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ORIGINAL ARTICLE

Scheduling with deteriorating jobs and past-sequence-dependent setup times

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Abstract In this paper, we presented a scheduling model in which the deteriorating jobs and the setup times are considered at the same time. Under the proposed model, the actual job processing time is a general function of the processing times of jobs already processed and its scheduled position, while the setup time is past-sequence-dependent. We provided the optimal schedules for some single-machine scheduling problems.

Keywords Scheduling · Deteriorating jobs · Past-sequence-dependent setup times · Single-machine

1 Introduction

In classical scheduling, the job processing times are assumed to be known and fixed. However, there are many real-life situations where a job processed later consumes more time than the same job processed earlier, which is known as "deteriorating jobs" scheduling in the literature. Examples can be found in scheduling maintenance or cleaning procedures, steel rolling operations, and fire fighting work. It was first introduced by Gupta and Gupta [1] and Browne and Yechiali [2]. Since then, many deteriorating jobs scheduling models and problems have been extensively studied from a variety of perspectives. Alidaee and Womer [3], Cheng et al. [4] and Gawiejnowicz [5] provided comprehensive reviews of different models and problems.

Recently, Ji and Cheng [6] considered the parallel-machine scheduling problem where the process service requests from various customers who are entitled to different levels of grade of service. They provided a fully polynomial-time approximation scheme for the makespan problem. Toksari and Guner [7] formulated a mixed nonlinear integer programming for a parallel-machine earliness/tardiness scheduling problem with simultaneous effects of learning and linear deterioration. They provided the optimal solutions for problems with 11 jobs and two machines. Later, Yin and Xu [8] studied the same problem further. Wu et al. [9] addressed a single-machine problem where the objective is to minimize the makespan under the piecewise linear deterioration model. They provided a branch-and-bound algorithm and two heuristic algorithms to search for the optimal solution and near-optimal solutions, respectively. Lee et al. [10] studied a single-machine problem to minimize the total completion time with deteriorating jobs and machine availability constraints. They derived the optimal and near-optimal solution for the problem of up to 20 jobs.

However, most of the research treats the setup time as a part of the job processing time. The research of deteriorating jobs scheduling with separate consideration of setup time is relatively limited. Wu et al. [11] studied the single-machine group scheduling problems where the setup time and the job processing times are simple linear functions of their starting times. They showed that the makespan and the total completion time problems remain polynomially solvable. Wang et al. [12] considered a single-machine problem with the deteriorating jobs and the group technology assumption. They showed that the makespan problem

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is polynomially solvable. Liu et al. [13] considered some two-agent scheduling problems with deteriorating jobs and group technology on a single machine. They proposed the optimal properties and presented the optimal polynomial-time algorithms for two different scheduling problems, respectively. Wei and Wang [14] considered two single-machine problems with the deteriorating jobs and the group technology assumption. They provided polynomial algorithms for problems to minimize the weighted sum of squared completion times and the weighted sum of squared waiting times, respectively. Lee et al. [15] showed that the makespan problem is polynomially solvable under the model where the actual job processing time is a function of the sum of processing times of jobs already processed. Recently, Koulamas and Kyparisis [16] presented the concept of “past-sequence-dependent” (p-s-d) setup times. They pointed out that in high-tech manufacturing, jobs are commonly processed in batches, e.g., a batch of jobs may consist of a group of electronic components mounted together on an integrated circuit board, whereby each batch incurs a setup time. They derived the optimal solutions for the makespan, the total completion time, and the total absolute differences in completion time problems. In addition to this un-readiness of components, Biskup and Herrmann [17] provided another example of wear-out of equipment (e.g., a drill), in which the sum of the processing times of the prior jobs adds to the processing time of the actual job. They showed that the maximum lateness, the maximum tardiness, and the total tardiness problems remain polynomially solvable under certain agreeable conditions. Wang et al. [18] dealt with some single-machine problems with setup time considerations where the processing time of a job is given as a function of its starting times and its position in a sequence. Under the consideration of the learning and deterioration effects, they showed that the makespan, the total completion time, and the sum of the kth power of job completion time problems can be solved in polynomial time, respectively. They also showed that the total weighted completion time problem, the maximum lateness problem, and the number of tardy jobs problem can be solved in polynomial time under certain conditions. Wang et al. [19] provided the optimal schedules for some single-machine scheduling problems with the consideration of learning effect and p-s-d setup times. They proved that the sum of the quadratic job completion time problem, the total waiting time problem, the total absolute differences in waiting time problem, and the sum of earliness penalties problem subject to no tardy jobs can be solved in polynomial time, respectively. Yin et al. [20] considered a model with p-s-d setup times and the learning effect. They provided the optimal schedules for some single-machine scheduling problems, such as the makespan, the sum of the kth power of completion time, and the total weighted completion time and the maximum lateness under certain agreeable conditions. Yin et al. [21] proposed a scheduling model with the consideration of the p-s-d setup times, the learning and deterioration effects. They derived the optimal solutions for the makespan, the total completion time problems. Moreover, they showed that the total weighted completion time, the maximum lateness, and the number of tardy jobs problems are polynomially solvable under certain agreeable conditions. With the consideration of the p-s-d setup times, the learning, and deterioration effects, Bakalke et al. [22] provided a tabu search and a genetic algorithm for the makespan problem.

In this paper, we study a deterioration model with p-s-d setup times. Under the proposed model, the actual job processing time is a general function of the normal processing times of jobs already processed and its scheduled position. The remainder of this paper is organized as follows. We formulate the problem in the next section. In Section 3, we derive the optimal solutions for some single-machine problems. We conclude the paper in the final section.

2 Problem formulation

There are n jobs ready to be processed on a single machine. For each job j, there is a normal processing time p_j, a weight $w_{j,}$ and a due date d_j. Inspired by Yin et al. [20], the actual processing time of job j is

$$p_{j[r]} = p_j f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) \text{ for } r = 1, 2, \cdots, n \tag{1}$$

if it is scheduled in the rth position where $p_{[k]}$ denotes the normal processing time of the job scheduled in the kth position in a sequence due to the deterioration effects. It is assumed that $f : (0, M) \times [1, \infty) \to [1, \infty)$ is a differentiable non-decreasing function with respect to each variable and $f_x(x, y_0) = \frac{\partial}{\partial x} f(x, y_0)$ is non-decreasing with respect to x for every fixed y_0. Moreover, it is assumed that $f_x(B + x, y) \geq f(B + x, y)/x$ for every x, y in domain and $0 \leq B \leq M$. In addition, the setup time is also taken into consideration. As in Koulamas and Kyparisis [16], the p-s-d setup time of job j if it is scheduled in the rth position of a sequence is as follows:

$$s_{j[1]} = 0 \text{ and } s_{j[r]} = b \sum\nolimits_{l=1}^{r-1} p_{[1]} \tag{2}$$

where b is a normalizing constant number with $0<b<1$, and $p_{[l]}$ denotes the normal processing time of a job if it is scheduled in the lth position. It is seen from the model that the actual job processing time will be prolonged if it is scheduled in a later position or at a later time since f is a non-decreasing function of both the variables. Throughout the paper, we use C_j, $L_j = C_j - d_j$ and $T_j = \max\{0, C_j - d_j\}$ to denote the completion time, the lateness and the tardiness of job j.

3 Single-machine problems

In this section, we will provide the optimal solutions for several single-machine problems under the proposed model. Before presenting the main results, we first state a lemma from Lai and Lee [23].

Lemma 1 *If $f_x(x,y_0)$ is non-decreasing with respect to x for every fixed y_0 and $f_x(B+x,y) \geq f(B+x,y)/x$ for every x, y and $0 \leq B \leq M$, then $x_1 f(B+x_2,y) - x_2 f(B+x_1,y) \leq 0$ for every $x_1 \geq x_2$, y and $0 \leq B \leq M$.*

Suppose that S and S' are two job schedules and the difference between S and S' is a pairwise interchange of two adjacent jobs i and j. That is, $S=(\pi, i, j, \pi')$ and $S'=(\pi, i, j, \pi')$, where π and π' each denote a partial sequence. Furthermore, we assume that there are $r-1$ jobs in π. In addition, let A denote the completion time of the last job in π. Under the proposed model, the completion times of jobs i and j in S are

$$C_i(S) = A + b\sum_{k=1}^{r-1} p_{[k]} + p_i f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) \tag{3}$$

and

$$C_j(S) = A + b\sum_{k=1}^{r-1} p_{[k]} + p_i f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) + b\left(\sum_{k=1}^{r-1} p_{[k]} + p_i\right) + p_j f\left(\sum_{k=1}^{r-1} p_{[k]} + p_i, r+1\right). \tag{4}$$

Similarly, the completion times of jobs j and i in S' are

$$C_j(S') = A + b\sum_{k=1}^{r-1} p_{[k]} + p_j f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) \tag{5}$$

and

$$C_i(S') = A + b\sum_{k=1}^{r-1} p_{[k]} + p_j f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) + b\left(\sum_{k=1}^{r-1} p_{[k]} + p_j\right) + p_i f\left(\sum_{k=1}^{r-1} p_{[k]} + p_j, r+1\right). \tag{6}$$

Property 1 Under the proposed model, the optimal schedule for the makepsan problem is obtained by the shortest processing time (SPT) rule.

Proof Suppose $p_j \geq p_i$. To show that S dominates S', it suffices to show that $C_j(S) \leq C_i(S')$. Taking the difference between Eqs. 4 and 6, we have

$$C_i(S') - C_j(S) = (p_j - p_i)\left(b + f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right)\right) + p_i f\left(\sum_{k=1}^{r-1} p_{[k]} + p_j, r+1\right) - p_j f\left(\sum_{k=1}^{r-1} p_{[k]} + p_i, r+1\right) \tag{7}$$

Substituting $B = \sum_{k=1}^{r-1} p_{[k]}$, $x_1=p_j$, $x_2=p_i$, and $y=r+1$ into Eq. 7, we have from Lemma 1 and $p_j \geq p_i$ that $C_j(S') \geq C_i(S)$. This completes the proof of Property 1.

Property 2 Under the proposed model, the optimal schedule for the total completion time problem is obtained by the SPT rule.

Proof The proof is omitted since it is similar to that of Property 1.

The weighted shortest processing time (WSPT) rule provides the optimal solution for the classical total weighted completion time problem. In the following, we will show that the result still holds if the job processing times and the weights are agreeable, i.e., $p_j/p_i \geq 1 \geq w_j/w_i$ for all jobs i and j.

Property 3 Under the proposed model, the optimal schedule for the total weighted completion time problem is obtained by the WSPT rule if the processing times and the weights are agreeable.

Proof Suppose that $p_j/p_i \geq 1 \geq w_j/w_i$. Since $p_i \leq p_j$, it implies from Property 1 that $C_j(S) \leq C_i(S')$. To show that S dominates S', it suffices to show that $w_iC_i(S) + w_jC_j(S) \leq w_jC_j(S') + w_iC_i(S')$. From Eqs. 3 to 6, we have Lemma 1 that

$$\begin{aligned}
&[w_jC_j(S') + w_iC_i(S')] - [w_iC_i(S) + w_jC_j(S)] \\
&= f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right)(w_i + w_j)(p_j - p_i) \\
&\quad + w_i\left(p_i f\left(\sum_{k=1}^{r-1} p_{[k]} + p_j, r+1\right) + b\sum_{k=1}^{r-1} p_{[k]} + bp_j\right) \\
&\quad - w_j\left(p_j f\left(\sum_{k=1}^{r-1} p_{[k]} + p_i, r+1\right) + b\sum_{k=1}^{r-1} p_{[k]} + bp_i\right) \\
&\geq f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right)(w_i + w_j)(p_j - p_i) \\
&\quad + w_i\left(p_i f\left(\sum_{k=1}^{r-1} p_{[k]} + p_j, r+1\right) - p_j f\left(\sum_{k=1}^{r-1} p_{[k]} + p_i, r+1\right)\right) \\
&\geq 0
\end{aligned}$$

since $w_i \geq w_j$ and $p_i \leq p_j$. This completes the proof.

In the following, we will show that the earliest due date (EDD) order provides the optimal solutions for the total tardiness, the maximum lateness, and the maximum tardiness problems if the job processing times and the due dates are agreeable, i.e., $d_i \leq d_j$ implies $p_i \leq p_j$ for all jobs i and j.

Property 4 Under the proposed model, the optimal schedule for the total tardiness problem is obtained by the EDD first rule if the job processing times and the due dates are agreeable.

Proof Suppose that $d_i \leq d_j$. It also implies $p_i \leq p_j$ since they are agreeable. The total tardiness of the first $r-1$ jobs are the same since they are processed in the same order. Since the makespan is minimized by the SPT rule (Property 1), the total tardiness of partial sequence π' in S will not be greater than that of π' in S'. Thus, to prove that the total tardiness of S is less than or equal to that of S', it suffices to show that $T_i(S) + T_j(S) \leq T_j(S') + T_i(S')$.

To compare the total tardiness of jobs i and j in S and in S′, we divide it into two cases. In the first case with $A + b\sum_{k=1}^{r-1} p_{[k]} + p_j f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) \leq d_j$, we have from Eqs. 3 to 6 that the total tardiness of jobs i and j in S and in S' are

$$T_i(S) + T_j(S) = \max\left\{A + b\sum_{k=1}^{r-1} p_{[k]} + p_i f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) - d_i, 0\right\} + \max\left\{A + b\sum_{k=1}^{r-1} p_{[k]} + p_i f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) + b\left(\sum_{k=1}^{r-1} p_{[k]} + p_i\right) + p_j f\left(\sum_{k=1}^{r-1} p_{[k]} + p_i, r+1\right) - d_j, 0\right\},$$

and $T_j(S') + T_i(S') = \max\left\{A + b\sum_{k=1}^{r-1} p_{[k]} + p_j f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) + b\left(\sum_{k=1}^{r-1} p_{[k]} + p_j\right) + p_i f\left(\sum_{k=1}^{r-1} p_{[k]} + p_j, r+1\right) - d_i, 0\right\}.$

Suppose that neither $T_i(S)$ nor $T_j(S)$ is zero. Note that this is the most restrictive case since it comprises the case that either one or both $T_i(S)$ and $T_j(S)$ are zero. From Property 1 and $d_i \leq d_j$, we have

$$\left\{T_j(S') + T_i(S')\right\} - \left\{T_i(S) + T_j(S)\right\} = (p_j - p_i)\left(f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) + b\right) + p_i f\left(\sum_{k=1}^{r-1} p_{[k]} + p_j, r+1\right) - p_j f\left(\sum_{k=1}^{r-1} p_{[k]} + p_i, r+1\right) + d_j - A - b\sum_{k=1}^{r-1} p_{[k]} - p_i f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) \geq 0.$$

Thus, $\left\{T_j(S') + T_i(S')\right\} - \left\{T_i(S) + T_j(S)\right\} \geq 0$ in the first case. In the second case that $A + b\sum_{k=1}^{r-1} p_{[k]} + p_j f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) > d_j$, the total tardiness of jobs i and j in S and in S′ are

$$T_i(S) + T_j(S) = \max\left\{A + b\sum_{k=1}^{r-1} p_{[k]} + p_i f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) - d_i, 0\right\} + \max\left\{A + b\sum_{k=1}^{r-1} p_{[k]} + p_i f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) + b\left(\sum_{k=1}^{r-1} p_{[k]} + p_i\right) + p_j f\left(\sum_{k=1}^{r-1} p_{[k]} + p_i, r+1\right) - d_j, 0\right\},$$

and $T_j(S') + T_i(S') = 2A + b\sum_{k=1}^{r-1} p_{[k]} + 2p_j f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right) + b\left(\sum_{k=1}^{r-1} p_{[k]} + p_j\right) + p_i f\left(\sum_{k=1}^{r-1} p_{[k]} + p_j, r+1\right) - d_i - d_j.$

Suppose that neither $T_i(S)$ nor $T_j(S)$ is zero. From Property 1, $d_i \leq d_j$ and $p_i \leq p_j$, we have

$$\{T_j(S') + T_i(S')\} - \{T_i(S) + T_j(S)\}$$
$$= (p_j - p_i)\left(b + 2f\left(\sum_{k=1}^{r-1} p_{[k]}, r\right)\right)$$
$$+ p_i f\left(\sum_{k=1}^{r-1} p_{[k]} + p_j, r+1\right) - p_j f\left(\sum_{k=1}^{r-1} p_{[k]} + p_i, r+1\right) \geq 0.$$

Thus, $\{T_j(S') + T_i(S')\} - \{T_i(S) + T_j(S)\} \geq 0$ in the second case. This completes the proof of Property 4.

Property 5 Under the proposed model, the EDD order yields the optimal schedule for the maximum lateness problem if the job processing times and the due dates are agreeable.

Property 6 Under the proposed model, the EDD order yields the optimal schedule for the maximum tardiness problem if the job processing times and the due dates are agreeable.

4 Conclusions

In this paper, we presented a scheduling model in which the deteriorating jobs and the setup times are considered at the same time. Under the proposed model, the actual job processing time is a general increasing function of the normal processing times of jobs already processed and its scheduled position, while the setup time is past-sequence-dependent. We showed that the SPT rule provides the optimal solutions for the makespan and the total completion time problems, the WSPT rule provides the optimal schedule for the total weighted completion time problem if the processing times and the weights are agreeable, and the EDD rule provides the optimal schedules for the total tardiness, the maximum lateness, and the maximum tardiness problems if the job processing times and the due dates are agreeable.

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