

Modeling blast furnace productivity using support vector machines

Abhijit Ghosh · Sujit K. Majumdar

Received: 3 December 2009 / Accepted: 15 June 2010 / Published online: 14 July 2010
© Springer-Verlag London Limited 2010

Abstract Productivity of a modern generation blast furnace was modeled with the help of a leading supervised learning tool viz. Support Vector Machines in the form of (1) minimum error, maximum margin classification function in binary setting of productivity classes (low/high) and (2) the class-specific regression functions for real values of productivity based on epsilon sensitive loss function and minimum regulated risk. The SVMs were trained with large number data-points each of which consisted of a setting of 21 critical input parameters of blast furnace, corresponding productivity value observed, and the productivity class (low/high) attributed. During the training session of the SVMs, the vectors of critical input parameters were required to be mapped into high-dimensional feature space via Radial basis kernel as function and the optimum SVM-RBF classifying function with chosen setting of its hyperparameters that had good generalization property was found using quadratic optimization. The SVM-RBF classifying function could be used to predict the class of productivity (low/high) for any given setting of the critical input parameters. Class-specific SVM-RBF regression models were also developed for both low as well as high-productivity classes and these models could be used to predict real value of productivity for any given setting of the critical input parameters. The SVM-RBF regression model fitted to the high-productivity class was subjected to constrained nonlinear optimization treatment to find the optimum setting of the critical input parameters that gave maximum productivity. The optimum setting of the critical parameters could be used as the target setting obtaining high productivity in the blast furnace.

Keywords Blast furnace productivity · Supervised learning · Support vector classification · Support vector regression · Nonlinear optimization · Maximum productivity

1 Introduction

Recurring spells of low productivity in a modern generation (having provision of pulverized coal injection facility) blast furnace in an integrated steel plant was the motivation behind the present study. The blast furnace (BF) that was designed to give higher productivity was commissioned about 5 years back. After attaining stability since commissioning, the BF did give increased productivity initially for a period of about 3 years. The problem of low productivity started thereafter. The problem was investigated by in-house teams of experts who reportedly used established mathematical models for simulating BF operations and recommended quite a few solutions. None of the solutions gave lasting improvement in BF productivity. Under the situation, the present study was undertaken and it was decided to use altogether a new approach involving one of the recent and leading supervised learning machines, viz. Support Vector Machines (SVM) for modeling the BF productivity. This approach was not yet used in BF although reports of application of machine learning approach appeared in diverse fields like, biomedical problems, magnetic resonance imaging, linear signal processing, speech recognition, image processing, wireless communication problems, spread spectrum receiver design, channel equalization, [1–13], and even for modeling burden layer thickness in blast furnace with neural network [14].

The use of a supervised learning machine like SVMs in the present study had quite a few practical implications. An enormous amount of rich information generated by the blast furnace year after year was almost left unused. The informa-

A. Ghosh (✉) · S. K. Majumdar
Indian Statistical Institute,
Kolkata, India
e-mail: abghosh35@gmail.com

S. K. Majumdar
e-mail: sujitm1@gmail.com

tion contained an enormous number of data-points each consisting of a setting of the critical input parameters $\{x_1, x_2, \dots, x_n\}$ of the blast furnace and the associated productivity $\{y_1, y_2, \dots, y_n\}$ values that were recorded on real time basis. To address the problem of recurring low productivity in the blast furnace, it was therefore ideal to explore the unknown dependency between the high dimensional vectors of the critical input parameters and the observed productivity y (i.e., output, scalar, or vector) by training a leading supervised learning machine, viz. Support Vector Machines with these available data-points arranged in suitable form (i.e., describing the BF productivity associated with each setting of the critical input parameters either in two classes viz. low/high productivity classes or, in terms of its real value). The necessity of using SVMs with high-speed capabilities and “learning” abilities of support vectors was all the more felt because, use of traditional models (thermodynamic and kinetic etc.) by the operating team did not provide any lasting solution to the problem of low productivity. It was contemplated that, the off-line-trained SVMs would bring out the unknown dependence (linear or nonlinear) between the two sets of variables (i.e., vectors of critical input parameters and class/real value of productivity) either in the form of optimal classification function $f_a(x, w) \sim y$ or minimum error regression function. The optimal SVM classification function with good generalization ability (ensured during training session) would be able to predict whether any given setting of the critical input parameters of the BF at any given time-point would be associated with high productivity or low productivity. Similarly, the SVMs could also be trained to express the unknown dependence between the vectors of critical input parameters and real value of productivity in the form of regression models for both low- and high-productivity classes. The class-specific regression model could then be used to predict the real value of BF productivity y corresponding to any given setting of the critical input parameters $\{x_1, x_2, \dots, x_n\}$ both in low- and high-productivity classes. Prediction of either the class of productivity (low/high) or real value of productivity for any $\{x_1, x_2, \dots, x_n\}$ would enable the operating team of the BF to work out suitable control strategy and to take appropriate preventive/corrective actions whenever needed (particularly when low productivity was predicted).

The second major implication of the present study was that the SVM regression model fitted to the high-productivity class could be subjected to nonlinear optimization treatment in constrained space of the critical input parameters (as defined by the observed/allowable lower and upper bounds of each critical input parameter) with a view to obtain the optimal setting of the critical input parameters that would most likely give maximum BF productivity. The optimum setting would serve as a good guideline for the BF operating team who could try to operate in the nearest

neighborhood of this optimum setting after having treated the same as the target setting.

The third major implication of the study was that, the off-line-trained SVMs could be implemented in control system hardware and embedded on any device for knowing real-time prediction on BF productivity based on on-line learning. In on-line learning, the SVMs would update (fine-tune) the current classification function or, the current class-specific regression functions with each new data-point $\{x_1, x_2, \dots, x_n; y_i\}$ and would be able to predict more accurately both the class (low/high) or the real value of BF productivity for any given setting of the critical input parameters. If the constrained nonlinear optimization algorithm is integrated with the control system of the blast furnace, it would also be possible to obtain the updated optimum setting of the critical parameters (i.e., target setting) associated with the maximum BF productivity.

Thus, the objectives of the present study were formulated as follows:

1. Finding an appropriate classification function by which it would be possible to predict with minimum error whether a given setting of the critical input parameters would be associated with high- or low-productivity class.
2. Finding the adequate SVM-regression models for the two productivity classes (low and high) such that, real value of the productivity can be predicted for a given setting of the critical input parameters within a class.
3. Determining the optimum setting of the critical input parameters corresponding to maximum productivity by subjecting the SVR-regression model of the high productivity class to constrained nonlinear optimization treatment.

2 Review of literature

A large number of mathematical models (both analytical and empirical) have been developed to characterize BF operations. These models are of the following types: heat and mass balance models, reaction-kinetic models and thermodynamic models, simulation models, neural network models etc. The thermodynamic and kinetic models are based on thermodynamic and kinetic characteristics of various processes of BF and expressed in terms of differential equations and difference equations which were solved against the boundary conditions obtained from chosen values of the process variables [15]. Muchi et al. [16] originally proposed a one-dimensional model which considered major chemical reactions and heat transfer and gave the distributions of process variables along furnace height. Many models were developed later extending this modeling technique. Chenl et al. [17] designed an expert system for continuously monitoring blast furnace performance that included a journal, in addition to the usual three major

components viz. the knowledge base, the global database, and the inference engine to account for the time complexities. Bilik et al. [18] developed a model for monitoring the performance of a blast furnace by combining a thermodynamic model (which estimated the minimal theoretically possible fuel rate in the condition of reaching the thermodynamic balance of wüstite reduction), a kinetic model (based on kinetic characteristics of the individual component of the iron-bearing burden), and a coke degradation model (which estimated the in-furnace changes of coke). Danloy et al. [19] modeled a blast furnace in steady state using the system of differential equations. The solutions were obtained by the finite differences method with the input data like, BF geometry and process data, the chemical and physical properties of the raw materials, the chemical composition of hot metal and slag, description of ore and coke layers, etc. Babich et al. [20] developed a model on the basis of the interrelations of material and heat balances equations. The model simulated the effect of coke, burden and blast parameters on the blast furnace productivity, hot metal heat and quality, slag volume and basicity, top gas volume, chemistry and temperature, etc. Nogami et al. [21] developed a multi-dimensional blast furnace operation simulator based on multi-fluid theory and reaction kinetics. The mathematical expression of reduction behavior of carbon composite agglomerates (CCB) was introduced into the blast furnace simulator and the effect of charging CCB to blast furnace and accompanying reduction in temperature were numerically examined. Dong et al. [22] developed a model to describe the behavior of fluid flow, heat and mass transfer, as well as chemical reactions in a BF in which gas, solid, and liquid phases affect each other through interaction forces, and their flows compete for the space available. Process variables that characterize the internal furnace state, such as reduction degree, reducing gas and burden concentrations, as well as gas and condensed phase temperatures were described quantitatively. Saxen [23] presented a BF model that described the steady-state operation of the furnace in one spatial dimension using real process data sampled at the steelworks. The measurement data were reconciled by an interface routine which yielded boundary conditions obeying the conservation laws of atoms and energy. Azadeh et al. [24] developed an integrated simulation model for a BF by considering all the major and detailed operations and interacting systems of the blast furnace. The model considered maintenance, repairs, quality control activities, systems' limitations and interaction with other systems, and introduced a set of optimizing alternatives through sensitivity analysis. Man-Sheng et al. [25] developed a multi-fluid blast furnace model that was used to investigate the performance of blast furnace under the condition of top gas recycling together with plastics injection, cold oxygen blasting and carbon composite agglomerate charging. Yagi et al. [26] developed another

multi-fluid blast furnace model and showed that the efficiency of blast furnace improved due to the decrease in heat requirements for solution loss, sinter reduction and silicon transfer reactions, and less heat outflow by top gas and wall heat loss. Jindal et al. [27] presented a reduced order BF model based on real time simulation of process variables for process monitoring, control and optimization. Pettersson et al. [14] developed a neural network-based model of the burden layer thickness in blast furnace that was estimated from a single radar measurement of the burden (stock) level inside the furnace. The model described the dependence between the layer thickness and key charging variables. An evolutionary algorithm was applied to train the network weights and connectivity by optimizing the model structure and parameters simultaneously. Xuegong Bi et al. [28, 29] developed a one-dimensional prediction model having considered increased blast temperature, oxygen enrichment of the blast, coal injection alone, coal injection combined with oxygen enrichment and changed coke quality. Matsuzaki et al. [30] developed the mathematical model of a blast furnace by combining models for material transfer, reaction and heat transfer of lumpy zone or cohesive zone. De Castro et al. [31] developed a comprehensive two-dimensional transient mathematical model to evaluate productivity, energy efficiency and transient phenomena for different injection rates of pulverized coal to the blast furnace. The model was built upon the conservation equations of mass, momentum, thermal energy for all phases, phase transformations and chemical reactions, moisture evaporation, reduction of iron oxides, solution loss, coke and pulverized coal combustion, silica reduction, gas phase reactions, etc.

Use of SVMs for modeling BF productivity in the present study was justified because there were no published reports on such applications.

3 Support vector machines—classification and regression

SVMs are quite a recent and leading supervised machine learning approach that constitutes very specific class of algorithms characterized by large margin hyperplanes, usage of kernels, geometrical interpretation of kernels as inner products in a feature space, absence of local minima, sparseness of the solution and capacity control obtained by acting on the margin, or on number of support vectors [32–41].

In binary classification setting, the SVMs learn from the training data $\{x_1, x_2, \dots, x_n\}$ that are vectors in some space $\subseteq \mathbb{R}^d$. Their labels are given $\{y_1, y_2, \dots, y_n\}$ where, y_i belongs to $y_i \in \{-1, 1\}$. The task of learning from training dataset can be formulated in the following way. Given a set of decision functions and data points of the training set drawn from an unknown distribution $P(x, y)$, a function

$f_a(x, w; w = \text{weight vector } w_n \text{ learned in training; } x \text{ training instances})$ is found that provides the minimum expected risk.

$$R(w) = \int |f_a(x, w) - y| P(x, y) dx dy \tag{1}$$

The function $f_a(x, w)$ usually belongs to a hypothesis space of candidate functions $H(f_a \in H)$ which very often include nonlinear kernel functions that are used to transform input data to a high-dimensional feature space in which the input data would become more separable compared with the original input space. The expected risk is a measure of how good a hypothesis is at predicting the correct level of y for a given setting of x .

If the data points in the training set are linearly separable into two classes, the goal of SVMs would be to find the best canonical hyperplane from among the set of canonical hyperplanes that correctly classify the data, the one with minimum norm, or equivalently the minimum $\|w\|^2$. The hypothesis space in this case is therefore the set of functions given by,

$$f(w, b) = \text{sign}(wx + b) \tag{2}$$

Then set of hyper-planes that satisfy the additional constraint $\min_{i=1,2,\dots,l} |wx_i + b| = 1$ are called canonical hyper-planes where, $\{x_1, x_2, \dots, x_n\}$ are the points in the dataset. Linear separability means that it is possible to find a pair (w, b) such that

$$wx_i + b \geq 1 \text{ for all } x_i \in \text{Class1} \tag{3}$$

$$wx_i + b \leq -1 \text{ for all } x_i \in \text{Class1} \tag{4}$$

Minimizing w^2 (in the case of linear separability) will be equivalent to finding the separating hyper-plane for which the distance (i.e., the margin) between the two convex hulls (i.e., the two classes of training data points), measured along a line perpendicular to the hyper-plane is maximized. To construct the maximal margin or, optimal separating hyper-plane, the vector x_i of the training set $\{(x_1, y_1), (x_2, y_2), \dots, (x_l, y_l)\}$; $x_i \in \mathbb{R}^n$ should be correctly classified into two different classes $y_i \in \{-1, 1\}$ using the smallest norm of the coefficients. This can be formulated as,

$$\min_{w,b} \Phi(w) = \frac{1}{2} w^2 \tag{5}$$

subject to $y_i(wx_i + b) \geq 1; i = 1, 2, \dots, l$

At this point the problem can be solved by Quadratic programming optimization technique. For solving this optimization, the dual problem may also be formulated and the technique of Lagrangian multiplier can be used. The Lagrangian is:

$$L(w, b, \Lambda) = \frac{1}{2} w^2 - \sum_{i=1}^l \lambda_i [y_i(wx_i + b) - 1] \tag{6}$$

where, $\Lambda = (\lambda_1, \lambda_2, \dots, \lambda_l)$ is the vector of non-negative Lagrange multipliers corresponding to the constraints $y_i(wx_i + b) \geq 1; i = 1, 2, \dots, l$.

The solution to this optimization problem is determined by a saddle point of this Lagrangian which has to be minimized w.r.t. w, b and maximized w.r.t. $\Lambda \geq 0$. Differentiating the Lagrangian function w.r.t., w and b , and solving the equations, w and b can be estimated and finally the decision function can be written as

$$f(w, b) = \text{sign} \left\{ \sum_{i=1}^l y_i \lambda_i^* (xx_i + b^*) \right\} \tag{7}$$

The extension to more complex non-linear decision surface is done by mapping the input variables x in a higher dimensional feature space via kernel function into vector of feature variables and by working with linear classification in that space. $x \rightarrow \phi(x) = \{a_1\phi_1(x), a_2\phi_2(x), \dots, a_n\phi_n(x), \dots\}$ where, $\{a_n\}_{n=1}^\infty$ are some real numbers $\{\phi_n\}_{n=1}^\infty$ are some real functions. The soft margin version of the SVMs is then applied substituting the variable x with the new feature space $\phi(x)$. Under the mapping, the solution a SVM has the following form

$$\begin{aligned} f(w, b) &= \text{sign} \left\{ \sum_{i=1}^l y_i \lambda_i^* (xx_i + b^*) \right\} \\ &= \text{sign} \left\{ \sum_{i=1}^l y_i \lambda_i^* (\phi(x)\phi(x_i) + b^*) \right\} \end{aligned} \tag{8}$$

A key property of the SVM is that the only quantities that are required to be computed are scalar products of the form $\phi(x) \cdot \phi(y)$. It is therefore convenient to introduce the kernel function K :

$$K(x, y) = \phi(x) \cdot \phi(y) = \sum_{n=1}^\infty a_n^2 \phi_n(x) \cdot \phi_n(y) \tag{9}$$

Using this quantity the solution for the decision surface will be of the form

$$f(w, b) = \text{sign} \left\{ \sum_{i=1}^l y_i \lambda_i^* (K(x, x_i) + b^*) \right\} \tag{10}$$

and the quadratic programming problem becomes:

$$\left. \begin{aligned} F(\Lambda) &= \Lambda \cdot 1 - \frac{1}{2} \Lambda \cdot D \cdot \Lambda \\ &\text{subject to} \\ &\Lambda \cdot y = 0 \\ &\Lambda \leq C \cdot 1 \\ &\Lambda \geq 0 \end{aligned} \right\} \tag{11}$$

where, D is a symmetric semi-positive definite l by l matrix with elements $D_{ij} = y_i y_j K(x_i, x_j)$.

In SVM regression, the input x is first mapped onto an m -dimensional feature space using some fixed non-linear mapping, and then a linear model is constructed in the feature space. The linear model is given by

$$f_a(x, w) = \sum_{j=1}^m w_j g_j(x) + b \tag{12}$$

where, $g_j(x); j = 1, 2, \dots, m$ denotes a set of nonlinear transformations and b is the bias term. The quality of estimation is measured by some loss function $l(y, f_a(x, w))$. SVM regression uses a new type of loss function, known as ε -insensitive loss function:

$$l(y, f_a(x, w)) = \begin{cases} 0 & \text{if } |y - f(x, w)| \leq \varepsilon \\ |y - f(x, w)| - \varepsilon & \text{otherwise} \end{cases} \tag{13}$$

The empirical risk is

$$R_{\text{emp}}(w) = \frac{1}{n} \sum_{i=1}^n l(y_i, f_a(x_i, w)) \tag{14}$$

The SVM regression is formulated as minimization of the following functional:

$$\left. \begin{aligned} &\text{Minimize } \frac{1}{2} w^2 + C \sum_{i=1}^n (\xi_i + \xi_i^*) \\ &\text{subject to} \\ &y_i - f(x_i, w) \leq \varepsilon + \xi_i^* \\ &f(x_i, w) - y_i \leq \varepsilon + \xi_i, i = 1, 2, \dots, n \\ &\xi_i, \xi_i^* \geq 0 \end{aligned} \right\} \tag{15}$$

The optimization problem can be transformed into a Dual problem and its solution is given by,

$$\left. \begin{aligned} &f(x) = \sum_{i=1}^{n_{SV}} (\alpha_i - \alpha_i^*) K(x_i, x) \\ &\text{subject to} \\ &0 \leq \alpha_i^* \leq C \\ &0 \leq \alpha_i \leq C \end{aligned} \right\} \tag{16}$$

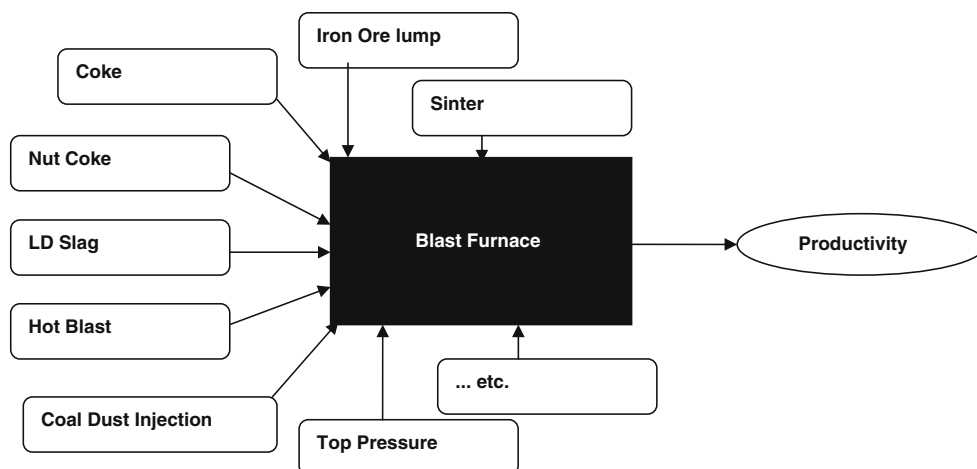
when, n_{SV} =the number of support vectors and the kernel function is

$$K(x_i, x) = \sum_{j=1}^m g_j(x) \cdot g_j(x_i) \tag{17}$$

It is well known that the SVM generalization performance (estimation accuracy) depends on a good setting of meta-parameters C , ε and kernel parameters. Parameter C determines the trade-off between model complexity (flatness) and the degree to which deviation larger than ε are tolerated in optimization formulation. Parameter ε controls the width of the ε -insensitive zone. The value of ε can affect the number of support vectors used to construct the regression function. The bigger ε , the fewer support vectors are selected.

4 Blast furnace and critical input parameters

A BF is essentially a moving bed reactor inside which five phases viz. gas, lump solids (iron ore, sinter, pellets and coke), liquids (pig iron and molten slag) and powders (tuyere injectants: pulverized coal, coke fines, or dust from the lump coke) interact with one another while consuming sinter, pellets, coke as resources, or raw materials. These five phases directly affect the BF productivity through numerous physical, chemical, physico-chemical, mechanical, and hydraulic processes, homogeneous and heterogeneous reactions which occur simultaneously affecting each other. Blast furnace converts iron oxides (ores) to hot metal (pig iron) that is used for steel making. Customarily, the BF Productivity was measured as per the standard definition adopted in steel plant w.r.t. coke consumption that goes as follows: (productivity=coke burning intensity/coke rate).



The critical input parameters which were most likely to influence the productivity of the blast furnace were tracked down based on survey of literature [18–20, 25–29] and expert views available within the organization. All the critical process parameters which were derived from basic input variables were: Coke rate, Iron ore lump, Coke, Nut coke, Sinter percentage, Coal dust injection, LD slag, Oxygen enrichment, Blast temperature, Coke ash, Coke M₁₀, Coke M₄₀, Coke CRI, Coke CSR, Average Al₂O₃ in slag, Total number of casting, Not-dry Cast, Raceway adiabatic flame temperature, Average Silicon content in hot metal, Solution loss Carbon and ETA CO.

The critical input parameters were required to be computed by using the following formulae.

$$\text{CokeRate} = \frac{\text{CokeActual} + \text{CokeNut}}{\text{ProductionTheoretical}} \times 1,000 \quad (18)$$

$$\text{Raceway Adiabatic Flame Temperature (RAFT)} = \left(-5.85 \times \left[\left\{ \left(\text{Steam} \times \frac{1000.0}{\text{BlastVolume} + (\text{O}_2)\text{Volume}} \right) \times 16.67 \right\} + 15 \right] + 43.7 \times \frac{(\text{O}_2)\text{Volume}}{\text{BlastVolume} + (\text{O}_2)\text{Volume}} \times 100 - 2.2 \times \left(\text{CDI} \times \frac{1000}{\text{ProductionTheoretical}} \right) \right) - \left[\frac{4100 \times \text{Tar}}{\{(\text{O}_2)\text{Volume} + \text{BlastVolume}\} \times (1440 - \text{BlastOff})} \right] \quad (23)$$

$$\text{Solution Loss Carbon} = \frac{(\text{TopGas})_C - c_{\text{meta}} - \text{Tuyer}_C}{\text{hmp}} \times 1,000 \quad (24)$$

Where,

$$c_{\text{meta}} = \left(\frac{12 \times (\text{HotMetal})_{\text{Pressure}}}{100} \right) \times \left(\frac{2}{28} \times \text{Si} + \frac{1}{55} \times \text{Mn} + \frac{2.5}{31} \times 0.15 \right) \quad (25)$$

$$(\text{HotMetal})_{\text{Pressure}} = \frac{\text{TopO}_2 - \text{TuyerO}_2}{0.392} \quad (26)$$

$$\text{ETA}_{\text{CO}} = \frac{\text{TopCO}_2}{\text{TopCO} + \text{TopCO}_2} \quad (27)$$

It was decided to consider all these critical input parameters for modeling the BF productivity.

5 Data collection and processing

Data on production/productivity and the corresponding settings of the critical input parameters of the blast furnace were routinely recorded by the Research and Control

where

$$\text{ProductionTheoretical} = \frac{\left((\text{Fe}_{\text{OreLump}} \times \text{Ore} \times 0.96) + \{ \text{Fe}_{\text{Sinter}} \times \text{Sinter} \} \right) + (\text{Iron}_{\text{LDslag}} \times \text{LDslag})}{\text{FeFactor}} \quad (19)$$

$$\text{FeFactor} = 100 - (\text{HM}_{\text{Si}} + \text{HM}_{\text{Mn}} + \text{HM}_{\text{S}} + \text{HM}_{\text{P}} + \text{HM}_{\text{Ti}} + \text{HM}_{\text{C}}) \quad (20)$$

$$\text{SinterPercentage} = \frac{\text{SinterActual} + (\text{LimeStone})_{\text{Reported}}}{\text{SinterActual} + \text{OreActual} + (\text{LimeStone})_{\text{Reported}}} \times 100 \quad (21)$$

$$(\text{O}_2)_{\text{Enrichment}} = \frac{79 \times (\text{O}_2)_{\text{Volume}}}{\text{BlastVolume} + (\text{O}_2)_{\text{Volume}}} \quad (22)$$

Laboratory of the organization in all the three shifts of a day. Every day, at the end of the third shift, the data thus collected were averaged to get average daily production, productivity and average values/levels of the variables related to raw materials, charging and process. These data were kept stored in a database made with FOXPRO by the Computer and Automation division of the organization. The same data were used to compute the values of the critical input parameters.

For the present study, the same database was used to collect required data for a continuous period of 2 years (2006–2008) and also for a period: (2003–2006) in discrete time domains. The period (2006–2008) pertained to the low productivity regime (as the blast furnace was endemically giving low productivity during this period) and the period (2003–2006) was termed as the high productivity regime. The values of the critical input parameters were computed using the same database.

The data were fed into Excel database and inappropriate data points (such as outliers, technically untenable data etc.) were excluded after consulting the experts in the field. The total number of data points considered for the study was 1344.

Scrutiny of the observed productivity values indicated that, it was quite logical to classify the data points into two classes, viz., High Productivity Class and Low Productivity Class as follows: High Productivity Class: Productivity > Median value of Productivity (=1.63) in the given period. Low Productivity Class: Productivity < Median value of Productivity (=1.63) in the given period.

Out of the total of 1,344 data points, High Productivity Class contained 746 and the Low Productivity Class had 598 data points.

A sample data sheet containing labeled productivity values and corresponding setting of the critical input parameters of the blast furnace is shown in Annexure-1.

A random sample containing 90% (=1,210) of the total number of data-points (1,344) was considered as the Training set and the remaining data-points constituted the Test set for SVM learning.

It was interesting to find that, most of the critical input parameters of the blast furnace were inter-correlated (as revealed by the correlation coefficient matrix) and majority of them had high auto correlation with different time lags (as revealed by ACF and PACF plots). Such a scenario was not conducive for taking to any statistical modeling approach which required distributional and other restrictive assumptions about the residuals to be satisfied. It was thus fully justified to use Support Vector Machines for modeling BF productivity because this approach did not require any distributional and other restrictive assumptions about the residuals.

6 SVM classification function

The dataset consisted of settings of the 21 critical input parameters (as described in section 4) as the input variable vector x_i corresponding to the observed class (low and high) of BF productivity, considered as the output variable $y_i; y_i=1,2$.

A training set consisting of 90% of the labeled data chosen at random from the total data set was considered for supervised learning by SVMs. Remaining 10% of the data was used for testing the fitted model. The SVMs mapped the data to a predetermined high-dimensional space via defined kernel function. Several inner-product kernels such as, linear, polynomial, radial basis function etc. were tried with a view to find the best classification function. As it may be seen from Table 1, the classification was most accurate when the Radial Basis function (RBF) of the form given in (Eq. 28) was used.

$$k(x, x_i) = \exp\left(-\frac{1}{2\sigma^2} \|x - x_i\|^2\right) \tag{28}$$

The fitted SVM classification function was:

$$\begin{aligned} C(x) &= \sum_{i=1}^n (\alpha_i - \alpha'_i) y_i k(x, x'_i) \\ &= \sum_{i=1}^n (\alpha_i - \alpha'_i) y_i \exp\left(-\frac{1}{2\sigma^2} \|x - x_i\|^2\right) \\ &= \sum_{i=1}^n \beta_i y_i \exp\left(-\frac{1}{2\sigma^2} \|x - x_i\|^2\right) \end{aligned} \tag{29}$$

where, $\beta'_i s, i = 1, \dots, n$ were obtained using the SVM Module in MATLAB. These $\beta'_i s, i = 1, \dots, n$ values are given in Annexure-2. In this case the value of σ^2 was taken as $\frac{1}{2 \times \text{gamma}}$.

It may be seen from Table 1 that, the highest level of accuracy (95.7983) was obtained with the fitted RBF Kernel in classifying the given dataset to the two productivity classes with the following parameter set.

$$C = 10, \text{ and } \gamma = 0.0001.$$

The kernel functions and/or, the parameter combinations within a kernel function for which the accuracy was not significant have not been shown in Table 1.

7 Support vector machine regression

Using the ε -insensitive loss function, the SVM regression models as function of critical input parameters were developed for the two (low and high) productivity classes by minimizing the empirical risk.

7.1 High Productivity Class

After having explored different SVR kernel functions with various parameter combinations within a kernel, the Radial Basis Function kernel (Eq. 30) gave the best fit regression model with a given combination of its parameters to the given dataset of the high productivity class in Table 2.

The fitted support vector regression model is given by

$$\begin{aligned} P(x) &= \sum_{i=1}^n (\alpha_i - \alpha'_i) k(x, x'_i) = \sum_{i=1}^n (\alpha_i - \alpha'_i) \exp\left(-\frac{1}{2\sigma^2} \|x - x_i\|^2\right) \\ &= \sum_{i=1}^n \beta_i \exp\left(-\frac{1}{2\sigma^2} \|x - x_i\|^2\right) \end{aligned} \tag{30}$$

where $\beta'_i s, i = 1, \dots, n$ were obtained using the SVM Module in MATLAB and prediction and are shown in Annexure-3.

The best parameter set obtained for the fitted Radial Basis Function was as follows:

$$C = 4.8, \text{ and } \gamma = 0.000001.$$

Table 1 The values of various hyperparameters of different kernels and the corresponding classification accuracy obtained

Polynomial function					Radial basis function			Linear function	
C	S	R	Degree	Accuracy ^a	C	Gamma (γ)	Accuracy ^a	C	Accuracy ^a
1e-007	0.00901	0.00501	4	92.437	10	0.0001	95.7983	1e-006	93.2773
1e-007	1e-005	1e-005	2	91.5966	100.5	0.0001	94.958	2e-006	94.1176
1e-007	1e-005	0.00101	5	90.483	0.1	4e-005	94.1176	8e-006	90.1875
1e-007	1e-005	0.00201	5	82.367	0.1	0.00013	92.437	7e-006	84.1688
1e-007	1e-005	0.00501	5	90.432	0.1	0.000146	91.5966	0.0002	94.1176
1e-007	1e-005	0.00701	5	88.764	0.1	0.000147	90.7563	0.00037	88.6884
1e-007	0.00101	0.00301	4	91.421	0.1	0.000151	89.916	0.00048	89.5643
1e-007	0.00101	0.00901	3	87.231	0.1	0.000156	88.2353	0.00059	82.1134
1e-007	0.00201	0.00501	2	81.598	0.1	0.000159	87.395		
1e-007	0.00201	0.00901	3	88.760	0.1	0.000161	86.5546		
1e-007	0.00301	0.00101	2	84.866	0.1	0.000167	85.7143		
					0.1	0.00017	84.8739		
					0.1	0.000174	84.0336		
					0.1	0.000183	84.0336		
					0.1	0.000155	89.0756		
					0.1	0.000185	82.3529		
					0.1	0.000192	80.6723		
					0.1	0.000184	83.1933		
					0.1	0.000193	79.8319		

^a If the predicted class of productivity of a data point was equal to the observed class of productivity of the data point in the test set, then accuracy was incremented by (1/no. of data points in test set)

The accuracy level with which the fitted SVR predicted the productivity value for a given setting of the critical input parameters of the blast furnace belonging to the high productivity class is given in Table 2.

7.2 Low Productivity Class

The Radial Basis Function gave the best fit regression model to the given dataset of low productivity class after

exploring with different kernel functions and various parameter combinations within a kernel. The SVM-RBF regression function is given in (Eq. 31).

$$\begin{aligned}
 P(x) &= \sum_{i=1}^n (\alpha_i - \alpha'_i) k(x, x'_i) = \sum_{i=1}^n (\alpha_i - \alpha'_i) \exp\left(-\frac{1}{2\sigma^2} \|x - x_i\|\right) \\
 &= \sum_{i=1}^n \beta_i \exp\left(-\frac{1}{2\sigma^2} \|x - x_i\|\right)
 \end{aligned}
 \tag{31}$$

Table 2 Prediction accuracy of different kernel fitted SVR in high-productivity class

Radial basis function			Polynomial function					Linear function	
C	Gamma	Accuracy ^a	C	S	R	D	Accuracy	C	Accuracy
4.8	0.000001	100	0.1	1	1	2	88.5906	0.1	81.2081
0.1	0.000001	99.3289	0.1	1	1	3	76.3758	1.8	85.2349
0.3	0.000001	92.3421	0.1	6	11	7	86.1324	1.5	79.8658
0.3	0.000008	93.3956	0.1	1	11	7	88.5842	0.9	82.5503
0.4	0.000009	89.8934	0.1	6	11	2	82.5463	1.0	83.2215
0.7	0.000004	84.3865	0.1	6	21	3	84.4683	0.6	84.5638
0.7	0.000009	88.9453						1.9	81.8792
0.9	0.000009	99.6574						2.0	69.1275
1.2	0.000004	92.1654							
1.4	0.000003	87.1276							
1.6	0.000005	88.7645							

^a If the productivity value as predicted by the fitted RBF kernel function for a given setting of the critical parameters was in the interval (0.95 productivity/1.05 productivity) of the test set, then the accuracy was incremented by (1/no. of data points in test set)

where $\beta'_i, i = 1, \dots, n$ were obtained using the SVM Module in MATLAB and $\frac{1}{2\sigma^2} = 0.0000045$ and these $\beta'_i, i = 1, \dots, n$ were estimated exactly the same way they were estimated for the SVM-RBF regression model of the high productivity class.

The best parameter set obtained for the Radial Basis Function was:

$$C = 45, \text{ and } \gamma = 0.0000045.$$

The accuracy level with which the fitted SVM-RBF regression model predicted the productivity value for a given setting of the critical input parameters in the low productivity Class is given in Table 3.

8 Model adequacy check

The observed productivity values were obtained from the blast furnace for 50 consecutive days in the post-study period corresponding to randomly chosen settings of the critical input parameters. The SVM-RBF regression models fitted to both low and high productivity classes were used to predict the productivity value corresponding to each of the 50 settings of the critical input parameters randomly chosen. Depending on whether the observed productivity value belonged to the high or, low productivity class, the corresponding SVM-RBF model was used for prediction. Assuming normality, the 95% lower and upper confidence bounds of the predicted productivity values were estimated as shown in Table 4. It may be seen that there is very good agreement between the observed and predicted values of BF productivity.

9 Optimal blast furnace productivity

The primary objective of the present study was to achieve maximum productivity in the blast furnace and hence the

Table 3 Prediction Accuracy with fitted SVR-RBF model for low-productivity class along with parameter combinations

C	Gamma(γ)	Accuracy ^a
45	4.5e-006	80.7018
0.4	1e-006	78.9474
0.2	2e-006	77.193
0.2	4e-006	75.4386
0.2	5e-006	73.6842
0.3	1e-006	71.9298
0.4	1.1e-005	70.1754
0.8	1e-005	68.4211
0.1	3e-005	66.6667

^a Accuracy level was computed as per the procedure described

SVR-RBF regression model fitted to the high productivity class was subjected to constrained non-linear optimization treatment for identifying the optimal setting of the critical input parameters that gave maximum productivity.

The optimization problem was formulated as follows:

$$\left. \begin{aligned} & \text{Max. } \sum_{i=1}^n \beta_i \exp \left(-\frac{1}{2\sigma^2} \|x - x_i\| \right) \\ & \text{Subject to} \\ & \max_{i=1, \dots, N} x_{ij} \leq x_j \leq \min_{i=1, \dots, N} x_{ij} \end{aligned} \right\} \quad (32)$$

The optimal setting of the critical input parameters in the high productivity class is given in Table 5.

The maximum productivity value of the blast furnace that was obtained corresponding to the optimum setting of the critical input parameters given in the Table 5 was: 2.0765 and this was quite high when compared to other productivity values obtained in the high productivity class.

10 Results and discussion

Highly complex nature of dependency between the vectors of the 21 critical input parameters and the productivity y (i.e.; output scalar or, vector) observed in the blast furnace was revealed during the training session of the Support Vector Machines with the given training set via quadratic optimization. It was evident that, the training set was not linearly separable into low and high productivity classes. Hence, the original space of the critical input parameters of blast furnace was effectively mapped into a high dimensional feature space via Radial Basis Kernel Function (RBF kernel) where the same training set was linearly separable and the linear separators in the high dimensional feature space corresponded to highly complex *non-linear* separators in original space of input vectors. The optimum (maximum margin and minimum risk) SVM-RBF classification function with reasonably good generalization property was developed on the basis of a trade-off done between the generalization ability and fitting to the training data. The classification function could successfully be used for predicting the class of productivity (low or, high) for any given setting of the 21 critical input parameters at any given time point and was thus immensely useful to the operating team of the blast furnace. Similarly, the class-specific adequate SVM-RBF regression models for BF productivity that were developed using the ϵ -insensitive loss function and minimizing the regulated risk function (i.e.; the distance between the input point set and the function.) for both low and high productivity classes were equally useful to the operating team for predicting the real value of the BF productivity corresponding to any given setting of critical BF parameters within a class.

Subjecting the SVM-RBF regression model of the high productivity class to constrained nonlinear optimization

Table 4 Observed and predicted values of BF productivity with 95% confidence bounds

Observed Productivity	Productivity Predicted By Fitted Model	Lower 95% confidence Bound	Upper 95% confidence Bound	Observed Productivity	Productivity Predicted By Fitted Model	Lower 95% confidence Bound	Upper 95% confidence Bound
1.44	1.368	1.512	1.45488	1.55	1.4725	1.6275	1.43877
1.31	1.2445	1.3755	1.29539	1.41	1.3395	1.4805	1.3783
1.19	1.1305	1.2495	1.13966	1.47	1.3965	1.5435	1.3345
1.36	1.292	1.428	1.34835	1.41	1.3395	1.4805	1.40387
1	0.95	1.05	1.19627	1.51	1.4345	1.5855	1.42634
1.32	1.254	1.386	1.32973	1.57	1.4915	1.6485	1.4686
1.6	1.52	1.68	1.46899	1.5	1.425	1.575	1.453
1.56	1.482	1.638	1.43212	1.39	1.3205	1.4595	1.39818
1.4	1.33	1.47	1.39955	1.44	1.368	1.512	1.41362
1.52	1.444	1.596	1.49214	1.53	1.4535	1.6065	1.42064
0.33	0.3135	0.3465	1.27308	1.49	1.4155	1.5645	1.4448
0.46	0.437	0.483	0.127434	1.41	1.3395	1.4805	1.42588
0.79	0.7505	0.8295	0.770252	1.41	1.3395	1.4805	1.41033
0.53	0.5035	0.5565	0.396362	1.42	1.349	1.491	1.40077
1.39	1.3205	1.4595	1.35107	1.43	1.3585	1.5015	1.39549
1.45	1.3775	1.5225	1.41011	1.44	1.368	1.512	1.44642
1.48	1.406	1.554	1.34514	1.46	1.387	1.533	1.43602
1.25	1.1875	1.3125	1.17155	1.41	1.3395	1.4805	1.41803
1.01	0.9595	1.0605	0.886303	1.3	1.235	1.365	1.30418
1.37	1.3015	1.4385	1.32807	0.67	0.6365	0.7035	0.770003
1.37	1.3015	1.4385	1.37471	1.48	1.406	1.554	1.47994
1.47	1.3965	1.5435	1.4395	1.58	1.501	1.659	1.52756
1.4	1.33	1.47	1.40662	1.52	1.444	1.596	1.53922
1.11	1.0545	1.1655	1.09345	1.56	1.482	1.638	1.52709
1.34	1.273	1.407	1.33276	1.26	1.197	1.323	1.2221

treatment was another challenging task because of its highly complex and nonlinear nature. However, using the SAS optimization module, convergence could be achieved and the optimum setting of the critical parameters that was obtained corresponding to the maximum productivity could be treated as the target setting by the operating team for obtaining high BF productivity.

Based on the value of the gradient objective function, the coke, coke rate, blast temperature emerged as very important factors insofar as their influence on the BF productivity were concerned and this was quite consistent with theory of BF. The present study also offered good scope for implementing the off-line-trained SVM-RBF classification model, class-specific regression models in control system hardware for on-line learning of SVM. In that case, the SVMs would update their current classification and regression models in response to each new data point $\{x_1, x_2, \dots, x_n; y\}$ considered in the training set and the prediction capability of these up-dated SVM models would continue to improve. Constrained nonlinear optimization software when integrated with the hardware would give updated optimum setting of the critical

parameters every time the SVM classification or, regression models are updated and thus optimum region of critical parameters rather than optimum point setting may be obtained. The task of devising an appropriate control scheme for the optimum region of critical parameter set would be easier than that required for the optimum point setting of the critical parameter set.

11 Implementation

The SVM-RBF classification model as well as the class-specific regression models developed for both low productivity and high productivity classes of the blast furnace were made available to its operating team for direct implementation into their online system so that they could readily predict the class or, real value of the BF productivity corresponding to a given setting as also the target setting of the critical process parameters. The task of implementing the off-line-trained SVM models in the hardware for BF control system was also taken up by the management.

Table 5 The optimum setting of the critical input parameters for maximum (point) productivity in high-productivity class

Number	Factor code	Factor name	Estimate	Gradient objective function
1	×1	Coke rate	459.1805	−0.00085
2	×2	Actual coke	1893	0.000486
3	×3	Nut coke	144	0.000123
4	×4	Actual ore	2135	0.000008629
5	×5	Sinter percentage	79.62545	0.000057521
6	×6	Coal dust injection (CDI)	224	0.000027648
7	×7	LD slag	0	−0.000064
8	×8	O ₂ enrichment (calculated)	0	−0.000001938
9	×9	Blast temperature	1,020	0.000113
10	×10	Ash	16.65377	−1.30E-11
11	×11	M ₁₀	7.666667	−0.000001035
12	×12	M ₄₀	83.33333	0.000014273
13	×13	CSR	69.8	0.000029591
14	×14	CRI	26.5	0.000002397
15	×15	Average Al ₂ O ₃ (SLG)	8.515052	2.21E-15
16	×16	Top pressure	1.15	0.00000062
17	×17	Total number of casting	12	0.000008572
18	×18	Not dry cast	0	−0.00000118
19	×19	Raft	1,949.632	3.01E-15
20	×20	HM average Si	0	−0.000001757
21	×21	Solution loss carbon	135.6709	0.000038456
22	×22	ETA CO	0.551559	−7.62E-09

12 Conclusion

- To address the problem of recurring spells of low productivity in a modern generation blast furnace, Support Vector Machines were trained with large number of labeled data points each consisting of a setting of 21 critical input parameters and the corresponding class (low/high) of productivity observed in the blast furnace with a view to develop optimum classification model. After mapping the vector space of the critical input parameters of the blast furnace into high dimensional feature space via Radial basis function (RBF) kernel, the optimum SVM-RBF classification model with given combination of its hyper-parameters that had good generalization property and low misclassification error, was developed. The SVM-RBF classification model could be effectively used to predict the productivity class for any given setting of the critical input parameters.
- Class-specific adequate SVM-RBF Regression models for BF productivity involving the 21 critical input parameters were also developed for both low and high productivity classes using the ε -insensitive loss function and minimizing the regulated risk function. The observed and the predicted values of productivity at randomly chosen settings of the critical input parameters agreed quite well. Early prediction of real value of productivity corresponding to any given setting of critical input parameters of the blast furnace with the help of the class-specific SVM-RBF regression models offered the operating team of the blast furnace the opportunity to decide on appropriate preventive/corrective action system.
- The optimum setting of the 21 critical parameters of blast furnace that gave maximum productivity was found by subjecting the SVM-RBF regression model fitted to the high productivity class to constrained nonlinear optimization treatment. The operating team of the blast furnace was given the option of treating the optimum setting of the critical input parameters as the target setting and operating within its nearest neighborhood with a view to obtain high productivity in the blast furnace.
- Implementation of the off-line-trained SVM-RBF classification and regression models in the hardware of the control system of the blast furnace along with integration of constrained nonlinear optimization software was recommended to the operating team to facilitate continuous on-line updating of the models and optimal setting of the critical input parameters with the arrival of new data-points.

Annexure-1

Sample data sheet

	A	B	C	D	E	F	G	H	I	J	K	L	M	O	Q	S	U	V	W	X	Y	Z	AA	AB	AC
1	Class	Productivity	Coke Rate	Actual Coke	Nut Coke	Actual Ore	Sinter Percentage	Coal Dust Injection (CDI)	LD Slag	O ₂ Enrichment (Calculated)	Blast Temperature	<----Coke Analysis-->					Average Al ₂ O ₃ (SLG)	Top Pressure	Total Number Of Casting	Not Dry Cast	RAFT	Hot Metal analysis	Solution Loss Carbon	EATA CO	
2												Ash	M ₁₀	M ₄₀	CSR	CRI							Average Si		
3	1	1.74	546.52	1572	102	1883	63.34	0	0	1.46	950	18.67	9.27	80.59	64.8	22.4	22.32	1	9	1	2065.18	0.87	75.33	0.47	
4	1	1.71	531.85	1525	78	1817	64.27	0	20	1.48	950	18.63	9.33	81.07	65	22.2	21.72	1	10	0	2063.24	0.8	70.67	0.46	
5	1	1.82	533.13	1621	85	1811	66.34	0	50	1.48	950	18.9	9.27	80.8	65	22.2	21.56	1	9	1	2073.12	0.49	75.43	0.45	
6	1	1.67	545.21	1533	71	1653	66.8	0	65	1.59	950	18.47	9.07	81.2	65.2	21.8	21.4	1	9	1	2153	0.62	78.45	0.45	
7	1	1.75	559.64	1638	84	1785	65.87	0	18	1.51	950	19.07	9.13	80.8	65.2	21.8	20.99	1	9	0	2147.78	0.52	86.94	0.45	
8	1	1.7	488.49	1364	100	1536	70.1	127	0	1.68	960	19.37	9.27	80.87	64.6	22.6	21.12	0.99	9	0	2060.92	0.89	77.83	0.47	
9	1	1.8	467.53	1404	79	1762	67.53	122	0	1.65	960	18.47	9.27	80.53	64.8	22.2	20.42	1	9	0	2060.11	0.83	82.94	0.47	
10	1	1.78	480.2	1418	86	1760	66.09	120	0	1.68	960	18.63	8.93	81.07	65.2	21.8	20.24	1	11	0	2129.83	0.71	90.85	0.47	
11	1	1.72	486.64	1371	104	1806	64.04	130	0	1.6	960	18.57	9.27	80.8	65	22.4	21.04	1	12	0	2096.99	0.85	89.58	0.48	
12	-1	1.18	542.57	1066	62	1234	64.8	0	24	1.23	938	18.17	9.27	80.67	64.8	22.4	22.77	1	6	0	2092.07	0.86	82.61	0.47	
13	-1	1.18	552.27	1081	60	1299	63.37	0	0	0.13	950	18.5	9.27	80.53	65.2	22.2	21.73	0.9	9	1	2053.51	1.1	58.86	0.5	
14	-1	1.59	550.34	1462	74	1608	65.85	0	6	0.94	960	18.6	9.2	80.87	65	22	22.12	1	9	0	2059.05	0.94	78.59	0.44	

Annexure-2

$\beta'_i s, i = 1, \dots, n$ values for support vector classification

#	A	B	C	D	E	F	G	H	I
1	1.77E-09	1.82E-09	0.202851	9.88E-09	3.57E-09	2.10E-09	6.85E-10	7.96E-09	1.10E-08
2	1.11E-09	3.62E-09	5.855914	1.28E-08	7.31E-09	2.16E-09	7.32E-10	10	6.50E-09
3	10	2.53E-09	2.46E-09	1.88E-09	0.232361	1.89E-09	7.31E-10	5.02E-09	1.01E-07
4	1.52E-09	1.64E-09	3.76E-09	2.35E-09	0.143204	1.85E-09	6.65E-08	4.01E-09	9.80E-10
5	1.36E-09	8.18E-09	2.06E-09	2.76E-09	3.75E-09	2.03E-09	10	10	1.49E-09
6	1.74E-09	2.01E-09	3.67E-09	2.35E-09	1.93E-09	1.58E-09	10	4.51E-09	10
7	1.46E-09	1.64E-09	8.82E-10	2.59E-09	3.29E-09	2.50E-09	7.45E-10	1.75E-09	4.75E-09
8	3.25E-09	2.79E-09	0.482103	3.06E-09	2.45E-09	2.11E-09	1.62E-09	6.81E-09	2.73E-09
9	3.83E-09	1.47E-09	1.32E-09	5.25E-09	2.07E-09	2.41E-09	5.353586	0.468756	10
10	1.79E-09	6.976919	4.41E-09	10	2.37E-09	2.52E-09	2.00E-09	2.385754	2.07E-09
11	1.84E-09	2.22E-09	4.96E-09	2.51E-09	3.42E-09	2.19E-09	1.15E-09	3.53E-08	2.87E-09
12	1.57E-09	2.72E-09	1.15E-08	3.37E-09	3.99E-09	4.05E-09	1.26E-09	2.88E-09	3.41E-09
13	4.15E-09	2.09E-09	3.84E-09	2.84E-09	7.87E-10	2.00E-09	1.79E-09	3.49E-09	10
14	1.19E-09	5.13E-09	1.52E-08	2.83E-09	10	2.60E-09	2.51E-09	4.69E-08	10
15	1.68E-09	1.57E-09	10	1.93E-09	1.64E-08	1.99E-09	6.694852	1.24E-08	10
16	1.77E-09	1.95E-09	2.76E-09	3.12E-09	5.35E-09	2.70E-09	10	3.01E-09	7.26E-09
17	3.81E-09	3.68E-09	2.30E-09	2.17E-09	1.490204	3.94E-09	1.25E-09	0.010595	10
18	3.29E-09	2.01E-09	2.66E-09	6.868539	3.078322	2.92E-09	10	10	5.07E-09
19	7.08E-09	4.41E-09	10	1.96E-09	3.34E-09	2.10E-09	1.92E-08	3.76E-08	4.26E-09
20	2.71291	10	10	7.57E-09	4.02E-09	5.467617	1.77E-09	1.34E-08	4.43E-09
21	2.72E-09	2.21E-09	7.40E-09	4.25E-09	3.79E-09	1.44E-09	1.12E-09	4.04E-09	3.99E-09
22	4.54E-09	1.84E-09	3.26E-09	3.33E-09	4.20E-09	2.19E-09	10	7.30E-09	5.27E-08
23	3.91E-09	1.72E-09	7.68E-09	3.95E-09	3.77E-09	4.14E-09	10	10	10
24	3.32E-09	1.51E-09	7.873344	2.96E-09	2.98E-09	3.20E-09	4.891444	3.47E-09	10
25	5.07E-09	3.82E-09	5.67E-09	2.18E-09	2.40E-09	2.36E-09	1.52E-09	2.232448	10

26	3.85E-09	3.93E-09	8.44E-09	2.87E-09	3.69E-09	1.20E-09	6.71E-09	10	2.49E-09
27	4.99E-09	4.01E-09	6.32E-09	3.16E-09	2.58E-09	2.27E-09	2.02E-09	3.74E-09	3.62E-08
28	5.10E-09	1.95E-09	4.46E-09	3.36E-09	3.98E-09	1.64E-09	6.08E-09	5.57E-09	0.320788
29	4.65E-09	3.90E-09	4.14E-09	2.97E-09	2.61E-09	1.22E-09	10	4.05E-09	10
30	0.745224	1.96E-09	1.07975	1.61E-09	2.22E-09	1.69E-09	9.599108	6.575893	1.94E-09
31	1.94E-09	6.26E-09	1.46E-08	2.14E-09	2.52E-09	3.576609	2.34E-09	2.50E-09	10
32	3.49E-09	6.33E-09	3.18E-09	6.598153	2.85E-09	0.500779	3.37E-09	1.57E-08	1.58E-09
33	2.54E-09	0.999999	6.30E-09	1.01E-09	3.50E-09	8.643946	1.00E-08	10	10
34	2.87E-09	6.61E-09	1.58E-08	2.92E-09	1.81E-09	5.358182	3.22E-09	1.48E-08	6.81E-09
35	3.50E-09	1.56E-08	3.72E-08	2.61E-09	10	2.338482	1.15E-09	10	2.53E-09
36	2.34E-09	2.97E-09	5.71E-08	3.66E-09	1.69E-09	5.575334	1.45E-08	0.167671	2.23E-09
37	2.01E-08	4.08E-09	2.219722	2.49E-09	2.68E-09	10	10	2.93E-09	1.49E-09
38	2.67E-09	7.35E-09	1.237857	2.13E-09	2.33E-09	2.12E-09	8.79E-09	9.240979	2.41E-09
39	2.69E-09	10	1.17E-08	2.47E-09	2.03E-09	9.88E-10	10	1.75E-09	2.83E-09
40	1.61E-09	2.85E-09	3.66E-08	10	1.99E-09	7.30E-09	5.21E-09	10	1.60E-09
41	2.01E-09	1.70E-09	4.19E-09	2.27E-09	1.98E-09	9.56E-10	8.53E-09	1.23E-08	5.71E-09
42	3.99E-09	2.84E-09	1.03E-07	10	1.87E-09	10	3.56E-09	10	2.38E-09
43	2.53E-09	10	1.96E-08	1.35E-09	1.74E-09	1.35E-09	10	2.84E-09	3.64E-08
44	2.18E-09	2.12E-09	3.06E-09	2.97E-09	1.69E-09	1.04E-09	10	2.70E-09	3.57E-09
45	10	2.40E-09	6.03E-09	2.80E-09	4.126362	5.89E-09	7.6404	9.37E-09	4.09E-09
46	4.60E-09	10	5.03E-09	5.13E-09	1.028594	8.71E-10	10	2.44E-08	5.420328
47	1.70E-09	2.47E-09	1.14E-08	2.93E-09	1.89E-09	7.38E-10	8.08E-09	6.230793	6.15E-09
48	2.51E-09	4.00E-09	3.996397	5.38E-09	1.96E-09	1.29E-09	10	1.81E-09	1.535575
49	10	3.54E-09	0.110781	7.79E-09	1.51E-09	7.34E-10	4.98E-08	4.17E-09	4.38E-09
50	2.51E-09	0.616977	7.17E-09	3.25E-09	1.47E-09	7.66E-10	7.93E-09	2.47E-09	3.74E-09

51	2.35E-09	6.61E-09	2.109116	6.15E-09	1.65E-08	8.63E-10	10	1.05E-09	4.25E-09
52	1.17E-09	10	2.04E-09	1.37E-09	1.15E-09	9.16E-10	2.50E-09	3.26E-09	4.85E-09
53	2.20E-09	10	10	3.36E-09	9.27E-10	10	10	1.74E-09	4.88E-09
54	1.62E-08	3.03E-09	7.874269	2.63E-09	6.35E-09	10	10	8.07E-10	10
55	10	2.89E-09	2.298881	3.92955	10	10	10	9.00E-10	1.14E-08
56	1.41E-08	1.84E-09	8.850161	3.25E-09	5.13E-09	4.36E-08	10	1.60E-08	0.394946
57	3.14E-09	1.38E-09	7.312436	10	2.38E-09	10	2.58E-09	10	3.90E-09
58	1.49E-09	1.64E-09	8.03E-09	6.44E-08	3.02E-09	0.986693	2.75E-09	1.30E-09	1.72E-09
59	3.83E-09	1.81E-09	3.57E-09	3.09E-09	2.94E-09	4.04E-09	10	1.48E-09	3.96E-09
60	4.84E-09	1.96E-09	5.791562	10	2.20E-09	1.179896	4.762078	2.12E-09	7.585566
61	3.25E-09	1.80E-09	2.34E-09	3.56E-08	1.92E-09	1.26E-09	1.36E-08	1.70E-09	10
62	10	9.54E-09	4.02E-08	10	10	5.67E-09	4.88E-09	10	7.03E-09
63	2.16E-09	2.34E-09	4.85E-09	10	7.98E-09	6.45E-09	1.58E-09	1.681274	10
64	1.53E-09	1.81E-09	3.53E-09	1.17E-09	1.625462	3.31E-09	2.00E-09	8.00E-10	7.90226
65	4.82E-09	1.59E-09	9.41E-10	7.23E-09	9.184217	2.27E-09	1.45E-09	2.43E-09	5.99E-09
66	1.11E-09	1.51E-09	1.161543	1.09E-08	9.470868	1.42E-09	5.20E-09	2.07E-09	2.32E-09
67	5.11E-09	2.77E-09	1.91E-08	4.16E-09	10	1.62E-08	4.88E-09	6.38E-08	1.28E-09
68	1.01E-08	1.19E-09	7.27E-09	8.953294	10	4.26E-09	1.60E-09	2.39E-09	3.36E-09
69	3.43E-09	8.24E-10	1.16E-09	5.54E-09	1.59E-09	2.26E-09	7.91E-10	3.42E-09	0.487631
70	2.66E-09	8.35E-10	1.26E-09	2.06E-08	1.96E-09	3.19E-09	5.09E-09	1.095551	2.88E-09
71	3.192042	8.08E-10	1.67E-09	2.95E-09	3.93E-08	4.751741	3.14E-09	1.79E-09	5.62E-09
72	1.14E-08	6.52E-10	1.92E-09	1.48E-09	2.04E-09	10	1.96E-09	9.38E-09	7.79E-10
73	4.06E-09	1.26E-08	2.10E-09	1.73E-09	1.563803	5.93E-09	2.68E-09	7.16E-09	5.30E-09
74	5.47E-09	2.39E-09	9.114587	1.22E-09	4.74E-09	2.69E-09	6.38E-09	2.50E-09	4.48E-09
75	3.52E-09	1.14E-09	4.32E-09	2.63E-09	2.26E-09	1.02E-09	1.326314	7.83E-09	4.80E-09

76	10	1.43E-09	9.81E-09	9.671005	10	10	2.26E-09	3.37E-08	10
77	5.40E-09	1.36E-09	1.91E-09	10	1.81E-09	4.39E-09	2.34E-09	2.76E-09	3.713678
78	7.706535	1.16E-09	7.50E-09	1.13E-09	1.77E-09	10	8.72E-10	3.05E-08	10
79	1.25E-08	7.92E-10	7.128165	8.70E-08	2.15E-09	1.70E-09	9.77E-10	5.432253	8.67E-10
80	3.55E-09	7.34E-10	10	1.53E-09	1.74E-09	5.21E-09	1.03E-09	3.62E-09	8.48E-10
81	10	6.38E-10	7.065246	4.161701	10	8.84E-10	3.76E-09	3.89E-09	2.70E-09
82	10	6.23E-10	10	1.00E-09	1.37E-09	1.02E-09	8.22E-10	1.04E-09	5.20E-10
83	1.47E-09	8.57E-10	8.07E-09	1.17E-09	1.54E-09	10	1.53E-09	4.49E-09	10
84	1.85E-09	5.73E-10	1.93E-09	2.51E-09	1.10E-09	1.79E-09	1.65E-09	2.86E-09	1.39E-09
85	8.046037	1.10E-09	1.30E-09	1.50E-09	2.33E-09	8.60E-10	1.03E-09	10	7.60E-10
86	1.22E-08	8.09E-10	1.02E-09	2.78E-08	1.38E-09	3.342367	3.86E-09	6.75E-10	5.989317
87	10	5.79E-10	1.49E-09	6.48E-09	1.95E-09	1.78E-08	9.58E-10	6.95E-10	10
88	6.37E-10	10	1.16E-09	7.42E-09	10	2.261546	1.77E-09	10	10
89	1.08E-09	9.82E-10	10	9.71E-09	4.76E-09	1.53E-08	10	3.60E-09	1.08E-09
90	7.68E-09	7.87E-10	7.56E-10	0.464726	1.09E-08	2.43E-09	10	1.43E-09	2.20E-09
91	7.70E-10	1.02E-09	7.35E-10	1.90E-09	7.59E-09	1.83E-09	8.58E-10	1.25807	10
92	6.18E-10	1.32E-09	1.71E-10	2.40E-09	6.286555	1.87E-09	1.69E-09	1.11E-09	1.61E-09
93	8.54E-10	6.77E-10	1.23E-09	4.83E-09	2.32E-09	10	8.67E-10	3.74E-09	1.26E-09
94	4.33E-09	7.93E-10	0.047159	5.83E-09	2.066E-09	9.258911	7.87E-10	6.41E-09	1.82E-10
95	3.83E-09	1.01E-09	7.920077	3.45E-09	1.71E-09	5.547287	9.92E-10	1.71E-08	1.85E-09
96	2.21E-09	6.63E-10	9.54E-09	1.39E-09	3.643121	5.96E-09	1.17E-09	10	8.52E-10
97	1.30E-09	6.63E-10	1.68E-09	3.11E-09	4.78E-09	3.10E-09	1.39E-09	1.42E-09	0.480883
98	10	8.66E-10	6.37E-10	1.96E-08	1.96E-09				

Annexure-3

$\beta'_i s, i = 1, \dots, n$ values for support vector regression, high-productivity class

	A	B	C	D	E	F	G	H	I
1	0.24759	-0.20747	-0.049	0.70601	0.20577	0.29664	0.13282	-0.51435	0.24504
2	-0.30692	-0.37251	0.30362	-0.52568	-0.35112	0.68932	-0.29883	-0.30173	0.12919
3	-0.07033	1.01868	0.23772	-0.43741	0.34154	0.49292	-0.31909	-0.12554	0.41947
4	0.1121	-0.47338	0.41985	-0.31606	0.58527	0.68931	-0.25452	-0.05117	0.2453
5	0.39426	1.8085	0.51119	-0.37136	0.25542	0.40953	0.01122	-0.36527	-0.03785
6	0.22118	-0.3694	0.34752	-0.39449	-0.01265	-0.86682	0.008	-0.12554	0.54486
7	-0.28778	-0.38156	0.92777	-0.17739	-0.21725	-0.98253	-0.06928	-0.26747	0.48343
8	-0.15444	-0.28793	0.16274	-0.26509	-0.01235	-0.91571	-0.00067	-0.34622	0.29206
9	0.10294	-0.25544	0.32427	-0.15031	0.02728	-0.78467	-0.38781	-0.60214	0.09546
10	0.07109	-0.24632	0.42655	0.15265	0.59177	-0.43547	0.08405	-0.29144	0.05305
11	-0.27503	-0.36059	0.38214	-0.46653	0.10604	-0.84143	0.24544	-0.04096	0.19935
12	0.18324	-0.65016	-0.05765	-0.1996	-0.34188	0.03854	-0.17482	-0.53091	0.164
13	-0.27661	-0.40357	0.30421	-0.30254	0.29593	0.0003	-0.17982	-0.14988	0.25467
14	-0.58345	-0.38879	0.26776	-0.14768	-0.02098	-0.6486	-0.30157	-0.45461	-0.12931
15	-0.32758	-0.10585	0.59599	-0.15448	0.21873	0.57119	-0.11525	0.12408	0.09506
16	-0.14483	-0.32706	0.3348	0.19749	0.18433	1.01634	-0.17417	0.15983	0.20773
17	0.0333	-0.32648	0.26531	-0.03232	-0.01634	0.71729	-0.21367	0.36821	0.71477
18	0.52472	0.11654	0.02664	-0.30683	0.09199	0.27462	0.05022	-0.36781	-0.11796
19	0.18824	-0.27871	-0.34808	-0.21115	-0.11954	0.00732	1.09187	-0.34922	-0.04736
20	0.04173	-0.06739	-0.06243	-0.30251	-0.27349	1.21733	-0.32122	-0.38721	0.35761
21	0.49184	-0.24517	-0.3522	-0.04563	-0.16368	0.41833	-0.43931	-0.34505	0.47826
22	0.15463	-0.29236	-0.07544	-0.15046	-0.47915	0.50531	-0.48952	-0.58973	-0.46296
23	1.21967	-0.42999	0.12252	-0.60811	-0.39847	0.46077	-0.44263	-0.32293	0.38597
24	0.63355	0.08379	0.70677	-0.10443	-0.43662	0.08922	-0.79353	-0.37441	0.08328
25	0.70767	-0.20418	0.20217	-0.3719	-0.30037	0.05588	-0.90985	-0.47324	0.19696
26	0.90475	-0.08098	0.42417	-0.4041	0.73088	-0.45986	-0.69849	-0.49419	-0.29366
27	0.2929	-0.29438	-0.06411	-0.12787	0.21824	-0.40474	1.36819	-0.61845	-0.46561
28	0.13914	-0.21668	0.42776	-0.26881	0.1902	-0.00373	-0.0183	-0.28533	0.68292
29	0.69752	-0.26093	0.25851	-0.11729	0.31369	0.55926	-0.03685	-0.30115	0.27134
30	0.66483	0.17294	-0.11155	-0.16789	0.9414	0.0886	0.80746	-0.04985	-0.49824
31	0.3733	0.12694	-0.03786	-0.2399	-0.21958	-0.27528	0.73943	0.09065	-0.04622
32	0.90679	0.17521	0.49068	0.0483	0.16926	0.05825	-0.42973	-0.22111	-0.13564
33	0.39188	-0.14195	0.71386	-0.01229	0.10306	-0.16442	0.30615	0.09282	-0.16805
34	0.05149	-0.16379	0.56259	-0.09642	0.05923	-0.18029	0.44773	-0.19078	0.3479
35	0.0572	-0.16898	0.72223	-0.26645	0.51194	-0.26099	0.06431	-0.05785	0.48803
36	0.8373	-0.31016	0.84973	0.03252	0.64145	-0.16874	-0.10103	0.36811	-0.37927
37	-0.30857	0.34719	-0.43935	0.08134	0.49702	-0.38781	0.19321	0.54284	-0.21709
38	-0.32514	0.37847	-0.06743	-0.23401	0.86174	-0.17732	0.34475	0.26338	-0.51748
39	0.6706	0.21599	-0.19524	0.02338	0.56166	-0.40393	-0.36054	0.34548	-0.05384
40	0.37464	0.49554	-0.71266	0.00876	0.46316	-0.66238	-0.18705	-0.22035	-0.16168
41	-0.366	0.18245	-0.21606	-0.09325	0.81753	0.08441	-0.45281	0.52603	0.01524
42	0.76951	0.42341	0.3489	1.05411	0.15464	0.50422	-0.44497	-0.13605	-0.13192
43	-0.09187	-0.21422	1.73699	-0.37368	0.80581	0.2021	-0.16804	0.34113	-0.25805
44	1.17556	0.47705	-0.2354	-0.22146	0.88511	-0.21559	-0.55829	0.2825	-0.24527
45	-0.5835	-0.0841	-0.22308	0.11915	0.57143	0.14129	-0.71862	0.22794	-0.40025
46	0.02102	0.12417	-0.27939	0.05116	0.02633	-0.01543	-0.50785	0.00065	0.07443
47	0.05663	-0.40232	-0.24395	-0.21655	0.42858	-0.18903	-0.49482	0.20811	0.70736
48	-0.78085	0.09572	-0.36711	-0.27036	0.47732	-0.18272	-0.5787	0.51133	0.46199
49	-0.66679	-0.09985	-0.03029	-0.35714	-0.00484	-0.21371	-0.42786	0.32785	0.47222
50	-0.64752	-0.27032	-0.4739	0.5438	0.74105	-0.09578	-0.49744	0.43808	0.53746
51	-0.30426	0.2102	0.02654	-0.23544	0.05872	-0.43261			
52	0.67497	0.13735	0.46462	-0.38482	0.14004	-0.34961			
53	0.13227	0.38418	0.22071	-0.2099	-0.0691	-0.59485			
54	0.78537	0.61727	-0.34702	-0.39541	-0.11521	-0.43349			
55	-0.39129	0.37925	0.03368	-0.532	0.51997	-0.21709			
56	0.11133	0.25083	0.08299	0.07194	0.51172	-0.53054			
57	0.33047	-0.26284	-0.11636	-0.34379	0.32717	-0.08784			
58	-0.38603	-0.06003	-0.05772	-0.31668	-0.20651	-0.39667			
59	-0.3391	-0.13954	-0.07953	-0.06728	-0.48347	0.08042			
60	0.8523	0.42515	-0.01798	0.26962	-0.08526	-0.66093			
61	0.32173	-0.43684	-0.25102	-0.15818	-0.44775	-0.47888			
62	-0.40766	-0.08171	-0.31409	-0.02902	0.25052	-0.2451			
63	0.0106	0.34517	-0.33077	0.07504	-0.55501	-0.42782			
64	0.34951	0.36148	0.19869	-0.2117	-0.18586	-0.3463			
65	0.37901	0.06602	-0.35002	-0.42166	-0.41132	-0.2184			
66	0.16181	-0.10816	0.08099	-0.50167	-0.46349	-0.51784			
67	0.33396	0.35266	0.17033	-0.50208	-0.35472	-0.51586			
68	0.57158	-0.04567	-0.17005	-0.38269	-0.22595				
69	0.97115	-0.24619	-0.10541	-0.23758	-0.57652				
70	0.78821	0.08812	-0.32372	-0.14582	-0.82526				
71	0.58486	-0.27952	0.44206	0.07496	-0.68325				
72	0.22343	-0.08951	0.11437	0.12046	-0.09508				
73	0.66598	0.03301	0.23092	0.17777	-0.00822				
74	0.67178	-0.1499	-0.15102	-0.3202	0.5189				
75	0.63891	-0.25894	-0.25533	-0.09935	-0.55367				
76	0.54358	-0.28103	-0.42466	-0.28988	-0.13031				

References

1. Chu F, Wang L (2003) Gene expression data analysis using support vector Machines. In: Proceedings of the 2003 IEEE International Joint Conference on Neural Networks, Portland, USA, 20–24 July 2003. pp 2268–2271
2. Alashwal H, Deris S, Othman RM (2006) One class SVMs for protein-protein interactions prediction. *Int J Biomed Sci* 1:120–127
3. Musiol J, Wieclawek MU (2008) Fuzzy support vector machines for gene expression data. Springer, Berlin
4. Ganapathiraju A (2002) Support vector machines for speech recognition. Mississippi State University, MS, USA
5. Camps-Valls G, Rojo-A'lvarez J L, Mart'inez-Ramo'n M (eds) (1996) Kernel methods in bioengineering. Signal and image processing. Idea Group, Hershey, PA, USA
6. Varshney PK, Arora MK (2004) Advanced image processing techniques for remotely sensed hyper-spectral data. Springer, London
7. Abe S (2005) Support vector machines for pattern classification. Springer, London
8. Zeng L, Quingwei L, Huiling X, Huafn C (2008) Support vector machines on functional MRI. Springer, the Netherlands
9. Fernandez-Getino Garcia MJ, Rojo-Alvarez JL, Alonso-Atienza F, Martinez-Ramon M (2006) Support vector machines for robust channel estimation in OFDM'. *IEEE Signal Process Lett* 13:397–400
10. Chen S, Sanmigan AK, Hanzo L (2001) Support vector machine multiuser receiver for DS-CDMA signals in multipath channels. *Neural Netw* 12:604–611
11. Chen S, Sanmigan A K, Hanzo L (2001) 'Adaptive multiuser receiver using a support vector machine technique. In: Proceedings of the IEEE Semiannual Vehicular Technology Conference, VTC 2001 Spring, Rhode, Greece. pp 604–608
12. Cao K, Shen H (2009) Scalable SVM processor and its application to nonlinear channel equalization. In: Proceedings of Pacific Asia conference on circuits, communications and systems, PACCS 09, Chengdu, China, 16–17 May 2009. pp 206–209
13. Ramon MM, Christodoulou C (2006) Support vector machines for antenna array processing and electromagnetic. *Synthesis lectures on computational electromagnetics*. Morgan and Claypool 1:1–120
14. Pettersson F, Hinnel J, Saxen H (2003) Evolutionary neural network modeling of blast furnace burden distribution. *Mater Manuf Process* 18:385–399
15. Omori I (1987) Blast furnace phenomena and modelling. Elsevier, London
16. Muchi I (1967) Mathematical model of blast furnace. *Trans ISIJ* 7:223–234
17. Chenl Y, Ricketts J, Heveziz J, Abramowitz H (1989) Expert system for blast furnace operation. In: Proceedings of 3rd international conference on industrial and engineering applications of artificial intelligence and expert systems, Tennessee, USA, vol. 1. pp 569–576
18. Bilik J, Kret J, Beer H (1990) Application of the simulating mathematical models for decreasing of the blast furnace fuel rate. *Hut listy* 4:225–230
19. Danloy G, Mignon J, Munnix R, Dauwels G, Bonte L (2001) A blast furnace model to optimize the burden distribution. In: 60th Iron-making Conference Proceedings, Baltimore, vol. 60. pp 37–48
20. Babich A, Senk D, Gudenau H W, Mavrommatis K, Spaniol O, Babich Y, Formosa A (2005) Visualisation of a mathematical model of blast furnace operation for distance learning purposes. *Revista De Metalurgia*, Vol. Extraordinario. pp 289–293
21. Nogami H, Chu M, Yagi J (2006) Numerical analysis on blast furnace performance with novel feed material by multi-dimensional simulator based on multi-fluid theory. *Appl Math Model* 30:1212–1228
22. Dong XF, Yu AB, Chew SJ, Zulli P (2010) Modeling of blast furnace with layered cohesive zone. *Metall Mater Trans B Proc Metall Mater Proc Sci* 41:330–349
23. Saxen H (1990) Blast furnace on-line simulation model. *Metall Mater Trans B* 21:913–923
24. Azadeh A, Ghaderi SF (2006) Optimization of an automatic blast furnace through integrated simulation modeling. *J Comput Sci* 2:382–387
25. Man-sheng YX, Shen F, Yagi J, Nogami H (2006) Numerical simulation of innovative operation of blast furnace based on multi-fluid model. *J Iron Steel Res Int* 13:8–15
26. Yagi J, Nogami H, Yu A (2006) Multi-dimensional mathematical model of blast furnace based on multi-fluid theory and its application to develop super-high efficiency operations. In: Proceedings of Fifth International Conference on CFD in the Process Industries CSIRO, Melbourne, 13–15 December 2006. pp 1–6
27. Jindal A, Pujari S, Sandilya P, Ganguly S (2007) A reduced order thermo-chemical model for blast furnace for real time simulation. *Comput Chem Eng* 31:1484–1495
28. Xuegong B, Torssell K, Wijk O (1992) Prediction of the blast furnace process by a mathematical model. *ISIJ Int* 32:481–488
29. Xuegong B, Torssell K, Wijk O (1992) Simulation of the blast furnace process by a mathematical model. *ISIJ Int* 32:470–480
30. Matsuzaki S, Nishimura T, Shinotake A, Kunitomo K, Naito M, Sugiyama T (2006) Development of mathematical model of blast furnace. *Shinnittetsu Giho* 384:81–88
31. De Castro JA, Nogami H, Yagi JI (2000) Transient mathematical model of blast furnace based on multi-fluid concept, with application to high PCI operation. *ISIJ Int* 40:637–646
32. Boser B E, Guyon I, Vapnik V (1992) A training algorithm for optimal margin classifiers. In: Proceedings of Fifth Annual Workshop on Computational Learning Theory, Pittsburgh. pp 144–152
33. Vapnik VN (1995) Chunking method (decomposition method) *The Nature of Statistical Learning Theory*. Springer Verlag Inc, New York
34. Osuna E, Freund R, Girosi F (1997) Support vector machines: training and applications. A.I.Memo 1602, C.B.C.L Paper No.144, Massachusetts Institute of Technology, USA
35. Drucker H, Burges C, Kaufman L, Smola A, Vapnik VN, (1997) Support vector regression machines. In: *Advances in Neural Information Processing Systems*, Vol. 9. MIT Press, Cambridge, MA. pp 155–161
36. Vapnik VN (1998) *Statistical learning theory*. John Wiley and Sons, New York
37. Cristianini N, Shawe-Taylor J (2000) *An introduction to support vector machines and other kernel based learning machines*. Cambridge University Press, Cambridge
38. Suykens JAK, Van Gestel T, De Brabanter J, De Moor B, Vandewalle J (2002) *Least squares support vector machines*. World Scientific Pub. Co., Singapore
39. Huang TM, Kecman V, Kopriva I (2006) *Kernel based algorithms for mining huge data sets, supervised, semi-supervised, and unsupervised learning*. Springer-Verlag, Berlin
40. Vapnik V, Kotz S (2006) *Estimation of dependences based on empirical data*. Springer, New York
41. Ingo S, Andreas C (2008) *Support vector machines*. Springer, New York