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ORIGINAL ARTICLE

The profiling of end mill and planing tools to generate helical surfaces known by sampled points

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Abstract Generation of helical surfaces with cylinder-frontal tools (end mill tool or grinding wheels) as so as with cylindrical tools (planing tools) are proceedings used in the small lot productions manufacturing or for reparations. In many situations, the surfaces to be generated are known by measuring on three-dimensional measuring machines. In this way, is possible to define a generatrix expressed by sampled points, which impose specifically algorithms for the end mill tool's profiling. In this paper, specifically proposed are algorithms for end mill tool and planing tool profiling, designated to generate helical surfaces known in discrete form. More is assumed that the helical surface's generatrix may be expressed only by few points, three or four, so the helical surfaces generatrix may be expressed by inferior degree Bezier polynomials using topological geometry. One of the goals of this paper is to compare the numerical results obtained by the proposed algorithm with the results obtained from profiling theoretically methods, for the same surfaces types, in order to proof the new method quality.

Keywords End mill tool · Planing tool · Helical surfaces

1 Introduction

There are multiple solutions when the helical surfaces, especially those which form flutes with identical anti-homologous flanks (the involute flanks of the teeth of helical gear, flanks of the modulus thread, and thread for ball circulating screw) are generated by milling with end mill tool [2, 6].

The enwrapping conditions of the helical surface with revolving tools [2, 5, 12] are defined, in an analytical form, as with the method of helical movement decomposition, the Nikolaev condition [3, 13, 14]. Also, for analytical form the complementary theorems were stated [6, 7], for numerical expression, by points cloud [8] or using graphical software [10].

These were made models of generating of surfaces measured point by point [9, 11], where, initially, they reconstruct the helical surface. In these cases, the number of points measured on surfaces is very large.

Also, in [4], we analyze only the issue of disk tool profiling, reciprocally enveloping with a helical surface known by sampled points (measured) by a reduced number of points known along the generatrix of these surface. The results of the proposed method were compared with data obtained using the fundamentals enveloping theorems.

The constructive simplicity of the tool which generates, in the cutting movement, a peripheral revolving surface, generating simultaneous the both gauge flank, the small number of the tool's teeth (often only two teeth) make the machining and use of this tool type to be more advantageous opposite a disk tool. Obviously, the productivity of machining using this tool (end mill) is more reduced, and, as follows, this solution should be economically used only for a small number of the same type pieces [16].

The profiling of this tool type often follows the same steps as for disk tool profiling, the particularities of the end mill tool imposing its profiling algorithm [3].

Exist situations when, for helical surfaces machined as unique pieces or for repairs, when the end mill realization is too expensive and, as follows, is imposed to choose a more

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simple method, the milling being replaced with the planing of helical surface flank, generating a cylindrical surface, reciprocally enwrapping with the helical surface, generating with planing a tool of the helical surface [15].

In the both situations, the surface to be generated may be known by a small-point number, often as results of the measuring of a generatrix of this (not necessary an in-plane generatrix) allowing a discrete expression for the helical surface to be generated.

In this paper, a methodology is proposed for end mill tool's profiling and planer tool for the case of the discrete expression with a small-point number (three or four points) of its generatrix profile, using Bezier polynomials approximation [1].

2 End mill tool profiling—methods

In many practical situations, we may know or measure only a small number of points along the generatrix of the surface to be generated. In these cases, the generatrix can be substituted by a small order (two or three) of Bezier polynomials [1] as illustrated in Fig. 1, where we have considered that the generatrix of the helical surface axis, $\vec{V}$ (Z axis) is:

$$X = P_X(\lambda); Y = P_Y(\lambda); Z = P_Z(\lambda), \tag{1}$$

where $\lambda \in [0,1]$, while $P_X(\lambda)$, $P_Y(\lambda)$, and $P_Z(\lambda)$ are Bezier polynomials used to approximate the generatrix G.

In the helical movement of G generatrix around axis $\vec{V}$ with helical parameter p, right helicoids,

$$\left\|\begin{matrix} X \\ Y \\ Z \end{matrix}\right\| = \left\|\begin{matrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{matrix}\right\| \cdot \left\|\begin{matrix} P_X(\lambda) \\ P_Y(\lambda) \\ P_Z(\lambda) \end{matrix}\right\| + \left\|\begin{matrix} 0 \\ 0 \\ p\varphi \end{matrix}\right\|, \tag{2}$$

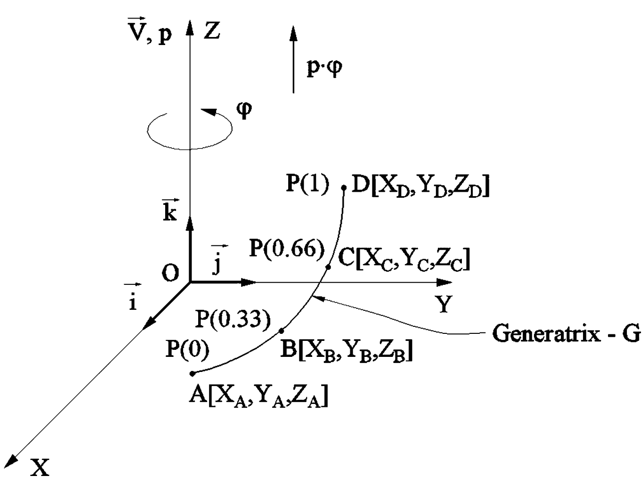

Fig. 1 Conventional points on generatrix (G) approximation by Bezier polynomials

The helical surface of constant pitch can be expressed as:

$$\Pi(\lambda,\varphi): \left| \begin{array}{l} X = P_X(\lambda)\cdot\cos\varphi - P_Y(\lambda)\cdot\sin\varphi; \\ Y = P_X(\lambda)\cdot\sin\varphi + P_Y(\lambda)\cdot\cos\varphi; \\ Z = P_Z(\lambda) + p\cdot\varphi, \end{array} \right. \tag{3}$$

Where λ and φ are variables parameters, φ rotation angle around Z axis.

The coordinates measured along the surface's generatrix are defined:

$$A[X_A, Y_A, Z_A];\ B[X_B, Y_B, Z_B];\ C[X_C, Y_C, Z_C]; D[X_D, Y_D, Z_D], \tag{4}$$

on the generatrix known in discrete form, see Fig. 1.

The λ parameter value is known only for a reduced number of values (3 or 4) which, in many cases, approximates sufficiently well, via Bezier polynomials, the helical surface generatrix. For second-order polynomials, one can express the generatrix as:

$$P \left| \begin{array}{l} P_X(\lambda) = \lambda^2 A_x + 2\lambda(1-\lambda)C_X + (1-\lambda)^2 B_X; \\ P_Y(\lambda) = \lambda^2 A_Y + 2\lambda(1-\lambda)C_Y + (1-\lambda)^2 B_Y; \\ P_Z(\lambda) = \lambda^2 A_Z + 2\lambda(1-\lambda)C_Z + (1-\lambda)^2 B_Z. \end{array} \right. \tag{5}$$

Similarly, for third-order polynomials, one can write:

$$P \left| \begin{array}{l} P_X(\lambda) = \lambda^3 A_X + 3\lambda^2(1-\lambda)B_X + 3\lambda(1-\lambda)^2 C_X + (1-\lambda)^3 D_X; \\ P_Y(\lambda) = \lambda^3 A_Y + 3\lambda^2(1-\lambda)B_Y + 3\lambda(1-\lambda)^2 C_Y + (1-\lambda)^3 D_Y; \\ P_Z(\lambda) = \lambda^3 A_Z + 3\lambda^2(1-\lambda)B_Z + 3\lambda(1-\lambda)^2 C_Z + (1-\lambda)^3 D_Z. \end{array} \right. \tag{6}$$

The coefficients A_X, A_Y, A_Z, B_X, B_Y, B_Z, C_X, C_Y, C_Z, D_X, D_Y, and D_Z can be determined using a fitting method such as least squares from the reduced number of the actual points on the curve for which the coordinates are known. In the cases below, we consider only a very small number of points such that direct determination of the coefficients is possible [1].

From Eqs. 3, 5, and 6, one can determine the approximate helical surface to be generated. Consequently, it is possible to use the fundamental theorems of surfaces enwrapping to determine the peripheral surface of the tool, which would generate by enwrapping the desired helical surface. The helical surface can be expressed generically as:

$$\Pi(\lambda,\varphi) \left| \begin{array}{l} X = \Pi_X(\lambda,\varphi); \\ Y = \Pi_Y(\lambda,\varphi); \\ Z = \Pi_Z(\lambda,\varphi). \end{array} \right. \tag{7}$$

In Eq. 7, $\Pi_X(\lambda,\varphi)$, $\Pi_Y(\lambda,\varphi)$,and $\Pi_Z(\lambda,\varphi)$ are:

$$\begin{array}{l} \Pi_X(\lambda,\varphi) = P_X(\lambda)\cdot\cos\varphi - P_Y(\lambda)\cdot\sin\varphi; \\ \Pi_Y(\lambda,\varphi) = P_X(\lambda)\cdot\sin\varphi + P_Y(\lambda)\cdot\cos\varphi; \\ \Pi_Z(\lambda,\varphi) = P_X(\lambda) + p\cdot\varphi, \end{array} \tag{8}$$

for a right cylindrical helix with constant pitch (p= constant).

The end mill tool's profiling problem, for generating by enwrapping a cylindrical helical surface with constant pitch, may be solved by more methods: the Nikolaev classical method [3] or complementary methods [6, 7], briefly presented in following.

- Nikolaev method

The method assumed to know the normal at helical surface presented in form (7).

The normal to the approximated helical surface $\Pi(\lambda, \varphi)$, can be written as

$$\vec{N}_\Pi = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \Pi'_X(\varphi) & \Pi'_Y(\varphi) & p \\ \Pi_X(\lambda) & \Pi_Y(\lambda) & \Pi'_Z(\lambda) \end{vmatrix}, \tag{9}$$

where $\Pi'_X(\varphi), \Pi'_X(\lambda)$ and the others denote derivative with respect to either of the independent parameters. In vector form, the normal can be written as

$$\vec{N}_\Pi = N_X \vec{i} + N_Y \vec{j} + N_Z \vec{k}. \tag{10}$$

The $X(\lambda)$, $Y(\lambda)$, and $Z(\lambda)$, see (1), polynomials will be identified regarding the known coordinates on G generatrix.

Starting from the (3) forms of the helical surface $\Pi(\lambda, \varphi)$, the parameters of the normal may be defined at the substitutive surface.

For the tool's axis definition, see Fig. 2,

$$\vec{A} = \vec{i} \tag{11}$$

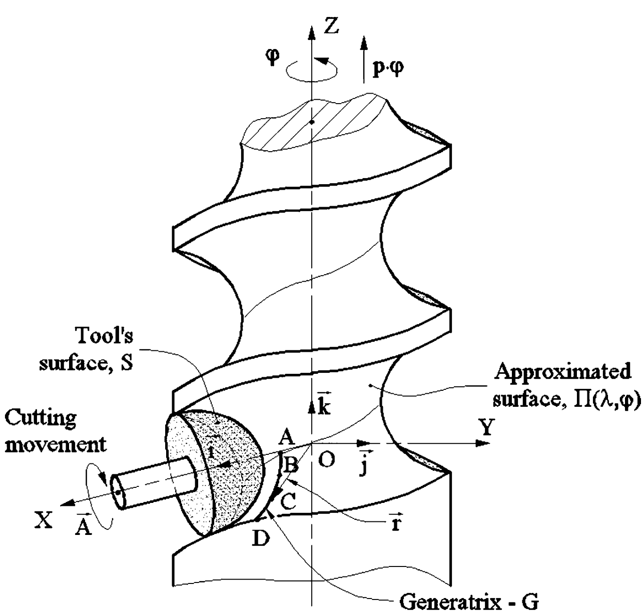

Fig. 2 End mill tool, reference system and tool's axis, $\vec{A}$

and for the position vector of the current point on the $\Pi(\lambda, \varphi)$, see (3), surface,

$$\vec{r} = X(\lambda, \varphi)\vec{i} + Y(\lambda, \varphi)\vec{j} + Z(\lambda, \varphi)\vec{k}, \tag{12}$$

the enwrapping condition [2, 3, 6], known as Nikolaev theorem,

$$\left| \vec{N_\Pi}, \vec{A}, \vec{r} \right| \leq q \tag{13}$$

with q—positive and very small (for example q=1×10^{-3}), may bring in form

$$|[Z(\lambda) + p \cdot \varphi] \cdot N_Y - [X(\lambda) \cdot \sin(\varphi)] \cdot N_Z| \leq q. \tag{14}$$

The equations assembly (7) and (14), determine, on the $\Pi(\lambda, \varphi)$, helical surface (3), the characteristic curve, in principle, in form:

$$C_\Pi \left| \begin{array}{l} X_{C_\Pi} = X(\lambda); \\ Y_{C_\Pi} = Y(\lambda); \\ Z_{C_\Pi} = Z(\lambda). \end{array} \right. \tag{15}$$

The characteristic curve C_Π is common for the helical surface $\Pi(\lambda, \varphi)$ and for peripheral primary surface of end mill tool—S revolution surface.

The axial section of end mill cutter, see Fig. 3, is determined starting from the characteristic curve (15), in the form:

$$S_A \left| \begin{array}{l} H = X(\lambda); \\ R = \sqrt{Y^2(\lambda) + Z^2(\lambda)}, \end{array} \right. \tag{16}$$

for λ and φ couples of values which meets the condition (14).

In all the presented algorithm stages, the profiles calculus is made only for three (4) considered points belonging to the profile.

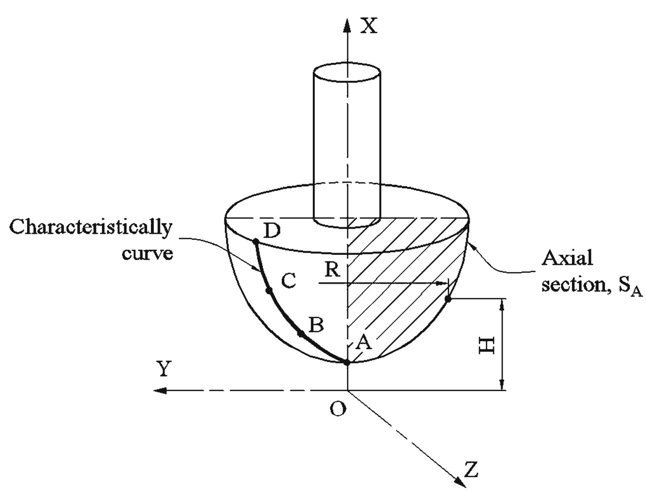

Fig. 3 Axial section of end mill tool—S_A

For the axial section (16), known in discrete form, for 3 (4) points on this, an approximation is made by a second-degree (third) Bezier polynomial, determining a representation form of this.

- The "minimum distance" method

The "minimum distance" method [6, 8] is based on a complementary theorem, which is stated thus: "the contact curve (characteristically curve) between a helical surface with constant pitch and a revolution surface is the geometrical place of points belonging to the helical surface, for which, in planes perpendicular to the revolution surface's axis (crossing planes), the distance to this axis is minimum".

In Fig. 4, defined are: $OO_1=b$, positioning constant; α is the angle between X_1 axis (the axis of end mill tool) and X axis (the axis of reference system joined with helical surface). Currently, $\alpha=0°$, $b=0$ (the two reference systems XYZ and $X_1Y_1Z_1$ are the same).

By the coordinate transformation (17), expressed is the helical surface in the tools' reference system, $X_1Y_1Z_1$ (see Fig. 4), which is a reference system associated with the end mill tool, with X_1 axis overlapped to the end mill tool's axis,

$$\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \left[\begin{pmatrix} X(\lambda,\varphi) \\ Y(\lambda,\varphi) \\ Z(\lambda,\varphi) \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix} \right] \tag{17}$$

with b and α as technological constants or, in principle, in form:

$$\Sigma_1 \left| \begin{aligned} X_1 &= X_1(\lambda,\varphi); \\ Y_1 &= Y_1(\lambda,\varphi); \\ Z_1 &= Z_1(\lambda,\varphi), \end{aligned} \right. \tag{18}$$

representing the Σ helical surface, in the $X_1Y_1Z_1$ reference system.

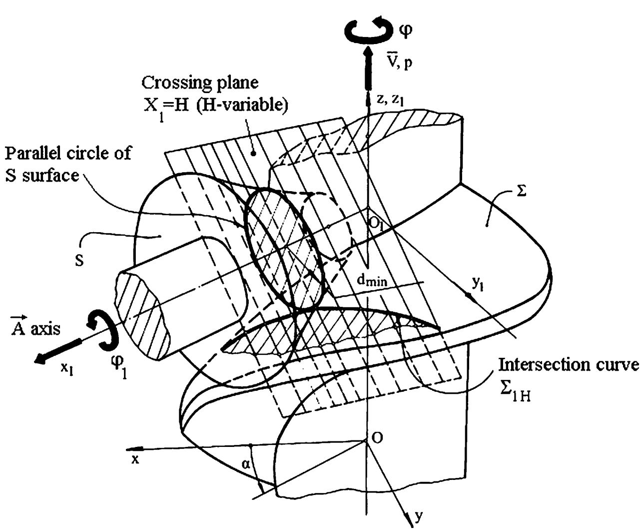

Fig. 4 End mill tool, peripherally surface

The intersection curve of surface (18) with planes $X_1=H$ (H arbitrary parameter), in principle, is:

$$\Sigma_{1H} \left| \begin{aligned} X_1 &= H; \\ Y_1 &= Y_1(\lambda); \\ Z_1 &= Z_1(\lambda). \end{aligned} \right. \tag{19}$$

As follows, in conformance with the above-stated theorem, the contact points between the revolution surface and the helical surface Σ_1 [Σ_{1H} curve, (19)], is determined from the condition that the distance to $\vec{A}$ axis, overlapped to X_1 axis,

$$d = \left(\sqrt{Y_1^2(\lambda) + Z_1^2(\lambda)} \right) \tag{20}$$

to be minimum, hence

$$Y_1 \cdot Y'_{1\lambda} + Z_1 \cdot Z'_{1\lambda} = 0. \tag{21}$$

The equations assembly (18) and condition (21), for different values for H, represent the characteristic curve on the Σ_1 surface, equivalent with the Eq. 15, with $Y'_{1\lambda}$ and $Z'_{1\lambda}$ as partial derivatives of Eq. 19 regarding the λ parameter.

- The in-plane-generating trajectories method

The method based on a complementary theorem [7] states that: "the revolution surface of end mill tool, reciprocally enwrapping with a helical cylindrical surface with constant pitch, is constituted by the circles family from the crossing planes of revolution surfaces, representing the enveloping of trajectories of helical surface's profiles in these planes, in the revolution movement around the tool's axis."

In this way, the profile Σ_{1H}, in the rotation movement around $\vec{A}$ axis (see Fig. 4),

$$\left\| \begin{matrix} X_1 \\ Y_1 \\ Z_1 \end{matrix} \right\| = \left\| \begin{matrix} 1 & 0 & 0 \\ 0 & \cos\varphi_1 & -\sin\varphi_1 \\ 0 & \sin\varphi_1 & \cos\varphi_1 \end{matrix} \right\| \cdot \left\| \begin{matrix} H \\ Y_1(\lambda) \\ Z_1(\lambda) \end{matrix} \right\| \tag{22}$$

describes the profile's family:

$$(\Sigma_{1H})_\varphi \left| \begin{aligned} X_1 &= H; \\ Y_1 &= Y_1(\lambda)\cos\varphi_1 - Z_1(\lambda)\sin\varphi_1; \\ Z_1 &= Y_1(\lambda)\sin\varphi_1 + Z_1(\lambda)\cos\varphi_1, \end{aligned} \right. \tag{23}$$

φ_1 variable parameters and H arbitrary parameter.

The enwrapping condition of the profile's family (23), is give by

$$\frac{Y'_{1\lambda}}{Y'_{1\varphi 1}} = \frac{Z'_{1\lambda}}{Z'_{1\varphi 1}}, \tag{24}$$

$Y_1(\lambda, \varphi_1)$, $Z_1(\lambda, \varphi_1)$ give by Eq. 23.

The (23) and (24) equations assembly determine the characteristically curve of the helical surface, Σ, and

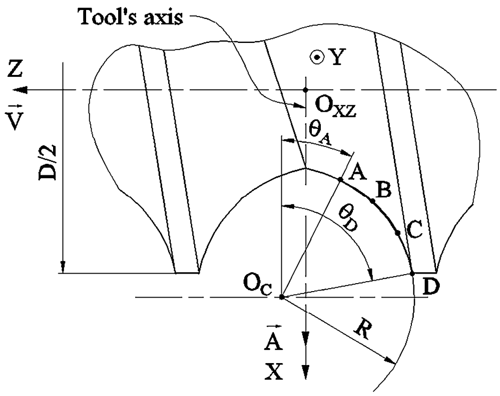

Fig. 5 Ball circulating screw (axial generatrix form)

revolution surface S—peripheral primary surface of end mill tool, equivalent with Eq. 15.

3 Applications

3.1 End mill tool for generating a worm with circular axial section

This surfaces type appears in construction of ball circulating screw or screws for traction with increased fatigue resistance.

They are presumed known (measured) the coordinates of four points on the helical surface generatrix profile, in analytical form; see Fig. 5 and Table 1.

The center coordinates of circle arc, which represent the discreetly known generatrix of the helical surface, are $O_C(X_{O_C}, 0, Z_{O_C})$.

The identification algorithm for the Bezier polynomial substitutive of the helical surface generatrix profile is presented in Table 1, for a 3rd-degree polynomial.

Note: As was previously shown, the p parameter value is known.

In fact, the measuring of points along the axial worm's profile may not respect conventional values for λ (0.33 and 0.66).

In this way, it is necessary to measure more points along the profile in order to approximate values of λ around conventional values; see Table 2 and Fig. 6. This mode to approximate the values of λ parameter was previously demonstrated in paper [4].

The algorithm is completed with specialized software in JAVA programming language. A specifically applet is presented in Fig. 7.

The dialog boxes allow the defining of: O_C center coordinates of axial profile; R radius value [millimeters]; external surface diameter, D [millimeters]; p helical parameter value [millimeters].

The applet allows drawing of the axial section of the end mill tool (H,R) and determination of profile coordinates and the method profiling error level against a theoretically method [3].

In applet, the helical surface, the end mill tool's axis and the normal at discreetly helical surface are presented.

In Fig. 8 and Table 3, the axial profile of end mill tool is presented, in a presentation approximated by Bezier polynomials, regarding the same profile determined by an analytical method. Also presented is the profiling error value regarding the profile determined by an absolutely rigorous analytical method, for: O_C= [52,0,0]; $\Theta_A = 0$; $\Theta_D = \frac{5\pi}{18}$; R=8 mm; D=60 mm; p=3.18 mm.

Table 1 Coefficients of 3rd Bezier polynomials for circle's arc profile

Θ	Primary profile	λ	Polynomial coefficients
Θ_A	$A\left\vert\begin{array}{l} X_A = X_{O_C} - R\cdot\cos(\Theta_A);\\ Y_A = 0;\\ Z_A = Z_{O_C} - R\cdot\sin(\Theta_A).\end{array}\right.$	0	$D_X = Y_A$ $D_Y = Z_A$
Θ_B	$\Theta_B = \Theta_A + \frac{1}{3}\cdot(\Theta_D - \Theta_A)$ $B\left\vert\begin{array}{l} X_B = X_{O_C} - R\cdot\cos(\Theta_B);\\ Y_B = 0;\\ Z_B = Z_{O_C} - R\cdot\sin(\Theta_B).\end{array}\right.$	$\frac{1}{3}$	$C_X = \frac{18\cdot X_C - 9\cdot X_B + 2\cdot X_A - 5\cdot X_D}{6}$ $C_Y = \frac{18\cdot Y_C - 9\cdot Y_B + 2\cdot Y_A - 5\cdot Y_D}{6}$
Θ_C	$\Theta_C = \Theta_A + \frac{2}{3}(\Theta_D - \Theta_A)$ $C\left\vert\begin{array}{l} X_C = X_{O_C} - R\cdot\cos(\Theta_C);\\ Y_C = 0;\\ Z_C = Z_{O_C} - R\cdot\sin(\Theta_C).\end{array}\right.$	$\frac{2}{3}$	$B_X = \frac{-5\cdot X_A + 2\cdot X_D + 18\cdot X_B - 9\cdot X_C}{6}$ $B_Y = \frac{-5\cdot Y_A + 2\cdot Y_D + 18\cdot Y_B - 9\cdot Y_C}{6}$
Θ_D	$D\left\vert\begin{array}{l} X_D = X_{O_C} - R\cdot\cos(\Theta_D);\\ Y_D = 0;\\ Z_D = Z_{O_C} - R\cdot\sin(\Theta_D).\end{array}\right.$	1	$A_X = X_D$ $A_Y = Y_D$

Table 2 Coordinates of actual points measured on profile

Measured points	A	B′	B″	C′	C″	D
X [mm]	44.000	44.269	44.315	45.208	45.320	46.853
Z [mm]	0.000	−2.070	−2.260	−4.229	−4.411	−6.124
λ average	λ=0	λ=0.345		λ=0.649		λ=1

The profiling error by the proposed method is relative small (1×10^{-2}mm, see Fig. 8); the tool profiled in this way may be used for the milling of this worm type.

The profiling error decreases with the increase of the substitution polynomials degree for the assembly of points measured on profile (4th degree or superior polynomials).

3.2 End mill tool for generating a teethed wheel with involute frontal section

For involute generatrix of circle with R_b radius, Fig. 9 is proposed as an approximation with four points along profile. We make the remark that points may be the result of a profile measuring on a 2D (3D) measuring machine; see Fig. 10.

Proposed for involute approximation are 3rd-degree polynomials (25), knowing the coordinates of points belong to the involute; see Table 4:

$$\begin{cases} X = A_X\lambda^3 + 3B_X\lambda^2(1-\lambda) + 3C_X\lambda(1-\lambda)^2 + D_X(1-\lambda)^3; \\ Y = A_Y\lambda^3 + 3B_Y\lambda^2(1-\lambda) + 3C_Y\lambda(1-\lambda)^2 + D_Y(1-\lambda)^3. \end{cases} \tag{25}$$

The angular values Θ_A and Θ_D, are defined for the flank of teethed wheel with m modulus and z teeth number (Fig. 11).

In Table 4, the identification algorithm for the 3rd degree polynomial is presented, substitutive for the involute generatrix, for a conventional form [1].

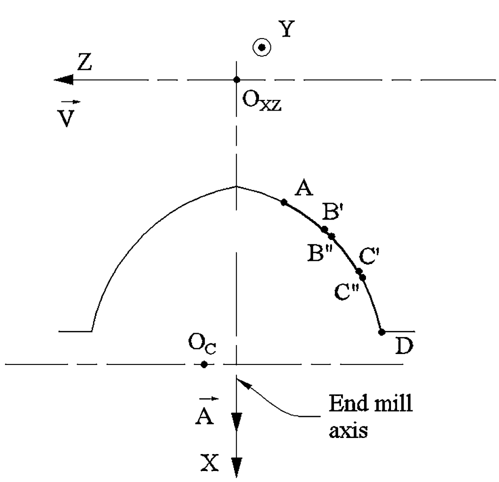

Fig. 6 Points measured on profile

In Table 5, the coordinates measured on the wheel to generate profile and the value of λ parameter are presented.

In Fig. 12 and Table 6 are presented the form and coordinates of axial section of tool and the errors of approximated profile regarding the theoretical profile for generating an involute gear with: m_t=4 mm; z=40 tooth; p=200 mm.

The proposed method is proof to be satisfactory from the point of view of tool geometry profiling precision. The point number is relatively small (four points known along the profile to be generated, in a frontal plane of this profile).

4 Planing tool's profiling (cylindrical tool)

The planing tool (the cylindrical tool) which generates in cutting movement a cylindrical surface reciprocally enveloping with a helical surface known by sampled points, may be profiled based on an algorithm similarly with those previously presented.

For an in-plane generatrix, defined in discrete form and approximated by a Bezier polynomial with small degree, as result of knowing a small-point number on the axial generatrix of the helical surface (three or four points), is accepted as expression form of helical surface presented in discrete form; see Eq. 7.

The planing tool's profiling problem for generating by enwrapping a cylindrical helical surface with constant pitch may be solved by more methods: the method of helical movement decomposition [3] or complementary methods [6, 7] presented in following.

- The method of helical movement decomposition

The kinematics of generation process includes the movement assembly:

- helical movement of blank (the movement with $\vec{V}$ axis and p helical parameter) around Z axis ($\vec{V}$, p);
- straight-line movement of the planning tool which described the cylindrical surface reciprocally enveloping with the helical surface to be generated (n=double stroke/min); see Figs. 13 and 14).

The characteristic curve, C_{Π}, on the Π helical surface expressed in discrete form, representing the tangent curve at the cylindrical surface (in Fig. 13, surface with generatrix

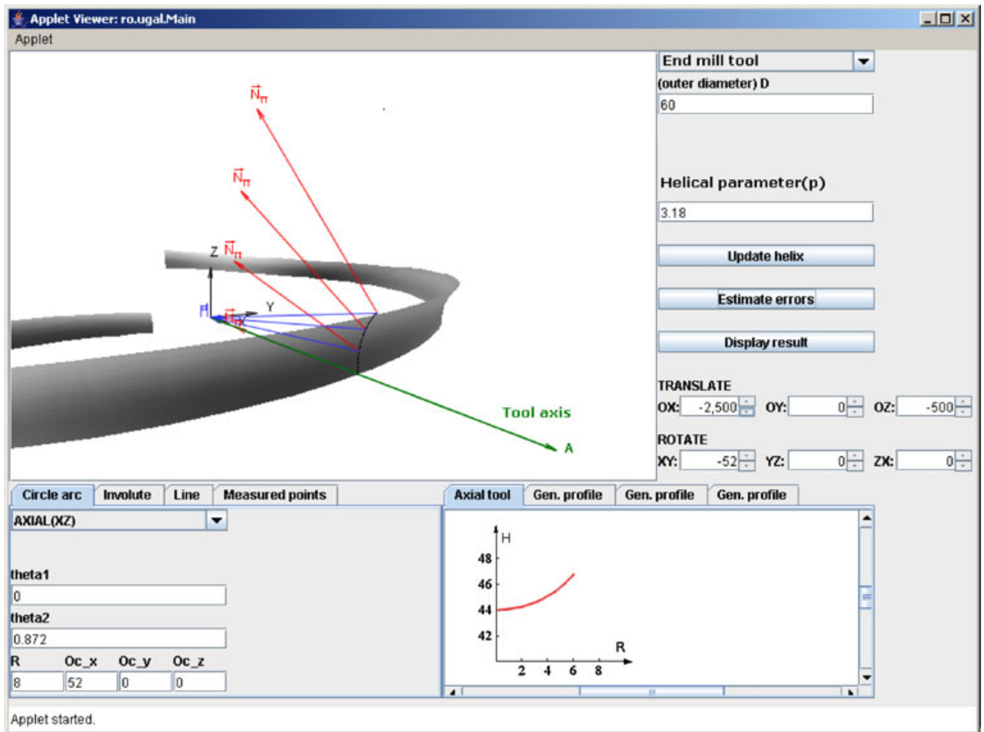

Fig. 7 Applet for end mill tool's profiling

perpendicularly on P_T plane and with C_Π as directrix) is defined based on a specifically enwrapping condition [2, 3]:

$$\vec{N}_\Pi \cdot \vec{t} = 0. \tag{26}$$

The P_T plane, crossing plane of the cylindrical surface, contains the X axis and admit as normal $\vec{t}$—the unitary vector of tangent at the helix with external diameter of helical surface Π.

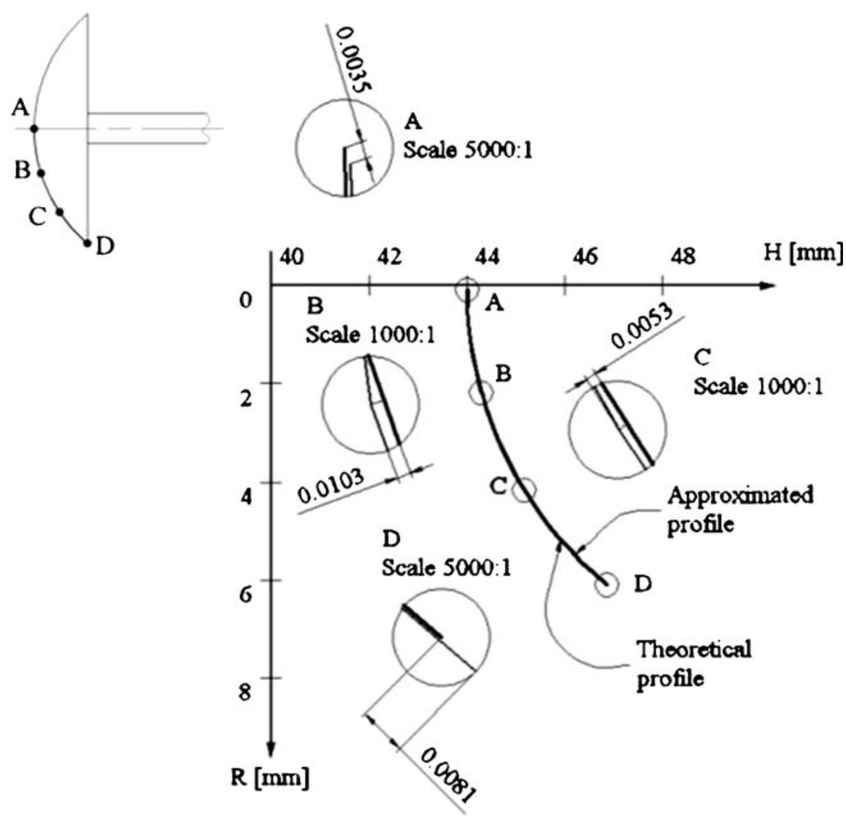

Fig. 8 The points on axial section of the mill end tool

Table 3 Axial section of tool profile and errors level

λ	Approximate tool profile		Theoretical tool profile		Error [mm]
	H [mm]	R [mm]	H [mm]	R [mm]	
0.0000	44.0004	0.0800	43.9992	0.0767	0.0035
	44.0113	0.4230	44.0059	0.4248	0.0057
	44.0371	0.7651	44.0287	0.7647	0.0085
	44.0778	1.1059	44.0681	1.1098	0.0105
	44.1332	1.4445	44.1222	1.4457	0.0110
	44.2032	1.7804	44.1930	1.7855	0.0114
	44.2878	2.1130	44.2772	2.1150	0.0107
0.345	44.29995	2.19472	44.33194	2.26621	0.0103
	44.3866	2.4416	44.3780	2.4472	0.0103
	44.4997	2.7656	44.4910	2.7681	0.0090
	44.6266	3.0844	44.6204	3.0904	0.0086
	44.7672	3.3974	44.7609	3.4005	0.0071
	44.9213	3.7040	44.9144	3.7044	0.0069
	45.0884	4.0037	45.0842	4.0076	0.0057
0.649	45.17382	4.15619	45.21797	4.21702	0.0053
	45.3285	4.3878	45.3242	4.3903	0.0050
	45.4606	4.5801	45.4582	4.5849	0.0053
	45.6650	4.8557	45.6613	4.8581	0.0044
	45.8811	5.1223	45.8759	5.1224	0.0052
	46.1084	5.3794	46.1063	5.3823	0.0036
	46.3466	5.6264	46.3430	5.6268	0.0036
	46.5951	5.8630	46.5954	5.8653	0.0023
1.0000	46.8536	6.0886	46.8476	6.0832	0.0081

In form (26), the Nikolaev condition specifically for this enwrapping problem type was marked:

$\vec{N}_\Pi$ normal at the helical surface expressed in discrete form

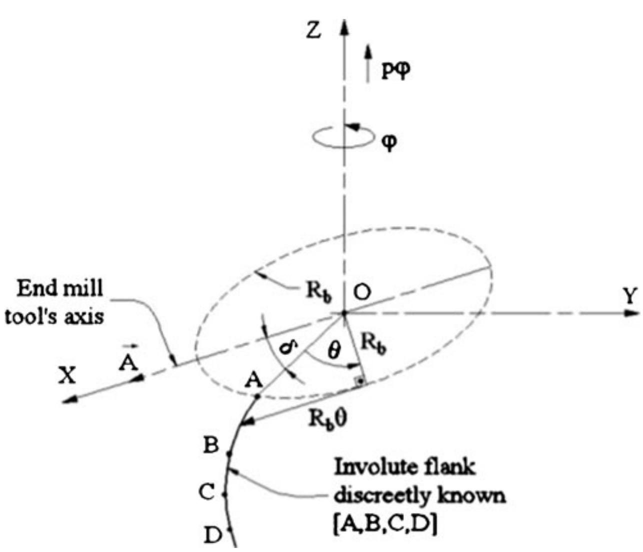

Fig. 9 Involute generatrix

$\vec{t}$ the unitary vector of the cylindrical surface

$$\vec{t} = \cos\alpha \cdot \vec{j} + \sin\alpha \cdot \vec{k}, \tag{27}$$

$$\alpha = \arctan\left(\frac{p}{D_E}\right) \tag{28}$$

where:

p is the helical parameter of surface (known);

D_E external diameter of the helical surface; see Fig. 16.

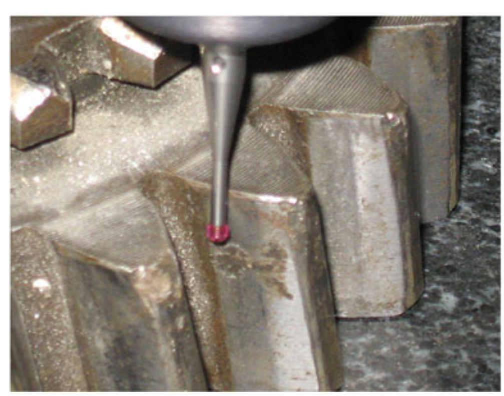

Fig. 10 Measuring on 3D measuring machine

Table 4 Coefficients of 3rd Bezier polynomials for involute arc profile of teethed wheel

Θ	Primary profile	λ	Polynomial coefficients
Θ_A	$X_A = R_b\cos(\delta+\Theta_A) + R_b\Theta_A\sin(\delta+\Theta_A)$ $Y_A = R_b\sin(\delta+\Theta_A) - R_b\Theta_A\cos(\delta+\Theta_A)$ $\Theta_A = \sqrt{\frac{R_i^2-R_b^2}{R_b^2}},\ R_i = \frac{m_t\cdot z}{2} - 1\cdot m_t,$ $R_b = \frac{m_t\cdot z}{2}\cdot\cos(\alpha_t),\ \alpha_t = 20^0,$ $\delta = \frac{1}{2}\left[\frac{\pi}{2} - z(\tan\alpha_t - \alpha_t)\right]\cdot m_t\cdot\cos\alpha_t$	0	$D_X = Y_A$ $D_Y = Z_A$
Θ_B	$\Theta_B = \Theta_A + \frac{1}{3}\cdot(\Theta_D - \Theta_A)$ $X_B = R_b\cos(\delta+\Theta_B) + R_b\Theta_B\sin(\delta+\Theta_B)$ $Y_B = R_b\sin(\delta+\Theta_B) - R_b\Theta_B\cos(\delta+\Theta_B)$	$\frac{1}{3}$	$C_X = \frac{18\cdot X_C - 9\cdot X_B + 2\cdot X_A - 5\cdot X_D}{6}$ $C_Y = \frac{18\cdot Y_C - 9\cdot Y_B + 2\cdot Y_A - 5\cdot Y_D}{6}$ $B_X = \frac{-5\cdot X_A + 2\cdot X_D + 18\cdot X_B - 9\cdot X_C}{6}$ $B_Y = \frac{-5\cdot Y_A + 2\cdot Y_D + 18\cdot Y_B - 9\cdot Y_C}{6}$
Θ_C	$X_C = R_b\cos(\delta+\Theta_C) + R_b\Theta_C\sin(\delta+\Theta_C)$ $Y_C = R_b\sin(\delta+\Theta_C) - R_b\Theta_C\cos(\delta+\Theta_C)$ $\Theta_C = \Theta_A + \frac{2}{3}(\Theta_D - \Theta_A)$	$\frac{2}{3}$	
Θ_D	$X_D = R_b\cos(\delta+\Theta_D) + R_b\Theta_D\sin(\delta+\Theta_D)$ $Y_D = R_b\sin(\delta+\Theta_D) - R_b\Theta_D\cos(\delta+\Theta_D)$ $\Theta_D = \sqrt{\frac{R_e^2-R_b^2}{R_b^2}},\ R_e = \frac{m_t\cdot z}{2} + 1\cdot m_t$	1	$A_X = X_D$ $A_Y = Y_D$

In this way, the (2) and (26) equations assembly represents the characteristic curve of the two surfaces: the helical surface, expressed in discrete form, and the cylindrical surface, as peripheral primary surface of the planing tool for helical surface generation.

In principle, the C_Π characteristic curve is expressed by its point coordinates:

$$C_\Pi = \left\| \begin{matrix} X_{C_\Pi,\lambda=0} & Y_{C_\Pi,\lambda=0} & Z_{C_\Pi,\lambda=0} \\ X_{C_\Pi,\lambda=\frac{1}{2}} & Y_{C_\Pi,\lambda=\frac{1}{2}} & Z_{C_\Pi,\lambda=\frac{1}{2}} \\ X_{C_\Pi,\lambda=1} & Y_{C_\Pi,\lambda=1} & Z_{C_\Pi,\lambda=1} \end{matrix} \right\|, \tag{29}$$

for the helical surface approximation with a 2nd-degree polynomial.

Being known as the cylindrical surface generatrix direction, $\overrightarrow{t}$ and the characteristic curve form, C_Π, expressed in discrete form, is defined the S surface for points where is defined, in form:

$$\overrightarrow{R} = \overrightarrow{r} + k\cdot\overrightarrow{t}, \tag{30}$$

where:

$$\overrightarrow{r} = X^{C_\Pi}\cdot\overrightarrow{i} + Y^{C_\Pi}\cdot\overrightarrow{j} + Z^{C_\Pi}\cdot\overrightarrow{k} \tag{31}$$

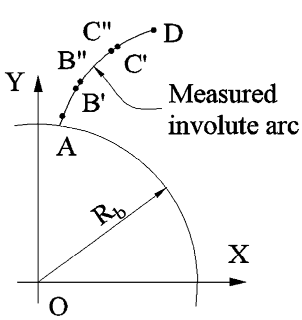

Fig. 11 Points measured on profile

vector of discrete points on the C_Π characteristic curve, k variable parameter.

The results, in principle, are the S surface coordinates,

$$S : \left| \begin{array}{l} X^S = X^{C_\Pi}; \\ Y^S = Y^{C_\Pi} + k\cos\alpha; \\ Z^S = Z^{C_\Pi} + k\sin\alpha, \end{array} \right. \tag{32}$$

which, by coordinate transforming:

$$X_1 = \omega_1(\alpha)\cdot X, \tag{33}$$

leads to:

$$S_{X_1Y_1Z_1} \left| \begin{array}{l} X_1 = X^S; \\ Y_1 = Y^S\cos\alpha + Z^S\sin\alpha; \\ Z_1 = -Y^S\sin\alpha + Z^S\cos\alpha, \end{array} \right. \tag{34}$$

representing the discrete cylindrical surface S in $X_1Y_1Z_1$ reference system; see Fig. 13.

The $X_1Y_1Z_1$ reference system is the reference system where the crossing plane of the cylindrical surface, P_T, is overlapped with X_1Z_1 plane.

The crossing section of discrete cylindrical surface (34) is obtained from condition

$$|Y_1| = q_1, \tag{35}$$

with q_1 arbitrary, positive, small $1\times10^{-2}...1\times10^{-3}$, in form:

$$S_{P_T} \left| \begin{array}{l} X_1 = X^S; \\ Z_1 = -Y^S\sin\alpha + Z^S\cos\alpha, \end{array} \right. \tag{36}$$

for k variable, see Fig. 14.

- The "minimum distance" method [6]

Table 5 Coordinates of actual points measured on involute's profile

Measured points	A	B′	B″	C′	C″	D
X [mm]	75.90574	78.3812	78.54254	80.98731	81.14147	83.6982
Y [mm]	3.783907	4.501682	4.56124	5.621388	5.697651	7.114187
λ average	λ=0	λ=0.34		λ=0.6501		λ=1

The complementary theorem specifically for "minimum distance" method is stated: "the characteristic curve of a helical cylindrical surface, with constant pitch, reciprocally enveloping with a cylindrical surface is the geometrical place of points belonging to helical surface for which the distance between the helical surface axis and the cylindrical surface's generatrix, in the contact plane, is minimum."

The contact plane is defined as a plane which contains the cylindrical surface's generatrix and is perpendicular to the plane determined by the unitary vectors of the helical surface's axis $\vec{V}$ and cylindrical surface's generatrix $\vec{t}$; see Fig. 15.

The Σ surface, (18), in the reference system $X_1Y_1Z_1$, is:

$$\begin{vmatrix} X_1 = X(\lambda, \varphi); \\ Y_1 = Y(\lambda, \varphi)\cos\alpha + Z(\lambda, \varphi)\sin\alpha; \\ Z_1 = -Y(\lambda, \varphi)\sin\alpha + Z(\lambda, \varphi)\cos\alpha, \end{vmatrix} \qquad (37)$$

α is the helix angle on the external diameter of the helical surface; see Eq. 28.

In-plane Z_1=H, H arbitrary parameter, the intersection curve of the helical surface with this plane is

$$\Sigma_{1H} \begin{vmatrix} X_1 = X_1(\lambda, H); \\ Y_1 = Y_1(\lambda, H)\cos\alpha + Z_1(\lambda, H)\sin\alpha. \end{vmatrix} \qquad (38)$$

According to the presented theorem, the minimum distance between the helical surface's axis and the cylindrical surface's generatrix in the contact plane is obtained for

$$X'_{1\lambda} = 0, \qquad (39)$$

equivalent to Eq. 26.

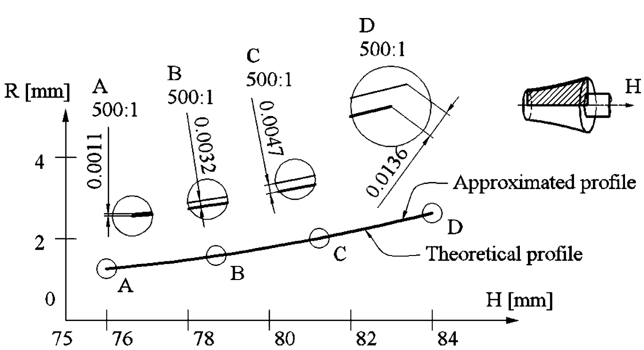

Fig. 12 Axial section of end mill tool

5 Application for planing tool for generation of helical surface with circular profile in axial plane

The helical surface is expressed in discrete form by an in-plane generatrix known by its points (Fig. 16).

Are knows the coordinates of points on the helical surfaces generatrix.

Is identified the 3rd degree Bezier polynomial for G generatrix approximation.

On the helical movement, $\vec{V}, p$ (see Fig. 16),

$$\begin{Vmatrix} X \\ Y \\ Z \end{Vmatrix} = \begin{Vmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} P_X(\lambda) \\ 0 \\ P_Z(\lambda) \end{Vmatrix} + \begin{Vmatrix} 0 \\ 0 \\ p\varphi \end{Vmatrix}, \qquad (40)$$

Table 6 Axial section of tool and error level

λ	Approximated tool profile		Theoretical tool profile		Error [mm]
	H [mm]	R [mm]	H [mm]	R [mm]	
0.0000	76.0000	−1.2667	76.0000	−1.2656	0.0011
	76.4046	−1.3030	76.4046	−1.3025	0.0005
	76.8088	−1.3436	76.8087	−1.3430	0.0005
	77.2125	−1.3881	77.2125	−1.3872	0.0010
	77.6159	−1.4364	77.6158	−1.4349	0.0016
	78.0188	−1.4882	78.0187	−1.4860	0.0022
	78.4212	−1.5434	78.4211	−1.5406	0.0028
0.3400	78.6866	−1.5816	78.6865	−1.5785	0.0032
	78.8232	−1.6019	78.8231	−1.5985	0.0034
	79.2247	−1.6635	79.2245	−1.6597	0.0038
	79.6257	−1.7282	79.6255	−1.7241	0.0041
	80.0263	−1.7960	80.0260	−1.7917	0.0043
	80.4263	−1.8668	80.4259	−1.8623	0.0045
	80.8257	−1.9405	80.8254	−1.9360	0.0046
0.6501	81.2246	−2.0172	81.2242	−2.0126	0.0047
	81.3522	−2.0424	81.3518	−2.0377	0.0047
	81.6230	−2.0968	81.6225	−2.0920	0.0048
	82.0207	−2.1793	82.0203	−2.1743	0.0051
	82.4179	−2.2647	82.4175	−2.2593	0.0054
	82.8144	−2.3529	82.8140	−2.3470	0.0059
	83.2103	−2.4440	83.2100	−2.4373	0.0067
	83.6055	−2.5379	83.6053	−2.5302	0.0077
1.0000	84.0000	−2.6346	83.9921	−2.6236	0.0136

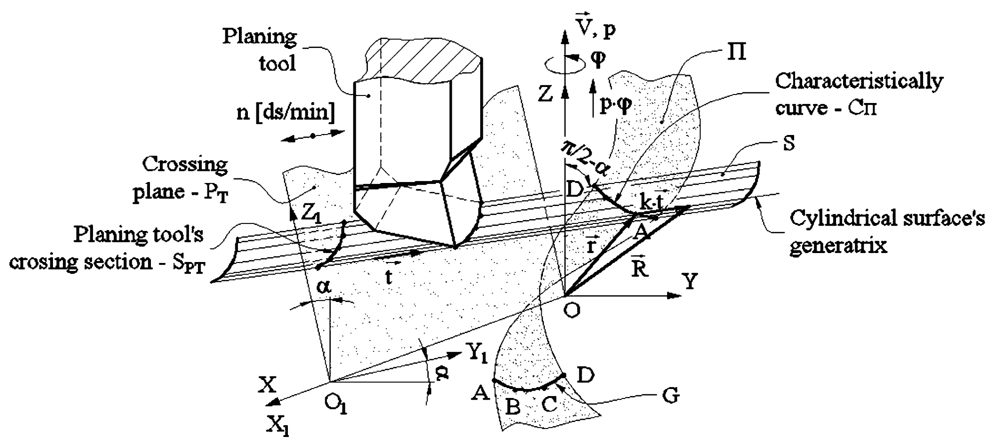

Fig. 13 Cylindrical surface and characteristic curve

for $P_Y(\lambda)=0$, on obtain the helical surface:

$$\Pi \left| \begin{array}{l} X(\lambda,\varphi) = P_X(\lambda)\cos\varphi; \\ Y(\lambda,\varphi) = P_X(\lambda)\sin\varphi; \\ Z(\lambda,\varphi) = P_Z(\lambda) + p\varphi, \end{array} \right. \tag{41}$$

with: $P_X(\lambda)$, $P_Z(\lambda)$ substituting polynomials for discrete generatrix;

φ variable angular parameter. The φ and λ values are determined, which satisfied the following condition

$$|N_Y\cos\alpha + N_Z\sin\alpha| \le q, \tag{42}$$

q positive and small-enough ($q=1\times10^{-3}$), with N_Y, N_Z; see (9) and (10).

In this way, the characteristic curve is determined on the S surface, helical surface discreetly expressed, and, from here, by (34), coordinates transforming, the crossing section form of the cylindrical surface S_{PT}; see (36) form.

In Fig. 17 and Table 7, the form and coordinates of the crossing section of cylindrical surface reciprocally enveloping with a worm with dimensional characteristics are presented:

- coordinates of points belong to axial section as presented in Table 2, for O_C=[52;0;0], R=8 mm, D_E=60 mm;
- helical parameter, p=3.18 mm.

We elaborate in Java programming language an applet (see Fig. 18) which allows determining of:

- the crossing section of cylindrical surface, X,Z;
- the planing tool's profile coordinates $[X_1,Z_1]$;
- the profiling error level against a fundamental analytical method (Nikolaev).

The numerical results are presented against the results obtained using a fundamental method for planing tool's profile calculus.

The level of maximum error is 0.01 mm, exactly enough for tool profiling.

It is obvious that the tool's profile precision is exactly enough, and, for certain surface types, the Bezier polynomials representation method for in-plane generatrix of surfaces may be an alternative to the analytical method for helical surface generating tool's profiling.

The proposed method is characterized by the fact that the point number is relatively small, three or four points, and the determination precision increasing may be obtained using substituting polynomials with superior degree.

The method has the advantage that allows the approach to the helical surface generation tool's profiling, starting from what is known of some points measured on these surface.

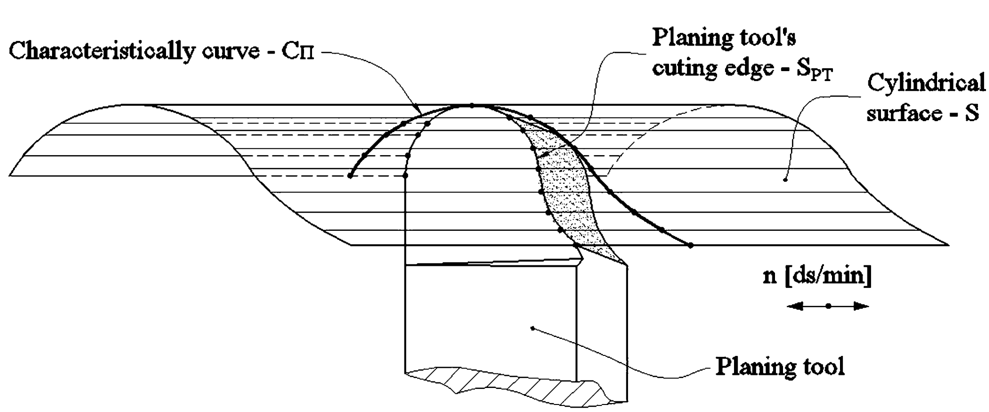

Fig. 14 Characteristic curve: S cylindrical surface, S_{PT}, tool's profile, C_Π characteristic curve at contact between surfaces S and Π

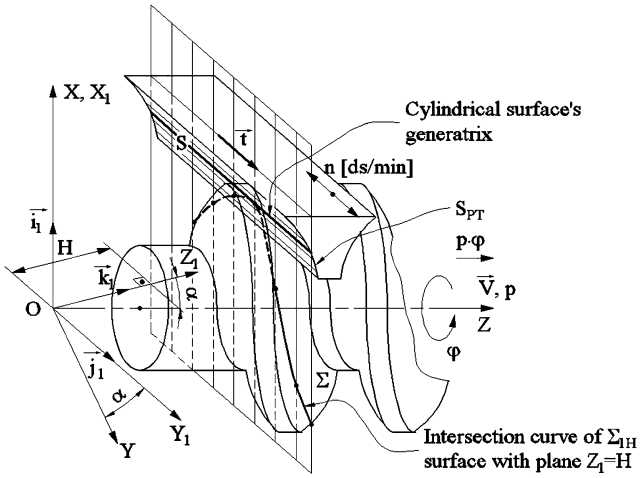

Fig. 15 Cylindrical surface enveloping of a helical surface

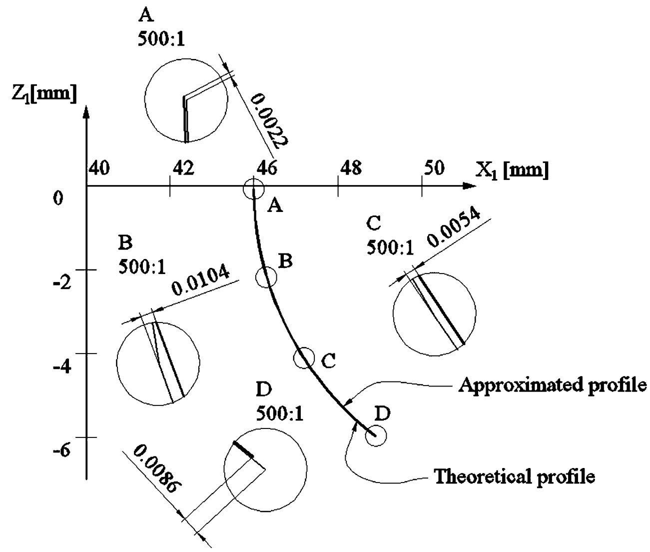

Fig. 17 Cylindrical tool's profile

6 Software considerations

Numerical values presented in this paper were obtained using a specially designed Java application; see Figs. 7 and 8. The application allows the end-user to manually enter or import from CSV files, an arbitrary set of measured coordinates of the piece surface—the helical generatrix.

These measured coordinates represent a topological model of the real surface. By this, the link is made between the surface's topological geometry and the tool's profiling practice.

The software automatically computes Bezier substitution polynomials of the piece profile, applies the enwrapping conditions, and determinates the tool axial section for end mill toll and, respectively, the crossing section for planing tool. For comparative results and error estimation, the end-user can also define a series of analytical profiles (circle arcs, straight lines, and involute curves). The user can select the tool type (end mill or planing tool), and the helical parameter of the surface can be modified. The error level of the proposed method is displayed in tabular form.

The software also displays bi-dimensional projections of helical curve generatrix and axial tool profile, and a three-dimensional representation of the approximated helical surface, characteristically curves, and normal vectors (as considered in the enwrapping condition). Numerical results—approximated helical generatrix, axial tool profile—can be exported to CSV files, in order to be used as topological model of tool.

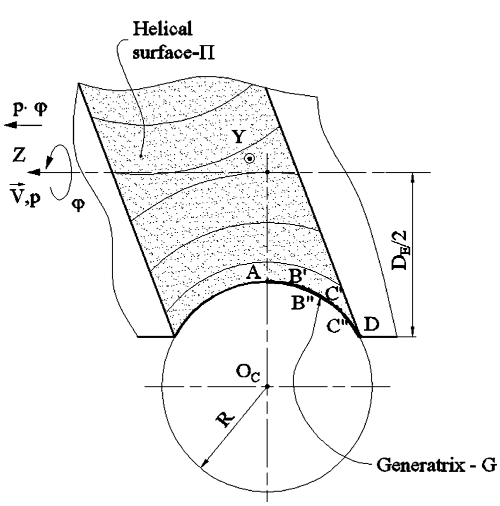

Fig. 16 Helical surface with circular generatrix in axial plane

7 Conclusions

The helical surfaces generating tool profiling method (end mill tool and cylindrical tool) is characterized by:

– the method is based on the reciprocally enveloping surface theory;
– the algorithm is applicable for helical surfaces known even for a small-point number (three or four points) which defined straight-line segments or curve arcs;
– the Bezier polynomial coefficients for discreetly known generatrix are presented in tables;
– the method may be used also in case of points known by measuring;
– the tool's profiling precision, by the proposed method, is equivalent with results obtained using an analytical method, the increasing of approximation polynomial degree increase the profile determination precision;
– the method is fast and easy to apply.

Table 7 Crossing section of tool profile and errors level

λ	Approx tool profile		Theoretical tool profile		Error [mm]
	X_1 [mm]	Z_1 [mm]	X_1 [mm]	Z_1 [mm]	
0.0000	44.0004	−0.0787	43.9993	−0.0768	0.0022
	44.0114	−0.4181	44.0063	−0.4208	0.0058
	44.0375	−0.7566	44.0294	−0.7568	0.0081
	44.0786	−1.0936	44.0693	−1.0980	0.0103
	44.1346	−1.4285	44.1241	−1.4301	0.0107
	44.2055	−1.7606	44.1956	−1.7660	0.0113
	44.2910	−2.0891	44.2808	−2.0916	0.0106
0.345	44.3038	−2.1706	44.3357	−2.2404	0.0104
	44.3911	−2.4136	44.3827	−2.4197	0.0104
	44.5056	−2.7332	44.4970	−2.7363	0.0091
	44.6342	−3.0475	44.6252	−3.0477	0.0089
	44.7767	−3.3557	44.7702	−3.3592	0.0074
	44.9328	−3.6572	44.9258	−3.6579	0.0071
	45.1024	−3.9513	45.0980	−3.9553	0.0058
0.649	45.1891	−4.1009	45.2339	−4.1603	0.0054
	45.3461	−4.3274	45.3418	−4.3295	0.0049
	45.4804	−4.5153	45.4779	−4.5193	0.0047
	45.6881	−4.7838	45.6845	−4.7851	0.0039
	45.9080	−5.0426	45.9074	−5.0464	0.0039
	46.1394	−5.2910	46.1376	−5.2921	0.0022
	46.3822	−5.5283	46.3789	−5.5268	0.0036
	46.6358	−5.7541	46.6365	−5.7545	0.0008
1.0000	46.8998	−5.9676	46.8942	−5.9611	0.0086

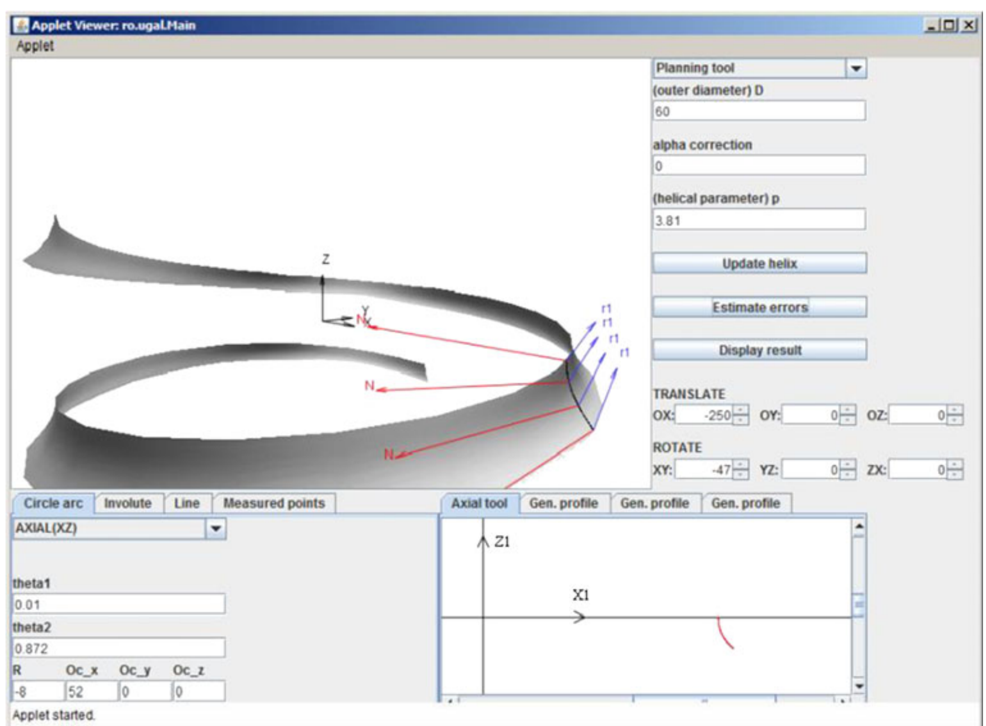

Fig. 18 Applet for cylindrical tool's profiling

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