

# Hydrodynamic analysis of partial film lubrication in the cold rolling process

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**Abstract** Lubrication in cold rolling plays an important part for process feasibility and process quality. The hydrodynamic process of lubrication is very complicated and affected by many material and process parameters. This paper examined partial lubrication in the cold rolling process. The average flow Reynolds equation for rolling lubrication was 2set, which considered the pressure-viscosity and average flow effects. Lubricating factors such as sidling, surface waviness, lubricant viscosity, surface roughness, and reduction ratio were investigated. The results of the lubrication equation show that sliding, lubricant viscosity, and surfaces roughness affect the values of rolling friction. Surface waviness and reduction ratio also influence both rolling pressure and rolling friction.

**Keywords** Lubrication · Average flow theory · Surface waviness · Rolling force · Rolling friction

## 1 Introduction

Rolling lubrication is one of the most complex problems in tribology because the rolling process is often accompanied by large pressure, sliding, plastic deformation, heating, and wearing. These problems make both theoretical and experimental studies in this field difficult. Further, there is currently no general theory that can address rolling

lubrication problems accurately. In the 1970s, a number of studies were conducted in the field of partial film lubrication [1]. Some researchers have applied these theories to metal working lubrication. For example, Greenwood and Tallian [1, 2] examined the partial film lubricating problems between two sliding rough surfaces, and they found that the total rolling pressure and total friction are shared by the fluid film and the surface asperity in partial film lubrication. Johnson also conducted similar studies wherein the random theory was used to describe surface asperity, and a model for film thickness calculation was provided [3]. Patir and Cheng developed the theory of hydrodynamic lubrication and proposed the famous average flow model for partial film lubrication [4, 5]. Liu and Tieu developed a thermal partial film lubrication model for cold rolling [6, 7]. Wang and Yu studied the non-steady-state rolling process with the partial film lubrication theory and obtained theoretical solutions for rolling pressure under sinusoidal back tension [8]. Sun et al. applied Patir and Cheng's theory to cold rolling lubrication and obtained a series of solutions to calculate inlet zone film thickness under different rolling conditions [9]. Lenard's research team have done various laboratory works for partial film lubrication in the process of metal forming [10, 11].

However, the features of lubrication depend not only on surface roughness but also on other factors such as relative sidling, surface waviness, lubricant viscosity, and reduction ratio, which can also affect the hydrodynamic state in rolling lubrication [2, 9, 12]. Therefore, partial film lubrication in the cold rolling process is discussed in this paper. The average flow Reynolds equation which considers both pressure-viscosity and average flow effects is set up in the sections below. The analysis includes the distributions of rolling pressure and friction under different factors such as relative sidling, surface waviness, surface roughness, lubricant viscosity, and reduction ratio.

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## 2 Lubricating equation for cold rolling

Studies have shown that partial film lubrication is the main form of lubrication in the rolling process [3]. The actual contact of rough surfaces in the cold rolling process is shown in Fig. 1.

The rolling direction is set as the  $x$ -axis in the analysis, whereas the direction perpendicular to the surface of the paper is set as the  $y$ -axis. Thus, the lubricating equation for cold rolling can be set up as follows [3, 4]:

$$\frac{\partial}{\partial x} \left( \frac{h_T^3}{12\eta} \frac{\partial \bar{p}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{h_T^3}{12\eta} \frac{\partial \bar{p}}{\partial y} \right) = \frac{u_1 + u_2}{2} \frac{\partial h_T}{\partial x} + \frac{\partial h_T}{\partial t} \quad (1)$$

$\bar{p}$  is the pressure in the contact zone,  $h_T$  is the actual thickness of the lubricating film,  $u_1$  and  $u_2$  are the speed of the rollers and the rolling sheet, respectively, and  $\eta$  is the viscosity of the lubricating fluid. Taking the pressure-viscosity effect of the lubricant into account, the viscosity of the disperse oil phase is given as follows:

$$\eta = \eta_0 e^{\xi P} \quad (2)$$

where  $\eta_0$ ,  $\xi$ , and  $P$  are the initial viscosity of the lubricant, the pressure coefficient of viscosity, and the total rolling pressure, respectively. The equation for calculating the actual film thickness can be given as

$$h_T = h + \delta_1 + \delta_2 \quad (3)$$

$h$  is the nominal film thickness or the gap between the surfaces of the smooth roller and the smooth sheet,  $\delta_1$  and  $\delta_2$  are the heights of surface asperities, and  $h$  can be given as [3]

$$h = h_0 \left( \frac{u_1 + u_2}{u_x + u_2} \right) \quad (4)$$

$u_x$  is the instantaneous tangential speed of the roller. Wilson derived an equation to calculate the entrance film thickness  $h_0$  [13, 14]:

$$h_0 = \frac{3\eta_0 \xi R (u_1 + u_2)}{x_0 (1 - e^{-\xi(\sigma_s - \sigma_t)})} \quad (5)$$

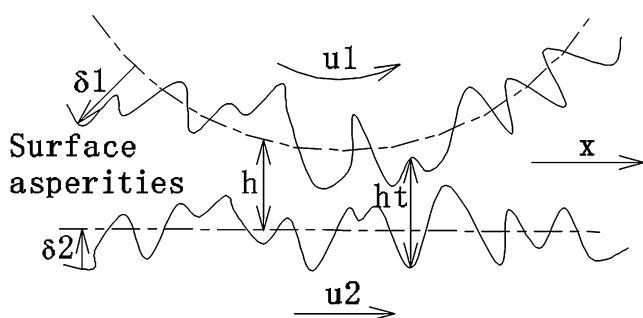


Fig. 1 Schematic of surface asperity contact

where  $\eta_0$ ,  $\xi$ , and  $R$  are the initial viscosity of the lubricant, the pressure coefficient of viscosity, and the radius of the roller, respectively;  $x_0$  is the length of the deformation zone; and  $\sigma_s$  and  $\sigma_t$  are the yield stress of the rolling sheet and the back tension, respectively.

### 2.1 Average flow parameters

To solve Eq. 1, Patir and Cheng's average flow model can be used. A rectangular cell  $\Delta x \times \Delta y$  is taken from the lubrication interface. Then the average flow parameters are added into Eq. 1. Thus, the original equation is transformed into the average flow lubrication equation [4, 5]:

$$\frac{\partial}{\partial x} \left( \frac{h^3}{12\eta} \phi_x \frac{\partial \bar{p}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{h^3}{12\eta} \phi_y \frac{\partial \bar{p}}{\partial y} \right) = \frac{U_1 + U_2}{2} \frac{\partial \bar{h}_T}{\partial x} + \frac{U_1 - U_2}{2} \delta \frac{\partial \phi_s}{\partial x} + \frac{\partial \bar{h}_T}{\partial t} \quad (6)$$

where  $\phi_x$  and  $\phi_y$  are the pressure parameters in the  $x$ - and  $y$ -directions,  $\phi_s$  is the shear parameter,  $\delta$  is the mean square value for surface roughness ( $\delta = \sqrt{\delta_1^2 + \delta_2^2}$ ), and  $\bar{h}_T$  is the value of average film thickness. From Greenwood and Williamson's formula for surface roughness,  $\bar{h}_T$  can be expressed as [15]:

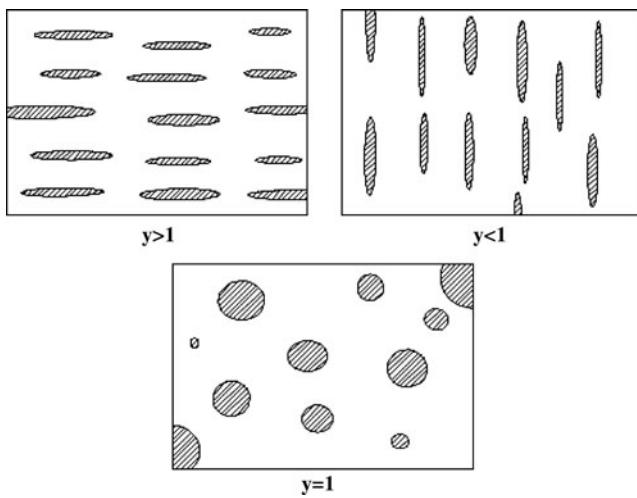
$$\bar{h}_T = \int_{-h}^{\infty} (h + z) f(z) dz \quad (7)$$

where  $\phi_x$ ,  $\phi_y$ , and  $\phi_s$  all depend on the values of  $h/\delta$  and the surface parameter  $\gamma$ . These parameters can only be calculated through the numerical method. The parameter  $\gamma$  reflects the feature of surface waviness of the rolling sheet. When  $\gamma > 1$ , the waviness of the sheet surface is distributed horizontally. When  $\gamma = 1$ , there is no difference between the distribution of the  $x$ -axis and  $y$ -axis on the sheet surface. When  $\gamma < 1$ , the surface waviness distribution becomes a vertical stripe. That is, a smaller  $\gamma$  means the waviness of the sheet surface is more vertical as shown in Fig. 2.

### 2.2 Simplified form of the lubricating equation

When the axial length of the roller is much greater than the length of the lubrication zone, the effects of edge leaking in the lubricating interface can be neglected. Thus, Eq. 5 is simplified as:

$$\frac{\partial}{\partial x} \left( \frac{h^3}{12\eta} \phi_x \frac{\partial \bar{p}}{\partial x} \right) = \frac{U_1 + U_2}{2} \frac{\partial \bar{h}_T}{\partial x} + \frac{U_1 - U_2}{2} \sigma \frac{\partial \phi_s}{\partial x} + \frac{\partial \bar{h}_T}{\partial t} \quad (8)$$



**Fig. 2** Surface directional feature versus  $\gamma$

To simplify Eq. 7 once more, two parameters  $U = (U_1 - U_2)/2$  and  $U_s = (U_1 + U_2)/2$  are adopted:

$$\frac{\partial}{\partial x} \left( \frac{h^3}{12\eta} \phi_x \frac{\partial \bar{p}}{\partial x} \right) = U \frac{\partial \bar{h}_T}{\partial x} + U_s \sigma \frac{\partial \phi_s}{\partial x} + \frac{\partial \bar{h}_T}{\partial t} \quad (9)$$

To facilitate the calculation of Eq. 8, a parameter  $\phi_c$  is used:

$$\phi_c = \frac{\partial \bar{h}_T}{\partial h} \quad (10)$$

Then Eq. 8 can be simplified a step further as follows:

$$\frac{\partial}{\partial x} \left( \frac{h^3}{12\eta} \phi_x \frac{\partial \bar{p}}{\partial x} \right) = U \phi_c \frac{\partial h}{\partial x} + U_s \sigma \frac{\partial \phi_s}{\partial x} + \phi_c \frac{\partial h}{\partial t} \quad (11)$$

### 2.3 Analysis of the dynamic feature in the rolling lubrication process

In the process of cold rolling, the pressure in the lubricating zone often goes beyond the yield stress of the work piece. Partial film lubrication and direct contact of surface asperity occur simultaneously. This is called partial film lubrication. The rolling force should be shouldered by the lubricating film and by surface asperity. Therefore, the total rolling pressure  $P$  contains the contact pressure  $P_c$  and the lubricating pressure  $P_h$ :

$$P = P_c + P_h \quad (12)$$

$P_c$  is the plastic contact pressure of surface asperity.  $P_c$  can be given by:

$$P_c = \sigma_s A_c \quad (13)$$

$\sigma_s$  is the yield stress of the rolling sheet,  $A_c$  is the real contact area of the rough surface, and  $A$  is the nominal

contact area. The lubricating pressure  $P_h$  can be expressed as:

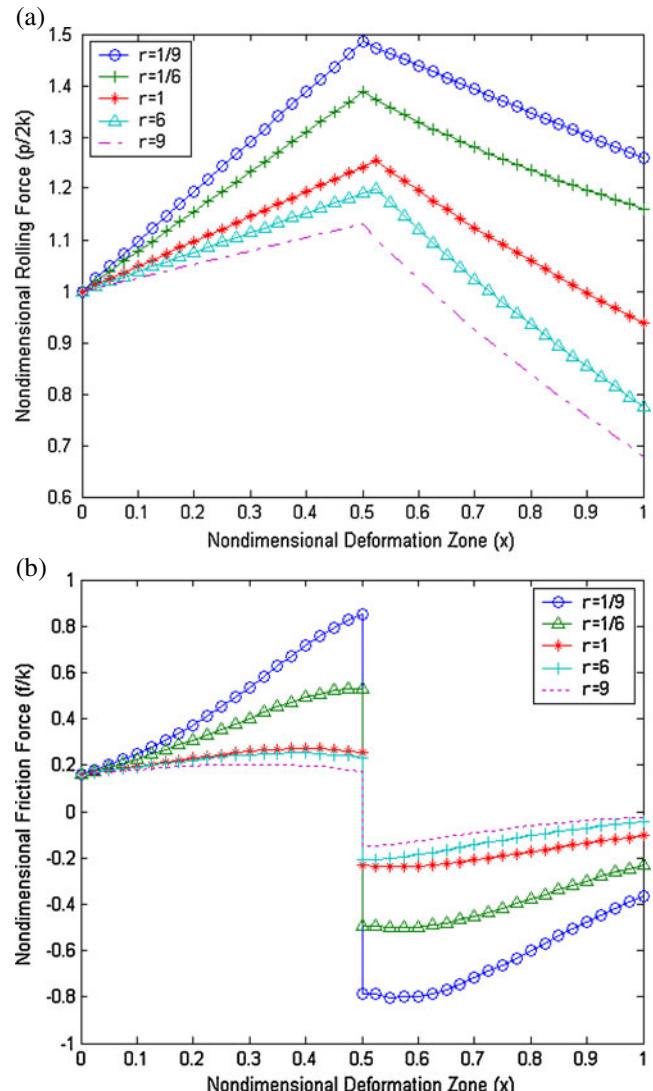
$$P_h = \bar{p}A(1 - A_c/A) \quad (14)$$

The real contact area ratio  $A_c/A$  can be obtained by the following integral [14]:

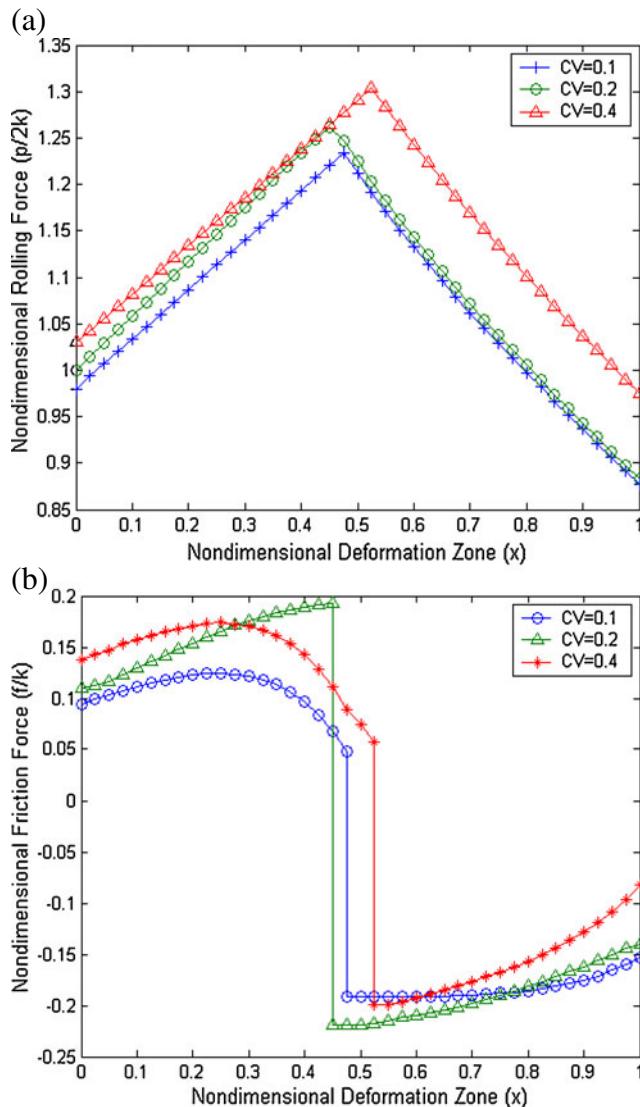
$$A_c/A = \int_h^\infty f(z)dz \quad (15)$$

According to the theory of adhesive friction, the friction stress on the roughness surface of the rolling sheet can be given by:

$$\tau_c = k \quad (16)$$



**Fig. 3** **a** Non-dimensional rolling force under varied surface waviness. **b** Non-dimensional friction stress under varied surface waviness



**Fig. 4** **a** Non-dimensional rolling force under different lubricant viscosity coefficient. **b** Non-dimensional friction stress under different lubricant viscosity coefficient

$k = \sigma/2$  is the shear yield stress of the rolling sheet. The friction stress of hydrodynamic lubrication can be expressed as:

$$\tau_h = \eta \frac{U_1 - U_2}{h} \quad (17)$$

The equation for total friction can be expressed as:

$$F = F_c + F_h = \tau_c A_c + \tau_h A (1 - A_c/A) \quad (18)$$

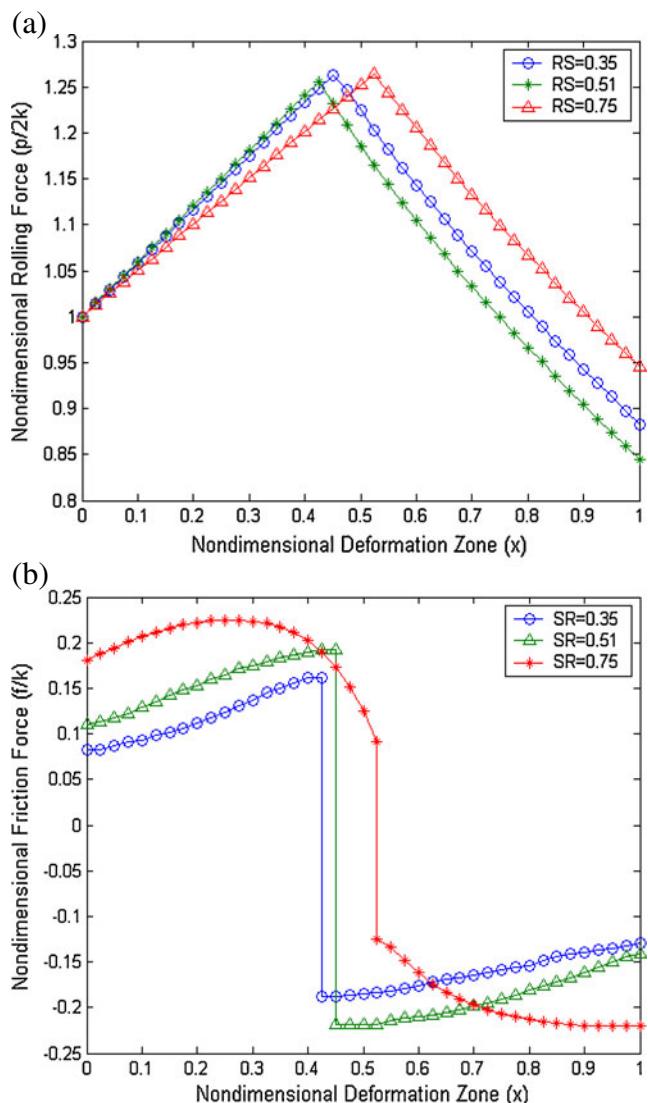
### 3 Example and results of analysis

According to relations 4 and 5,  $h$  is obtained. Equation 7 was solved using the initial value of  $h$  by the integrating method of Runge Kutta. To solve the Reynolds Eq. 11 and to find  $\bar{p}$ ,

the central difference method was used. All of the equations were solved by numerical calculation on Matlab. The distribution of rolling pressure and the friction stress in the deformation zone are studied in detail. Data for analysis were obtained from the experiment of rolling aluminum sheets on a two-roll mill with a roll diameter of 130 mm, lubricant viscosity of 0.2 Pas, surface roughness of 0.25  $\mu\text{m}$ , rolling speed of  $\mu_1=0.32$  m/s, thickness of aluminum sheet of 1.0 mm, and yield stress of aluminum of 120 MPa.

#### 3.1 Calculation results under different surface waviness values

The surface waviness of the rolling sheets affects the distribution of the fluid lubricant in the deformation zone. Therefore, surface waviness can also change the rolling



**Fig. 5** **a** Non-dimensional rolling force under different surface roughness. **b** Non-dimensional friction stress under different surface roughness

force and the rolling friction. The values of waviness parameters  $\gamma$  varied ( $\gamma=1/9, 1/6, 1, 6$ , and  $9$ ), whereas the inlet velocity was fixed in the calculation (0.95-fold of the rolling velocity). The coefficient of lubricant viscosity was 0.2 Pas, and the ratio of rolling reduction was 20%. The distribution graphs of non-dimensional rolling force and rolling friction are shown in Fig. 3a, b.

### 3.2 Calculation results under different viscosity values

The rolling pressure in the deformation zone is quite high. Thus, the pressure–viscosity effect of the lubricant cannot be neglected. This has a significant influence on rolling lubrication. The parameter of surface waviness was unchanged ( $\gamma=1$ ), whereas the coefficient of lubricant viscosity varied (0.1, 0.2, and 0.4 Pas) in the calculation. The inlet velocity was 0.95-fold of the rolling speed, and the ratio of rolling reduction was 20%. The distribution graphs of non-dimensional rolling force and rolling friction are shown in Fig. 4a, b.

### 3.3 Calculation results under different roughness values

The relationship between surface roughness and rolling lubrication is quite complex. The value of rolling surface roughness varied as 0.25, 0.45, and 0.71  $\mu\text{m}$ , whereas the parameter of surface waviness was unchanged ( $\gamma=1$ ). The coefficient of lubricant viscosity was 0.2 Pas. The inlet rolling velocity was 1.05-fold of the rolling speed, and the ratio of rolling reduction was 20%. The distribution graphs of non-dimensional rolling force and rolling friction are shown in Fig. 5a, b.

## 4 Conclusions

1. The values of  $\gamma$  (surface waviness parameter) substantially affect the values of rolling force and rolling friction. A smaller  $\gamma$  causes a smaller rolling stress, whereas a smaller  $\gamma$  also means a higher rolling friction. That is, the rolling friction is higher when the surface waviness appears as a vertical stripe. On the contrary, it is smaller when the surface waviness appears as a horizontal stripe.
2. The viscosity coefficient of the lubricant substantially affects the values of rolling force and rolling friction. The rolling force and rolling friction are higher when a relatively large viscosity coefficient of the lubricant is used in the rolling process. A higher viscosity coeffi-

cient also means that the neutral plane is closer to the export.

3. Surface asperity has little effect on the value of the rolling force, but it can affect the results of rolling friction. A higher value of surface roughness means a higher rolling friction. It also indicates that the neutral plane is closer to the export.

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