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ORIGINAL ARTICLE

Study on error compensation of machining force in aspheric surfaces polishing by profile-adaptive hybrid movement–force control

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Abstract The hybrid movement–force control policy has been widely used to deal with the coupling of the movement control subsystem and the force control subsystem in a compliant control system by a Jacobi matrix while its position and/or posture changing. But the Jacobi matrix, which is used to decouple the coupling of movement–force control, could not restrain the disturbance of the errors of position and posture to the force control. For their great uncertainty and non-linearity, the errors of position and posture in the movement controlling place lots of troubles to the force controlling in the engineering applications. In this paper, a kind of profile-adaptive compliant control policy is added to reduce the errors of position and posture of the polishing tool and to restrain their disturbance to the machining force in a hybrid movement–force control system. The new policy has been used to polish aspheric surfaces by CNC machine tools of two axes. Experimental results show that it could compensate the errors of machining force and improve surface quality obviously.

Keywords Error compensation · Compliant polishing · Hybrid movement–force control · Profile-adaptive · Aspheric surfaces

1 Introduction

In the processing of compliant control, the coupling between movement control and force control usually confuses the involved researchers because the stiffness coefficient matrix for force controlling would be changed by the movement control when the position and posture have been changed. According to the early research works [1, 2], the Jacobi matrix could be used to decouple the coupling of movement control and force control, but it could not restrain the disturbance of the movement control to the force control. For the errors of position and posture are of the great uncertainty and high non-linearity, the disturbance of movement control to force control could hardly be reduced effectively in the real applications for a long time. In the past decades, lots of research works [3–6] are taken into the issue by algorithms studying and software developing in order to compensate the errors of force control, but it still seems to be unsatisfactory.

In the processing of aspheric surfaces polishing, the errors of the infeed movement, which are resulted from the CNC multi-axis interpolation theoretically, are the main disturbance to the machining force control [7–10]. In this paper, a hardware system is placed to adapt the position and posture of polishing tool changing automatically. In this way, the profile-adaptive compliant control system could reduce the disturbance of the infeed movement to machining force control. Simulating experiments and polishing experiments testify that this system could compensate the errors of machining force and improve surface quality efficaciously.

2 Compliant control system for aspheric surfaces polishing

Aspherics is one of the most important parts in the military industry, the aeronautics and astronautics industry and some high-tech industries. Aspheric surfaces always have to be

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polished or grinded after being cut by NC machine tools to reach the required high precision and low surface roughness. Lots of great research works on measurement, control, and machining have been taken for aspheric surfaces cutting, grinding, and polishing [10–14]. In order to avoid impacts between polishing tool and workpiece being compliant, the machining force in the normal direction of the cut points in the aspheric surfaces should be controlled in real time in the whole polishing processing [15–19]. Based on an ordinary CNC lathe and the two-axis interpolation theory, this paper develops a precision polishing system for aspheric surfaces by the hybrid movement–force control [20–22], in which a torque servo shaft is installed in order to control the machining force, as shown in Fig. 1.

The above polishing system includes three parts. The first one is the cutting movement subsystem, which is composed of 1 and 2. The cutting movement is driven by the spindle of CNC lathe. The second one is the infeed subsystem, made up of 4 and 5. Besides moving in two vertical line directions, the infeed subsystem is used to servo the posture angle θ according to the machining force F_n in the aspheric surfaces polishing, which is quite different from the one of an ordinary CNC lathe. The third one is the machining force control subsystem, of 6, 7, 8, and 9. A step motor, the device 6, transmits the required torque to 7, and through 8 to 9. A torque sensor that is installed on 7 shall detect the torque when something is wrong. When the torque changes from the required value, the step motor would be driven to adjust the machining force.

3 Displacement errors of position and posture

The dynamics model of the polishing tool, the subassembly 4 in Fig. 1, is shown in Fig. 2. In the figure, the posture angle θ is defined the complementary angle to the tangent angle at the cutting point of the machined surfaces. When an aspheric surface of Eq. 1 is polished, the polishing tool is moved along with the machined surface and the machining force F_n needs to be controlled in the normal direction at the cutting point. Therefore, the infeed subsystem is used to fulfill servo movements in two vertical line directions as well as the posture angle θ. As to an aspheric surface of Eq. 1, the trajectory of the infeed (x_p, y_p) is obtained as Eq. 2.

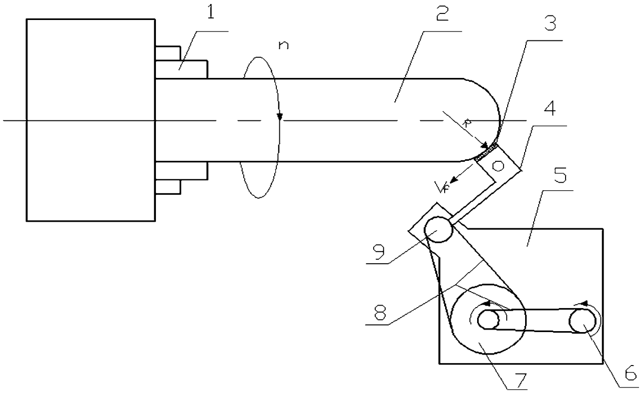


Fig. 1 Polishing system with compliant control. *1* Turning chuck. *2* Workpiece. *3* Polishing saucer. *4* Polishing tool. *5* Tool post. *6* Electromotor shaft. *7* Torque servo device. *8* Synchronous belt. *9* Revolution shaft

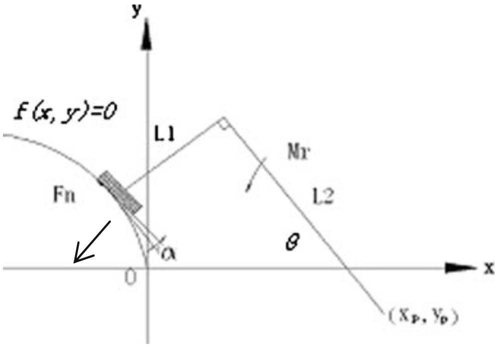


Fig. 2 Displacement errors of position and posture

$$f(x,y) = 0, \tag{1}$$

$$\left.\begin{aligned} x_p &= x + L_2\cos\theta + L_1\sin\theta \\ y_p &= y - L_2\sin\theta + L_1\cos\theta \\ \theta &= \tfrac{\pi}{2} - a\tan\left(\tfrac{dy}{dx}\right) \end{aligned}\right\}, \tag{2}$$

where L_1=37.14 mm and L_2=70 mm are defined in Fig. 2. When the three variables, x_p, y_p, and θ, take the three-axis interpolation operations, the displacement errors will change the programming cutting point (x, y) to the real cutting point (x_1, y_1), and it results in the displacement errors of position and posture of polishing tool. α is defined as the angle between the two tangent lines at (x, y) and at (x_1, y_1) on the aspheric surface. By Fig. 2 and Eq. 2, there are

$$\left\{\begin{aligned} dx &= dx_p + (L_2\sin\theta - L_1\cos\theta)d\theta \\ dy &= dy_p + (L_2\cos\theta + L_1\sin\theta)d\theta \end{aligned}\right., \tag{3}$$

$$\left\{\begin{aligned} \Delta x &= x_1 - x \\ \Delta y &= y_1 - y \end{aligned}\right., \tag{4}$$

$$\alpha = a\tan\left(\frac{dy}{dx}\right) - a\tan\left(\frac{dy_1}{dx_1}\right), \tag{5}$$

where (x, y) and (x_1, y_1) belong to Eq. 1. By Eq. 5, any error of the trajectory of the infeed (x_p, y_p) and error of the posture angle θ would result in the position (x, y) different form the one of (x_1, y_1), and then $\alpha \neq 0$.

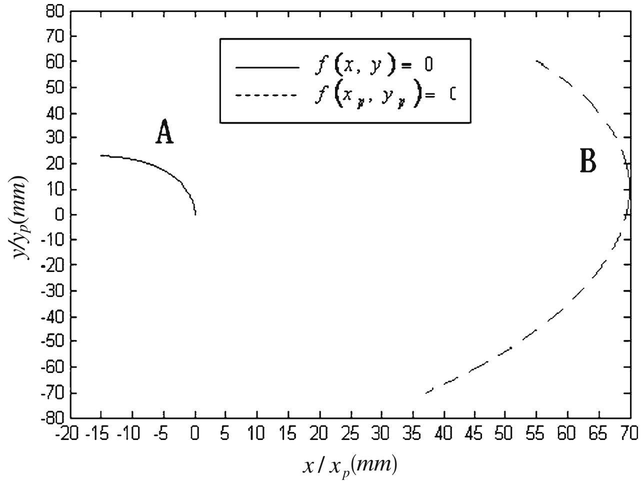


Fig. 3 Machining trajectories

Take a certain surface for example. An aspheric surface in the axial section plane is an elliptic equation as the following Eq. 6:

$$\frac{(x+15)^2}{15^2} + \frac{y^2}{23^2} = 1, \tag{6}$$

where, $-15 \leq x \leq 0$. The curve from Eq. 6 is shown as the curve A in Fig. 3. Assuming $a(x) = (1,216x^2 + 36,480x + 476,100)^{\frac{1}{2}}$, $b(x) = 2x + 30$, $c(x) = (-x^2 - 30)^{\frac{1}{2}}$, the trajectory of infeed (x_p, y_p) could be obtained as:

$$\begin{cases} x_p = x + \dfrac{42{,}711b(x)}{50a(x)} + \dfrac{2{,}100c(x)}{a(x)} \\ y_p = \dfrac{23c(x)}{15} + \dfrac{5{,}571c(x)}{5a(x)} - \dfrac{1{,}610b(x)}{a(x)} \end{cases}, \tag{7}$$

where, $-15 \leq x \leq 0$. When $a(x)$, $b(x)$, and $c(x)$ are substituted into Eq. 7, the trajectory of infeed could be obtained as the curve B in Fig. 3. That is, when the trajectory of the polishing saucer is the curve A, the trajectory of infeed of the machine tool is the curve B.

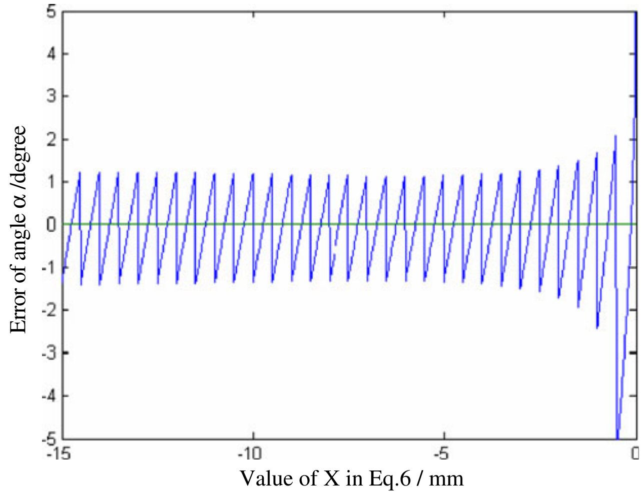


Fig. 4 α changes by two-coordinate interpolation

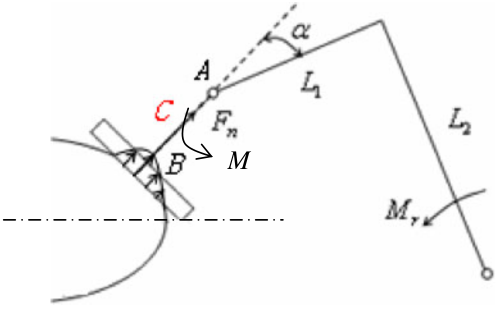


Fig. 5 Polishing tool with a hinge

If only two variables, x_p and y_p, take two-coordinate interpolation operation at 1,500 cutting points for a section curve of Eq. 6 by the CNC interpolator in a kind of CAM software, the famous MasterCAM, the displacement errors of the position and posture of the polishing tool would change the programming cutting point to the real cutting point at every point. It results in α fluctuating. Taking one form every 30 points of the above 1,500 points, the fluctuating curve of α is shown in Fig. 4.

4 Machining force describing and profile-adaptive solution

Define $\overrightarrow{\delta} = (\overrightarrow{x_1 - x}) + (\overrightarrow{y_1 - y})$. When the torque servo device outputs a moment M_r in Fig. 2, then

$$F_n = \frac{M_r}{L_2 + \delta} \tag{8}$$

In all kinds of control system, δ could hardly be measured because δ would be changed both in value and in direction by the posture angle α all the time. Furthermore, the non-

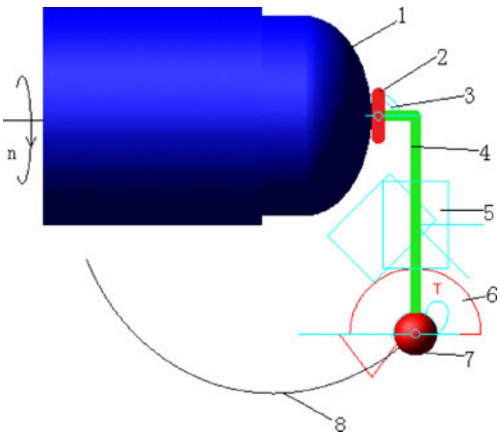
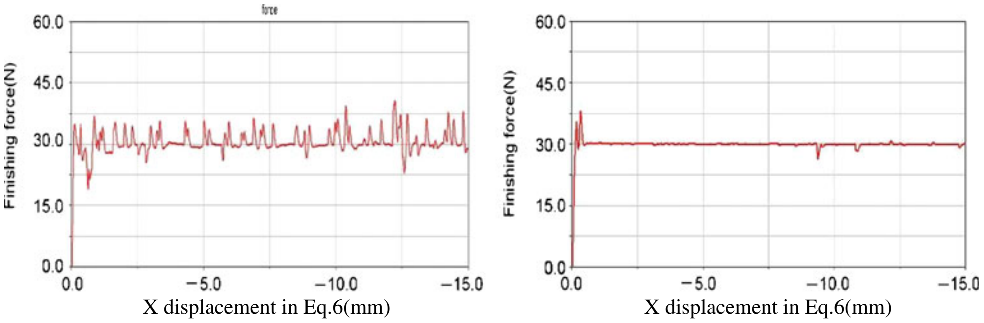
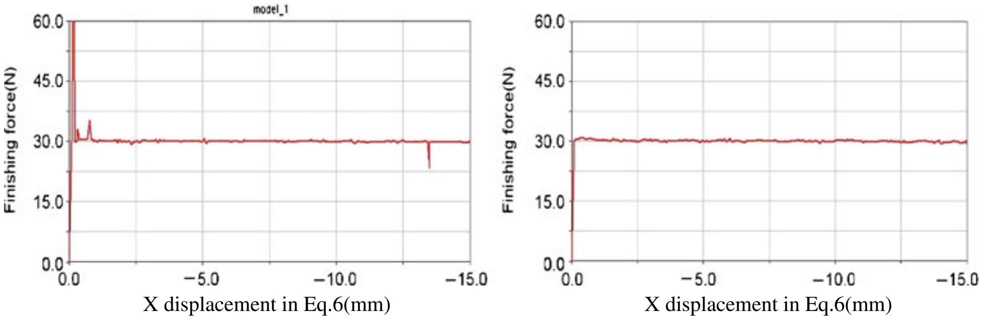


Fig. 6 System structure in ADAMS. *1* Workpiece. *2* Polishing saucer. *3* Hinge. *4* Rod. *5* Planar joints. *6* Drive torque. *7* Torque servo device and turret. *8* Trajectory of turret

Fig. 7 Machining force curves with or without hinge



(a) No Hinge (b) Hinge at point A



(c) Hinge at point C (d) Hinge at point B

linear relationship between δ and α has not yet been modeled mathematically for a long time. The disturbance of α to δ results in the disturbance of α to the machining force F_n. In this way, the infeed movement control disturbs the machining control in the processing of aspheric surfaces polishing.

In order to restrain the disturbance, a hinge is fixed at the point A, B, or C on the rod of the polishing tool to make the polishing saucer rotate freely in order to adapt the slope and curvature changing along with the surface of the machined workpiece, which is shown in Fig. 5. In this figure, point A is the middle point of L_1 in Fig. 2, point B the fixed point of polishing saucer, and point C the middle point of A and B.

Supposing the machining pressure P_n at a certain cutting point (x_n, y_n), and the distance δ_n between (x_n, y_n) and (x, y), so

$$F_n = \iint p_n dxdy, \tag{9}$$

$$M = \iint p_n \delta_n dxdy, \tag{10}$$

where the machining pressure P_n at very cutting point (x_n, y_n) is vertical to the plane of polishing saucer, so the machining force F_n is vertical to the plane of polishing saucer, too. With a hinge keeping the saucer rotate freely, M=0. And according to Fig. 5, there is

$$M_r = M + F_n \cdot L_2 \cos\alpha + F_n \cdot L_1 \sin\alpha, \tag{11}$$

and,

$$F_n = \frac{M_r}{L_2 \cos\alpha + L_1 \sin\alpha}, \tag{12}$$

where, the value of α could be measured easily by angle sensors. By Eq. 12, all displacement errors of movement control are included in α and could be used to control the machining force by the torque servo.

Table 1 Parameters of experiments

Speed of cutting movement	30 m/min
Speed of infeed	0.06 mm/r
Material of abrasive	Diamond
Position of hinge	Point C
Aim of force	30 N
Polishing saucer 1#	Wool pad
Polishing saucer 2#	Gasbag

5 Simulation experiments and structure optimizing with ADAMS

In order to verify the efficiency and rationality of the above solution, and to make an optimization of the position of hinge, the simulation analysis is carried out with the dynamic analysis software ADAMS. A virtual prototype aiming at the analysis of the profile-adaptive polishing system is established in Fig. 6. The hinge could be fixed at point A, point B, and point C in Fig. 5. If the hinge restrains the angle α from fluctuating, the machining force F_n should change at a small range. With a hinge fixed at the three points, the machining force F_n is simulated, respectively, while the infeed subsystem is doing its job. In contrast, the machining force F_n is also simulated when no hinge is fixed in the polishing tool. The results of the simulation experi-

Table 2 Polishing procedures

	Step 1				Step 2
	Metallographic sandpaper				Polishing paste
Size of abrasive	400#	1000#	2000#	3000#	W0.5
Polishing times	3 times	3 times	3 times	3 times	10 times

ments are shown in Fig. 7. The curves (a), (b), (c), and (d) are the curves of machining force when there are no hinge and a hinge is set at the points of A, C, and B separately. The curve (a) fluctuates at a range of about 20 to 40 N. As to the curves (b), (c), and (d) in the figure, the machining force F_n is not fluctuating remarkably rather than it does in the curve (a). Furthermore, F_n is almost stable in all the work time when the hinge is fixed at point B. It seems that, the moment of inertia of polishing tool influences its machinery performances.

In this way, the polishing tool with a hinge could change its posture to fit the changing of slope and curvature at all cutting points of aspheric surfaces to keep the machining force F_n stable. And when the hinge is fixed at point B, the polishing system is good at controlling the machining force F_n. By the way, point B is the point near the polishing saucer.

6 Experiments on machining force

With the parameters being defined in the following Table 1 and by polishing procedures as Table 2, the experiments on machining force are taken according to the following proceeding:

(1) From the trajectory of infeed, $f\left(x_p, y_p\right) = 0$ in Fig. 3, calculating cutting data and NC code with CAD/CAM software, such as Mastercam.
(2) Running the infeed subsystem to make the polishing saucer meet the aspheric surface.
(3) Starting up the torque servo.
(4) Executing the machine tool according to the NC codes to fulfill the polishing processing.

When the aspheric surface of Eq. 6 is polished by saucers of wool pad and of gasbag, the signal of machining force F_n is measured and shown in Fig. 8.

The relative error of F_n is defined as:

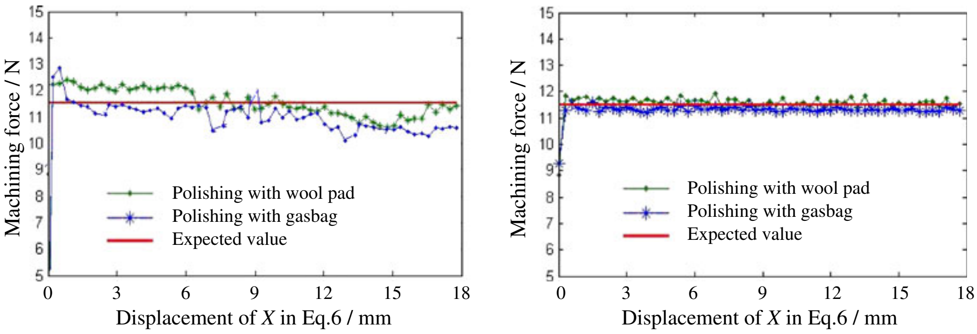
$$\text{Relative error} = \frac{\max(F_n) - \min(F_n)}{\text{Excepted}(F_n)} \times 100\% \tag{13}$$

The relative error of F_n is about 25% in Fig. 8a, but it is less than 7% in Fig. 8b. In this way, the new system could restrain the errors of machining force efficaciously.

7 Polishing experiments on an aspheric surface

To study its performances of the profile-adaptive polishing system, polishing experiments on an aspheric surface are taken according to Table 2 with the parameters as in Table 1. And the polishing results are shown in Fig. 9 and Table 3 by the profile-adaptive polishing system with a hinge fixed at point C. Polishing experiments is also taken under the same condition by a conventional polishing tool without any hinge and its results are shown in Table 3. The profile of the polished workpiece is an aspheric surface of steel no. 45 that is defined in Eq. 6. As shown in Fig. 9, machining conditions in zone 1, zone 2, and zone 3 are different from each other in the machining force controlling.

Fig. 8 Machining force measured curves with or without hinge



(a) Without hinge

(b) With a hinge at point C

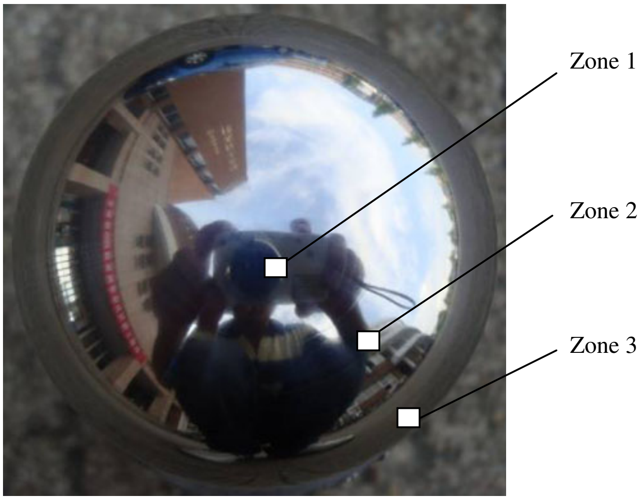


Fig. 9 Photo of an aspheric surface after being polished

Measuring the surface roughness, Ra, of the polishing surface at the three zones, polishing results are shown in Table 3.

Relative difference is defined as

$$\mathrm{Relativedifference} = \frac{\max(|\mathrm{Ra}_1 - \mathrm{Ra}_2|, |\mathrm{Ra}_2 - \mathrm{Ra}_3|, |\mathrm{Ra}_3 - \mathrm{Ra}_1|)}{\left(\frac{\mathrm{Ra}_1+\mathrm{Ra}_2+\mathrm{Ra}_3}{3}\right)} \times 100\% \quad (14)$$

In Table 3, the relative difference of Ra by the conventional polishing tool is more than 50%, and the other one by profile-adaptive polishing tool is about 10%. That is, the material removal of the workpiece by profile-adaptive polishing system is more uniform than that by the conventional one. Compared with the conventional polishing system, the profile-adaptive one is fixed a rotating structure to adapt the profile of workpiece. In this way, the new polishing system could improve surface quality obviously.

8 Conclusions

For the great uncertainty and high non-linearity of the relationship between movement control and force control in compliant control systems, the disturbance of the errors of the movement control to the force control troubles the real engineering applications for a long time. By the profile-adaptive policy, a variable, which is easy to be modeled and measured, is added to embody the errors of position and posture in a hybrid movement–force control system. With a hinge making the executor adapt the profile of the workpiece, the errors of machining force could be compensated efficaciously in aspheric surfaces polishing by measuring the turning angle.

In the processing of aspheric surfaces precision machining, no mistakes are allowed to lower the control performance of the machine tools. When a CNC machine tool polishes a aspheric surface with the infeed of x, y, and θ, the displacement errors from the three-coordinate interpolation operations make the programming trajectory of the infeed different from the required one, and it results in the errors of position and posture of the polishing tool. With a hinge making the polishing saucer turning freely along with the machining surface, the profile-adaptive polishing tool could adapt timely the profile of the workpiece to reduce the errors of machining force. Therefore, the new polishing system would improve surface quality obviously. Simulation experiments show that the polishing system could reduce the errors of machining force by changing the posture of the polishing saucer to fit the profile of the machining aspheric surfaces. Measuring experiments on machining force show that the profile-adaptive compliant control system could restrain the errors of machining force efficaciously too. Under the same conditions as the simulation experiments and the measuring experiments on machining force, polishing experiments on an aspheric surface show that the material removal of the workpiece by profile-adaptive polishing tool is more uniform than that by the conventional one.

Table 3 Surface roughness of an aspheric surface after polished

	Cutoff wavelength	Evaluation length	By profile-adaptive polishing tool			By conventional polishing tool		
			Ra (μm)	Rz (μm)	Ry (μm)	Ra (μm)	Rz (μm)	Ry (μm)
Zone 1	0.25 mm	5*Cutoff	0.059	0.340	0.517	0.043	0.244	0.296
Zone 2	0.25 mm	5*Cutoff	0.053	0.380	0.563	0.069	0.565	0.740
Zone 3	0.25 mm	5*Cutoff	0.055	0.376	0.524	0.075	0.495	0.603
Difference of Ra			10%	–	–	51%	–	–

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