad i = 1, 2, \dots, r \quad (1)$$

where r is the number of possible processing routes for producing part j ; i, j are the subscripts for route and part respectively and F_{ij} is the flow time of route i in producing part j . The routing efficiency on producing part type j can be calculated as the average of the total efficiency values of the routes is as follows:

$$E_j = \frac{1}{r} \sum_{i=1}^r e_{ij} \tag{2}$$

3.1.2 Routing versatility

Routing versatility implies that the more the number of routes available for producing a part, the more the flexibility of the system is. Furthermore, the greater the similarity of performance outcomes of the routes set, the more flexible is the system. The entropy approach proposed by Chang et al. [12] satisfies these two features of flexibility. Thus, using the entropy approach for measuring single-machine flexibility, the routing versatility in a multi-machine FMS is determined as follows. The normalised value of routing efficiency of part j produced by route i denoted as α_{ij} is computed as

$$\alpha_{ij} = \frac{e_{ij}}{\sum_{i=1}^r e_{ij}} \tag{3}$$

The versatility of route i in producing part j denoted as V_{ij} is calculated as follows.

$$V_{ij} = -\alpha_{ij} \log_{10} \alpha_{ij} \tag{4}$$

The routing versatility of the system in producing part type j with r routes is R_j which is computed as follows.

$$R_j = \sum_{i=1}^r V_{ij} \tag{5}$$

3.1.3 Routing variety

Routing variety is defined as the difference among the routes of producing a specific part. Generally, the greater the differences among the routes, the higher the degree of variety and hence the higher the degree of flexibility. The difference between two routes can be calculated as the ratio of the number of different machines visited to the total number of machines visited in the two alternative routes.

Thus, the similarity function s_{il} is expressed as follows.

$$S_{il} = \frac{R_i \cap R_l}{m} \tag{6}$$

where R_i and R_l denote the two sets of machines, which are both capable of producing the particular part for a pair of

routes i and l , and m denotes the total number of machines visited within route i or l . The numerator in the similarity function denotes the common machines within routes i and l . Thus, the difference function d_{il} is expressed as

$$d_{il} = 1 - s_{il} \tag{7}$$

Routes i and l will show no difference when the machines in both the routes are all the same; whereas the difference will be at its maximum when there are no common machines visited between the two routes. If there are r routes in the feasible routes set of a part type j , there will be $r(r-1)$ pair wise comparisons. Hence, the total difference of the routes set for part type j , D_j is obtained from the following equation.

$$D_j = \frac{1}{r(r-1)} \sum_{l=1}^r \sum_{i=1}^r d_{il} \tag{8}$$

3.1.4 Routing flexibility of the system

The routing flexibility (RF) of the system in producing k part types is obtained as

$$RF = \frac{1}{k} \sum_{j=1}^k E_j \times R_j \times D_j \tag{9}$$

4 Development of the simulation model

The various aspects pertaining to the development of the simulation model are identified as follows:

4.1 Physical configuration

A typical FMS is considered for investigation in the present study. The FMS consists of six different (non-identical) machines with local input and output buffers, two automatic-guided vehicles (AGVs) as the material handling system for part transportation and a load/unload station as shown in Fig. 1.

4.2 Modelling routing flexibility

The machines in the system perform 15 different operations. An operation can be performed on alternative machines depending upon the level of routing flexibility present in the system. The system can be set at five different levels of routing flexibility. The various routing flexibility levels (RFLs) considered in the present study are summarised in Table 1.

As shown in Table 1, RFL 1 denotes the situation wherein each operation has exactly two alternative

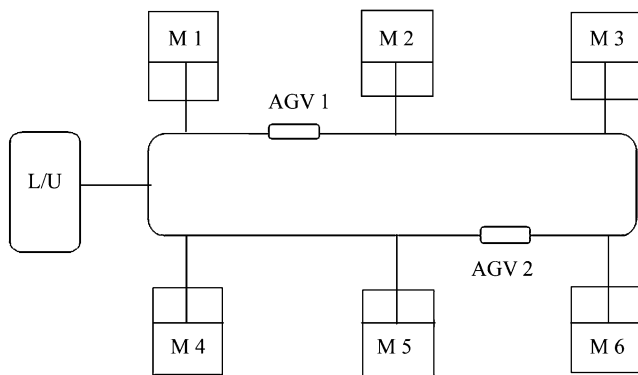


Fig. 1 Physical configuration of FMS

machines. RFL 4 represents the situation wherein each operation has exactly one alternative machine. The other three flexibility levels (RFL 2, RFL 3, and RFL 5) correspond to three different combinations of alternative machines for operations. It can be noted that in the present study, there are 15 types of operations that can be processed using six machines. Five different combinations of alternative machines (resulting in five different routing flexibility levels) have been considered for analysis. It is possible to consider more alternative machines for each operation. The various possible combinations of alternative machines for operations will lead to a very large number of routing flexibility levels. Since the objective of the present study is to evaluate the routing flexibility of FMS, only five levels have been considered. Based on these different levels of routing flexibility, the operation-machine compatibility data are shown in Table 2.

In the present study, it is considered that the processing time on the secondary machine has been increased by 10% of the processing time on the primary machine and the processing time on the tertiary machine is increased by 20% of the processing time on the primary machine. It can be noted that the alternative machines for an operation cannot be more efficient than the primary machine. The increased processing time on alternative machines represent the combined change in the characteristics such as machine efficiency, machine setup time, tool changing time etc. The case of no increase in processing times on alternative machines has also been considered in the simulation experiments.

4.3 Part data

Ten different part types are considered for processing in the FMS. Orders for part types to be produced arrive at the system randomly. Arrival of an order for a part type

among the ten part types is equally likely. The details regarding the orders for part types are generated as described below:

- The interarrival time of orders follows an exponential distribution with a mean of 20 min.
- The number of operations for each part type is uniformly distributed in the range 4–6.
- The processing time for an operation on the primary machine is uniformly distributed in the range 10 to 20 min.
- The operation type of an operation is uniformly distributed in the range 1 to 15.

4.4 Operational configuration

For the FMS considered, orders for part types arrive randomly and the FMS is run continuously. In the simulation model, the initial status of the system is assumed to be empty and idle with the first order arrival event scheduled to occur at time zero. The order can belong to any one of the ten part types with the same likelihood. After identifying the part type associated with the order arrived, the attributes associated with the part type such as the number of operations, the processing sequence of operations, machines for the operations and processing times for the operation are determined from the part data file. The raw part is loaded onto the pallet for subsequent launching into the system based on *part launching rule* (shortest processing time (SPT)). The parts are then routed to the machines for processing. When more than one machine is available for performing an operation, the machine on which the operation can be finished earliest is chosen, i.e., the part routing rule

Table 1 Routing flexibility levels and their definitions

Routing flexibility level (RFL)	Definition
RFL 1	Each operation has exactly 2 alternative machines
RFL 2	For some operations, there is 1 alternative machine and for the other operations, there are 2 alternative machines
RFL 3	For some operations, there are 2 alternative machines; for some other operations, there is 1 alternative machine; and for some other operations, there are no alternative machines
RFL 4	Each operation has exactly one alternative machine
RFL 5	For some operations, there is one alternative machine and for the other operations, there are no alternative machines

Table 2 Operation-machine compatibility data

	Operation type	Routing flexibility level (RFL)				
		RFL 1	RFL 2	RFL 3	RFL 4	RFL 5
O_1		M_1, M_5, M_2	M_1, M_5, M_2	M_1, M_5, M_2	M_1, M_5	M_1, M_5
O_2		M_6, M_5, M_3	M_6, M_5, M_3	M_6	M_6, M_5	M_6
O_3		M_5, M_6, M_4	M_5, M_6	M_5, M_6, M_4	M_5, M_6	M_5, M_6
O_4		M_4, M_6, M_5	M_4, M_6	M_4	M_4, M_6	M_4
O_5		M_3, M_2, M_4	M_3, M_2, M_4	M_3, M_2	M_3, M_2	M_3, M_2
O_6		M_6, M_4, M_1	M_6, M_4, M_1	M_6, M_4, M_1	M_6, M_4	M_6, M_4
O_7		M_6, M_3, M_2	M_6, M_3	M_6, M_3	M_6, M_3	M_6, M_3
O_8		M_5, M_2, M_3	M_5, M_2, M_3	M_5, M_2, M_3	M_5, M_2	M_5, M_2
O_9		M_4, M_1, M_6	M_4, M_1	M_4, M_1	M_4, M_1	M_4, M_1
O_{10}		M_3, M_1, M_5	M_3, M_1, M_5	M_3, M_1, M_5	M_3, M_1	M_3, M_1
O_{11}		M_2, M_4, M_6	M_2, M_4, M_6	M_2	M_2, M_4	M_2
O_{12}		M_1, M_3, M_2	M_1, M_3	M_1, M_3	M_1, M_3	M_1, M_3
O_{13}		M_1, M_4, M_3	M_1, M_4	M_1, M_4	M_1, M_4	M_1, M_4
O_{14}		M_2, M_5, M_4	M_2, M_5	M_2	M_2, M_5	M_2
O_{15}		M_3, M_6, M_1	M_3, M_6, M_1	M_3	M_3, M_6	M_3

considered is earliest finishing time with alternatives (EFTA). This routing rule aids in minimising the throughput time (flow time) of parts. The parts are then released to the machines according to the part routing rule as soon as AGVs are available. The parts waiting at the input buffer of a machine are selected for processing based on part sequencing rule (shortest processing time (SPT)). After a machine completes processing an operation of a part, the machine releases the part on the output buffer and places an AGV call to remove the part. If an AGV is available, then it travels to the machine and picks up the part. If all the operations of the part are completed, the finished part will be unloaded at the load/unload station. Else, the machine for the next operation in the part-route is identified using the part routing rule and the AGV takes the part to the machine. After an AGV unloads a part at the input buffer of a machine, it tries to attend the pending calls. If there are no calls to be met, then an attempt is made to find whether any raw part can be launched into the system.

4.5 Assumptions

The following are the assumptions made regarding the operational aspects of the FMS for developing the simulation model:

- Processing times of operation of part types include setup times and tool changing times and are independent of the sequence followed.
- The transportation time for the AGVs is proportional to the distance travelled.

- AGVs after completing the task assigned can remain near the machines or at the load/unload station as the case may be.
- After completing any material transfer, an AGV tries to attend the pending calls. The calls for AGVs can be of two types: (1) Providing an input to a machine, and (2) removing the finished part from the output buffer of a machine. The second type of call is given priority so that machine blocking can be avoided.
- Upon job completion at any machine, if more than one AGV is available to transfer the part to the next machine or the load/unload station, the one closest to the current machine is selected.
- The machines and AGVs are continuously available.

4.6 Structure of the simulation model

In the present study, a discrete-event simulation model that can capture the logic of the different levels of routing flexibility and the operational decisions of the FMS has been developed using the C programming language. The discrete-event model views the FMS as consisting of entities, their associated attributes and files which contain entities with common characteristics. The entities in the FMS are parts, machines and AGVs. The operation of the FMS is conceptualised as a succession of events centering on the parts to be processed. The appropriate events are suitably generated for capturing the dynamics that are taking place in the system. The simulation model is structured in a modular way

consisting of a number of modules each of which performing a specific role. The modules included in the simulation model are as follows:

- Main program
- Data generation module
- Event routines module
- Output module
- File processing module
- Initialisation module
- Timing module
- Scheduling module
- Random sample generation module

The salient features of the modules such as event routines module, scheduling module, and output module are presented in the following subsections.

4.6.1 Event routines module

This module contains the subroutines that deal with the following events that characterise the operation of the FMS.

- Arrival of an order for a part type.
- A raw part is loaded on AGV at the load/unload station.
- AGV unloads a part at the input buffer of a machine.
- Machine starts processing an operation of a part.
- Machine finishes processing an operation of a part.
- Machine unloads a part at its output buffer.
- AGV picks up a part from the output buffer of a machine.
- AGV unloads a finished part at the load/unload station.
- Empty AGV reaches the load/unload station.

4.6.2 Scheduling module

This module contains subroutines to deal with the scheduling decisions such as part launching, part routing and part sequencing as described below.

- The part launching decision involves selecting a part to be loaded on a pallet at the load/unload station for launching into the system. For making this decision, the SPT rule is used. i.e. the part with the shortest total average processing time is selected for launching.
- The part routing decision necessitates selecting the machine for performing an operation of a part. The scheduling rule used is EFTA. When more than one machine is available for performing an operation, the machine on which the operation can be finished earliest is chosen.

The finishing time of an operation consists of the following components:

- Sum of the processing time of the operations of parts waiting in the queue of the machine that is capable of processing the operation (workload of the machine).
- Remaining processing time of the machine for completing its current operation.
- Processing time of the operation of the part that is considered for assignment.
- Transportation time involved in moving the part from current location to the machine.

Thus, EFTA denotes the earliest time at which the operation (to be selected) will be completed on the machine if the operation is assigned to the machine.

- The part sequencing decision is required for selecting a part to be processed on a machine from among the parts waiting at the input buffer of the machine. SPT is the scheduling rule used for the part sequencing decision. i.e. the part with the shortest processing time for the imminent operation is chosen for processing.

4.6.3 Output module

This module performs the task of consolidating the output of the simulation model to present the results such as mean flow time, mean tardiness, percentage of tardy parts, mean utilisation of machines, mean utilisation of AGVs and the average queue size at machines. The flow time for each route of each part is also computed.

The simulation model has been subjected to a multi-level verification and validation exercise.

5 Experimentation

Using the simulation model as a test-bed for experimentation, a number of experiments have been conducted. The objective of the experimentation is to evaluate the system performance and the routing flexibility of the FMS when the system is set at different levels of routing flexibility. Ten scenarios have been considered for experimentation. Scenarios 1 to 5 correspond to the five routing flexibility levels RFL 1 to RFL 5, respectively, under the condition of penalty for processing times on alternative machines (denoted as case 1), whereas scenarios 6 to 10 deal with the five routing flexibility levels RFL 1 to RFL 5 respectively under the condition of no penalty for processing times on alternative machines (denoted as case 2).

The first stage in the simulation experimentation involves determining the end of the initial transient period (identification of the steady state). For this purpose, Welch's procedure described in Law and Kelton [52] is used. In a pilot simulation study conducted for the FMS considered in the present research, it was found that the system reached steady state when 200 parts were completed. Hence, in the simulation experiments for the scenarios, ten replications are performed for each scenario. The simulation for each replication is run for the completion of 1,200 parts. Parts are numbered on arrival at the system and the simulation outputs from parts numbering 1 to 200 are discarded. The outputs for the remaining 1,000 parts (parts numbering 201 to 1,200) are used for the computation of the performance measures.

In a steady state simulation, each simulation run can be divided into two phases: an initialisation phase (initial transient period) followed by a data-collection phase. Davies and O'Keefe [53] suggest that the data-collection period in steady state simulation should be set at least equal to the initial transient period. Law and Kelton [52] state that for a given sample size, it is preferable to have a smaller number of replications and a larger run length. Rangsaritratamee et al. [54] take the data-collection period as four times the length of the initial transient period. Jayamohan and Rajendran [55] consider the data-collection period as three times the length of the initial transient period. In the present study, data-collection period is taken as five times the length of the initial transient period. Hence, the run length has been fixed as six times the length of the initial transient period. The required number of replications for the simulation experiments is determined using the method suggested by Banks et al. [56]. This method involves determining the number of replications

for a specified error and significance level. It requires conducting a pilot simulation study with three or five replications. Using this pilot study, the number of replications required for the simulation experiments is obtained.

For the purpose of computation of routing flexibility of each part type, the route followed and the flow time of parts following the route are determined for each replication and for each scenario. The values of routing efficiency, routing versatility, routing variety and routing flexibility are determined for each part type for case 1 and case 2.

6 Illustration of the computation of routing flexibility

For the purpose of illustration, the computation of the attributes of routing flexibility for part type 1 is described as follows. Scenario considered: 1; routing flexibility level, $RFL=1$; case 1 (penalty for processing times on alternative machines). From the simulation output, the route followed by the part and the flow time of the part on each route is obtained. A sample of the simulation results for ten parts of part type 1 is given in columns 1 to 4 in Table 3.

6.1 Computation of routing efficiency

The routing efficiency of individual part on each of the routes is computed as follows. Routing efficiency of part 1 is computed using Eq. 1 as $e_{1j}=0.96$. Similarly, the routing efficiency of parts 2 to 10 is computed. These values are shown in column 5 in Table 3. When more than one part follows the same route (for example, parts 2 and 3), the routing efficiency of the route is calculated as the average

Table 3 Sample data for illustration

Part no.	Route no.	Route	Mean flow time	Routing efficiency of individual part on route	Routing efficiency of individual route	Normalised routing efficiency	Routing versatility
1	1	2, 1, 1, 2, 5	95.31	0.96	0.96	0.23	0.15
2	2	2, 3, 1, 2, 4	167.20	0.55	0.50	0.12	0.11
3		2, 3, 1, 2, 4	205.50	0.45			
4	3	2, 5, 5, 3, 4	162.05	0.57	0.51	0.12	0.11
5		2, 5, 5, 3, 4	204.10	0.45			
6	4	3, 3, 2, 2, 4	151.31	0.61	0.55	0.13	0.12
7		3, 3, 2, 2, 4	185.35	0.49			
8	5	5, 1, 1, 2, 4	91.59 ^a	1.00	0.91	0.22	0.14
9		5, 1, 1, 2, 4	110.94	0.83			
10	6	5, 5, 5, 6, 4	129.53	0.71	0.71 0.69 ^b	0.22	0.13 0.76 ^c

^a Minimum mean flow time

^b Routing efficiency of part type 1

^c Routing versatility of part type 1

Table 4 ANOVA results for two-factor analysis for performance measures

Source of variation	<i>F</i> ratio for performance measures					
	Mean flow time	Mean tardiness	Percentage of tardy parts	Mean utilisation of machines	Mean utilisation of AGVs	Mean queue length
Main effects						
A. Routing flexibility level	27.20 ^a	53.36 ^a	81.84 ^a	1.16	32.84 ^a	61.89 ^a
B. Case (with/without penalty)	67.74 ^a	48.35 ^a	83.47 ^a	35.54 ^a	13.60 ^a	19.68 ^a
Interactions between A and B	75.51 ^a	15.62 ^a	38.11 ^a	1.33	1.83	0.99

^a Denotes *F* ratio significant at 5% significance level

of the routing efficiency of the individual parts. In a similar manner, the routing efficiency of other routes for part type 1 are calculated and these results are shown in Column 6 in Table 3. Hence, the routing efficiency of part type 1 is obtained as the average of the routing efficiency of the individual routes 1 to 5 using Eq. 2 as $E_I=0.69$.

6.2 Computation of routing versatility

At first, the normalised routing efficiency of each route is computed and then the versatility of each route is determined. For example, the normalised routing efficiency of route 1 for part type 1 is determined using Eq. 4 as $\alpha_{11}=0.23$. Similarly, the normalised routing efficiency of other routes 2, 3, 4, 5, and 6 are calculated. These values are shown in column 7 in Table 3. Routing versatility of part type 1 is computed using Eq. 5 as $v_{11}=0.15$. In a similar manner, routing versatility of the other routes 2, 3, 4, 5, and 6 are computed and these results are shown in column 8 in Table 3. Thus, the routing versatility of part type 1 is determined as the sum of the routing versatility of the individual routes.

6.3 Computation of routing variety

This requires the computation of similarity function between every pair of routes and then the difference function. Thus, using equation (6), the similarity function which denotes the similarity between the routes for routes 1 and 2 of part type 1, s_{12} is determined as follows.

$R_1 = \{2, 1, 1, 2, 5\}$; $R_2 = \{2, 3, 1, 2, 4\}$; $m=5$; $R_1 \cap R_2 = 3$. Hence, $S_{12}=0.6$

The difference function is computed using Eq. 7 as $d_{12}=0.4$. In a similar manner, d_{13} , d_{14} , d_{15} , and d_{16} are computed for route 1 of part type 1. It can be noted that d_{11} has no significance. This process is repeated for routes 2 to 6 of part type 1. The routing variety of route 1 for part type 1 with reference to all other routes, D_{r1} is the average of the difference functions for route 1. This is obtained as $D_{r1}=0.68$. Similarly, D_{r2} , D_{r3} , D_{r4} , D_{r5} , and D_{r6} are computed. The routing variety of part type 1, D_1 is the average of the routing varieties of routes 1 to 6 and is obtained as $D_1=0.626$.

6.4 Computation of routing flexibility

Routing flexibility of part type 1 is computed as the product of the routing efficiency, routing versatility and routing variety values. Thus, routing flexibility of part type 1 = 0.3283. Similarly, the routing flexibility of the remaining part types are computed for different levels of flexibilities. The routing flexibility of the system is computed using the Eq. 9.

7 Results and discussion

The analysis of the results obtained for the FMS operating under each of the five levels of flexibility are presented in this section. At first, the results obtained for the performance measures such as mean flow time, mean tardiness, percentage

Table 5 Multiple-comparison test results for performance measures: main effect—routing flexibility level

Routing flexibility level	Mean flow time	Mean tardiness	Percentage of tardy parts	Mean utilisation of machines	Mean utilisation of AGVs	Mean queue length
RFL 1	151.93 ^a	5.54 ^a	9.33 ^a	82.38 ^a	72.53 ^a	3.19 ^a
RFL 2	155.40 ^b	6.55 ^b	10.40 ^b	82.25 ^a	73.28 ^a	3.33 ^a
RFL 3	168.45 ^c	9.15 ^c	15.50 ^c	82.37 ^a	77.75 ^{b,c}	3.69 ^b
RFL 4	172.60 ^d	10.50 ^d	17.65 ^d	81.59 ^a	78.20 ^c	4.31 ^c
RFL 5	226.40 ^e	57.00 ^e	32.00 ^e	80.60 ^a	76.65 ^c	6.26 ^d

For each performance measure, values with the same letter are not found significantly different from each other by statistical test

Table 6 Multiple-comparison test results for performance measures: main effect—case

Case	Mean flow time	Mean tardiness	Percentage of tardy parts	Mean utilisation of machines	Mean utilisation of AGVs	Mean queue length
Case 1	190.59 ^a	22.09 ^a	21.09 ^a	83.73 ^a	76.43 ^a	4.47 ^a
Case 2	159.32 ^b	13.40 ^b	12.86 ^b	79.95 ^b	74.93 ^b	3.84 ^b

For each performance measure, the values for case 1 and case 2 are found significantly different from each other by statistical test

of tardy parts, mean utilisation of machines, mean utilisation of AGVs and the average queue size at machines are analysed. Then, the results for flexibility measures are analysed. Due to space limitations, the graphical analysis of simulation results are not presented in the paper.

The simulation results for the performance measures and flexibility measures are subjected to statistical analysis using the analysis of variance (ANOVA) procedure in order to study the effect of experimental factors. The routing flexibility level and the case (with penalty/without penalty for processing time on alternative machines) are the two factors and hence, two-factor ANOVA has been carried out. In conducting statistical analysis, the simulation results for the performance measures/flexibility measures pertaining to each replication have been accommodated in each treatment combination (cell). ANOVA- F test has been carried out to determine whether the treatment means are significantly different from each other. The least significant difference (LSD) method is used for performing pair wise comparisons in order to determine which means differ from other means. The null hypothesis (H_0) is that all means are equal. The alternative hypothesis (H_1) is that at least two means are significantly different. All the tests are conducted at 5% level of significance. Values that are not significantly different are grouped. The results obtained and their analysis are presented in the following sections.

7.1 Analysis of performance measures

Table 4 shows the ANOVA results for the performance measures.

It is observed that the main effect, routing flexibility level is significant for the performance measures such as mean flow time, mean tardiness, percentage of tardy parts, mean utilisation of AGVs and mean queue length. The main effect, case (with/without penalty) is significant for all the performance measures. The interaction effect is found to be significant for the measures such as mean flow time, mean tardiness and percentage of tardy parts. In order to determine which means for the routing flexibility level and case (main effects) are significantly different from which others, the LSD method of multiple-comparison tests is conducted.

Table 5 provides the results obtained using the LSD test for the routing flexibility level.

As shown in Table 5, each routing flexibility level forms a unique significant group labelled 'a, b, c, d, and e' for the performance measures such as mean flow time, mean tardiness and percentage of tardy parts. Since there is no statistical significance in the mean utilisation of machines, all the routing flexibility levels form one group labelled 'a'. Three groups labelled 'a, b, and c' are formed for mean utilisation of AGVs and four groups labelled 'a, b, c, and d' for mean queue length. Hence, it is evident that the routing flexibility level has a significant effect on the performance of the system. The routing flexibility level RFL 1, in which each operation has exactly two alternative machines, leads to the best values for the performance measures. As expected, the performance measure values increase when the routing flexibility of the system is decreased.

The results obtained using the LSD test for the case are shown in Table 6.

The present study reveals that the performance of the system evaluated using various measures for case 1 (without penalty for processing time on alternative machines) are significantly better than that for case 2 (with penalty for processing time on alternative machines). There is a substantial difference in the values of performance measures between case 1 and case 2. Since the interaction effect is found to be significant for the measures such as mean flow time, mean tardiness and percentage of tardy part, graphical plots are also obtained for these measures. The interaction plot for mean flow time is shown in Fig. 2. Due to space limitations, the plots for the other measures are not presented. The pattern of variation for these measures is found to be similar to that for mean flow time.

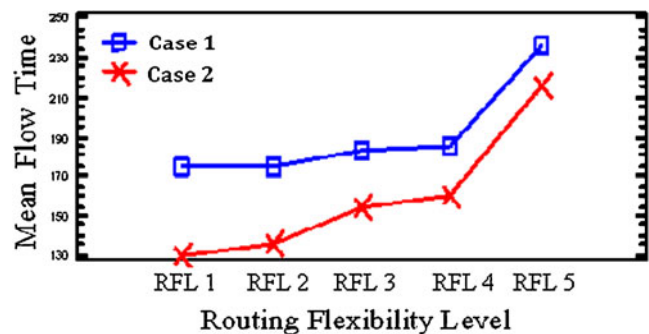


Fig. 2 Interaction plot for mean flow time

Table 7 ANOVA results for two-factor analysis for flexibility measures

Source of variation	<i>F</i> ratio for flexibility measures			
	Routing efficiency	Routing versatility	Routing variety	Routing flexibility
Main effects				
A. Routing flexibility level	48.63 ^a	201.98 ^a	100.32 ^a	234.07 ^a
B. Case (with/without penalty)	75.75 ^a	21.25 ^a	11.00 ^a	53.33 ^a
Interactions ^a				
	71.63 ^a	97.52 ^a	15.67 ^a	91.53 ^a

^a Denotes *F* ratio significant at 5% significance level

As shown in Fig. 2, mean flow time values for the routing flexibility levels RFL 1 and RFL 2 are closer even though the values are significantly different from each other. This is due to the fact that RFL 1 denotes the situation wherein each operation has two alternative machines whereas with the system operation at RFL 2, for some operations, there is one alternative machine and for the other operations, there are two alternative machines. A similar pattern of variation in mean flow time is observed for the routing flexibility levels RFL 3 and RFL 4.

7.2 Analysis of flexibility measures

The ANOVA results for the flexibility measures are shown in Table 7.

It is observed that the main effects, routing flexibility level and case, and the interaction effects are significant for all the flexibility measures. The LSD method of multiple-comparison tests is conducted. The results thus obtained are presented in Tables 8 and 9 for the main effects, routing flexibility level and case, respectively.

As shown in Table 8, each routing flexibility level forms a unique significant group labelled ‘*a, b, c, d, and e*’ for each flexibility measure. Hence, it is evident that the routing flexibility level has a significant effect on each of the flexibility measures of the system. The routing flexibility level RFL 1, in which each operation has exactly two alternative machines, leads to the best values for the flexibility measures. When the routing flexibility of the system decreases, the flexibility measure values also decrease. From Table 9, it is found that the flexibility of the system evaluated using various measures for case 1 (without penalty for processing time on alternative machines) are significantly

better than that for case 2 (with penalty for processing time on alternative machines). Since the interaction effect is found to be significant for all the flexibility measures, graphical plots are also obtained for these measures. Figure 3 shows the interaction plot for routing efficiency. The plots for the other flexibility measures are not included here due to space limitations. The pattern of variation for these measures is found to be similar to that for routing efficiency.

As expected, the routing efficiency measure decreases when the flexibility of the system decreases. For both case 1 and case 2, the routing efficiency values for the routing flexibility level RFL 5 are closer even though the values are significantly different from each other. This is because when the system operates with the flexibility level RFL 5, for some operations, there is one alternative machine and for the other operations, there are no alternative machines. Thus, the penalty for processing time on alternative machines is found to have very little effect on flexibility measures for RFL 5 compared with the other flexibility levels.

For both the cases 1 and 2, each of the flexibility measures for the system show a decreasing trend as the flexibility level varies in the following manner:

$$RFL\ 1 > RFL\ 2 > RFL\ 3 > RFL\ 4 > RFL\ 5.$$

This ranking is found to be consistent with the flexibility that arises out of the available options of machines for various operations.

8 Validation of routing flexibility measures using fuzzy logic approach

Fuzzy logic uses fuzzy set theory which incorporates imprecision and subjectivity into the model formulation

Table 8 Multiple-comparison test results for flexibility measures: main effect—routing flexibility level

Routing flexibility level	Routing efficiency	Routing versatility	Routing variety	Routing flexibility
RFL 1	0.6450 ^a	1.6905 ^a	0.6280 ^a	0.6740 ^a
RFL 2	0.6205 ^b	1.5350 ^b	0.5615 ^b	0.4418 ^b
RFL 3	0.5915 ^c	1.2875 ^c	0.4470 ^c	0.2810 ^c
RFL 4	0.5750 ^d	1.1275 ^d	0.3363 ^d	0.2190 ^d
RFL 5	0.5405 ^e	0.6445 ^e	0.2030 ^e	0.0753 ^e

For each flexibility measure, values with the same letter are not found significantly different from each other by statistical test

Table 9 Multiple-comparison test results for flexibility measures: main effect—case

Case	Routing efficiency	Routing versatility	Routing variety	Routing flexibility
Case 1	0.5720 ^a	1.2384 ^a	0.4377 ^a	0.2894 ^a
Case 2	0.6170 ^b	1.2756 ^b	0.4326 ^b	0.3870 ^b

For each flexibility measure, the values for case 1 and case 2 are found significantly different from each other by statistical test

and solution process. Fuzzy set theory provides a natural platform to model fuzzy relationships such as ‘low’, ‘average’, ‘high’, etc. Fuzzy set theory uses a multi-valued membership function to denote membership of an object in a class rather than the classical binary true or false values used by the classical set. A fuzzy set is a set containing elements that have varying degrees of membership in the set. The valuation set is allowed to be real interval (0, 1). This idea is in contrast with classical or crisp sets because members of a crisp set would not be members unless their membership was full or complete [43]. A fuzzy set *A* of the universe *X* is characterised by a membership function μ_A , which takes its value in interval (0, 1). The closer the value of $\mu_A(X)$ is to 1, the more *X* belongs to *A*. Fuzzy inference maps an input space to an output space. The primary mechanism for doing this is a list of if–then statements called rules, which are expressed in the following form.

IF (antecedent) THEN (consequent).

Figure 4 shows the fuzzy logic model used in the present study.

In general, there are three steps in fuzzy inference system: (1) fuzzification (2) fuzzy inference and (3) defuzzification.

1. Fuzzification

This converts inputs into their fuzzy representations. This is done as follows:

- a. Identification of fuzzy variables: a fuzzy variable is one which has uncertain values or blurred boundaries. The fuzzy variables defined for the assessment of RF are routing efficiency (RE), routing versatility (RV), and routing variety (RT). These variables take linguistic values such as extremely low, very low, etc. that are frequently used by managers and researchers to quantify flexibility.
- b. Defining the membership functions: defining the membership function for a variable involves specifying the universe of discourse and choosing the required number of linguistic labels to fully cover the universe of discourse. Shapes of the membership functions chosen are trapezoidal and triangular as shown in Fig. 5.

Table 10 shows the membership functions for the linguistic variables.

2. Fuzzy inference

This consists of developing the fuzzy rule base. The

rule base consists of a set of fuzzy propositions and is derived from the existing knowledge of the system. A fuzzy proposition or a statement establishes a relationship between input fuzzy sets and output fuzzy sets.

IF *X* is *A* and *Y* is *B* THEN *Z* is *C*

This is implemented via multi-antecedent fuzzy IF–THEN rules which are conditional statements that relates the observations concerning the allocated types (IF part) with the value of flexibility measure (THEN part). An example of such a rule is given below:

IF routing efficiency is Extremely Low (EL) AND routing versatility is Very Low (VL) AND routing variety is Low (L) THEN Routing Flexibility is Very Low (VL).

The fuzzy rule base thus obtained is shown in Table 11.

3. Defuzzification

Defuzzification is the process of obtaining a crisp value out of fuzzy values which are determined during the fuzzy inference. In the present study, centre-of-area defuzzification method is used to obtain the crisp value of routing flexibility.

8.1 Illustration for the computation of routing flexibility using fuzzy logic

In the present study, there are nine linguistic variations for each of the three variables involved.

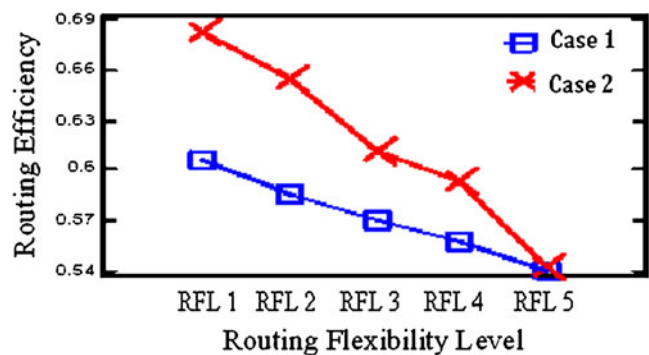
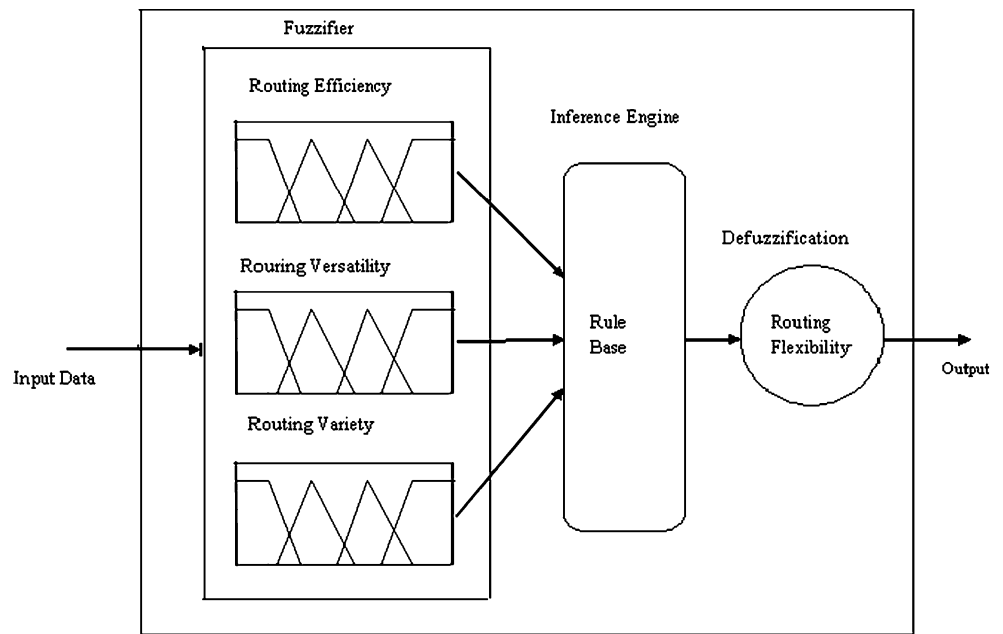


Fig. 3 Interaction plot for routing efficiency

Fig. 4 Fuzzy logic model



The membership functions for the linguistic variables are denoted by $\mu_T: X \rightarrow [0, 1]$ where: $T = \{EL, VL, L, SL, A, SH, H, VH, and EH\}$. By representing the discrete membership functions of the linguistic values with $\mu_T(x)/x, x \in X$, where $\mu_T(x)$ is the membership grade of point x , we have $EL = [1/0, 1/0.1, 0.75/0.15, 0.5/0.2, 0.25/0.25, and 0/0.3]$ where, for example, $1/0$ means that 0 belongs to Extremely Low with membership grade 1. Similarly, the membership functions for VL, L, SL, A, SH, H, VH, and EH are obtained. For the purpose for illustrating the fuzzy logic-based approach for the computation of routing flexibility, the case of routing flexibility level 1 (RFL 1) is considered. The observation O is given by O : RE is H and RV is VH and RT is SH.

By denoting routing flexibility as RF, the observation O can be written as follows: O : RE is H and RV is VH and RT is SH or more simply as O : H AND VH AND SH.

The rule with which observation O matches best is IF (RE is H AND RV is VH AND RT is SH), THEN (RF is H), or compactly $H \text{ AND } VH \text{ AND } SH \rightarrow H$ (10)

By taking the convex combination of the union (\cup) and intersection (\cap) for the antecedent of the fuzzy rule (Eq. 10) given above, we have

$$\mu_{H \text{ AND } VH \text{ AND } SH}(x) = (1 - \gamma)\mu_{H \cap VH \cap SH}(x) + \gamma \mu_{H \cup VH \cup SH}(x), x \in X, \gamma \in [0, 1] \tag{11}$$

where γ is the grade of compensation (0.4 in the present study). $H \text{ AND } VH \text{ AND } SH = [0.4/0.6, 0.45/0.65, 0.7/0.7, 0.45/0.75, 0.4/0.8]$. Then, the normalised membership of the observation O is computed. The implication operator

selected is a function of the conjunction $\mu_{H \text{ AND } VH \text{ AND } SH}(x), x \in X$, and the consequent $\mu_H(y), y \in Y$ over $X \times Y$, which in the membership domain is given by

$$L_{H \text{ AND } VH \text{ AND } SH \rightarrow H}(x, y) = L \rightarrow (x, y) = (1 - \mu_{H \text{ AND } VH \text{ AND } SH}(x)) \cup \mu_H(y) \tag{12}$$

From Eq. 12, the relation matrix is computed as shown below:

$$L \rightarrow = \begin{bmatrix} 0.6 & 0.6 & 0.75 & 1 & 0.75 & 0.6 \\ 0.55 & 0.55 & 0.75 & 1 & 0.75 & 0.55 \\ 0.3 & 0.5 & 0.75 & 1 & 0.75 & 0.5 \\ 0.55 & 0.55 & 0.75 & 1 & 0.75 & 0.55 \\ 0.6 & 0.6 & 0.75 & 1 & 0.75 & 0.6 \end{bmatrix}$$

The value of routing flexibility is inferred by applying the compositional rule of inference [51], which is the most frequently used approximate reasoning method. It is described by the following inference pattern: O : H AND VH AND SH (observation)

Rule: H AND VH AND SH \rightarrow H (existing knowledge)

RFL 1: $O \circ L \rightarrow$ (conclusion)

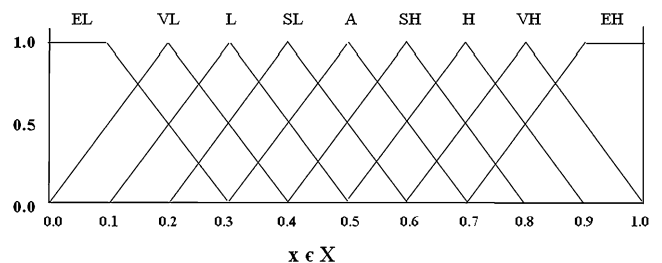


Fig. 5 Membership function of the linguistic values

Table 10 Membership functions and linguistic values

Membership function	Linguistic values
(0.0, 0.1, 0.1)	Extremely low
(0.0, 0.2, 0.4)	Very low
(0.1, 0.3, 0.5)	Low
(0.2, 0.4, 0.6)	Slightly low
(0.3, 0.5, 0.7)	Average
(0.4, 0.6, 0.8)	Slightly high
(0.5, 0.7, 0.9)	High
(0.6, 0.8, 1.0)	Very high
(0.7, 0.9, 1.0)	Extremely high

Where \circ denotes the max–min composition as $RFL\ 1 = \max(O\ L \rightarrow)$ which gives the membership function of routing flexibility as given below.

$$RFL\ 1 = [0.5714/0.55, 0.5714/0.6, 0.75/0.65, 0.999/0.7, 0.75/0.75, 0.5714/0.8].$$

By applying the centre-of-area defuzzification method, the crisp value of routing flexibility for level 1 obtained by defuzzification (denoted as def RFL 1) is determined as

$$\text{follows: def RFL 1} = \frac{\sum_{i=1}^6 x_i \mu_F(x_i)}{\sum_{i=1}^6 \mu_F(x_i)}$$

where x_i is the numerical value for the membership function i and $\mu_F(x_i)$ is the degree of membership at which membership function i was scaled. Thus,

$$\text{def RFL 1} = \frac{0.5714 \times 0.55 + 0.5714 \times 0.6 + 0.75 \times 0.65 + 0.999 \times 0.7 + 0.75 \times 0.75 + 0.5714 \times 0.8}{0.5714 + 0.5714 + 0.75 + 0.999 + 0.75 + 0.5714} = 0.679$$

In a similar manner, the routing flexibility values for the other flexibility levels are obtained as follows: RFL 2=0.600; RFL 3=0.500; RFL 4=0.400; RFL 5=0.321

Thus, the routing flexibility values for the system show a decreasing trend as the flexibility level varies in the following manner: RFL 1>RFL 2>RFL 3>RFL 4>RFL 5. This ranking is the same as that obtained in section 7.2. For case 2 also, the routing flexibility values are computed using the fuzzy logic approach. Here also, the ranking obtained is the same as that given above.

9 Conclusions

The present paper has dealt with the evaluation of the routing flexibility of an FMS with the dynamic arrival of part types for processing in the system. A typical FMS configuration is chosen for detailed study. The system is set at five different levels of routing flexibility. Operations of part types can be processed on alternative machines depending upon the level of routing flexibility present in the system. Two cases have been considered with respect to

Table 11 The fuzzy rule base

Routing efficiency	Routing versatility	Routing Variety									
		EL	VL	L	SL	A	SH	H	VH	EH	
EL	EL	EL	EL	VL	VL	L	L	SL	SL	A	A
–	VL	EL	VL	VL	L	L	SL	SL	A	A	A
–	L	VL	VL	L	L	SL	SL	A	A	A	SH
–	–	–	–	–	–	–	–	–	–	–	–
VL	–	–	–	–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–	–	–	–	–
EH	–	–	–	–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–	–	–	–	–
–	SH	SH	SH	SH	SH	H	H	H	VH	VH	VH
–	H	SH	SH	SH	H	H	H	H	VH	VH	VH
–	VH	SH	SH	H	H	H	VH	VH	VH	VH	EH
–	EH	SH	H	H	H	VH	VH	VH	VH	EH	EH

Due to space limitations, the complete Table is not presented

the processing times of operations on alternative machines. A discrete-event simulation model has been developed to describe the operation of the chosen FMS. The performance of the system under various levels of routing flexibility is determined using various measures. The simulation results have been subjected to statistical analysis. Routing flexibility level and case (with/without penalty for processing time on alternative machines) have significant effect on the system performance.

Mean flow time is used as the performance measure for the evaluation of routing flexibility. The routing flexibility of the system has been evaluated using the attributes such as routing efficiency, routing versatility and routing variety. For both the cases 1 and 2, each of the flexibility measures for the system show a decreasing trend as the flexibility level varies in the following manner: $RFL\ 1 > RFL\ 2 > RFL\ 3 > RFL\ 4 > RFL\ 5$.

The ranking thus obtained has been validated using the routing flexibility values determined based on fuzzy logic approach. Fuzzy logic provides a method for synthesizing the attributes such as routing efficiency, routing versatility and routing variety which are not homogeneous.

The flexibility levels chosen for experimentation in the present research include a maximum of two alternative machines for an operation. Further experimentation is required for analysing situations involving more options for operations and for varying penalty levels for processing of operations on alternative machines. In the present study, time aspect is only considered in the measurement of routing efficiency and hence routing flexibility. However, efficiency of a manufacturing system may also depend on the cost of processing on alternative machines. This aspect needs further investigation.

In the present study, machines and AGVs are assumed to be available continuously. However, in practice, FMSs are subjected to interruptions such as machine failures and AGV breakdowns. To model these interruptions, it is necessary to specify the time between failures and the repair times. When a machine breakdown occurs, parts waiting/under processing can be rerouted to alternative machines available if any. The coding of the simulation model involves incorporating the necessary logic to represent the working of the system. These interruptions may lead to increase in flow time and tardiness of parts. Hence, there is a need for further research to evaluate flexibility measures for the experimental conditions that consider system disruptions such as breakdowns of machines and AGVs.

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