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ORIGINAL ARTICLE

Multi-objective approaches to balance mixed-model assembly lines for model mixes having precedence conflicts and duplicable common tasks

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Abstract The existence of common tasks for model mixes is the main characteristic of the mixed-model assembly lines. The decision problem considered in this study is how some common tasks can be duplicated to improve the efficiency of a mixed-model assembly line under the assumption that some of the precedence relationships among common tasks for different products are conflicting. This decision problem is called by the authors as "mixed-model assembly line balancing with precedence conflicts and duplicable common tasks (MALB-CD)." Although precedence conflicts have been mentioned in some of the earlier studies, to the best knowledge of the authors, this is the first study that deals with precedence conflicts by mathematical modeling. In the first step of this study, a new binary mathematical model with single objective for MALB-CD is developed where the single-objective is to minimize the number of workstations. Three goals relevant to MALB-CD are then incorporated into this single-objective model to give rise to two pre-emptive goal programming models, one with precise and the other with fuzzy goals. Minimizing the number of workstations, the cycle time and the total cost required to duplicate common tasks are the goals in question. The proposed mathematical models are illustrated and validated by means of a number of numerical illustrations.

Keywords Assembly line balancing · Mixed-model assembly lines · Integer programming · Goal programming · Fuzzy goal programming

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1 Introduction

Assembly lines are one of the most important elements of mass production systems commonly utilized in small product variety and high volume industries such as automotive, electronics, and machinery so as to improve productivity. An assembly line consists of several successive workstations in which a set of tasks for one or more product types are performed. Assembly lines can be classified as "single-model" and "mixed-model" with respect to the number of different products assembled on an assembly line. On a single-model assembly line, only one product type is produced. However, different products having similar assembly processes or different models of a product type are assembled on mixed-model assembly lines. In today's competitive market conditions, manufacturers are obligated to produce a wide range of product types to meet the diversified customer expectations. Therefore, mixed-model assembly lines rather than single-model assembly lines are utilized in many industries.

One of the most important problems in assembly line management is to group tasks into several successive workstations. This problem is called the assembly line balancing (ALB). Essentially, the ALB problem entails assigning tasks into workstations by optimizing one or more performance measures while satisfying some restrictions. In general, minimizing the number of workstations on the assembly line is one of the well-known performance measures. This performance measure requires that workload of a workstation cannot exceed the pre-determined cycle time (cycle time constraints) and that all precedence relationships among tasks are satisfied (precedence constrains). Precedence relationships among tasks are illustrated using a "precedence diagram." In addition, each task should be assigned to at least and at most one workstation (assignment constraints).

In line with the classification of assembly lines given above, ALB problems can also be classified as single-model assembly line balancing (SALB) and mixed-model assembly line balancing (MALB). The basic version of ALB problems, the SALB problem, was first studied by Salveson [1] and then by many researchers. The literature on SALB is too excessive to be presented here, but detailed reviews of the SALB studies can be found in Baybars [2], Ghosh and Gagnon [3], Erel and Sarin [4], and Becker and Scholl [5].

MALB problems differ from SALB problems in some respects. The MALB problem assumes that more than one product type is assembled on the same mixed-model assembly line with diverse task parameters and precedence relationships for each product type. Different product types may have common tasks, and the completion times of these common tasks may not be equal for each product type [6]. Although mixed-model assembly lines are more widespread than single-model assembly lines, the literature on MALB is relatively small compared to the literature on SALB. Most of the MALB studies have adopted the methodology where the MALB problem is transformed into a SALB problem by adjusting completion times of the tasks and/or by combining the individual precedence relationship diagrams of different product types. The resulting SALB problem is solved by using a SALB algorithm; the obtained solution is then smoothed to reduce the possible model imbalance. A combined precedence diagram is the common diagram that represents two or more precedence diagrams of different products. An adjusted task time is the weighted average of task times for different products [7]. Two example precedence diagrams for two different products are shown in Fig. 1.

As shown in Fig. 1, tasks 1, 3, 5, and 6 are common in two different products. The precedence relationships among these common tasks are not conflicting in two different precedence diagrams. For example, task 3 is a predecessor of task 6 for both products. Since no precedence conflicts exist in two precedence diagrams, the combined precedence diagram can be constructed as shown in Fig. 2.

In most of the MALB procedures found in the literature, a restriction is required to ensure that a common task should be assigned to at least and at most one workstation. That is, common tasks of different products must be assigned to the same workstation. However, Roberts and Villa [7], Bukchin et al. [8], and Bukchin and Rabinowitch [9] have relaxed this restriction and assigned a common task for multiple products to different workstations.

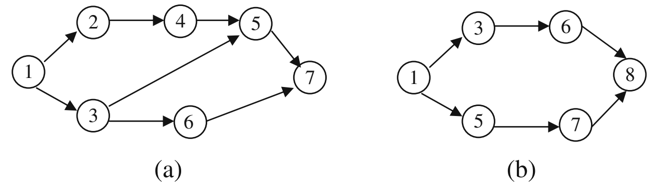

Fig. 1 Precedence diagrams for (**a**) product 1, (**b**) product 2

Although a task of a particular product should be assigned to a single workstation, common tasks may be assigned to different workstations. This relaxation is referred to as task duplication. Some additional costs such as costs of machinery and tool duplication result from task duplication. Assigning a common task to more than one workstation may reduce the number of workstations and increase efficiency especially when no significant establishment cost or additional workspace is required. Establishment cost of a new workstation may generally be greater than the cost of duplicating some common tasks. Bukchin and Rabinowitch [9] have developed an integer formulation and proposed a branch-and-bound algorithm to solve the MALB with duplicable common tasks. Their objectives are to minimize (a) the total cost of workstation utilization and (b) task duplication for a given cycle time for each product. Essentially, these two objectives conflict with each other. In other words, the more tasks that are duplicated, the less workstations are required. This is due to the relaxation of the assignment constraints.

Bukchin and Rabinowitch's [9] formulation is originally a multi-objective model that makes a trade-off between the number of workstations and the total establishment cost required to duplicate the common tasks. Assembly line managers prefer such multi-objective approaches and compromise solutions rather than using single-objective approaches and optimizing a single objective. There are several multi-objective approaches in SALB and MALB literature [10–15]. The multi-objective ALB approaches generally consider the number of workstations and cycle time as conflicting objectives.

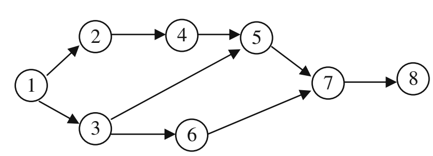

Fig. 2 The combined precedence diagram of two products

All of the above mentioned MALB studies assume that there are some common tasks among different products and there is no existing conflict among the precedence relationships of the products. Suppose that tasks a and b are common to two different products. It is widely assumed by the producers that if task a is a predecessor of task b in the first product, then task a cannot be a successor of task b in the second product. However, in practice, this may not always be possible due to the “conflicting precedence relationships” caused by design differences of similar products. In other words, different designs of similar products may require different sequences of assembly operations because of these precedence conflicts.

As far as the authors know, the conflicting precedence relationships have been mentioned by Ahmadi and Wurgaft [16], Fokkert and de Kok [17], and Boysen et al. [18] but have not been formulated to date. Conflicts in precedence relationships are an important barrier for a synchronized flow in mixed-model assembly lines and should be considered on its own in balancing.

Although additional examples can be found in many industries, a brief example of conflicting precedence relationships in practice can be presented here. It is taken from a solar thermal panel producer located in Konya/Turkey. A panel is the main part of a solar thermal collector which collects solar radiation for water heating. The company produces two different but similar types of panels called FIN and Full-Plate on the same line. The line is running with an average cycle time of 3.5 min. Basic materials of a panel are copper tubes and copper selective surface sheets. A panel consists of eight or ten riser tubes and two manifold tubes. In FIN-type panel production, selective sheets are first welded on riser tubes, and each riser tube is then soldered on manifold tubes. However, in Full-Plate-type panel production, riser tubes are first welded on manifold tubes, and a full-plate sheet is then welded on riser tubes. The list of tasks required to complete the assembly of these panels are given in Table 1.

Table 1 shows that most tasks are common to the two panel types, but their precedence relationships are different. The precedence diagrams of two types of solar panels are given in Fig. 3:

As shown in Fig. 3, tasks 8 and 10 are conflicting in two different panels. Since the company makes mixed-model production, task 10 is duplicated in assembly line using the second welding machine. On the other hand, it is clear from Fig. 3 that a regular combined precedence diagram for the two products cannot be obtained.

This study focuses on balancing mixed-model assembly lines with duplicable common tasks and conflicting precedence relationships, which we have called MALB-CD. In the first step of this study, a new binary mathematical model for MALB-CD is developed where the single objective is to

Table 1 Solar panel assembly tasks

Task no.	Task description	FIN	Full-Plate
1	Cutting riser tubes to length	✓	✓
2	Cutting selective sheets (surface)	✓	✓
3	Cutting manifold tubes to length	✓	✓
4	Rinsing riser tubes with solvent	✓	✓
5	Hole-punching manifold tubes	✓	✓
6	Soldering of fittings on manifold tubes	✓	✓
7	Brushing of fittings	✓	✓
8	Welding selective sheets on riser tubes	✓	✓
9	Sheets surface treatment	✓	–
10	Soldering riser tubes on manifold tubes	✓	✓
11	In-process hydrostatic leak-proofing test	✓	✓
12	Painting manifold tubes	✓	–
13	Final hydrostatic leak-proofing test	–	✓

minimize the number of workstations. Three conflicting goals relevant to MALB-CD are then incorporated into this single-objective model to give rise to two pre-emptive goal programming models with conflicting goals. One of them is the precise (traditional) goal programming (GP) model and the other is the fuzzy goal programming (FGP) model. Minimizing the number of workstations, the cycle time, and the total cost required to duplicate common tasks are the three conflicting goals common to GP and FGP. GP is a flexible and pragmatic multi-criteria decision-making methodology which is the most suitable for application to many decisional contexts. Indeed, a decision model with multiple criteria and a complex constraint set is tractable only by means of the formulation of a GP model [19].

The proposed two goal programming approaches are different from the approaches in the existing studies in terms of the conflicting precedence relations, multiple objectives that are considered, and the decision environments. Both goal programming approaches take into consideration the most frequently used three conflicting objectives in the literature. On the other hand, the proposed

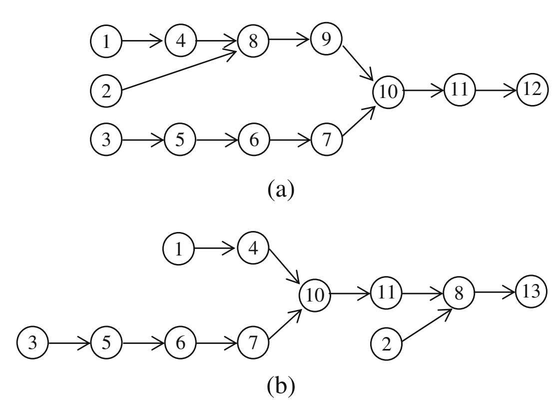

Fig. 3 Precedence diagrams of FIN (**a**) and Full-Plate (**b**) solar panels

approaches are the first goal programming approaches to MALB with the relaxation of task assignment in precise and fuzzy decision environments.

Determining aspiration levels for the objectives precisely may be difficult in the real world decision problems, as the MALB-CD may take place in imprecise, uncertain, or vague environments. The proposed approaches require decision-makers to determine their priorities for the three conflicting objectives. With these two approaches, we hope to provide useful and flexible decision-making tools for assembly line managers who have to make decisions in precise or in vague management environments. On the other hand, to the best knowledge of the authors, this is the first study that formulates mixed-model assembly line balancing problems with precedence conflicts. The authors are of the opinion that owing to their features expressed above, the proposed goal programming approaches will efficiently satisfy the requirements in the real world mixed-model applications as well as the expectations of assembly line managers.

The remainder of the paper is organized as follows: A binary formulation for MALB-CD is proposed in Section 2. Based on the proposed binary formulation, a GP and a FGP formulation is given in Sections 3 and 4, respectively. The proposed binary formulation and the two-goal programming formulations are validated and analyzed using a number of numerical illustrations in Section 5. Concluding remarks are given in Section 6.

2 A binary formulation for MALB-CD

In this section, a binary formulation for MALB-CD is proposed. The proposed formulation uses the concepts of Bukchin and Rabinowitch [9], and it has also some similarities with their model. The following conditions of MALB-CD are assumed to be satisfied:

- Task completion times associated with each product are known and deterministic.
- Common tasks among the products exist, and they do not need to have the same completion times.
- There may be conflicting precedence relationships among the tasks of different products.
- Some common tasks can be duplicated by assigning them to more than one workstation.

The following notation is used in the proposed binary formulation.

Indices

p	Product
i, r, s	Task
j	Workstation

Parameters and sets

P	Set of products
T_p	Set of tasks of product p
N	Total number of tasks; $N = \sum_{p \in P} \lVert T_p \rVert$
TD	Set of common tasks that can be duplicated
TND	Set of common tasks that cannot be duplicated
W	Set of workstations
$K_{\max}$	Maximum number of workstations
C	Cycle time
AC	Available capital that can be invested for task duplication
NP_i	Total number of products that require task i
PR_p	Set of precedence relationships of product p
$(r,s) \in \mathrm{PR}_p$	A precedence relationship; r is an immediate predecessor of s
t_{ip}	Deterministic completion time of task i of product p
c_i	Establishment cost of task i

Variables

x_{pij}	1, if task i of product p is assigned to workstation j; 0, otherwise
u_j	1, if workstation j is utilized; 0, otherwise
y_{ij}	1, if common task $i \in$TND is assigned to workstation j
z_{ij}	1, if common task $i \in$TD is assigned to workstation j; 0, otherwise

The proposed binary formulation is presented below:

$$\text{Min} \sum_{j \in W} u_j \tag{1}$$

$$\sum_{j \in W} x_{pij} = 1 \qquad \forall p \in P \quad \forall i \in T_p \tag{2}$$

$$\sum_{p \in P} x_{pij} - \mathrm{NP}_i y_{ij} = 0 \qquad \forall i \in \mathrm{TND} \quad \forall j \in W \tag{3}$$

$$\sum_{j \in W} (K_{\max} - j + 1)\left(x_{prj} - x_{psj}\right) \geq 0 \quad \forall p \in P \quad \forall (r,s) \in \mathrm{PR}_p \tag{4}$$

$$\sum_{i \in T_p} t_{ip} x_{pij} \leq C \qquad \forall p \in P \quad \forall j \in W \tag{5}$$

$$\sum_{p \in P} \sum_{i \in T_p} x_{pij} - N u_j \leq 0 \qquad \forall j \in W \tag{6}$$

$$\sum_{p \in P} x_{pij} - \mathrm{NP_i} z_{ij} \leq 0 \qquad \forall i \in \mathrm{TD} \quad \forall j \in W \tag{7}$$

$$\sum_{i \in \mathrm{TD}} \sum_{j \in W} c_i z_{ij} - \sum_{i \in \mathrm{TD}} c_i \leq \mathrm{AC} \tag{8}$$

The minimization of the number of workstations is the most commonly used performance measure in the existing SALB and MALB procedures. Therefore, the objective function of this binary formulation is also based on the minimization of the total number of workstations required on the mixed-model assembly line.

Equation 2 ensures that each task is assigned to at least and at most one workstation. All tasks of all products including common tasks are assigned to a workstation by Eq. 2. That is, all common tasks can be duplicated.

However, assembly line managers may desire to assign some common tasks to the same workstation. For example, if a common task requires a specific experience, then this common task should be assigned to the same operator. Therefore, we add Eq. 3 to the model so as to assure that some common tasks of different products are assigned to the same workstation.

Equation 4 satisfies the precedence relationships among the tasks for each product. Precedence relationships of different products are considered independently. Therefore, precedence conflicts among common tasks will result in task duplications.

Equation 5 assures that the workload of a workstation does not exceed the cycle time of mixed-model assembly line.

Equation 6 determines whether or not a workstation is utilized. If any task is assigned to workstation j, then the variable u_j will be 1; otherwise, it will be 0.

Equation 7 is required to determine whether or not the duplicable task i is assigned to workstation j. If common task i is assigned to workstation j, then the variable z_{ij} will be 1. The sum of z_{ij} values for any common task represents the number of different workstations to which common task i is assigned.

An "establishment cost" (c_i) may be required to establish task i on the mixed-model assembly line. Nevertheless, some manual tasks may be very simple, and they may not require any establishment cost. However, the cost of duplicating machinery tasks usually consists of the purchasing cost of the necessary machines if they are not already available. The tasks that are not common or common but not duplicable should be assigned only once. Therefore, the assembly line manager should spend the establishment cost of these tasks only once. In addition, duplicable common tasks should also be assigned at least once, but these tasks may become duplicated and be assigned more than once. In this case, the decision-maker should spend an additional cost for task duplication. The decision-maker desires that the amount of this additional cost should not exceed the available capital allotted for the task duplication. Equation 8 ensures that the task duplication cost does not exceed the available capital (AC).

The above binary formulation minimizes the number of workstations for a given cycle time and an available task duplication capital.

The MALB-CD problem considered in this study also has a multi-objective nature. In addition to the number of workstations and the cycle time, the MALB-CD problem motivates us to consider the amount of task duplication cost as an additional important goal. Hence, in this study, we focus on developing two GP approaches for MALB-CD problem. The proposed GP approaches aim at finding compromise solutions for the above three conflicting goals, namely the number of workstations, the cycle time, and the total duplication cost for the common tasks. The first GP approach (precise GP) is developed for crisp decision environments where the decision-maker is able to determine his/her target values precisely. The second GP approach (fuzzy GP) is developed for uncertain/vague decision environments where the decision-maker is unable to determine his/her target values precisely. These two proposed GP approaches are not rivals but alternatives for each other.

3 Precise goal programming approach for MALB-CD

GP technique was first introduced by Charnes and Cooper [20] in order to deal with multi-objective optimization problems. A general model of GP with a set of aspiration levels g_k (k=1,2,...,n) for the n goals can be formulated as follows [21, 22]:

$$\begin{aligned}
&\text{Minimize } \sum_{k=1}^{n} (d_k^+ + d_k^-)\\
&\text{Subject to } f_k(x) - g_k = d_k^+ - d_k^-, \quad k = 1,\ 2,\ \ldots,\ n,\\
&\qquad x \in F \ (F \text{ is a feasible set}),\ d_k^+, d_k^- \geq 0,\ k = 1,\ 2,\ \ldots,\ n,
\end{aligned}$$

where $d_k^+ = \max(0, f_k(x) - g_k)$ indicates the positive deviation and $d_k^- = \max(0, g_k - f_k(x))$ indicates the negative deviation from the target value of goal k.

The above GP formulation assumes that decision-makers are able to define their target values precisely. In this section, we propose a precise GP formulation for the MALB-CD based on the binary formulation presented in Section 2. The proposed GP formulation includes some rigid constraints which are not appropriate for being considered as the goals, and it includes three goal constraints.

Rigid Constraints Assignment constraints in Eqs. 2 and 3, precedence constraints in Eq. 4, workstations constraints in Eq. 6, and establishment constraints in Eq. 7 of the binary formulation presented in Section 2 are transferred in their original form to the proposed GP formulation. These constraints are not appropriate for being considered as goal constraints.

The first goal of the proposed GP formulation is the number of workstations and the relevant goal constraint is Goal 1 (G_1): Number of Workstations

$$\sum_{j\in W} u_j + d^- - d^+ = \mathrm{gw} \tag{9}$$

where gw is the target value for the number of workstations, d^- is the negative, and d^+ is the positive deviational variable for the number of workstations. The decision-maker desires that the number of workstations should not exceed gw.

The cycle time of the mixed-model assembly line is considered as the second goal, and the relevant goal constraint is formulated as follows:
Goal 2 (G_2): Cycle Time

$$\sum_{i\in Tp} t_{ip}x_{pij} + e_{pj}^- - e^+ = \mathrm{gc} \qquad \forall p\in P \quad \forall j\in W \tag{10}$$

where gc is the target value for the cycle time, e_{pj}^- is the negative deviational variable of workload of workstation j for product p, and e^+ is the positive deviational variable for the cycle time. Equation 10 is a modified form of the Eq. 5 (cycle time constraints) in the binary formulation. The decision-maker desires that the cycle time should not exceed gc.

The amount of cost required to duplicate the common tasks is considered as the third goal and formulated as follows:
Goal 3 (G_3): Total Duplication Cost

$$\sum_{i\in \mathrm{TD}}\sum_{j\in W} c_i z_{ij} - \sum_{i\in \mathrm{TD}} c_i + f^- - f^+ = \mathrm{gd} \tag{11}$$

where gd is the target value for the total cost required to duplicate the common tasks, f^- is the negative, and f^+ is the positive deviational variable for the total cost. Equation 11 is a modified form of the Eq. 8 (constraints for task duplication cost) in the binary formulation. The decision-maker desires that the total cost of task duplication should not exceed gd.

The objective function of the proposed GP formulation aims at minimizing the sum/total of the over achievements in the above-mentioned three goals:

$$\text{Minimize } d^+ + e^+ + f^+ \tag{12}$$

A shortcoming of the GP formulation is its tendency to produce solutions that are not Pareto efficient. That is, a GP formulation may produce inefficient solutions if the target values are set too high within the context of ALB problems. However, if such a situation occurs, the solution can be projected onto a Pareto efficient solution in an appropriate manner [19]. To achieve Pareto efficient solutions, the negative deviational variables are added to the objective function of the proposed GP model as follows:

$$\text{Minimize } d^+ + e^+ + f^+ - 0.001d^- - 0.001\sum e_j^- - 0.001f^- \tag{13}$$

The above GP formulation can be solved using either weighted or pre-emptive GP approach. In this study, we adopt pre-emptive GP approach that needs decision-makers to determine a priority order for the above three goals. Weights p_1, p_2, and p_3 (p_1>>>p_2>>>p_3) must then be assigned to d^+, e^+, and f^+ in line with the determined priority order.

4 Fuzzy goal programming approach for MALB-CD

The aim of GP is to minimize the positive deviations from the aspiration levels determined by decision-maker(s). However, determining the aspiration levels for the objectives precisely or certainly is usually difficult in the real world decision problems. Especially, the real life applications often take place in imprecise, uncertain, or vague environments. Fuzzy set theory, first introduced by Zadeh [23], is a better means for modeling the imprecision of human thought. Early formulation of fuzzy programming for solving the multi-objective linear programming problems has been proposed by Zimmermann [24]. Narasimhan [25] has introduced the initial FGP model and determined imprecise aspiration levels of the goals in a fuzzy environment using the triangular linear membership functions in the solution procedure. Thereupon, many studies of FGP models have been presented in the related literature [26–29].

For the FGP, a solution set x is obtained where g_k is the aspiration level of the kth goal, $f_k(x)\tilde{\geq} g_k \ (or\, f_k(x)\tilde{\leq} g_k)$, $k = 1,\ 2,\ \ldots,\ n$

subject to $x\in F$, (F is a feasible set)

$f_k(x)\tilde{\geq}(\tilde{\leq})g_k$ indicates that the achievement level of the kth fuzzy goal may be approximately greater than or equal to (approximately less than or equal to) the aspiration level g_k Using Zimmerman's [24] approach, the FGP is formulated as follows: (FGP)

$$\begin{aligned}&\text{Maximize } \lambda\\ &\text{Subject to } \lambda - \mu_k(f_k(x)) \leq 0, \quad k = 1,\ 2,\ \ldots,\ n,\\ &x\in F, (F \text{ is a feasible set}),\end{aligned}$$

where λ is an additional continuous variable; μ_k ($f_k(x)$) is a membership function of the kth goal and is defined as follows:

$$\mu_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \geq g_k \\ \dfrac{f_k(x) - l_k}{g_k - l_k} & if\ l_k < f_k(x) < g_k \\ 0 & if\ f_k(x) \leq l_k \end{cases}$$

or as

$$\mu_k(f_k(x)) = \begin{cases} 1 & if\ f_k(x) \leq g_k \\ \dfrac{u_k - f_k(x)}{u_k - g_k} & if\ g_k < f_k(x) < u_k \\ 0 & if\ f_k \geq u_k \end{cases}$$

where l_k (or u_k) is the lower (or upper) tolerance limit for the kth fuzzy goal $f_k(x)\tilde{\geq}\ (\tilde{\leq})g_k$.

Chang [26] has proposed a methodology for FGP that converts binary FGP (BFGP) to a standard version of FGP using the linearization strategies and the binary membership functions. This approach is very suitable for handling multi-objective optimization problems like MALB-CD. Hence, we have selected this methodology for the development of a FGP approach for the MALB-CD problem. The mathematical formulation of BFGP is as follows [29]:(BFGP)

$$\begin{aligned}&f_k(x)\cdot b_k\tilde{\geq} g_k\cdot b_k\ (\text{or}\, f_k(x)\cdot b_k\tilde{\leq} g_k\cdot b_k)\quad k=1,2,\ldots,n,\\&\text{Subject to } x\in F(F \text{ is a feasible set}),\\&b_k\in R_k(x),\ k=1,\ 2,\ \ldots,\ n\end{aligned}$$

Chang [29] has proposed the following model to solve the above BFGP problem:

$$\begin{aligned}\text{Minimize }\ & d_k^-,\quad k=1,2,\ldots,n\\ \text{Subject to }\ & L_k f_k(x)b_k - L_k^0 b_k + d_k^- - d_k^+ = 1,\\ & \qquad k=1,2,\ldots,n\quad \text{for}\quad f_k(x)\tilde{\geq} g_k\\ & I_k^0 b_k - I_k f_k(x) b_k + d_k^- - d_k^+ = 1,\\ & \qquad k=1,2,\ldots,n\quad \text{for}\quad f_k(x)\tilde{\leq} g_k\\ & b_k\in R_k(x),\quad k=1,2,\ldots,n,\end{aligned}$$

where $L_k=\frac{1}{g_k-l_k}$, $L_k^0=L_k l_k$, $I_k=\frac{1}{u_k-g_k}$, and $I_k^0=I_k u_k$. The rigid constraints given in precise GP formulation are included in their original form in the proposed BFGP formulation. The same three goals of precise GP are considered in a fuzzy environment. Fuzzy goals of the proposed BFGP formulation with their aspiration levels and their membership functions are explained as follows:

The first fuzzy goal concerns the number of workstations that is desired to be approximately less than or equal to the lower limit for the number of workstations $\left(\underline{\text{gw}}\right)$:

Fuzzy Goal 1 ($f_1(x)$): Fuzzy Number of Workstations

The decision-maker desires that the number of workstations must be equal to or less than $\underline{\text{gw}}$. This can be formulated as follows:

$$f_1(x)=\sum_{j\in W} u_j \tilde{\leq} \underline{\text{gw}} \tag{14}$$

The linear membership function for the fuzzy number of workstations goal is given in Fig. 4.

In Fig. 4 $\overline{\text{gw}}$ is the upper limit for this fuzzy goal. According to Fig. 4, if the number of workstations is equal to $\underline{\text{gw}}$, the value of membership function will be 1 which implies that this goal is fully achieved. If the number of workstations is between $\underline{\text{gw}}$ and $\overline{\text{gw}}$, the value of membership function will vary between 0 and 1 which implies that this goal is level achieved. This goal is not

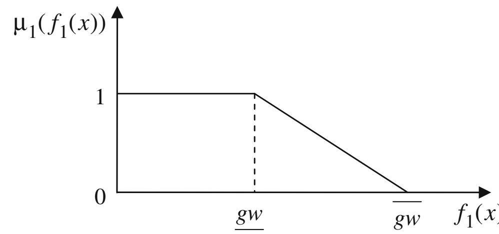

Fig. 4 Membership function of the fuzzy number of workstations goal

achieved when the number of workstations is equal to or greater than $\overline{\text{gw}}$.

The second fuzzy goal is associated with the cycle time of mixed-model assembly line. We desire that the cycle time must be equal to or less than lower limit for the cycle time ($\underline{\text{gc}}$).

Fuzzy Goal 2 ($f_2(x)$): Fuzzy Cycle Time

Fuzzy cycle time goal can be formulated as follows:

$$f_2(x)=\sum_{i\in T_p} t_{ip}x_{pij}\tilde{\leq}\underline{\text{gc}}\ \ \forall p\in P\ \ \forall j\in W \tag{15}$$

The linear membership function for fuzzy cycle time goal is given in Fig. 5.

The third fuzzy goal is for the total cost required to duplicate common tasks. Equation 8 needs to be modified for being used in a fuzzy goal program. We transfer the constant cost $\sum_{i\in \text{TD}} c_i$ to the right-hand side of the equation as follows:

$$\sum_{i\in \text{TD}}\sum_{j\in W} c_i z_{ij} \leq \sum_{i\in \text{TD}} c_i + \text{AC} \tag{16}$$

The constant cost $\sum_{i\in \text{TD}} c_i$ is always incurred once the common tasks in TD are established, but some common tasks in TD can be duplicated, and this naturally will entail additional cost. Based on Eq. 16, the decision-maker desires that the sum of establishment costs of the common tasks in *TD* and the total cost of task duplication must be equal to or less than $\underline{\text{gd}}$.

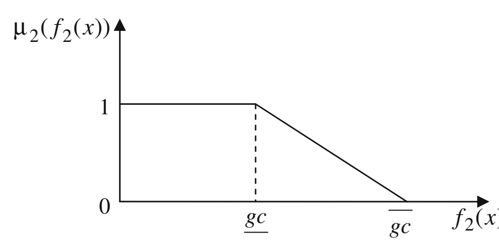

Fig. 5 Membership function of the fuzzy cycle time goal

Fuzzy Goal 3 ($f_3(x)$): Fuzzy Total Cost

Total cost should be equal to or less than $\underline{\mathrm{gd}}$. This goal can be formulated as follows:

$$f_3(x) = \sum_{i \in \mathrm{TD}} \sum_{j \in W} c_i z_{ij} \tilde{\leq} \underline{\mathrm{gd}} \tag{17}$$

The linear membership function for the fuzzy total cost goal is given in Fig. 6.

Chang's [29] linearization methodology requires the specification of the linearization parameters for each fuzzy goal. These linearization parameters can be specified as

- D^0 Linearization parameter for the number of workstation goal
- E^0 Linearization parameter for the cycle time goal
- F^0 Linearization parameter for the total cost goal

Equation 18 is the linearized formulation of the number of workstations goal:

$$D^0 - D\left(\sum_{j \in W} u_j\right) + d^- - d^+ = 1 \tag{18}$$

where $D = \frac{1}{\overline{\mathrm{gw}} - \underline{\mathrm{gw}}}$ and $D^0 = D\overline{\mathrm{gw}}$

Equation 19 is the linearized formulation of the cycle time goal:

$$E^0 - E\left(\sum_{i \in Tp} t_{ip} x_{pij}\right) + e^- - e^+_{mj} = 1 \quad \forall p \in P \quad \forall j \in W \tag{19}$$

where $E = \frac{1}{\overline{\mathrm{gc}} - \underline{\mathrm{gc}}}$ and $E^0 = E\overline{\mathrm{gc}}$

Equation 20 is the linearized formulation of the total cost goal:

$$F^0 - F\left(\sum_{i \in \mathrm{TD}} \sum_{j \in W} c_i z_{ij}\right) + f^- - f^+ = 1 \tag{20}$$

where $F = \frac{1}{\overline{\mathrm{gd}} - \underline{\mathrm{gd}}}$ and $F^0 = F\overline{\mathrm{gd}}$

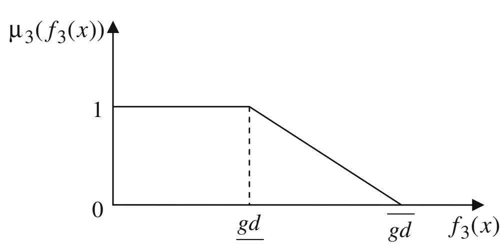

Fig. 6 Membership function of the fuzzy total cost goal

The objective function of the proposed BFGP formulation aims at minimizing the sum/total of the under achievements in the three fuzzy goals:

$$\text{Minimize} \quad d^- + e^- + f^- \tag{21}$$

where d^-, e^-, and f^- are under achievements of the number of workstation, the cycle time, and the total cost goals, respectively.

As in the precise GP formulation, the objective function of the proposed BFGP formulation is also arranged to achieve Pareto efficient solutions. The positive deviational variables are added to the objective function of the proposed BFGP model as follows:

$$\text{Minimize} \quad d^- + e^- + f^- - 0.001d^+ - 0.001\sum e^+_j - 0.001f^+ \tag{22}$$

Based on the priorities determined by the decision-maker, the above GP and BFGP formulations are solved with pre-emptive approach. Weights p_1, p_2, and p_3 ($p_1 >>> p_2 >>> p_3$) are assigned to d^-, e^-, and f^- in line with the determined priority order.

5 Numerical illustrations

In this section, the proposed approaches are illustrated and their computational requirements are assessed on a data set. In the first stage, a two-product problem is generated as example and solved to illustrate the proposed approaches. Task data of the example is given in Table 2:

As shown in Table 2, tasks 1, 2, 3, 4, and 5 are common to both products. We suppose that task 3 cannot be duplicated, while tasks 1, 2, 4, and 5 can. Table 2 shows that at least 22

Table 2 Task data of the example

Task	Immediate successors		Completion times (t_{ip})		Establishment costs (c_i)
	Product 1	Product 2	Product 1	Product 2	
1	2,3	3,4	3	2	0
2	4	8	5	6	3
3	5,6	5	4	5	2
4	5	2	3	5	4
5	7	8	2	1	4
6	7	–	1	–	1
7	–	–	5	–	3
8	–	–	–	4	5

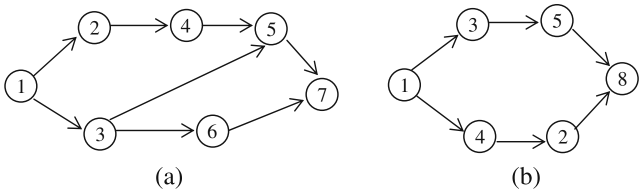

Fig. 7 Precedence diagrams for (**a**) product 1, (**b**) product 2

cost units are required to establish all of the tasks in this mixed-model assembly line. However, assembly line manager will have to use a limited available capital (AC), if some duplicable common tasks are to be duplicated. Table 2 also shows the existence of one precedence conflict in this example problem. Task 4 is the immediate successor of task 2 for product 1 while task 2 is the immediate predecessor of task 4 for product 2. The precedence diagrams of the two products are given in Fig. 7.

The illustrative example is first solved in a single-objective form using the proposed binary formulation to minimize the number of workstations for a given cycle time and an available capital. The problem is solved considering four levels of the cycle time (7, 8, 9, and 10 time units) and three levels of the available capital (3, 7, and 11 cost units), and results are given in Table 3.

Due to the precedence conflict between tasks 2 and 4 for the two different products, one of these tasks is duplicated in all solutions. As an example, task assignments for C=10 and AC=11 are given in Table 4.

Table 4 shows that task 2 of product 1 is completed in workstation 1 while task 2 of product 2 is completed in workstation 3. The layout of the example mixed-model assembly line for the solution given in Table 4 is illustrated in Fig. 8.

The single-objective results given in Table 3 make it evident that it is necessary to handle the MALB-CD

Table 3 Single-objective solutions of the example (K_{max}=5)

Cycle time (C)	Available capital (AC)	Number of workstations	Duplicated tasks	Duplication cost used from AC
7	3	5	2	3
	7	4	2, 4	7
	11	4	2, 4	7
8	3	4	2	3
	7	4	2, 5	7
	11	4	4, 5	8
9	3	4	2	3
	7	4	2	3
	11	4	4	4
10	3	3	2	3
	7	3	2	3
	11	3	2, 4	7

Table 4 Task assignments for C=10 and AC=11 (K_{max}=5)

	Workstation 1	Workstation 2	Workstation 3
Tasks assigned for product 1	1, 2	3, 4, 5	6, 7
Workload for product 1	8	9	6
Tasks assigned for product 2	1, 4	3, 5	2, 8
Workload for product 2	7	6	10

problem with multiple objectives. Compare, for example, the solution for C=7; AC=11, and the solution for C=9; AC=11. Four workstations are utilized for both solutions spending 7- and 4-unit duplication costs, respectively. This means that the decision-maker can reduce the cycle time from 9 to 7 by spending 7−4=3 more cost units.

The illustrative example is then solved using the proposed precise GP formulation. Suppose that the decision-maker's precise goals with their priority levels are as follows:

The number of workstations should not exceed 3 (gw=3)
The cycle time should not exceed 9 time units (gc=9)
The total cost should not exceed 7 cost units (gd=7)

The model is solved in accordance with the priority sequence given above. The values of positive deviational variables are found as d^+=0, e^+=1, and f^+=0. These values mean that the number of workstations and the total cost goals are satisfied while the cycle time goal is not satisfied. The value e^+=1 implies that the cycle time of the line exceeds the target value by 1 time unit. Task assignments in the solution of GP model is given in Table 5.

Table 5 shows that the cycle time of the line is 10 time units. Only task 2 is duplicated by assigning them to two different workstations. A total of 3 cost units are spent for these duplications.

We also performed a sensitivity analysis to determine the effects of different priority orders on the solutions. For this purpose, the illustrative example is solved for the remaining five different priority orders of the goals. The precise aspiration levels for the goals are again determined as gw=3, gc=9, and gd=7, respectively. The results of the sensitivity analysis are summarized in Table 6.

Table 6 shows that the total cost goal is satisfied in all priority orders. For a given 3 cost units of available capital, the decision-maker should make a choice between two alternatives. The first alternative is three workstations and 10 time units of cycle time while the second one is four workstations and 8 time units of cycle time. That is, if the

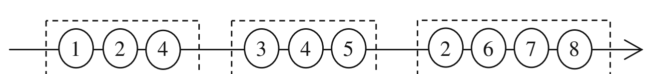

Fig. 8 Line layout for the single-objective solution of the example (C=10 and AC=11)

Table 5 Precise GP solution of the example (K_{max}=4)

Workstation	Product 1		Product 2	
	Tasks	Workload	Tasks	Workload
1	1, 3, 6	8	1, 3	7
2	2, 4, 5	10	4, 5	6
3	7	5	2, 8	10

Table 7 BFGP solution of the example (K_{max}=4)

Workstation	Product 1		Product 2	
	Tasks	Workload	Tasks	Workload
1	1, 2, 3	12	1, 3, 4	12
2	4, 5, 6, 7	11	2, 5, 8	11

decision-maker accepts running the line with one more time unit of cycle time, the fourth workstation will not be required.

The same example is also solved using the proposed BFGP approach with fuzzy aspiration levels of the goals. First of all, lower and upper limits for the goals should be defined to solve the problem using BFGP approach. These limits are defined as follows:

The number of workstations utilized should approximately be less than or equal to 4 ($\underline{gw} = 2$) with an upper limit of $\overline{gw} = 4$.
The cycle time of line should approximately be less than or equal to 8 time units ($\underline{gc} = 8$) with the upper limit of $\overline{gc} = 11$time units.
The total cost should be approximately less than or equal to 17 cost units ($\underline{gd} = 17$) with an upper limit of $\overline{gd} = 21$.

Note that 11 cost units of lower and upper limits here is the constant establishment cost of the common tasks in TD.

The model is solved in accordance with the above priority sequence. The values of the negative deviational variables are found as d^-=0, e^-=1, and f^-=0.25. These values mean that the number of workstations goal is fully achieved; total cost goal is level achieved with the membership value of 1−0.25=0.75 while the cycle time goal is not achieved. The value e^-=1 implies that cycle time of the line is greater than $\overline{gc} = 11$. Task assignments in the solution of BFGP model is given in Table 7.

Table 7 shows that the cycle time of the mixed-model line is 12 time units. Tasks 2 and 4 are duplicated by assigning them to two different workstations. A total of 7 cost units are spent for these duplications.

We also performed a sensitivity analysis to determine the impacts of different priority orders on the solutions. For this purpose, the illustrative example is also solved for the remaining five different orders of the fuzzy goals. The aspiration levels for the fuzzy goals are determined as $\underline{gw} = 2$, $\overline{gw} = 4$; $\underline{gc} = 8$, $\overline{gc} = 11$; and $\underline{gd} = 17$, $\overline{gd} = 21$, respectively. The results of the sensitivity analysis are summarized in Table 8.

Table 8 shows that the full satisfaction for all goals at the same time is not achieved in any order of the goals. The total cost goal is level achieved in only the first scenario where it is in the third priority level and fully achieved in all of the other scenarios. The number of workstations goal is fully achieved only when the cycle time goal is not achieved (scenarios 1, 2, and 5). Similarly, the cycle time goal is fully achieved only when the number of workstations goal is not achieved (scenarios 3, 4, and 6). In terms of the given aspiration levels, level achievement of these two goals is not observed. The solutions for the second and fifth scenarios have resulted in two workstations with 13 time units of cycle time and no task duplication. In these two solutions, neither task 2 nor task 4 is duplicated, since these two conflicting tasks are assigned to the same workstation. These results mean that 7 units of duplication cost is required to reduce the cycle time from 13 to 12 time units in the case of two workstations.

The illustrative example given above showed that the proposed models are valid. However, the proposed models should also be validated for large-sized problems. For this purpose, eight large-sized problems are solved and the

Table 6 Sensitivity analysis of the example problem by changing the order of precise goals (K_{max}=4)

Scenarios	Priorities	d^+	e^+	f^+	Unsatisfied goal	Number of workstations	Cycle time	Duplicated tasks	Duplication cost used from AC
1	$G_1{>>>}G_2{>>>}G_3$	0	1	0	G_2	3	10	2	3
2	$G_1{>>>}G_3{>>>}G_2$	0	1	0	G_2	3	10	2	3
3	$G_2{>>>}G_1{>>>}G_3$	1	0	0	G_1	4	8	2	3
4	$G_2{>>>}G_3{>>>}G_1$	1	0	0	G_1	4	8	2	3
5	$G_3{>>>}G_1{>>>}G_2$	0	1	0	G_2	3	10	2	3
6	$G_3{>>>}G_2{>>>}G_1$	1	0	0	G_1	4	8	2	3

Table 8 Sensitivity analysis of the example problem by changing the order of fuzzy goals ($K_{\max}$=4)

Scenarios	Priorities	d^-	e^-	f^-	Number of workstations	Cycle time	Duplicated tasks	Duplication cost used from AC
1	$f_1(x)>>>f_2(x)>>>f_3(x)$	0	1	0.25	2	12	2, 4	7
2	$f_1(x)>>>f_3(x)>>>f_2(x)$	0	1	0	2	13	–	–
3	$f_2(x)>>>f_1(x)>>>f_3(x)$	1	0	0	4	8	2	3
4	$f_2(x)>>>f_3(x)>>>f_1(x)$	1	0	0	4	8	2	3
5	$f_3(x)>>>f_1(x)>>>f_2(x)$	0	1	0	2	13	–	–
6	$f_3(x)>>>f_2(x)>>>f_1(x)$	1	0	0	4	8	2	3

behaviors as well as the computational requirements of the proposed models are analyzed. For conducting the tests, eight test problems with different number of products and different number of tasks are generated. The number of products ranges from 2 to 5. The total number of unique tasks (UT) in mixed-model assembly lines is selected as 10 and 12 for the two-product problems; 14 and 16 for the three-product problems; 18 and 20 for the four-product problems; and finally 25 and 30 for the five-product problems. On the other hand, the number of conflicting pairs of tasks (NC) is selected to be one for the two-product problems, two for the three-product problems, three for the four-product problems, and four for the five-product problems. The completion times and the establishment costs of the tasks are randomly generated with u.d. [1, 10] and u.d. [0, 7], respectively. A summary of the problem parameters is given in Table 9.

Each problem is solved in terms of six different orders of the goals by the proposed precise and fuzzy GP approaches. All problems are solved using XPRESS Solver Engine Version 9.5 on an IntelCore2 Duo 2.00 GHz and 2 GB RAM computer. The time allowed to solve the problems is limited to 1 h. Solution times in minutes and seconds (ST) and the number of branch-and-bound iterations (IT) for the problems, for which optimal solutions are obtained, are given in Tables 10 and 11. If the optimal solution of any problem cannot be obtained, it is labeled as “optimal solution not found” (o.s.n.f.). The results for the precise and fuzzy goals are given in Tables 10 and 11, respectively.

Both of the tables show that one of the goals is always unsatisfied depending on the order of the goals. In other words, no solution has been found where all of the goals are achieved. The results show that optimal solutions are obtained within a reasonable duration for the problems for which the number of tasks ranges from 10 to 20. However, the optimal solutions of 25- and 30-task problems cannot be obtained in some orders of the goals within the limited duration. Significant changes occur in the solution times when the order of the goals is altered. Comparison of the 25-task problems with the 30-task problems in both of the GP solutions indicates that the capability of achieving optimal solutions depends not only on the size of the problems but also on the order of the goals.

6 Conclusions

Although most assembly lines utilized in practice are arranged to produce multiple products, less research has been done in the field of the MALB problem than in the field of the SALB problem. The literature on MALB problem consists of two basic approaches, namely combined precedence diagram and adjusted task times. The use of combined precedence diagrams forces the researchers to make the assumption that a common task should be assigned to a single workstation. It should also be mentioned that it is impossible to construct a regular combined precedence diagram if there are conflicting

Table 9 Summary of problem parameters

No.	P	UT	NC	gw	gc	gd	$\underline{gw}$	$\overline{gw}$	$\underline{gc}$	$\overline{gc}$	$\underline{gd}$	$\overline{gd}$
1	2	10	1	3	10	11	3	5	9	12	20	30
2	2	12	1	3	13	8	3	5	12	16	26	32
3	3	14	2	4	12	18	3	5	11	15	34	43
4	3	16	2	3	16	9	2	5	15	17	25	30
5	4	18	3	4	11	16	2	5	9	12	33	43
6	4	20	3	4	13	13	3	5	12	15	28	41
7	5	25	4	3	20	30	3	6	19	22	76	86
8	5	30	4	4	23	15	3	6	20	26	59	74

Table 10 Numerical results for precise goals

Problem no.	G_1>>>G_2>>>G_3					G_1>>>G_3>>>G_2					G_2>>>G_1>>>G_3					G_2>>>G_3>>>G_1					G_3>>>G_1>>>G_2					G_3>>>G_2>>>G_1				
	d^+	e^+	f^+	ST	IT	d^+	e^+	f^+	ST	IT	d^+	e^+	f^+	ST	IT	d^+	e^+	f^+	ST	IT	d^+	e^+	f^+	ST	IT	d^+	e^+	f^+	ST	IT
1	0	2	0	00:13	361	0	2	0	00:08	606	1	0	0	00:02	14	1	0	0	00:01	5	0	4	0	00:05	376	1	0	0	00:02	5
2	0	1	0	00:11	1,368	0	1	0	00:09	880	1	0	0	00:09	1,304	1	0	0	00:09	1,333	0	2	0	00:04	201	1	0	0	00:04	259
3	0	1	0	03:13	29,462	0	1	0	01:07	8,728	1	0	0	03:26	32,068	1	0	0	02:26	19,627	0	6	0	00:33	1,572	1	0	0	00:19	697
4	0	1	0	00:56	3,388	0	1	0	00:29	1,542	1	0	0	00:42	4,761	1	0	0	00:44	3,697	0	3	0	00:15	437	1	0	0	00:23	1,091
5	0	2	0	02:19	6,949	0	2	0	05:09	12,356	1	0	0	03:42	12,155	1	0	0	02:01	7,923	0	4	0	00:39	571	1	0	0	01:58	4,924
6	0	2	0	02:16	7,029	0	2	0	03:19	8,446	1	0	0	01:27	4,137	1	0	0	02:00	4,880	0	4	0	00:35	641	1	0	0	00:48	1,169
7	0	4	0	24:45	142,639	0	4	0	17:02	76,074	1	0	0	52:00	213,376	o.s.n.f.					o.s.n.f.					o.s.n.f.				
8	o.s.n.f.					o.s.n.f.					o.s.n.f.					o.s.n.f.					0	6	0	13:22	6,309	2	0	0	14:50	14,669

Table 11 Numerical results for fuzzy goals

Problem no.	$f_1(x)$>>>$f_2(x)$>>>$f_3(x)$					$f_1(x)$>>>$f_3(x)$>>>$f_2(x)$					$f_2(x)$>>>$f_1(x)$>>>$f_3(x)$					$f_2(x)$>>>$f_3(x)$>>>$f_1(x)$					$f_3(x)$>>>$f_1(x)$>>>$f_2(x)$					$f_3(x)$>>>$f_2(x)$>>>$f_1(x)$				
	d^-	e^-	f^-	ST	IT	d^-	e^-	f^-	ST	IT	d^-	e^-	f^-	ST	IT	d^-	e^-	f^-	ST	IT	d^-	e^-	f^-	ST	IT	d^-	e^-	f^-	ST	IT
1	0	1	0.3	00:11	352	0	1	0	00:07	643	1	0	0	00:04	149	1	0	0	00:03	135	0	1	0	00:07	511	1	0	0	00:03	131
2	0	0.5	0	00:09	1,203	0	0.5	0	00:06	450	0.5	0	0	00:07	899	0.5	0	0	00:08	745	0	0.75	0	00:04	257	0.5	0	0	00:05	265
3	0	1	0.11	00:19	993	0	1	0	00:33	1,630	1	0	0	00:33	2,896	1	0	0	00:21	1,780	0	1	0	00:20	715	1	0	0	00:13	389
4	0	1	0.2	00:13	434	0	1	0	00:10	343	0.67	0	0	00:40	3,031	0.67	0	0	00:33	2,115	0.33	1	0	00:24	1,202	1	0	0	00:17	1,036
5	0	1	0	00:27	790	0	1	0	00:21	442	1	0.67	0.7	04:03	16,228	1	0.67	0.7	03:04	10,849	0	1	0	01:02	1,306	1	1	0	00:47	1,439
6	0	1	0.54	01:09	2,248	0	1	0.08	01:20	1,859	1	0	1	03:22	12,803	1	0	1	02:48	12,364	0	1	0	00:59	1,365	1	0	0	00:44	566
7	0	1	0	22:05	140,427	0	1	0	18:.30	86,931	o.s.n.f.					o.s.n.f.					o.s.n.f.					o.s.n.f.				
8	o.s.n.f.					0	1	0	11:47	5,408	o.s.n.f.					o.s.n.f.					0	1	0	26:31	10,443	1	0	0	28:32	10,995

precedence relationships between the common tasks. In this study, we assumed that the common tasks may have some precedence conflicts and that they can be assigned to more than one workstation. These relaxations result in a new version of assembly line balancing, i.e., mixed-model assembly line balancing with precedence conflicts and duplicable common tasks (MALB-CD). Furthermore, compared with earlier researches on MALB, the MALB-CD problem is more appropriate for representing the real-world mixed-model production environments. In the first step of this study, a new binary formulation is proposed for the MALB-CD. The proposed formulation minimizes the number of workstations for a given cycle time and available capital to duplicate the common tasks. Since the MALB-CD problem is a multi-objective rather than a single-objective optimization problem, we have also developed two GP approaches to balance a mixed-model assembly line, one with precise and the other with fuzzy goals. The proposed approaches give decision-makers an important opportunity to optimize their mixed-model assembly lines in line with their preferred priorities. On the other hand, the proposed GP formulations are capable of achieving Pareto efficient solutions. The validation and computational requirements of the proposed approaches are evaluated by a number of numerical illustrations. The NP-Hard nature of assembly line balancing problems is well known. Hence, the MALB-CD problem is also NP-Hard. Consideration of stochastic task completion times and development of effective heuristics are among the future works to be done on line balancing. Developing formulations for the mixed-model U-line balancing with precedence conflicts and duplicable common tasks is also among the future works. Furthermore, as the sequencing problem of products is tightly related to the line balancing problem in mixed-model assembly lines, a simultaneous consideration of line balancing and model sequencing problems in the case of duplicable common tasks and precedence conflicts should also be mentioned among the future works.

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