Int J Adv Manuf Technol (2010) 50:793–800
DOI 10.1007/s00170-010-2537-2

ORIGINAL ARTICLE

Extension of the ELECTRE method for decision-making problems with interval weights and data

Behnam Vahdani · Amir Haji Karim Jabbari · Vahid Roshanaei · Mostafa Zandieh

Received: 25 June 2009 / Accepted: 18 January 2010 / Published online: 20 February 2010

Abstract Recent research is recognizing that multicriteria decision making should take into account the concepts of uncertainty, risk, and confidence. In some cases, precise determination of the exact value of the attributes is difficult. Consequently, to surmount this potential problem, their values are considered as fuzzy and intervals. The elimination and choice translating reality (ELECTRE) is one of the most widely used methods to rank a set of alternatives versus a set of criteria to reflect the decision maker's preference. What is customary in almost all recent papers regarding ELECTRE is the utilization of fuzzy concepts in decision-making process. In this paper, we propose a novel algorithmic ELECTRE method which assimilates the concepts of interval weights and data for the facilitation of evaluation of a set of alternatives against a set of criteria in this situation. Finally, the executive procedure of our proposed ELECTRE method is illustrated by applying it to the problem of supplier selection.

B. Vahdani (✉)
Department of Industrial and Mechanical Engineering,
Islamic Azad University Qazvin branch,
Qazvin, Iran
e-mail: b.vahdani@gmail.com

A. H. K. Jabbari
Master of Business Administration Department,
Mahan Higher Education Institute,
Tehran, Iran

V. Roshanaei
Department of Industrial Engineering,
Amirkabir University of Technology,
424 Hafez,
Tehran, Iran

M. Zandieh
Faculty of Industrial Engineering,
K.N. Toosi University of Technology,
Tehran, Iran

Keywords Multiple criteria decision making · ELECTRE · Interval data · Interval weights · Supplier selection

1 Introduction

Multicriteria decision making is a well-known branch of decision making. It is a branch of a general class of operations research models which deal with decision problems under the presence of a number of decision criteria. This major class of models is very often called multicriteria decision making (MCDM). This class is further divided into multiobjective decision making and multiattribute decision making (MADM) [2]. There are several methods in each of the above categories. Triantaphyllou [15] extensively compares, both theoretically and empirically, real-life MCDM issues and makes the reader aware of quite a number of surprising "abnormalities" with some of these methods. Priority based, outranking, distance-based, and mixed methods are also applied to various problems. Each method has its own characteristics, and the methods can also be classified as deterministic, stochastic, and fuzzy methods. There may be combinations of the above methods. Depending upon the number of decision makers, the methods can be classified as single or group decision-making methods. Decision making under uncertainty and decision support systems are also prominent decision-making techniques [4]. Entania and Tanaka [5] use the interval approach for obtaining interval evaluations which are suitable for handling uncertain data. Since the given comparisons are ratio measures and too large intervals are not useful information the intervals should be normalized, and their redundancy should be reduced. Furthermore, they introduce interval probability which fills the role of interval normalization instead of crisp normalization in the estimations at each hierarchy. Then, as a final decision, the interval global weights reflecting the decision maker's uncertain judgments as their widths without redundancy are obtained.

A series of MCDM models uses what is known as "outranking relations" to rank a set of alternatives. The elimination and choice translating reality (ELECTRE) method and its derivatives play a prominent role in this group. The ELECTRE approach was first introduced in [1]. It is a comprehensive evaluation approach in which it also tries to rank alternatives, each one of which is described in terms of a number of criteria. The main idea is the proper utilization of what is called "outranking relations." Soon after the introduction of the first version known as ELECTRE I [6], this approach was evolved into a number of other variants. Today, the most widely used versions are known as ELECTRE II [8, 11, 13] and ELECTRE III [12, 14]. Mustajoki et al. [9] utilized intervals in the simple multiattribute rating technique and SWING weighted methods. Halouani et al. [7] proposed two new multicriteria 2-tuple group decision methods called "Preference Ranking Organisation Method for Enrichment Evaluation Multi Decision maker 2-Tuple-I and II" (PROMETHEE-MD-2 T-I and II). They integrated their procedure with both quantitative and qualitative information in an uncertain context. Opricovic and Tzeng [10] extended VIKOR method for solving MCDM problems with conflicting and noncommensurable criteria; assuming that compromising is acceptable for conflict resolution, the decision maker wants a solution that is the closest to the ideal, and the alternatives are evaluated according to all established criteria. Moreover, this proposed method is compared with three multi-criteria decision-making methods: TOPSIS, PROMETHEE, and ELECTRE. Zavadskas et al. [16] considered the application of grey relations methodology to defining the utility of alternatives and offered a multiple criteria method of complex proportional assessment of alternatives with grey relations for analysis. In this model, the parameters of the alternatives are determined by the grey relational grade and expressed in terms of intervals. Dembczyński et al. [3] proposed Dominance-based Rough Set Approach to deal with multiple criteria classification (also called multiple criteria sorting or ordinal classification with monotonicity constraints), where assignments of objects may be inconsistent with respect to dominance principle. The presented methodology preserves well-known properties of rough approximations, such as rough inclusion, complementarity, identity of boundaries, and precisiation. Moreover, the meaning of the precisiation property is extended to the considered case. Furthermore, they presented a way to reduce decision tables and to induce decision rules from rough approximations.

Under many conditions, exact data are inadequate to model real-life situations. Therefore, these data may have some structures such as bounded data, ordinal data, interval data, and fuzzy data. In this paper, owing to the fact that, in some cases, precise determination of the exact value of the attributes is difficult, and consequently, their values are considered as intervals. We extend the concept of ELECTRE to develop a methodology for solving MCDM problems with interval weights and data.

The rest of the paper is organized as follows: In Section 2, we shortly review the basic calculation (algebraic operations) of intervals. Then, we briefly introduce the original ELECTRE method in Section 3. In Section 4, we first introduce MCDM problems with interval weights and data. Then, we present an algorithm to extend ELECTRE method which deals with interval weights and data. In Section 5, we illustrate our proposed algorithmic method by applying it to an example. Section 6 consists of conclusions and future work.

2 Interval arithmetic

Prior to the describing ELECTRE method with interval weights and data, we first have a short review on interval definition, and then we explain how functions and algebraic operations work on intervals.

Definition The interval is a bounded subset of real numbers. Formally:

$$(X = [a, b] \quad \text{is an interval}) \Leftrightarrow (X = \{x \in R | a \leq x \leq b\}), \quad (1)$$

where $a, b \in R$ (set of all real numbers); in particular a, b, or even both of them may be infinite.

Geometrically, interval is just a section of a real line, uniquely determined by its own endpoints. The set of all intervals is commonly denoted as IR. Lower and upper endpoint of interval X is usually referred to as $\underline{X}$ and $\overline{X}$, respectively. Intervals with property $\underline{X} = \overline{X}$ are called *thin* (or *degenerate*). Any of them contains exactly one real number and can thus be formally identified with this very number. Three basic real-valued functions defined on intervals (i.e., of type $IR \mapsto R$) are

$$1.\ Width : w(X) = |X| \stackrel{\text{def}}{=} \left|\overline{X} - \underline{X}\right| \quad (2)$$

$$2.\ Center : \text{mid}(X) \stackrel{\text{def}}{=} \frac{1}{2} \left|\underline{X} + |\overline{X}\right| \quad (3)$$

$$3.\ \text{Absolute } value : |X| = \max\left\{|\underline{X}|, \left|\overline{X}\right|\right\} \quad (4)$$

Algebraic operations on intervals are defined in such a way that their results always contain every possible outcome of the corresponding algebraic operation on real numbers. More specifically, the result of $X \Diamond Y$ is again an interval, Z, with property

$$X \Diamond Y = Z = \{z = x \Diamond y | x \in X, y \in Y\}, \quad (5)$$

where $\Diamond$ belongs to the set $\{+, -, \times, \div\}$. One can easily prove that arithmetic operations on intervals can be expressed in terms of ordinary arithmetics on their endpoints:

$$X + Y = [\underline{X} + \underline{Y}, \overline{X} + \overline{Y}] \qquad X - Y = [\underline{X} - \overline{Y}, \overline{X} - \underline{Y}] \quad (6)$$

$$X \times Y = \begin{bmatrix} \min(\underline{X} \times \underline{Y}, \underline{X} \times \overline{Y}, \overline{X} \times \underline{Y}, \overline{X} \times \overline{Y}), \\ \max(\underline{X} \times \underline{Y}, \underline{X} \times \overline{Y}, \overline{X} \times \underline{Y}, \overline{X} \times \overline{Y}) \end{bmatrix} \quad (7)$$

$$X \div Y = \begin{bmatrix} \min(\underline{X} \div \underline{Y}, \underline{X} \div \overline{Y}, \overline{X} \div \underline{Y}, \overline{X} \div \overline{Y}), \\ \max(\underline{X} \div \underline{Y}, \underline{X} \div \overline{Y}, \overline{X} \div \underline{Y}, \overline{X} \div \overline{Y}) \end{bmatrix} \quad (8)$$

with an extra condition for division: $0 \notin Y$.

One of the ideas behind interval analysis is that it is a kind of language which designates real computations, in the sense that an interval $[a, b]$ represents any real number $r \in [a, b]$. The quality of this representation is based upon the interval *width*—i.e., $b - a$. Good interval functions $F{:}IR \rightarrow IR$ used to designate real functions $f{:}R \rightarrow R$ are those which preserve the order of set inclusion and consequently the error quality. Those functions $F{:}IR \rightarrow IR$ have the following property:

$$x \in [a, b] \Rightarrow f(x) \in F([a, b]) \quad (9)$$

This property captures the main requirement of interval analysis, the correctness of interval methods, namely, it is enough to compute with intervals to obtain the resulting real number $f(x) \in F([a, b])$. This required property can be observed in [5].

In this paper among available functions, "Minimum" and "Maximum" functions are used, and we study them from now on. Referring to above definitions, minimum and maximum of real number intervals are calculated as follows:

$$A = [\underline{a}, \overline{a}] \quad , \quad B = [\underline{b}, \overline{b}]$$

$$\min(A, B) = [\min(\underline{a}, \underline{b}), \min(\overline{a}, \overline{b})]$$

$$\max(A, B) = [\max(\underline{a}, \underline{b}), \max(\overline{a}, \overline{b})]$$

then

$$\begin{aligned} &A < B \Leftrightarrow \max(A, B) = B \quad \text{OR} \\ &A < B \Leftrightarrow \min(A, B) = A \end{aligned} \quad (10)$$

Also two real intervals are sorted in the following manner:

$$[\underline{a}, \overline{a}] \leq [\underline{b}, \overline{b}] \Leftrightarrow \underline{a} \leq \underline{b}, \overline{a} \leq \overline{b} \quad (11)$$

Hereinafter, we use the definitions presented in this section for all algebraic operations which are used in this paper.

3 The ELECTRE

ELECTRE is a multicriteria decision-making procedure that can be used when a set of alternatives must be ranked according to a set of criteria reflecting the decision maker's preferences. Relationships between alternatives and criteria are described using attributes referred to the aspects of alternatives that are relevant according to the established criteria. In multicriteria decision problems, although logical and mathematical conditions required to determine an optimum do not exist, a solution representing a good compromise according to the conflicting criteria established can be individuated. ELECTRE method is based upon the pseudocriteria. A pseudocriterion allows, by using proper thresholds, to take into account the uncertainty and ambiguity that can affect the evaluation of the performance, so that, if the difference in the performance of two alternatives is minimal, according to a certain criterion, such alternatives can be considered indifferent according to that criterion. Another peculiarity which differentiates ELECTRE from other methodologies is that it is not compensative, which means that a very bad score in one objective function is not compensated by good scores in other objectives. In other words, the decision maker will not choose an alternative if it is very bad compared to another one, even on a single criterion. This occurs if the difference between the values of an attribute of two alternatives is greater than a fixed veto threshold.

The ELECTRE method is based on the concordance and discordance indices defined as follows. We start from the data of the decision matrix and assume here that the sum of the weights of all criteria (w_i) equals to 1. For an ordered pair of alternatives (A_j, A_k), the concordance index c_{jk} is the sum of all the weights for those criteria where the performance score of A_j is least as high as that of A_k, i.e.,

$$c_{jk} = \sum_{i:a_{ij} \geq a_{ik}} w_i, \quad j, k = 1, 2, ..., n, \quad j \neq k. \quad (12)$$

The computation of the discordance index d_{jk} is a bit more complicated: d_{jk}=0 if a_{ij}>a_{ik}, i=1, 2,...,m, i.e., the discordance index is zero if A_j performs better than A_k on all criteria. Otherwise,

$$d_{jk} = \max_{i=1,2,\ldots,m} \frac{a_{ik} - a_{ij}}{\max\limits_{j=1,2,\ldots,n} a_{ij} - \min\limits_{j=1,2,\ldots,n} a_{ij}}, \quad j,k = 1,2,\ldots,n, \quad j \neq k, \tag{13}$$

i.e., for each criterion where A_k outperforms A_j, the ratio is calculated between the difference in performance level between A_k and A_j and the maximum difference in score on the criterion concerned between any pair of alternatives. The maximum of these ratios (which must lie between 0 and 1) is the discordance index. A concordance threshold c^* and discordance threshold d^* are then defined such that $0<d^*<c^*<1$. Then, A_j outranks A_k if the $c_{jk}>c^*$ and $d_{jk}<d^*$, i.e., the concordance index is above and the discordance index is below its threshold, respectively. This outranking defines a partial ranking on the set of alternatives. Consider the set of all alternatives that outrank at least one other alternative and are themselves not outranked. This set contains the promising alternatives for this decision problem. Interactively changing the level thresholds, we also can change the size of this set.

4 ELECTRE method with interval weights and data

Suppose A_1, A_2,..., A_m are m possible alternatives among which decision makers have to choose; C_1, C_2,...,C_n are criteria with which alternative performance are measured; x_{ij} is the rating of alternative A_i with respect to criterion C_j and is not known exactly and only we know is $x_{ij} \in [x^l_{ij}, x^u_{ij}]$, besides the weights of criteria cannot be calculated exactly and we can just consider an interval for them as $w_i \in [w^l_i, w^u_i]$. In this situation, an MCDM problem with interval weights and data can be concisely expressed in format of one matrix as in Table 1.

4.1 The proposed algorithmic method

A systematic approach to extend the ELECTRE to the interval weights and data is proposed in this section.

Table 1 MCDM problem with interval weights and data

Alternatives	C_1 $[w^l_1, w^u_1]$	C_2 $[w^l_2, w^u_2]$	…	C_n $[w^l_n, w^u_n]$
A_1	$[x^l_{11}, x^u_{11}]$	$[x^l_{12}, x^u_{12}]$	…	$[x^l_{1n}, x^u_{1n}]$
A_2	$[x^l_{21}, x^u_{21}]$	$[x^l_{22}, x^u_{22}]$	…	$[x^l_{1n}, x^u_{1n}]$
⋮	⋮	⋮	⋮	⋮
A_m	$[x^l_{m1}, x^u_{m1}]$	$[x^l_{m1}, x^u_{m1}]$	…	$[x^l_{mn}, x^u_{mn}]$

Step 1. Convert the decision matrix into a normalized matrix using Eqs. 14 and 15

$$n^l_{ij} = \frac{x^l_{ij}}{\sqrt{\sum\limits_{i=1}^{m} \left(x^l_{ij}\right)^2 + \left(x^u_{ij}\right)^2}}, \quad j = 1,2,\ldots,m. \quad i = 1,2,\ldots,n, \tag{14}$$

$$n^u_{ij} = \frac{x^u_{ij}}{\sqrt{\sum\limits_{i=1}^{m} \left(x^l_{ij}\right)^2 + \left(x^u_{ij}\right)^2}}, \quad j = 1,2,\ldots,m. \quad i = 1,2,\ldots,n. \tag{15}$$

Now $[n^l_{ij}, n^u_{ij}]$ is normalized interval of interval $[x^l_{ij}, x^u_{ij}]$. The abovementioned normalization method is to preserve the property that the ranges of normalized interval numbers fall within the [0, 1].

Step 2. Determine the weighted normalized matrix

Referring to Eq. 6 presented in Section 2 and with regard to $W_i = [w^l_i, w^u_i]$, $w^l_i \geq 0$, we can construct the weighted normalized interval decision matrix as

$$v^l_{ij} = w^l_i n^l_{ij}, \qquad J = 1,2,\ldots,m, \quad i = 1,2,\ldots,n, \tag{16}$$

$$v^u_{ij} = w^u_i n^u_{ij}, \qquad J = 1,2,\ldots,m, \quad i = 1,2,\ldots,n, \tag{17}$$

then

$$V_{ij} = [v^l_{ij}, v^u_{ij}] \tag{18}$$

where w^l_i, w^u_i is the lower and upper weight of the ith attribute or criterion, and $\frac{\sum_{j=1}^{n} [w^l_j, w^u_j]}{2} = 1$.

Step 3. Specify concordance and discordance interval sets

Specify concordance and discordance interval sets for each interval pairs of k and l alternatives k, l=1, 2,...,m; l≠k. The set of available interval indicators (J={j|j=1, 2,...,n}) is divided into two different sets of concordance interval set (S_{kl}) and discordance interval set (D_{kl}). Concordance set (S_{kl}) of alternatives A_k and A_l consists of all indicators which for each of them, A_k is superior to A_l, namely,

$$S_{kl} = \{j | [v^l_{kj}, v^u_{kj}] \geq [v^l_{lj}, v^u_{lj}]\} \tag{19}$$

Vice versa, the complementary subset named discordance set is a set of indicators that for each of them, we have

$$D_{kl} = \{j | [v^l_{kj}, v^u_{kj}][v^l_{lj}, v^u_{lj}]\} = J - S_{kl} \tag{20}$$

(the interval $[v^l_{ij}, v^u_{ij}]$ is given with positive utility).

Step 4. Calculate the concordance interval matrix

Possible values of concordance interval set are measured by means of available interval weights of concordance indexes which exist in that set. In other words, concordance interval index is equivalent to the sum of interval weights $\left(w_j^l, w_j^u\right)$ for those indexes which form the set (S_{kl}). Thus, concordance interval index $\left[I_{k,l}^l, I_{k,l}^u\right]$ between A_l, A_k is as follows:

$$I_{k,l} = [I_{k,l}^l, I_{k,l}^u] = \sum_{j \in S_{k,l}} [w_j^l, w_j^u] \quad ; \quad \frac{\sum_{j=1}^{n} [w_j^l, w_j^u]}{2} = 1 \tag{21}$$

Concordance interval index $\left[I_{k,l}^l, I_{k,l}^u\right]$ is reflective of relative significance of A_k with respect to A_l such that $0 \leq \frac{[I_{k,l}^l, I_{k,l}^u]}{2} \leq 1$. The high value of $\left[I_{k,l}^l, I_{k,l}^u\right]$ shows both the superiority and concordance of A_k with regard to A_l. Therefore, obtained value of indexes $\left[I_{k,l}^l, I_{k,l}^u\right]$ (k, l=1, 2,...,m; l≠k) constructs the asymmetrical concordance interval matrix (I) as follows:

$$I = \begin{vmatrix} - & [I_{1,2}^l, I_{1,2}^u] & [I_{1,3}^l, I_{1,3}^u] & \cdots & [I_{1,m}^l, I_{1,m}^u] \\ [I_{2,1}^l, I_{2,1}^u] & - & [I_{2,3}^l, I_{2,3}^u] & \cdots & [I_{2,m}^l, I_{2,m}^u] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ [I_{m,1}^l, I_{m,1}^u] & [I_{m,2}^l, I_{m,2}^u] & \cdots & [I_{m(m-1)}^l, I_{m(m-1)}^u] & - \end{vmatrix}$$

Step 5. Calculate the discordance interval matrix

Discordance interval index (matched with set $D_{k,l}$) contrary to the index $\left[I_{k,l}^l, I_{k,l}^u\right]$ indicates that A_k is strongly superior with respect to A_l. The index $[NI_{k,l}^l, NI_{k,l}^u]$ is calculated using the members of matrix V (weighted scores) for each element of discordance interval set (D_{kl}) as follows:

$$NI_{k,l} = [NI_{k,l}^l, NI_{k,l}^u] = \frac{\max_{j \in D_{k,l}} \left| [v_{kj}^l, v_{kj}^u] - [v_{lj}^l, v_{lj}^u] \right|}{\max_{j \in J} \left| [v_{kj}^l, v_{kj}^u] - [v_{lj}^l, v_{lj}^u] \right|} \tag{22}$$

With respect to Eq. 4, discordance matrix for all pairwise comparisons of alternatives converts into a matrix with absolute numbers which is

$$NI = \begin{vmatrix} - & NI_{1,2} & NI_{1,3} & \cdots & NI_{1,m} \\ NI_{2,1} & - & NI_{2,3} & \cdots & NI_{2,m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ NI_{m,1} & NI_{m,2} & \cdots & NI_{m(m-1)} & - \end{vmatrix}$$

It deserves to note that the available values of I and NI differs noticeably with each other, yet they have complementary relationship such that interval matrix I is illustrative of the weights $\left(w_j^l, w_j^u\right)$ resulted from concordance indexes, and asymmetrical matrix NI reflects the high relative difference of $\bar{v}_{ij}^l = w_i^l \bar{n}_{ij}^l, \bar{v}_{ij}^u = w_i^u \bar{n}_{ij}^u$ for each discordance indexes.

Step 6. Specify the effective concordance matrix

The values of indexes $\left[I_{k,l}^l, I_{k,l}^u\right]$ of concordance matrix have to be compared against a threshold value so that the superiority chance of A_k with respect to A_l are better judged. In the case when $\left[I_{k,l}^l, I_{k,l}^u\right]$ exceeds from a minimum threshold $\overline{I} = \left[\overline{I}^l, \overline{I}^u\right]$, this chance increases. Besides, we can calculate the average of each arbitrary index $\left[\overline{I}^l, \overline{I}^u\right]$ from concordance indexes in the following manner:

$$\overline{I} = \left[\overline{I}^l, \overline{I}^u\right] = \sum_{k=1}^{m} \sum_{l=1}^{m} [I_{k,l}^l, I_{k,l}^u] / m(m-1) \tag{23}$$

Then based on minimum threshold $\left[\overline{I}^l, \overline{I}^u\right]$, we construct a boolean matrix F which has elements 0 and 1 as:

$$\begin{cases} f_{kl} = 1 & \text{if} \quad [I_{kl}^l, I_{kl}^u] \geq [\overline{I}^l, \overline{I}^u] \\ f_{kl} = 0 & \text{if} \quad [I_{kl}^l, I_{kl}^u] < [\overline{I}^l, \overline{I}^u] \end{cases} \tag{24}$$

Then each element 1 in matrix F (effective concordance matrix) indicates that it is an effective and dominant alternative against the other alternatives.

Step 7. Specify the effective discordance matrix

Elements $NI_{k,l}$ from discordance matrix as done in step 6 should be evaluated with respect to a threshold value. This threshold value ($\overline{NI}$) is calculated with the following formula.

$$\overline{NI} = \sum_{k=1}^{m} \sum_{l=1}^{m} N_{k,l} / m(m-1) \tag{25}$$

Then we construct a boolean matrix G (effective discordance matrix) such that

$$\begin{cases} g_{kl} = 1 & \text{if} \quad NI_{k,l} \leq \overline{NI} \\ g_{kl} = 0 & \text{if} \quad NI_{k,l} > \overline{NI} \end{cases} \tag{26}$$

Unit elements in matrix G indicate dominance relations among alternatives.

Step 8. Specify effective and outranking matrix

Common elements ($h_{k,l}$) construct outranking matrix (H) for making decision from matrix F and matrix G with the following formula.

$$h_{k,l} = f_{k,l} \cdot g_{k,l} \tag{27}$$

Step 9. Eliminate the less attractive alternatives

Outranking matrix (H) indicates the order of relative superiority of alternatives. This means that if $h_{k,l}$=1, A_k is superior to A_l in terms of both concordance and discordance indexes. However, A_k might be still dominated by other alternatives. Therefore, the condition which makes A_k an effective alternative is as follows:

$$\begin{cases} h_{k,l} = 1 & \text{for at least one unit element} \quad \text{for} \quad l = 1, 2, ..., m; k \neq l \\ h_{k,l} = 0 & \text{for all } i \quad \text{for} \quad l = 1, 2, ..., m; i \neq k; i \neq l \end{cases} \tag{28}$$

In the situations where these two conditions are not simultaneously fulfilled, we can simply identify the effective alternatives from matrix (H). For this purpose, we can eliminate those columns of (H) which at least possess a unit element (1) from matrix (H) because those columns are dominated by other row or rows. It is noteworthy that the threshold values of $\overline{I}$ and $N\overline{I}$ which were presented in stages 6 and 7 are approximate and are used to facilitate creating a criterion to select the best alternative among existing alternatives. So long as the Eq. 28 is not true for any of the alternatives, we can increase $\overline{I}$ and reduce $N\overline{I}$ until the above condition is satisfied to come up with the best alternative. Moreover, if Eq. 28 is true for any of the alternatives, we can reduce $\overline{I}$ and increase $N\overline{I}$ until come up with the best alternative.

5 Numerical example

In this section, we work out a numerical example to illustrate the ELECTRE method for decision-making problems with interval weights and data. In this case study, three suppliers are compared with respect to five decision criteria defined as follows:

1. Profitability of supplier (c_1)
2. Relationship closeness (c_2)
3. Technological capability (c_3)
4. Conformance quality (c_4)
5. Conflict resolution (c_5)

Interval weights of five decision criteria and interval data resulted from appraising each alternative against each criterion shown in Table 2.

In sum, we elaborate on how our algorithm is going to determine the most preferable alternative among all possible alternatives when interval weights and data are taken into consideration.

Step 1. Construct the interval normalized decision matrix by making use of Eqs. 14 and 15. The obtained results are reported in Table 3

Step 2. Construct the interval weighted normalized decision matrix by making use of Eqs. 16 and 17. The obtained results are reported in Table 4

Step 3. Specify the concordance and discordance interval set by making use of Eqs. 19 and 20. The concordance and discordance internal set is as follows:

$$S_{k,l} = S_{A_1A_2} = S_{1,2} = \{1,5\} \qquad D_{k,l} = D_{1,2} = \{2,3,4\}$$
$$S_{1,3} = \{1,5\} \qquad D_{1,3} = \{2,3,4\}$$
$$S_{2,1} = \{2,3,4\} \qquad D_{2,1} = \{1,5\}$$
$$S_{2,3} = \{2,4,5\} \qquad D_{2,3} = \{1,3\}$$
$$S_{3,1} = \{1,2,3,4\} \qquad D_{3,1} = \{5\}$$
$$S_{3,2} = \{1,3\} \qquad D_{3,2} = \{2,4,5\}$$

Step 4. Calculate the concordance interval matrix by making use of Eq. 21

Table 2 The interval decision matrix of three alternatives

Alternatives	C_1		C_2		C_3		C_4		C_5	
	[0.08,0.16]		[0.25,0.38]		[0.21,0.33]		[0.11,0.22]		[0.09,0.17]	
	x_{1j}^l	x_{1j}^u	x_{2j}^l	x_{2j}^u	x_{3j}^l	x_{3j}^u	x_{4j}^l	x_{4j}^u	x_{5j}^l	x_{5j}^u
A_1	7	8	5	9	6	8	6	8	8	9
A_2	6	8	7	9	7	8	8	9	7	9
A_3	7	8	6	9	7	9	7	9	6	9

Table 3 The Interval normalized decision matrix

Alternatives	C_1		C_2		C_3		C_4		C_5	
	[0.08,0.16]		[0.25,0.38]		[0.21,0.33]		[0.11,0.22]		[0.09,0.17]	
	n_{1j}^l	n_{1j}^u	n_{2j}^l	n_{2j}^u	n_{3j}^l	n_{3j}^u	n_{4j}^l	n_{4j}^u	n_{5j}^l	n_{5j}^u
A_1	0.3876	0.4430	0.2661	0.4790	0.3239	0.4319	0.3098	0.4131	0.4040	0.4545
A_2	0.3223	0.4430	0.3725	0.4790	0.3779	0.4319	0.4131	0.4647	0.3535	0.4545
A_3	0.3876	0.4430	0.3193	0.4790	0.3779	0.4859	0.3614	0.4647	0.3030	0.4545

Regarding to interval weights available in Table 2 as W={(0.08,0.16), (0.25,0.38),(0.21,0.33),(0.11,0.22), (0.09,0.17)}and with respect to Eq. 21, we have:

$$I = \begin{vmatrix} - & [0.17, 0.33] & [0.17, 0.33] \\ [0.57, 0.93] & - & [0.45, 0.77] \\ [0.65, 1.09] & [0.29, 0.49] & - \end{vmatrix}$$

Step 5. Calculate the discordance interval matrix by making use of Eq. 22

Considering the weighted matrix presented in Table 4. For instance, discordance index $NI_{2,1}$ are calculated as follows:

$$NI_{2,1} = [NI_{2,1}^l, NI_{2,1}^u] = \frac{\max\limits_{j\in D_{2,1}}\{|V_{2,j} - V_{1,j}|\}}{\max\limits_{j\in J}\{|V_{2,j} - V_{1,j}|\}}$$

$$= \frac{\max\{|V_{21} - V_{11}|, |V_{25} - V_{15}|\}}{\max\{|V_{21} - V_{11}|, |V_{22} - V_{12}|, |V_{23} - V_{13}|, |V_{24} - V_{14}||V_{25} - V_{15}|\}}$$

$$= \frac{\max\{|[v_{21}^l, v_{21}^u] - [v_{11}^l, v_{11}^u]|, |[v_{25}^l, v_{25}^u] - [v_{15}^l, v_{15}^u]|\}}{\max\{|[v_{21}^l, v_{21}^u] - [v_{11}^l, v_{11}^u]|, |[v_{22}^l, v_{22}^u] - [v_{12}^l, v_{12}^u]|, |[v_{23}^l, v_{23}^u] - [v_{13}^l, v_{13}^u]|, |[v_{24}^l, v_{24}^u] - [v_{14}^l, v_{14}^u]|, |[v_{25}^l, v_{25}^u] - [v_{15}^l, v_{15}^u]|\}}$$

$$= \frac{\max\{|[-0.0443, 0.0398]|, |[-0.0454, 0.0409]|\}}{\max\{|[-0.0443, 0.0398]|, |[-0.0889, 0.1155]|, |[-0.0632, 0.0745]|, |[-0.0454, 0.0682]|, |[-0.0454, 0.0409|\}}$$

$$= \frac{\max\{0.0443, 0.0454\}}{\max\{0.0443, 0.1155, 0.0745, 0.0682, 0.0454\}} = \frac{0.0454}{0.1155}$$
$$= 0.3931$$

The remaining indexes $NI_{k,l}$ are calculated in this way. So, we have

$$NI = \begin{vmatrix} - & 1 & 1 \\ 0.3931 & - & 0.7925 \\ 0.4329 & 1 & - \end{vmatrix}$$

Table 4 The interval weighted normalized decision matrix

Alternatives	C_1		C_2		C_3		C_4		C_5	
	v_{1j}^l	v_{1j}^u	v_{2j}^l	v_{2j}^u	v_{3j}^l	v_{3j}^u	v_{4j}^l	v_{4j}^u	v_{5j}^l	v_{5j}^u
A_1	0.0310	0.0708	0.0665	0.1820	0.0680	0.1425	0.0340	0.0908	0.0363	0.0772
A_2	0.0265	0.0708	0.0931	0.1820	0.0793	0.1425	0.0454	0.1022	0.0318	0.0772
A_3	0.0310	0.0708	0.0798	0.1820	0.0793	0.1603	0.0397	0.1022	0.0272	0.0772

Step 6. Specify effective concordance matrix by making use of Eqs. 23 and 24

$$\overline{I} = \frac{[0.17, 0.33] + [0.17, 0.33] + [0.57, 0.93] + [0.45, 0.77] + [0.65, 1.09] + [0.29, 0.49]}{6} = \frac{[2.3, 3.94]}{6} = [0.3833, 0.6566]$$

For matrix F, we have:

$$F = \begin{vmatrix} - & 0 & 0 \\ 1 & - & 1 \\ 1 & 0 & - \end{vmatrix}$$

Step 7. Specify effective discordance matrix by making use of Eqs. 25 and 26

$$\overline{NI} = \frac{4.6185}{6} = 0.7697$$

Matrix G is obtained as follows:

$$G = \begin{vmatrix} - & 0 & 0 \\ 1 & - & 0 \\ 1 & 0 & - \end{vmatrix}$$

Step 8. Determine the outranking matrix H by making use of Eq. 27

$$H = \begin{vmatrix} - & 0 & 0 \\ 1 & - & 0 \\ 1 & 0 & - \end{vmatrix}$$

Step 9. Eliminate the less attractiveness alternatives

As can be seen from matrix H, the first column has at least one unit element, and therefore, we can eliminate it. So alternative A_2 is placed in the first rank, and alternatives A_3 and A_1 are placed in the second and third ranks, respectively.

6 Conclusion

In some cases, precise determination of the exact value of the attributes is difficult, and their values are considered as intervals; therefore, in this paper, we proposed a novel algorithmic ELECTRE method for the evaluation of alternatives against criteria. This ELECTRE method made use of interval weights and data. Finally, the executive procedure of our proposed ELECTRE method was illustrated by applying it to the problem of supplier selection. As a direction for future research, it could be interesting to apply the proposed method to other ELECTRE methods such as ELECTRE II and III. Moreover, another clue for future research is applying intuitionistic fuzzy sets [4] instead of interval weights and data for the problem considered.

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