

# Determination of optimum parameters with multi-performance characteristics in laser drilling—A grey relational analysis approach

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**Abstract** Laser drilling is increasingly becoming the method of choice for precision drilling for variety of components. However, a number of defects such as spatter, recast, heat-affected zone (HAZ), and taper limit the application. Elimination of these defects is the subject of intense research. This paper presents a grey relational optimization approach for the determination of the optimum process parameters which minimize the HAZ and hole circularity and maximize material removal rate in a Pulsed Nd:YAG laser micro-drilling in high carbon steel within existing resources. The input process parameters considered are pulse width, number of pulses, assist gas (oxygen) flow rate, and its supply pressure. A higher resolution-based  $L_{25}$  orthogonal array has been used for conducting the experiments. The designed experimental results are used in grey relational analysis and the weights of the quality characteristics are determined optimizing the parameters. On the basis of optimization results, it has been found that the optimal parameter level gives a small HAZ, fine hole, and maximum material removal rate. Subsequently, the results are also verified and found appropriate by running confirmation tests.

**Keywords** Hole circularity · Heat-affected zone · Laser micro-drilling · Grey relational analysis

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## 1 Introduction

Laser-drilling technique has progressed remarkably to become the method of choice for meeting a vast majority of potential industrial requirements for micro hole drilling in diverse components such as watches, turbine blades, circuit boards, etc. [1]. Beam-energy-based material processes are of benefit in the processing of difficult-to-process materials with stringent design requirements, complex shape, and unusual size of work piece. In beam machining processes, the thermal source of energy for processing is achieved by focusing a beam of photons, electrons, +ve ions, or plasma on to the work piece.

Pulsed laser drilling has progressed remarkably over the years to become an essential tool for micro-hole drilling in many components used in the technologically advanced industries. The basic material removal mechanism in laser drilling is based on the absorption of laser energy from a series of laser pulses at the same spot [2, 3]. Material is melted and ejected to form a hole. The use of laser micro-drilling or micro-machining in manufacturing industry can be attributed to several advantages like high production rate, applicable to both conductive and non-conductive materials, no mechanical damage or tool wear due to non-contact processing, improved product quality, low material wastage, low production cost, small heat-affected zone (HAZ), and ecologically clean technology. A pulsed Nd:YAG laser produces high intensity infrared radiation at a wavelength of 1.06  $\mu\text{m}$ , power ranging from 500 to 12,000 W. Due to its short wavelength (compared with  $\text{CO}_2$  lasers), it enables processing of highly reflective materials with less laser power [4]. Taper formation and production of non-circular holes are characteristic of the laser micro-drilling operation, as laser machining is based on the interaction of a laser beam with inherent focusing character-

istics [5]. But it is desirable to make the drilled holes circular without taper. Modeling of the process is required to be able to control these two important characteristics. Developing a physical model for laser percussion drilling is very complicated since a large number of parameters control the process. Some prior studies have used statistical models to analyze and determine optimal parameter setting of the process.

Kuar et al. [6] experimentally investigated the influence of laser machining parameters on the heat affected zone thickness and phenomena of tapering during CNC-pulsed Nd:YAG laser micro-drilling of zirconium oxide ( $ZrO_2$ ) and performed parametric analysis through response surface methodology (RSM). In another study, Kuar et al. [7] investigated the effect of several laser machining parameters on HAZ thickness and taper of the micro drilled holes on alumina–aluminum composite. Ghoreishi et al. [8] employed a statistical model to analyze and compare hole taper and circularity in laser percussion drilling on stainless steel and mild steel. Jackson and O'Neill [9] investigated the interaction phenomena of Q-switched, diode-pumped Nd:YAG laser using different wavelengths on M2 tool steel. Yilbas and Yilbas [10] used a statistical method to investigate the effects of the variation of single-pulse laser drilling parameters on the hole geometry for Nimonic75-workpiece material by using a full factorial design to identify the main and first-order interaction effects on the hole quality in single-pulse drilling including re-solidified material, taper, barreling, inlet cone, exit cone, surface debris, and mean hole diameter. Yilbas [11] examined four materials nickel, tantalum, EN-58-B, and titanium to obtain laser drilling speed using a statistical analysis. In another study, Yilbas [12] conducted drilling experiments on three materials, stainless steel, nickel, and titanium, using single-pulsed laser beam. It has been shown that the extent of taper formation during laser percussion drilling of thin sections can be significantly reduced by suitable control of laser variables. His work has further revealed that optimization of parameters like focal position, pulse energy, pulse width and pulse frequency can effectively control recast layer formation inside the drilled hole. In this context, it can be suggested that the required peak power should be preferably obtained by appropriate control of pulse energy and pulse duration. Bandhopadhyay et al. [13] investigated the influence of the process variables on hole diameter and taper angle of drilled holes produced on thick IN 718 and Ti-6Al-4V sheets by Nd:YAG laser. French et al. [14] used two level factors in Nd:YAG laser percussion drilling to find the significant factors from a list of 17 factors. The main effects of factors and first- and second-order interactions were analyzed, and it was found that pulse shape, energy, peak power, focal position, gas pressure, and Nd:YA Glaser rod were the most significant influences on the

hole taper and circularity. In addition to statistical analysis of hole taper, some recent efforts have also been made to control hole taper via the development of drilling techniques [15, 16]. Almeida et al. [17] investigated the effects of Nd:YAG laser machining on quality and formation of phases in the cut surface on commercially pure titanium (grade 2) and Ti-6Al-4V (grade 5) sheets. The process parameters were investigated using factorial. Normally, a high peak power is desirable to promote material removal by vaporization rather than melting, and the high vapor pressure generated in the process also serves to efficiently eject molten material, thereby suppressing formation of recast layer. Studies by Yeo et al. [18] and results reported by Chen et al. [19] indicate that excessively long pulse duration or a very high pulse energy can both lead to deformation of parent material structure as well as adversely influence taper and recast layer formation. Other research efforts have established that a shorter pulse duration minimizes the recast layer and reduces micro-cracking at the sidewalls of the laser-drilled hole [20, 21], with a pulse duration of 0.1–2.5 ms being proposed as being most suitable for deep hole drilling [22]. There has also been an initiation of efforts aimed at tailoring the pulse pattern to enhance hole quality, and this has yielded encouraging results [23, 24]. Several methods involving application of anti-spatter coatings [18, 25, 26] as well as drilling in transparent media [22] have also been experimented with to minimize spatter formation.

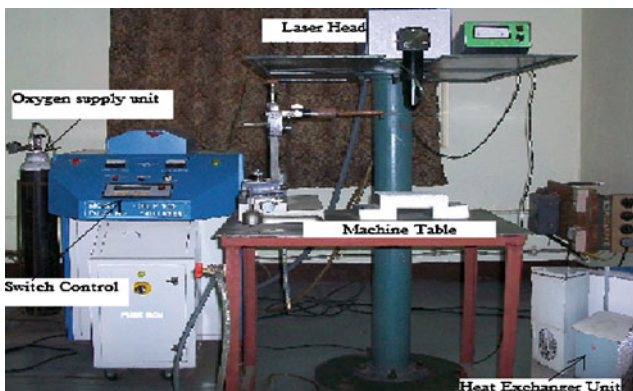
Researchers have investigated the effect of laser cutting parameters on performance of laser micro-drilling quality and also suggested the optimal parameter ranges during cutting of conventional sheet metals and also few advanced sheet materials like ceramics, composites and super alloys by varying the one factor at a time. But this technique requires a large number of experimental runs. To overcome this problem, few researchers have also applied the design of experiment approach for deciding the optimal parameter levels during study of laser beam cutting process performance. The Taguchi method (TM) is a statistical approach for the purpose of designing and improving product quality. It uses orthogonal array to set up the experiment for the advantage of less number of experiments, and optimizes the process parameters by the analysis of signal-to-noise ratio response table and response graph. The optimal process parameters can improve the robustness of products, so the Taguchi method is also called parameter design. In recent years, the TM has become a powerful tool for improving the productivity during research and development stage so that quality products can be produced quickly and at low cost.

Some researchers have concentrated on achieving multiple quality characteristic at a time as a function of different appropriate level of input parameter settings. Dubey et al. [27–29] applied TM, orthogonal array with

principle component analysis and TM, and RSM to optimize multiple quality characteristics during pulsed Nd:YAG laser cutting of different thin sheets. Some researchers also tried to solve the problem with multiple quality characteristics by the grey relational analysis (GRA). In 1982, Deng first proposed grey relational analysis to fulfill the crucial mathematical criteria for dealing with a poor, incomplete, and uncertain system [30]. In GRA, black represents having no information and white represents having all information. A grey system has a level of information between black and white. It avoids the inherent shortcomings of conventional, statistical methods and only requires a limited data to estimate the behavior of an uncertain system. It also provides an efficient solution to the uncertain, multi-input, and discrete data problem. The main function of the GRA is to indicate the relational degree between two sequences by using discrete measurement method the distance. GRA can be effectively recommended as a method for optimizing the complicated interrelationships among multiple performance characteristics [31, 32].

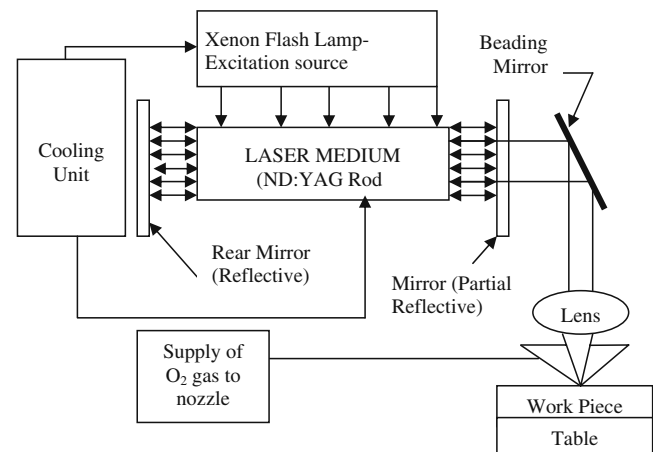
## 2 Experimental procedure

The experiments of laser drilling were conducted using a 100-W-pulsed Nd:YAG laser beam system (SIL-100 model, SI Laser, Pune, India) with wavelength  $1.06 \mu\text{m}$  (Figs. 1, 2). Oxygen was used as the assist gas, and it is supplied at a pressure of  $2 \text{ Kg/cm}^2$ . The assist gas was passed through a conical nozzle co-axially with the laser beam. The laser beam was focused using a lens of focal length 50 mm. The stand-off distance (SOD) was set at 1.0 mm. The work piece was a high carbon steel Domex C67 thin sheet, the chemical composition of which is given in Table 1. Nozzle diameter, focal length of lens, nozzle stand-off distance, and work piece material thickness were kept constant throughout the experimentation. Because of



Photographic view of pulsed ND:YAG Laser drilling system

Fig. 1 Photographic view of pulsed ND:YAG laser drilling system



Schematic representation of ND:YAG Laser Beam Drilling Process

Fig. 2 Schematic representation of ND:YAG laser beam drilling process

large number of independent parameters that control the laser micro-drilling process, a parametric study was conducted by Biswas et al. [33] in order to determine which parameters should be considered for optimization. In this case, the pulse width was varied keeping pulse rate constant, and it was found that power output was remain constant for range of pulse width and the experiments were conducted in those range of pulse with. Therefore the optimization of the laser micro-drilling process was obtained by varying only the pulse width, pulse rate, assist gas flow rate, and assist gas supply pressure in this study.

During experimentation, some errors may be induced due to system sensitivity and other process-related activities. Oxygen as assist gas acts as an additional heat source and helps in improving the rate of machining. The formation of oxides like  $\text{Cr}_2\text{O}_3$  formed during the process also supports this argument. In this context, it is relevant to mention that material removal during micro-drilling occurs as a result of both vaporization and melting. The molten metal is ejected out from the entry side by the recoil pressure generated by material vaporization and it can erode the wall of hole. This results in greater interaction between hole wall and the melt and may lead to higher hole diameter. Higher flow rate of assist gas increases the material removal rate as more gas will be available for oxidation during machining process but high gas line pressure may increase the entry side diameter.

Further, at higher pulse frequency less or no time is available for the molten material to re-solidify and as a result the hole diameter tend to increase. Also at higher pulse width and low pulse frequency material removal rate, width of HAZ and hole diameter may be affected. The SOD between the laser head and the workpiece is mechanical which may induce some backlash error in the gauges leading to variation in hole size. Also, the beam spot may not fall exactly on the workpiece surface due to this variation.

**Table 1** Nominal chemical composition of Domex C67 steel

| C   | Si   | Mn  | P     | S     | Cr  | Mo    | V    | Fe      |
|-----|------|-----|-------|-------|-----|-------|------|---------|
| 0.7 | 0.35 | 0.7 | 0.025 | 0.025 | 0.3 | 0.015 | 0.01 | Balance |

In multiobjective optimization, the loss in some quality characteristics is always expected as compared to a single objective optimization but the overall quality always improves. In the present case, performance characteristics like the hole circularity, small HAZ, and high material removal rate (MRR) have improved considerably.

The summary of experimental conditions is listed in Table 2. The experimental results after laser micro-drilling were evaluated in terms of the following measured machining performances: (1) hole diameter; (2) width HAZ; (3) MRR. Each parameter was measured five times and average value taken for a more accurate reading. The hole diameter and width of HAZ of laser micro-drilling surfaces were measured using a OLYMPUS GX41 ( $\times 100$ ) microscope. In order to achieve best drilling quality, Taguchi's experimental design, an efficient plan, was used for conducting experiments (Fig. 3). For this purpose, a  $L_{25}$  orthogonal array was used for experiment (Table 3). The experimental results are summarized in Table 3.

In order to perform the experiment, the following steps are followed:

- Step 1: In order to perform the experiment work-piece of specific dimension was taken and it was cut into five pieces, and surface of each work-piece specimen was grinded using surface grinding machine. Then, it was polished in the surface polishing machine, using emery paper of different grades like—400, 220, 100, 80, 0/1, 0/2, 0/3, 0/4.
- Step 2: On a sample work piece of same material laser drilling was done by keeping the pulse rate constant and varying pulse width. Output power was measured using the wattmeter. Pulse width range for which output power remains constant was determined from the experiment.
- Step 3: Drilling of microholes was carried out by focusing laser light on the specimen one by one by varying its pulse width, number of pulses, assist gas flow rate and pressure subsequently. For each pulse width from the previously determined range, the pulse rate was varied in the range 1 to 5 with variation of assist

gas pressure and flow rate according to taguchi  $L_{25}$  array, and holes were drilled and the output power was measured simultaneously.

- Step 4: The work pieces were etched with the solution of natal (dehydrated ethyl alcohol ( $C_2H_5OH$ ) and concentrated nitric acid ( $HNO_3$ ) 10%  $v/v$ ).The post-etched work pieces were again polished with fine-grade emery paper.
- Step 5: After getting completely smooth surface, the specimens were kept under the electronic microscope which was interfaced to the computer through a data cable. The digital camera was used to capture the photographs of each hole. Using the Material-plus software the holes and HAZ diameter measurement were measured.

### 3 Grey relational analysis

In grey relational analysis, black represents having no information and white represents having all information. A grey system has a level of information between black and white [34]. This analysis can be used to represent the grade of correlation between two sequences so that the distance of two factors can be measured discretely. In the case when experiments are ambiguous or when the experimental method cannot be carried out exactly, grey analysis helps to compensate for the shortcoming in statistical regression [35]. Grey relation analysis is an effective means of analyzing the relationship between sequences with less data and can analyze many factors that can overcome the disadvantages of statistical method [36].

#### 3.1 Data pre-processing

In GRA, when the range of sequences is large or the standard value is large, the functions of factors are neglected. However, if a factor's measured unit, goals, and directions are different, the grey relational analysis might produce incorrect results. Therefore, original exper-

**Table 2** Laser micro-drilling factors and their levels

| Symbol | Cutting factors      | Level 1 | Level 2 | Level 3 | Level 4 | Level 5 |
|--------|----------------------|---------|---------|---------|---------|---------|
| A      | Pulse width          | 1,700   | 1,800   | 1,900   | 2,000   | 2,100   |
| B      | No. of pulses        | 1       | 2       | 3       | 4       | 5       |
| C      | Assist gas pressure  | 2       | 3       | 4       | 5       | 6       |
| D      | Assist gas flow rate | 1       | 2       | 3       | 4       | 5       |

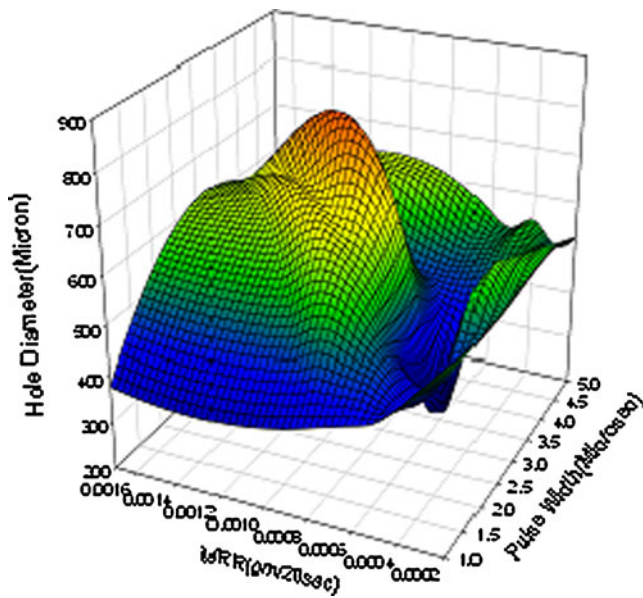


Fig. 3 Behavior of performance parameters—Hole diameter and MRR with respect to pulse width

Experimental data must be pre-processed to avoid such effects. Data pre-processing is the process of transforming the original sequence to a comparable sequence. For this purpose, the experimental results are normalized in the range of zero and one, the process is called grey relational generating. Three different types of data normalization according to whether we require the lower is better, the higher is better, or nominal the best. The normalization is taken by three approaches. If the target value of original sequence is infinite, then it has a characteristic of “the-larger-the-better” (e.g. benefit). The original sequence can be normalized as follows [37];

$$x_i^*(k) = \frac{x_i^0(k) - \min x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \tag{1}$$

If the expectancy is the-smaller-the-better (e.g. cost and defects), then the original sequence should be normalized as follows:

$$x_i^*(k) = \frac{\max x_i^0(k) - x_i^0(k)}{\max x_i^0(k) - \min x_i^0(k)} \tag{2}$$

Table 3 Experimental layout using an L<sub>25</sub> orthogonal array and multi-performance results

| Experiment no. | A Pulse width (μs) | B Pulse/s | C Assist gas Pressure (Kg/Cm <sup>2</sup> ) | D Assist gas flow rate (Lit/Min) | Diameter (μm) | HAZ (μm) | MRR (g/20 s) |
|----------------|--------------------|-----------|---|----------------------------------|---------------|----------|--------------|
| 1              | 1                  | 1         | 2   | 1                                | 390.99        | 490.23   | 0.0008       |
| 2              | 1                  | 2         | 3   | 2                                | 438.57        | 542.96   | 0.0004       |
| 3              | 1                  | 3         | 4   | 3                                | 436.38        | 664.04   | 0.0005       |
| 4              | 1                  | 4         | 5   | 4                                | 418.61        | 572.19   | 0.0006       |
| 5              | 1                  | 5         | 6   | 5                                | 518.22        | 634.75   | 0.0004       |
| 6              | 2                  | 1         | 3   | 3                                | 485.91        | 699.19   | 0.0003       |
| 7              | 2                  | 2         | 4   | 4                                | 532.63        | 772.41   | 0.0007       |
| 8              | 2                  | 3         | 5   | 5                                | 423.21        | 607.52   | 0.0004       |
| 9              | 2                  | 4         | 6   | 1                                | 469.7         | 715.25   | 0.0006       |
| 10             | 2                  | 5         | 2   | 2                                | 488.29        | 643.90   | 0.0003       |
| 11             | 3                  | 1         | 4   | 5                                | 347.24        | 405.45   | 0.0006       |
| 12             | 3                  | 2         | 5   | 1                                | 533.43        | 853.41   | 0.0004       |
| 13             | 3                  | 3         | 6   | 2                                | 534.23        | 698.90   | 0.0003       |
| 14             | 3                  | 4         | 2   | 3                                | 425.24        | 665.03   | 0.0006       |
| 15             | 3                  | 5         | 3   | 4                                | 282.06        | 334.48   | 0.0005       |
| 16             | 4                  | 1         | 5   | 2                                | 553.06        | 799.28   | 0.0003       |
| 17             | 4                  | 2         | 6   | 3                                | 465.28        | 686.11   | 0.0005       |
| 18             | 4                  | 3         | 2   | 4                                | 589.14        | 682.48   | 0.0017       |
| 19             | 4                  | 4         | 3   | 5                                | 465.61        | 731.99   | 0.0007       |
| 20             | 4                  | 5         | 4   | 1                                | 495.36        | 685.24   | 0.0005       |
| 21             | 5                  | 1         | 6   | 4                                | 498.58        | 638.88   | 0.0003       |
| 22             | 5                  | 2         | 2   | 5                                | 545.35        | 723.26   | 0.0004       |
| 23             | 5                  | 3         | 3   | 1                                | 542.65        | 727.56   | 0.0006       |
| 24             | 5                  | 4         | 4   | 2                                | 521.31        | 649.17   | 0.0002       |
| 25             | 5                  | 5         | 5   | 3                                | 518.92        | 642.33   | 0.0005       |

**Table 4** Sequences of performance characteristic after data preprocessing

| Expt No.<br>Reference<br>sequence | Diameter (in μm)<br>1.00000 | HAZ (in μm)<br>1.00000 | MRR (g/20 s)<br>1.00000 |
|-----------------------------------|-----------------------------|------------------------|-------------------------|
| 1                                 | 0.64527159                  | 0.69986318             | 0.6                     |
| 2                                 | 0.490328253                 | 0.598250246            | 0.866666                |
| 3                                 | 0.497459945                 | 0.364923978            | 0.8                     |
| 4                                 | 0.555327602                 | 0.541922803            | 0.733333                |
| 5                                 | 0.23094959                  | 0.421367044            | 0.866666                |
| 6                                 | 0.336166471                 | 0.297188445            | 0.933333                |
| 7                                 | 0.184023707                 | 0.156090417            | 0.666666                |
| 8                                 | 0.540347792                 | 0.473840402            | 0.866666                |
| 9                                 | 0.388954018                 | 0.266240148            | 0.733333                |
| 10                                | 0.328416048                 | 0.403734608            | 0.933333                |
| 11                                | 0.787742608                 | 0.863237816            | 0.733333                |
| 12                                | 0.181418523                 | 0                      | 0.866666                |
| 13                                | 0.178813339                 | 0.297747288            | 0.933333                |
| 14                                | 0.533737137                 | 0.363016206            | 0.733333                |
| 15                                | 1                           | 1                      | 0.8                     |
| 16                                | 0.117493813                 | 0.104310793            | 1                       |
| 17                                | 0.403347662                 | 0.322394157            | 0.8                     |
| 18                                | 0                           | 0.32938932             | 0                       |
| 19                                | 0.402273023                 | 0.233981462            | 0.666666                |
| 20                                | 0.305392732                 | 0.324070684            | 0.8                     |
| 21                                | 0.294906865                 | 0.41340836             | 0.933333                |
| 22                                | 0.142601277                 | 0.25080454             | 0.866666                |
| 23                                | 0.151393774                 | 0.242518259            | 0.733333                |
| 24                                | 0.220887065                 | 0.393579095            | 0.933333                |
| 25                                | 0.228670053                 | 0.302449271            | 0.8                     |

However, If the expectancy is nominal-the-best (e.g., the age), if there is a definite target value to be achieved, the original sequence will be normalized in the form

$$x_i^*(k) = 1 - \frac{|x_i^0(k) - x^0|}{\max x_i^0(k) - x^0} \tag{3}$$

or the original sequence can be simply normalized by the most basic methodology, i.e. let the values of original sequence be divided by the first value of sequence:

$$x_i^k = \frac{x_i^0(k)}{x_i^0(1)} \tag{4}$$

where  $x_i^k(k)$  is the value after the grey relational generation (data pre-processing),  $\max x_i^0(k)$  is the largest value of  $x_i^0(k)$ ,  $\min x_i^0(k)$  is the smallest value of  $x_i^0(k)$  and  $x_i^0$  is the desired value.

### 3.2 Grey relational coefficient and grey relational grade

Following data pre-processing, a grey relational coefficient is calculated to express the relationship between the ideal and actual normalized experimental results. The grey relational coefficient can be expressed as follows [38]:

$$\zeta_i(k) = \frac{\Delta_{\min} + \zeta \cdot \Delta_{\max}}{\Delta_{0i} + \zeta \cdot \Delta_{\max}} \tag{5}$$

where  $\Delta_{0i}(k)$  is the deviation sequence of the reference sequence  $x_0^*(k)$  and the compatibility sequence  $x_i^*(k)$  namely;

$$\begin{aligned} \Delta_{0i}(k) &= \left\| x_0^*(k) - x_i^*(k) \right\| \\ \Delta_{\min}(k) &= \max_{\forall j \in i} \min_{\forall k} \left\| x_0^*(k) - x_j^*(k) \right\| \\ \Delta_{\max}(k) &= \max_{\forall j \in i} \max_{\forall k} \left\| x_0^*(k) - x_j^*(k) \right\| \end{aligned}$$

$\zeta$  is distinguish or identification coefficient:  $\zeta \in [0, 1]$ ,  $\zeta = 0.5$  is generally used. After obtaining the grey relational coefficient, the average of the grey relational coefficient is

**Table 5** The deviation sequences

| Deviation sequence | $\Delta_{0i}(1)$ | $\Delta_{0i}(2)$ | $\Delta_{0i}(3)$ |
|--------------------|------------------|------------------|------------------|
| Exp. No.1          | 0.354728         | 0.300136         | 0.4              |
| Exp. No.2          | 0.509671         | 0.401749         | 0.133334         |
| Exp. No.3          | 0.502540         | 0.635076         | 0.2              |
| Exp. No.4          | 0.444672         | 0.458077         | 0.266667         |
| Exp. No.5          | 0.769050         | 0.578632         | 0.133334         |
| Exp. No.6          | 0.663833         | 0.702811         | 0.066667         |
| Exp. No.7          | 0.815976         | 0.843909         | 0.333334         |
| Exp. No.8          | 0.459652         | 0.526159         | 0.133334         |
| Exp. No.9          | 0.611045         | 0.733759         | 0.266667         |
| Exp. No.10         | 0.671583         | 0.596265         | 0.066667         |
| Exp. No.11         | 0.212257         | 0.136762         | 0.266667         |
| Exp. No.12         | 0.818581         | 1                | 0.133334         |
| Exp. No.13         | 0.821186         | 0.702252         | 0.066667         |
| Exp. No.14         | 0.466262         | 0.636983         | 0.266667         |
| Exp. No.15         | 0                | 0                | 0.2              |
| Exp. No.16         | 0.882506         | 0.895689         | 0                |
| Exp. No.17         | 0.596652         | 0.677605         | 0.2              |
| Exp. No.18         | 1                | 0.670610         | 1                |
| Exp. No.19         | 0.597726         | 0.766018         | 0.333334         |
| Exp. No.20         | 0.694607         | 0.675929         | 0.2              |
| Exp. No.21         | 0.705093         | 0.586591         | 0.066667         |
| Exp. No.22         | 0.857398         | 0.749195         | 0.133334         |
| Exp. No.23         | 0.848606         | 0.757481         | 0.266667         |
| Exp. No.24         | 0.779112         | 0.606420         | 0.066667         |
| Exp. No.25         | 0.771329         | 0.697550         | 0.2              |

**Table 6** Grey relational coefficient and Grey relational grade

| Expt no. | Grey relational coefficient |          |          | Grey relational grade | Order |
|----------|-----------------------------|----------|----------|-----------------------|-------|
|          | Diameter                    | HAZ      | MRR      |                       |       |
| 1        | 0.584981                    | 0.624893 | 0.555555 | 0.588476              | 5     |
| 2        | 0.495210                    | 0.554478 | 0.789472 | 0.613053              | 3     |
| 3        | 0.498733                    | 0.440499 | 0.714285 | 0.551172              | 13    |
| 4        | 0.529284                    | 0.521878 | 0.652173 | 0.567778              | 11    |
| 5        | 0.393995                    | 0.463550 | 0.789472 | 0.549005              | 14    |
| 6        | 0.429614                    | 0.415692 | 0.882352 | 0.575886              | 8     |
| 7        | 0.379946                    | 0.372049 | 0.599999 | 0.450664              | 24    |
| 8        | 0.521022                    | 0.487253 | 0.789472 | 0.599249              | 4     |
| 9        | 0.450026                    | 0.405265 | 0.652173 | 0.502488              | 20    |
| 10       | 0.426773                    | 0.456094 | 0.882352 | 0.588406              | 6     |
| 11       | 0.701993                    | 0.785222 | 0.652173 | 0.713129              | 2     |
| 12       | 0.379195                    | 0.333333 | 0.789472 | 0.500666              | 21    |
| 13       | 0.378447                    | 0.415886 | 0.882352 | 0.558895              | 12    |
| 14       | 0.517457                    | 0.439760 | 0.652173 | 0.536463              | 15    |
| 15       | 1                           | 1        | 0.714285 | 0.904761              | 1     |
| 16       | 0.361662                    | 0.358245 | 1        | 0.573302              | 10    |
| 17       | 0.455933                    | 0.424590 | 0.714285 | 0.531602              | 16    |
| 18       | 0.333333                    | 0.427127 | 0.333333 | 0.364597              | 25    |
| 19       | 0.455487                    | 0.394939 | 0.599999 | 0.483475              | 22    |
| 20       | 0.418547                    | 0.425195 | 0.714285 | 0.519342              | 18    |
| 21       | 0.414905                    | 0.460154 | 0.882352 | 0.585803              | 7     |
| 22       | 0.368351                    | 0.400257 | 0.789472 | 0.519360              | 17    |
| 23       | 0.370753                    | 0.397620 | 0.652173 | 0.473515              | 23    |
| 24       | 0.390896                    | 0.451907 | 0.882352 | 0.575051              | 9     |
| 25       | 0.393289                    | 0.417519 | 0.714285 | 0.508364              | 19    |

normally taken as the grey relational grade. The grey relational grade is defined as follows:

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n \zeta_i(k) \tag{6}$$

However, since in real application, the effect of each factor on the system is not exactly same. Equation 6 can be modified as:

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n w_k \zeta_i(k) \quad \sum_{k=1}^n w_k = 1 \tag{7}$$

Where  $w_k$  represents the normalized weighing value of factor  $k$ . Given the same weights, Eqs. 6 and 7 are equal.

In the grey relational analysis, the grey relational grade is used to show the relationship among the sequences. If the two sequences are identical, then the value of grey relational grade is equal to 1. The grey relational grade also indicates the degree of influence that the comparability sequence could exert over the reference sequence. Therefore, if a particular comparability sequence is more important than the other comparability sequences to the reference sequence, then the grey relational grade for that

**Table 7** Response table for the grey relational grade

| Cutting parameters | Average grey relational grade by factor level |           |                       |                       |                       |          |
|--------------------|---|-----------|-----------------------|-----------------------|-----------------------|----------|
|                    | Level 1                                       | Level 2   | Level 3               | Level 4               | Level 5               | Max–min  |
| A                  | 0.5738968                                     | 0.5433386 | 0.642782 <sup>a</sup> | 0.4944636             | 0.5324186             | 0.148319 |
| B                  | 0.607319                                      | 0.514264  | 0.509485              | 0.5330506             | 0.613975 <sup>a</sup> | 0.104490 |
| C                  | 0.516589                                      | 0.554567  | 0.627694 <sup>a</sup> | 0.540326              | 0.547722              | 0.111100 |
| D                  | 0.517974                                      | 0.566086  | 0.541904              | 0.610708 <sup>a</sup> | 0.529715              | 0.092734 |

<sup>a</sup> Optimal level

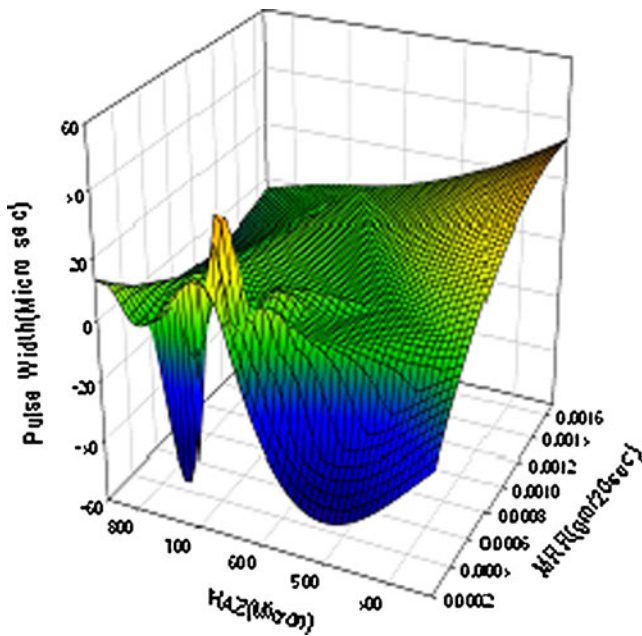


Fig. 4 Behavior of performance parameters—HAZ and MRR with respect to pulse width

comparability sequence and reference sequence will be higher than other grey relational grades [15]. The grey relational algorithm involves the following steps.

- Step 1: Normalize the experimental results of each performance characteristic.
- Step 2: Compute the value of  $\Delta_{0i}(k) = \|x_0^*(k) - x_i^*(k)\|$ ,  $\Delta_{\min}(k) = \max_{\forall j \in i} \min_{\forall k} \|x_0^*(k) - x_j^*(k)\|$  and  $\Delta_{\max}(k) = \max_{\forall j \in i} \max_{\forall k} \|x_0^*(k) - x_j^*(k)\|$  to calculate the grey relational coefficient.
- Step 3: Calculate the grey relational grade by the mean value of grey relational coefficient.
- Step 4: Compute the response table and response graph for each level of the laser drilling parameters.
- Step 5: Recognize the noticeable variable factors and selects the optimal levels of drilling parameter.
- Step 6: Run confirmation test and verify.

**4 Analysis and interpretation of experimental results**

In the present study, the hole diameter, width of HAZ, and MRR for different pulsed width and pulse rate are measured, and the experimental runs are listed in Table 3. Typically, lower values of the hole diameter, width of HAZ, and higher MRR as the target values are desirable. Therefore, the data sequences have a smaller the better characteristic for hole diameter and HAZ and larger the better characteristics for MRR.

The values of the hole diameter, width of HAZ, and MRR are set to be the reference sequence,  $x_i^*(k)$ ,  $k=1-3$ . Moreover, the results of 25 experiments were the comparability sequences  $x_0^*(k)$ ,  $i=1,2,\dots,25$ ,  $k=1-3$ . Table 4 lists all of the sequences following data pre-processing using Eq. 2 for hole diameter and HAZ and Eq. 1 for MRR.

The deviation sequences;  $\Delta_{0i}$ ,  $\Delta_{\max}(k)$ , and  $\Delta_{\min}(k)$  for  $i=1,2,\dots,25$ ,  $k=1-3$  can be calculated as follows:

$$\Delta_{01}(1) = |x_0^*(1) - x_1^*(1)| = |1 - 0.64527159| = 0.354728$$

$$\Delta_{02}(2) = |x_0^*(2) - x_1^*(2)| = |1 - 0.69986318| = 0.300136$$

$$\Delta_{03}(3) = |x_0^*(3) - x_1^*(3)| = |1 - 0.6| = 0.4$$

The results for all deviation sequence are given in Table 5.

Using Table 5, the  $\Delta_{\max}$  and  $\Delta_{\min}$  can be found as follows;

$$\Delta_{\max} = \Delta_{18}(1) = \Delta_{12}(2) = \Delta_{18}(3) = 1$$

$$\Delta_{\min} = \Delta_{15}(1) = \Delta_{15}(2) = \Delta_{16}(2) = 0$$

The distinguishing coefficient  $\zeta$  can be substituted for the grey relational coefficient in Eq. 5. If all the process parameters have equal weighting,  $\zeta$  is 0.5. Table 6 lists the

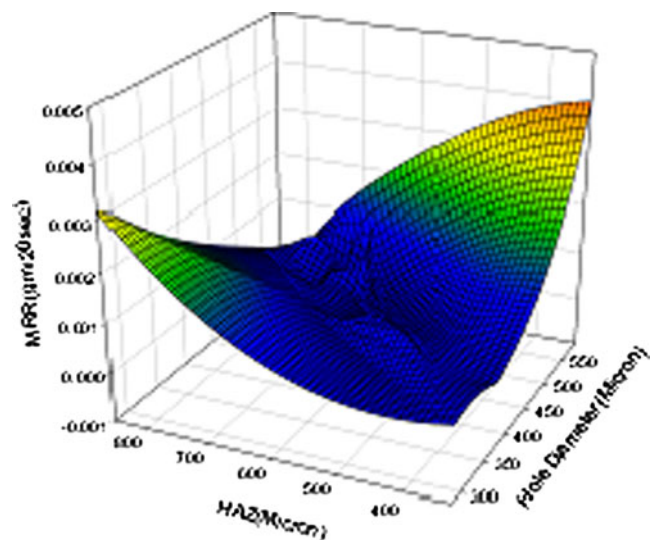
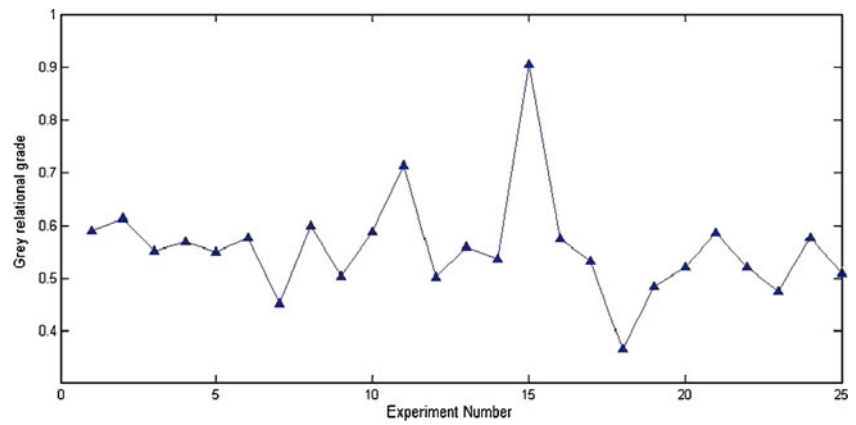


Fig. 5 Behavior of performance parameters—hole diameter, HAZ, and MRR



Fig. 6 Grey relational grade



grey relational coefficient and grade for each experiment of the  $L_{25}$  orthogonal array by applying Eqs. 5 and 6.

It is clearly observed from Table 6 and Fig. 6 that the laser micro-drilling parameters setting of experiment no. 15 has the highest grey relation grade. Thus, the fifteenth experiment gives the best multi-performance characteristics among the 25 experiments.

The response table of Taguchi method is employed here to calculate the average grey relational grade for each factor level. The average grey relational grade is computed by grouping the relational grades by considering the factor level for column in the QA and then by taking average of these values [37]. For example, the grey relational grades for factors A and B at level 1 can be calculated as follows:

$$\gamma_{A1} = (1/5)(0.588476 + 0.613053 + 0.551172 + 0.567778 + 0.549005)$$

$$\gamma_{B1} = (1/5)(0.588476 + 0.575886 + 0.713129 + 0.573302 + 0.585803)$$

Using the same method, calculations were performed for each factor level and response table was generated, as shown in Table 7.

Since the grey relational grades represented the level of correlation between the reference and the comparability sequences, the larger grey relational grade means the comparability sequence exhibits a stronger correlation with the reference sequence. Therefore, the comparability sequence has a larger value of grey relational grade for the diameter of the hole, width of HAZ, and MRR. The behavior of performance parameters and their interrelationship are presented in Fig. 4 and Fig. 5. Based on this premise, this study selects the level that provides the largest average response. In Table 7,  $A_3$ ,  $B_5$ ,  $C_3$ , and  $D_4$  show the largest value of grey relational grade for

factors A, B, C, and D, respectively. Therefore,  $A_3B_5C_3D_4$  is the condition for the optimal parameter combination of the laser cutting process. Restated, laser pulse width is 1,900  $\mu\text{m}$ , pulse rate is 5/s, assist gas pressure 4  $\text{kg}/\text{cm}^2$ , and assist gas flow rate 4 L/min. It is possible to find the difference between the initial and optimal machining performances in Fig. 8. The influence of each drilling parameter can be more clearly presented by means of the grey relational grade graph. The grey relational grade graph shows the change in the response, when the factors go for their levels 1–5. The response graph for the drilling parameters of the laser micro-drilling process is presented in Fig. 9. In this graph, the greater values give the low diameter of the hole, width of HAZ, and optimum MRR dashed line indicates the value of the total mean of the grey relational grade (Figs. 6, 7, 8, 9).

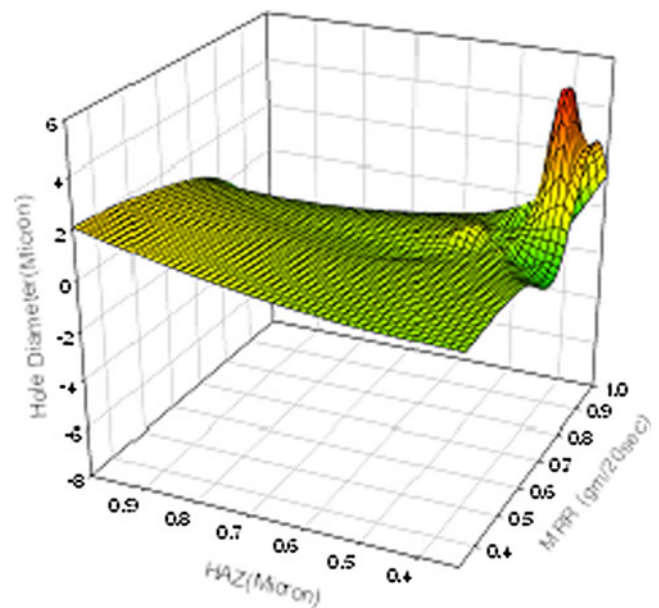
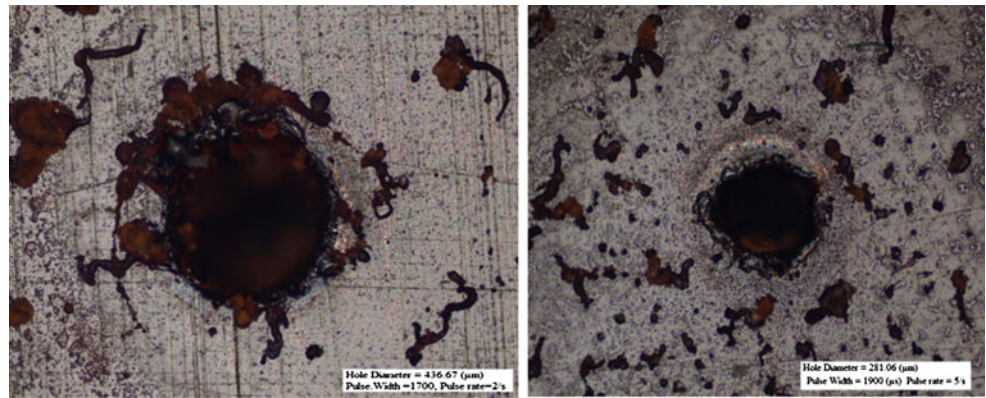


Fig. 7 Grey relational coefficient of performance parameters

**Fig. 8** Comparison between initial hole and optimized\* hole



P.W =1700, Pulse rate=2/s(initial)  
 Assist gas pre.= 2kg/cm<sup>2</sup>  
 Flow rate= 1 lit/Min

P.W =1900, Pulse rate=5/s\*(optimized)  
 Assist gas pre.= 4kg/cm<sup>2</sup>  
 Flow rate= 4 lit/Min

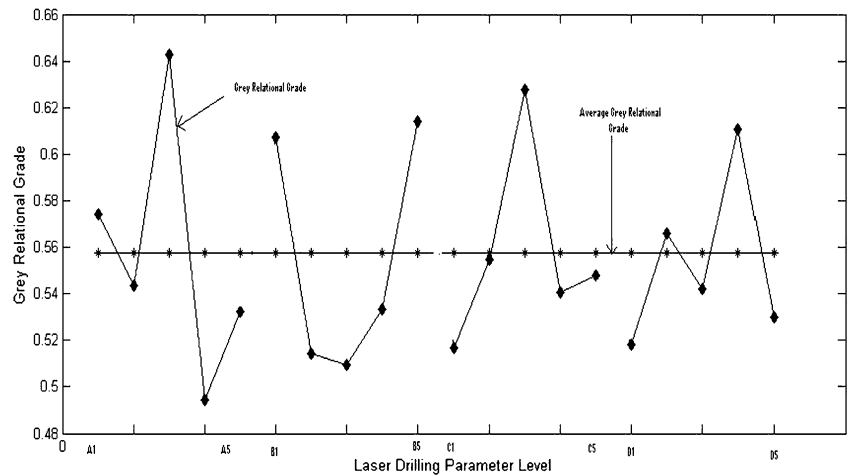
When the last column of Table 7 was compared, it is observed that the difference between the maximum and minimum value of the grey relational grade for factor A is bigger than factor Cs, Bs, and Ds. This indicates that the laser pulse width has stronger effect on the multi-performance characteristics than pulse rate factor. If the number of machining Parameters increases, the importance of the controllable factors on the multi-performance characteristics will be determined by ordering max–min grade relational values.

**5 Conclusion**

This study applies the grey relational analysis to optimize the laser drilling process with the multiple-performance criteria. The response table and response

graph for each level of the laser micro-drilling parameters are obtained from the grey relational grade and find out the optimal levels of laser micro-drilling parameters. Grey relational analysis for the work piece hole diameter, HAZ width, and MRR obtained from the Taguchi method converts optimization of the multiple-performance characteristics into optimization of a single performance characteristic, i.e., the grey relational grade. As a result, optimization of complicated multiple-performance characteristics is greatly simplified through this approach. It is shown that the performance characteristics of the laser drilling process such as hole diameter, HAZ width, and MRR are improved simultaneously by using the method proposed herewith. Through this study, the noticeable variable factors are effectively recognized to decrease try–error time and consuming cost in the state of increasing quality and reduce production costs.

**Fig. 9** Effect of laser drilling parameters on multi-performance



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