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ORIGINAL ARTICLE

Single-machine group scheduling with linearly decreasing time-dependent setup times and job processing times

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Abstract This paper considers single-machine scheduling problems with group technology (GT). We consider the case of group setup times and job processing times are a decreasing function of their starting time. We first prove that the makespan minimization problem remains polynomially solvable under the general decreasing linear deterioration. We then prove that the total weighted completion time minimization problem remains polynomially solvable under the proportional decreasing linear deterioration.

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1 Introduction

Traditional machine scheduling problems usually involve jobs with constant, independent processing times. In practice, however, we often encounter settings in which the job processing times increase or decrease over time. Researchers have formulated this phenomenon into different models and solved different problems for various criteria. Applications of these models can be found, among others, in fire fighting, emergency medicine, machine maintenance, and radar science.

Browne and Yechiali [1] considered a single-machine scheduling problem in which the processing times of the jobs are linear deterioration functions of their starting times. They showed that this problem can be solved optimally. Mosheiov [2] considered the single-machine problem that all the jobs are characterized by a common positive basic processing time. Based on this basic assumption, he proved that the optimal schedule to minimize flowtime is symmetric and has a V-shaped property with respect to the increasing rates of deterioration. Mosheiov [3] considered the single-machine scheduling problem with the following objective functions: makespan, total flow time, total weighted completion time, total lateness, maximum lateness and maximum tardiness, and number of tardy jobs. When the values of the basic processing times equal zero, all these problems can be solved polynomially. Sundararaghavan and Kunnathur [4] considered the single-machine scheduling problem in which the processing time is a binary function of a common start-time due date. The jobs will incur processing time penalties for starting after the due date, and the objective is to minimize the sum of the weighted completion times. Three special cases of this problem can be solved optimally.

Chen [5], Hsieh and Bricker [6], and Mosheiov [7] considered scheduling linear deteriorating jobs on a group of parallel identical machines. Chen [5] considered minimizing the flow time, while Hsieh and Bricker [6] and Mosheiov [7] studied makespan minimization. Mosheiov [8] considered the computational complexity of the flow shop, open shop, and job shop makespan minimization problems with simple linear deteriorating jobs. Kononov and Gawiejnowicz [9] considered the makespan minimization problem on dedicated machines.

By contrast, the other model assumes that the processing time of a job is a decreasing function of its starting time. An application of this model is the so-called "learning effect." This model was introduced by Ho et al. [10]. Ng et al. [11] considered three single-machine scheduling problems with a decreasing linear model of the job processing times, where the objective function is to minimize total completion time, and two of the problems are solved optimally. A pseudopolynomial time algorithm was constructed to solve the third problem using dynamic programming. Bachman et al. [12] considered the single-machine scheduling problem with start-time-dependent job processing times. They proved that the problem of minimizing total weighted completion time is NP-hard. They also considered some special cases. Wang and Xia [13] considered scheduling problems under a special type of linear decreasing deterioration. They presented optimal algorithms for single-machine scheduling to minimize makespan, maximum lateness, maximum cost, and number of late jobs, respectively. For the two-machine flow shop scheduling problem to minimize makespan, they proved that the optimal schedule can be obtained by Johnson's rule. If the processing times of the all operations are equal for each job, they proved that the flow shop scheduling problem can be transformed into a single-machine scheduling problem. Wang [14] considered the general, no-wait, and no-idle flow shop scheduling problem with deteriorating jobs, respectively. They assumed that the processing time is a decreasing function of its starting time. They also assumed that the normal processing time is proportional to its decreasing rate, and some dominant relationships between the machines hold. They showed that polynomial algorithms exist for the problems to minimize makespan or weighted sum of completion time. When the objective is to minimize maximum lateness, the solution of the classical version may not hold. Extensive surveys of scheduling models and problems concerning start-time-dependent job processing times can be found in Alidaee and Womer [15] and Cheng et al. [16]. More recent papers that have considered scheduling jobs with deterioration effects include Wu, Lee, and Shiau [17], Lee, Wu, Wen, and Chung [18], Lee, Wu, and Chung [19], Lee and Wu [20], Lee, Wu, and Liu [21], Gao, Huang, and Wang [22], Wang and Wang [23], Huang, Wang, and Wang [24], Wu, Shiau, and Lee [25], Wu and Lee [26], Wang, Lin, and Shan [27], and Wang, Gao, Wang, and Wang [28].

Recently, an important class of scheduling problems is characterized by the group technology assumption, i.e., the jobs are classified into groups by the similar production requirements, no machine setups are needed between two consecutively scheduled jobs from the same group, although an independent setup is required between jobs of different groups. In group technology, it is conventional to schedule continuously all jobs from the same group. Many advantages have been claimed through the wide applications of group technology. For instance, changeover between different parts are simplified, thereby reducing the costs involved; parts spend less time waiting, which results in less work-in-process inventory; parts tend to move through production in a direct route, and hence, the manufacturing lead time is reduced; and the variability of tasks is reduced, and hence, worker training is simplified [29–31]. Hence, the scheduling in group technology environment results in a new stream of research (Potts and Van Wassenhove [32]).

However, to the best of our knowledge, apart from the recent paper of Wu, Shiau, and Lee [25], Wu and Lee [26], Wang, Lin, and Shan [27], and Wang, Gao, Wang, and Wang [28], the scheduling problems with deteriorating jobs under the group technology with the assumption that both the setup times and the job processing times are functions of their starting times has not been investigated. Wu, Shiau, and Lee [25] and Wu and Lee [26] studied the deteriorating jobs scheduling problems under the group technology with the assumption that both the setup times and the job processing times are functions of their starting times. They showed that the makespan and the total completion time problems remain polynomially solvable under the simple linear deterioration and linear deterioration model. Wang, Lin, and Shan [27] considered single machine scheduling with deteriorating jobs and group technology assumption. They showed that the makespan minimization problem and the total weighted completion time minimization problem remain polynomially solvable under the proportional linear deterioration. Wang, Gao, Wang, and Wang [28] studied the single machine scheduling problem with deterioration jobs and group technology assumption at the same time. They showed that the makespan minimization problem remains polynomially solvable under the general linear deterioration.

Generally, two types of models are used to describe this kind of process. The first type is devoted to problems in which the job processing time is characterized by a non-decreasing function, and the second type concerns problems in which the job processing time is given by a non-increasing function. In this paper, we study the latter group of problems, i.e., single-machine scheduling problems with decreasing time-dependent job processing times under the group technology assumption. The objective functions are to minimize the makespan and the total weighted completion time, respectively. The remaining part of the paper is organized as follows: In the next section, a precise formulation of the problem with decreasing job processing times is given. The problems of minimizing the makespan and the total weighted completion time are given in Sections 3 and 4, respectively. The last section contains some conclusions.

2 Problem formulation

Assume that there are n jobs $[J_1, J_2, \ldots, J_n]$, which are grouped into f groups, and these n jobs are to be processed on a single machine, each of which is available at time zero. Jobs are processed one by one in groups on the machine and a setup time is required if the machine switches from one group to another, we assume that the processing of a job may not be interrupted. Jobs in the same group are processed consecutively. Let n_i be the number of jobs belonging to group G_i; thus, $n_1 + n_2 + \ldots + n_f = n$; J_{ij} denotes the jth job in group G_i, $i = 1, 2, \ldots, f$; $j = 1, 2, \ldots, n_i$. In addition,

$$p_{ij} = a_{ij} - b_{ij}t \tag{1}$$

denotes the actual processing time of job J_{ij} if its start time is t, where $a_{ij} \geq 0$ is the normal processing time of job J_{ij} and $b_{ij} \geq 0$ is the decreasing rate of job J_{ij}. As in the above general decreasing linear deterioration model, we also assume that the setup time of group G_i is a general decreasing linear deterioration model, i.e.,

$$s_i = c_i - d_i t, \tag{2}$$

where $c_i \geq 0$ is the normal setup time of group G_i, $d_i \geq 0$ is its decreasing rate, and t is its start time. It is assumed that the decreasing rates satisfy the following conditions:

$$0 < b_{ij} < 1,\ 0 < d_i < 1,\ d_j \left(\sum_{i=1}^{f} \sum_{j=1}^{n_i} (c_i + a_{ij}) - c_j \right) < c_j$$

$$\text{and } b_{ij} \left(\sum_{i=1}^{f} \sum_{j=1}^{n_i} (c_i + a_{ij}) - a_{ij} \right) < a_{ij}.$$

The first two conditions ensure that the decrease of each job processing time and each group setup time are less than one unit for every unit delay in its starting moment. The last two conditions ensure that all job processing times and all setup times are positive in a feasible schedule (see also [10, 11] for detailed explanations). The objectives are to minimize the makespan and the total weighted completion time, respectively.

For a given schedule π, $C_{ij} = C_{ij}(\pi)$ represents the completion time of job J_{ij} in group G_i. $C_{\max} = \max\{C_{ij} | i = 1, 2, \ldots, f;\ j = 1, 2, \ldots, n_i\}$ and $\sum w_{ij} C_{ij}$ represent makespan and total weighted completion time of a given schedule, respectively. In the remaining part of the paper, all the problems considered will be denoted using the three-field notation schema $\alpha|\beta|\gamma$ introduced by Graham et al. [33].

3 Makespan minimization problem

Lemma 1 *Ho et al.* [10] *The schedule obtained by the non-increasing order of* $\frac{a_j}{b_j}$ *is optimal for the problem* $1|p_j = a_j - b_j t|C_{\max}$. *If the staring time of the first job is* $t_0 \geq 0$, *the makespan is*

$$C_{\max}(t_0|J_1, J_2, \ldots, J_n) = \sum_{j=1}^{n} a_j \prod_{i=j+1}^{n} (1 - b_i) + t_0 \prod_{i=1}^{n} (1 - b_i).$$

Stem from Lemma 1, we can easily obtain the following theorem.

Theorem 1 *For the problem* $1|p_{ij} = a_{ij} - b_{ij}t, s_i = c_i - d_i t, GT|C_{\max}$, *if the schedule of groups is given, then the optimal schedule that jobs within each group can be obtained by scheduling the jobs in non-increasing order of* $\frac{a_{ij}}{b_{ij}}$, $i = 1, 2, \ldots, f$; $j = 1, 2, \ldots, n_i$.

So without loss of generality, we suppose the schedule of each group is given according the order of Theorem 1. Now we consider the order of groups. Once the optimal order of the jobs within each group is achieved, every group can be seen as a compound job. Now we only consider two groups of jobs.

Let $G_1 : J_{11}, J_{12}, \ldots, J_{1n_1}$; $G_2 : J_{21}, J_{22}, \ldots, J_{2n_2}$. Without loss of generality, stem from Theorem 1, we assume that G_1: $\frac{a_{11}}{b_{11}} \geq \frac{a_{12}}{b_{12}} \geq \ldots \geq \frac{a_{1n_1}}{b_{1n_1}}$; G_2: $\frac{a_{21}}{b_{21}} \geq \frac{a_{22}}{b_{22}} \geq \ldots \geq \frac{a_{2n_2}}{b_{2n_2}}$.

Theorem 2 *For the problem* $1|p_{ij} = a_{ij} - b_{ij}t, s_i = c_i - d_i t, GT|C_{\max}$, *if*

$$\rho(G_1) \geq \rho(G_2),$$

where

$$\rho(G_i) = \frac{\sum_{j=1}^{n_i} a_{ij} \prod_{k=j+1}^{n_i}(1 - b_{ik}) + c_i \prod_{k=1}^{n_i}(1 - b_{ik})}{1 - (1 - d_i)\prod_{k=1}^{n_i}(1 - b_{ik})},$$

$$i = 1, 2,$$

then, it is optimal to process the group G_1 *before the group* G_2.

Proof If the order is G_1, G_2. Then, for G_1:

$$s_1 = t_0 + c_1 - d_1 t_0 = c_1 + t_0(1 - d_1).$$

$$C_{1n_1} = \sum_{j=1}^{n_1} a_{1j} \prod_{k=j+1}^{n_1}(1 - b_{1k}) + (c_1 + t_0(1 - d_1))\prod_{k=1}^{n_1}(1 - b_{1k}).$$

For G_2:

$$s_2 = C_{1n_1} + c_2 - d_2 C_{1n_1} = c_2 + (1 - d_2)\left[\sum_{j=1}^{n_1} a_{1j} \prod_{k=j+1}^{n_1}(1 - b_{1k}) + (c_1 + t_0(1 - d_1))\prod_{k=1}^{n_1}(1 - b_{1k})\right].$$

$$\begin{aligned}
C_{\max}(t_0|G_1, G_2) &= C_{2n_2} \\
&= \sum_{j=1}^{n_2} a_{2j} \prod_{k=j+1}^{n_2}(1 - b_{2k}) + S_{21}\prod_{k=1}^{n_2}(1 - b_{2k}) \\
&= \sum_{j=1}^{n_2} a_{2j} \prod_{k=j+1}^{n_2}(1 - b_{2k}) + c_2\prod_{k=1}^{n_2}(1 - b_{2k}) \\
&\quad + (1 - d_2)\prod_{k=1}^{n_2}(1 - b_{2k})\left[\sum_{j=1}^{n_1} a_{1j} \prod_{k=j+1}^{n_1}(1 - b_{1k}) + (c_1 + t_0(1 - d_1))\prod_{k=1}^{n_1}(1 - b_{1k})\right].
\end{aligned}$$

If the order is G_2, G_1. Then

$$\begin{aligned}
C_{\max}(t_0|G_2, G_1) &= \sum_{j=1}^{n_1} a_{1j} \prod_{k=j+1}^{n_1}(1 - b_{1k}) + c_1\prod_{k=1}^{n_1}(1 - b_{1k}) \\
&\quad + (1 - d_1)\prod_{k=1}^{n_1}(1 - b_{1k})\left[\sum_{j=1}^{n_2} a_{2j} \prod_{k=j+1}^{n_2}(1 - b_{2k}) + (c_2 + t_0(1 - d_2))\prod_{k=1}^{n_2}(1 - b_{2k})\right].
\end{aligned}$$

$$\begin{aligned}
&C_{\max}(t_0|G_1, G_2) - C_{\max}(t_0|G_2, G_1) \\
&= \left(\sum_{j=1}^{n_2} a_{2j} \prod_{k=j+1}^{n_2}(1 - b_{2k}) + c_2\prod_{k=1}^{n_2}(1 - b_{2k})\right) \times \left(1 - (1 - d_1)\prod_{k=1}^{n_1}(1 - b_{1k})\right) \\
&\quad - \left(\sum_{j=1}^{n_1} a_{1j} \prod_{k=j+1}^{n_1}(1 - b_{1k}) + c_1\prod_{k=1}^{n_1}(1 - b_{1k})\right) \times \left(1 - (1 - d_2)\prod_{k=1}^{n_2}(1 - b_{2k})\right)
\end{aligned}$$

If

$$\frac{\sum_{j=1}^{n_1} a_{1j} \prod_{k=j+1}^{n_1}(1 - b_{1k}) + c_1 \prod_{k=1}^{n_1}(1 - b_{1k})}{1 - (1 - d_1)\prod_{k=1}^{n_1}(1 - b_{1k})} \geq \frac{\sum_{j=1}^{n_2} a_{2j} \prod_{k=j+1}^{n_2}(1 - b_{2k}) + c_2 \prod_{k=1}^{n_2}(1 - b_{2k})}{1 - (1 - d_2)\prod_{k=1}^{n_2}(1 - b_{2k})}$$

then

$C_{\max}(G_1, G_2) - C_{\max}(G_2, G_1) \le 0.$

This completes the proof. □

Theorem 2 has the transitivity; hence, for the case of f groups, the order of groups can be obtained similarly.

From Theorem 1 and Theorem 2, the problem $1|p_{ij} = a_{ij} - b_{ij}t, s_i = c_i - d_i t, GT|C_{\max}$ can be solved by the following algorithm:

Algorithm 1

Step 1. *Jobs in each group scheduled in non-increasing order of* $\frac{a_{ij}}{b_{ij}}$, *i.e.,*

$$\frac{a_{i(1)}}{b_{i(1)}} \ge \frac{a_{i(2)}}{b_{i(2)}} \ge \ldots \ge \frac{a_{i(n_i)}}{b_{i(n_i)}}, i = 1, 2, \ldots, f.$$

Step 2. *Calculate* $\rho(G_i) = \frac{\sum_{j=1}^{n_i} a_{i(j)} \prod_{k=j+1}^{n_i}(1-b_{i(k)})+c_i \prod_{k=1}^{n_i}(1-b_{i(k)})}{1-(1-d_i)\prod_{k=1}^{n_i}(1-b_{i(k)})}$, $i = 1, 2, \ldots, f$.

Step 3. *Groups scheduled in non-increasing order of* $\rho(G_i)$, *i.e.,* $\rho(G_1) \ge \rho(G_2) \ge \ldots \ge \rho(G_f)$.

Theorem 3 *For the problem* $1|p_{ij} = a_{ij} - b_{ij}t, s_i = c_i - d_i t, GT|C_{\max}$, *Algorithm 1 generates an optimal solution.*

Proof The proof can be easily obtained by Theorem 1 and Theorem 2. □

Obviously, the optimal schedule in a certain group G_i can be obtained in $O(n_i \log n_i)$ and the optimal group schedule can be obtained in $O(f \log f)$. It is easy to show that $\sum_{i=1}^{f} O(n_i \log n_i) \le O(n \log n)$. Hence, the total time for Algorithm 1 is $O(n \log n)$. In addition, we demonstrate the result of Theorem 3 in the following example.

Example 1 Assume $n = 7$, $f = 3$, the normal processing times and decreasing rates for each group are given in Table 1.

Table 1 An illustrative example

Group	G_1		G_2		G_3		
J_{ij}	J_{11}	J_{12}	J_{21}	J_{22}	J_{31}	J_{32}	J_{33}
a_{ij}	25	28	24	20	32	27	25
b_{ij}	0.05	0.06	0.01	0.03	0.05	0.08	0.02
c_i	29		25		26		
d_i	0.05		0.01		0.05		

Solution According to Algorithm 1, we solve Example 1 as follows:

Step 1: In group G_1, the optimal job sequence is $J_{11} \to J_{12}$. In group G_2, the optimal job sequence is $J_{21} \to J_{22}$. In group G_3, the optimal job sequence is $J_{33} \to J_{31} \to J_{32}$.

Steps 2 and 3: $\rho(G_2) = 1364.8000 > \rho(G_1) = 510.3660 > \rho(G_3) = 372.7279$.

Therefore, the optimal group sequence is $G_2 \to G_1 \to G_3$. Hence, the optimal schedule is $[J_{21} \to J_{22}] \to [J_{11} \to J_{12}] \to [J_{33} \to J_{31} \to J_{32}]$, and the optimal value of the makespan is 193.2457.

4 Total weighted completion time minimization problem

Since the complexity of the problem $1|p_j = a_j - b_j t|\sum w_j C_j$ is NP-hard (Bachman et al. [12]), in this section, we consider the case of proportional decreasing linear deterioration, that is, $b_{ij} = b a_{ij}$, $d_i = b c_i$, i.e., $p_{ij} = a_{ij}(1 - bt)$, $s_i = c_i(1 - bt)$.

Lemma 2 *Wang and Xia* [13] *The schedule obtained by any order is optimal for the problem* $1|p_j = a_j(1 - bt)|C_{\max}$. *If the staring time of the first job is* $t_0 \ge 0$, *the makespan is*

$$C_{\max}(t_0|J_1, J_2, \ldots, J_n) = \left(t_0 - \frac{1}{b}\right)\left(\prod_{i=1}^{n}(1 - b a_i)\right) + \frac{1}{b}.$$

Theorem 4 *For the problem* $1|p_{ij} = a_{ij}(1 - bt), s_i = c_i(1 - bt), GT|\sum w_{ij} C_{ij}$, *the optimal schedule satisfies:*

1. *The job sequence in each group is in non-decreasing order of* $\frac{a_{ij}}{w_{ij}(1-b a_{ij})}$, *i.e.,*

$$\frac{a_{i(1)}}{w_{i(1)}(1 - b a_{i(1)})} \le \frac{a_{i(2)}}{w_{i(2)}(1 - b a_{i(2)})} \le \cdots \le \frac{a_{i(n_i)}}{w_{i(n_i)}(1 - b a_{i(n_i)})}, i = 1, 2, \ldots, f;$$

2. *The groups are arranged in non-decreasing order of*

$$\frac{1 - (1 - b c_i)\prod_{j=1}^{n_i}(1 - b a_{i(j)})}{(1 - b c_i)\sum_{k=1}^{n_i} w_{i(k)} \prod_{j=1}^{k}(1 - b a_{i(j)})}.$$

Proof In the same group, the result of 1 can be easily obtained by using simple interchanging technique.

Next, we consider case 2. Let π and π' be two job schedules where the difference between π and π' is a pairwise interchange of two adjacent groups G_i and

G_j, that is, $\pi = [S_1, G_i, G_j, S_2]$, $\pi' = [S_1, G_j, G_i, S_2]$, where S_1 and S_2 are partial sequences. Furthermore, we assume that t denote the completion time of the last job in S_1. To show π dominates π', it suffices to show that $C_{jn_j}(\pi) \le C_{in_i}(\pi')$ and $\sum w_{ij}C_{ij}(\pi) \le \sum w_{ij}C_{ij}(\pi')$. Under π, from Lemma 2, the completion time for the kth job in group G_i is

$$C_{i[k]}(\pi) = \left(t - \frac{1}{b}\right)(1 - bc_i)\prod_{l=1}^{k}(1 - ba_{i(l)}) + \frac{1}{b},$$

and the completion time for the kth job in group G_j is

$$C_{j[k]}(\pi) = \left(t - \frac{1}{b}\right)(1 - bc_i)(1 - bc_j) \prod_{l=1}^{n_i}(1 - ba_{i(l)})\prod_{l=1}^{k}(1 - ba_{j(l)}) + \frac{1}{b}. \quad (3)$$

Under π', the completion times of the kth job in groups G_j and G_i are

$$C_{j[k]}(\pi') = \left(t - \frac{1}{b}\right)(1 - bc_j)\prod_{l=1}^{k}(1 - ba_{j(l)}) + \frac{1}{b}.$$

and the completion time for the kth job in group G_j is

$$C_{i[k]}(\pi') = \left(t - \frac{1}{b}\right)(1 - bc_j)(1 - bc_i) \prod_{l=1}^{n_j}(1 - ba_{j(l)})\prod_{l=1}^{k}(1 - ba_{i(l)}) + \frac{1}{b}. \quad (4)$$

From Eqs. 3 and 4, we have

$$C_{jn_j}(\pi) = C_{in_i}(\pi')$$

and

$$\sum w_{ij}C_{ij}(\pi) - \sum w_{ij}C_{ij}(\pi')$$
$$= \left(t - \frac{1}{b}\right)(1 - c_i)\sum_{k=1}^{n_i} w_{i(k)}\prod_{l=1}^{k}(1 - ba_{i(l)}) \times \left(1 - (1 - bc_j)\prod_{l=1}^{n_j}(1 - ba_{jl})\right)$$
$$- \left(t - \frac{1}{b}\right)(1 - bc_j)\sum_{k=1}^{n_j} w_{j(k)}\prod_{l=1}^{k}(1 - ba_{j(l)}) \times \left(1 - (1 - bc_i)\prod_{l=1}^{n_i}(1 - ba_{il})\right)$$

If

$$\frac{1 - (1 - bc_i)\prod_{l=1}^{n_i}(1 - ba_{il})}{(1 - bc_i)\sum_{k=1}^{n_i} w_{i(k)}\prod_{l=1}^{k}(1 - ba_{i(l)})} \le \frac{1 - (1 - bc_j)\prod_{l=1}^{n_j}(1 - ba_{jl})}{(1 - bc_j)\sum_{k=1}^{n_j} w_{j(k)}\prod_{l=1}^{k}(1 - ba_{j(l)})}$$

from $t - \frac{1}{b} \le 0$, we have

$$\sum w_{ij}C_{ij}(\pi) - \sum w_{ij}C_{ij}(\pi') \le 0.$$

This completes the proof. □

From Theorem 4, the problem $1|p_{ij} = a_{ij}(1 - bt), s_i = c_i(1 - bt), GT|\sum w_{ij}C_{ij}$ can be solved by the following algorithm:

Algorithm 2

Step 1. *Jobs in each group scheduled in non-decreasing order of* $\frac{a_{ij}}{w_{ij}(1-ba_{ij})}$, *i.e.,*

$$\frac{a_{i(1)}}{w_{i(1)}(1 - ba_{i(1)})} \le \frac{a_{i(2)}}{w_{i(2)}(1 - ba_{i(2)})} \le \ldots \le \frac{a_{i(n_i)}}{w_{i(n_i)}(1 - ba_{i(n_i)})}, i = 1, 2, \ldots, f.$$

Step 2. *Calculate* $\rho(G_i) = \frac{(1-bc_i)\prod_{j=1}^{n_i}(1-ba_{i(j)})-1}{(1-bc_i)\sum_{k=1}^{n_i} w_{i(k)}\prod_{j=1}^{k}(1-ba_{i(j)})}$, $i = 1, 2, \ldots, f$.

Step 3. *Groups scheduled in non-decreasing order of* $\rho(G_i)$, *i.e.,*

$$\rho(G_1) \le \rho(G_2) \le \ldots \le \rho(G_f).$$

Obviously, the total time for Algorithm 2 is $O(n\log n)$. In addition, we demonstrate the result of Theorem 4 in the following example.

Example 2 Assume $n = 7$, $f = 3$, $b = 0.01$. The normal processing times, decreasing rates, and weights for each group are given in Table 2.

Table 2 An illustrative example

Group	G_1		G_2		G_3		
J_{ij}	J_{11}	J_{12}	J_{21}	J_{22}	J_{31}	J_{32}	J_{33}
a_{ij}	5	18	2	12	16	7	15
w_{ij}	1	7	2	6	5	8	2
c_i	2		5		6		

Solution. According to Algorithm 2, we solve Example 2 as follows:

Step 1: In group G_1, the optimal job sequence is $J_{12} \to J_{11}$. In group G_2, the optimal job sequence is $J_{21} \to J_{22}$. In group G_3, the optimal job sequence is $J_{32} \to J_{31} \to J_{33}$.

Steps 2 and 3: $\rho(G_3) = -0.0315 < \rho(G_1) = -0.0283 < \rho(G_2) = -0.0267$.

Therefore, the optimal group sequence is $G_3 \to G_1 \to G_2$. Hence, the optimal schedule is $[J_{32} \to J_{31} \to J_{33}] \to [J_{12} \to J_{11}] \to [J_{21} \to J_{22}]$, and the optimal value of the total weighted completion time is 1,170.949.

5 Conclusions

In this paper, we considered two single-machine scheduling problems with decreasing linear deterioration and group technology assumption. By decreasing linear deterioration and group technology assumption, we mean that the group setup times and job processing times are both described by decreasing function of starting time. We showed that the makespan minimization problem remains polynomially solvable. For the total weighted completion time minimization problem, we showed that the problem remains polynomially solvable under the assumption that the job deterioration is a function that is proportional to a linear function of time.

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