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ORIGINAL ARTICLE

A study of gate location optimization of plastic injection molding using sequential linear programming

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Abstract A gate location is one of the most important design variables controlling the product quality of injection molding. In this paper, the numerical simulation of injection mold filling process is combined with the design optimization method to find the optimum gate location to achieve balanced flow. The objective function is expressed in terms of the difference between the maximum and minimum times of boundary filling. The coordinates of gate are chosen as design variables, and a constraint is employed to limit the clamp force lower than the reference value. The optimization problem is solved with the sequential linear programming algorithm, and design sensitivities are evaluated via the finite difference approximation. Finally, numerical examples are given to demonstrate the effect of proposed methods.

Keywords Injection molding · Gate location · Optimal design · Flow balance

1 Introduction

Injection molding is by far the most popular process for the production of plastic parts. As the injection molding production is dominated by complex process dynamics, it is difficult to fully understand and predict the final part quality that is related to various molding parameters. In the past three decades, the numerical simulation of injection molding has been greatly developed by clear understanding characteristics of flowing and heat transfer of polymer melts to predicate the quality characteristics of injection-molded parts without actually fabricating a mold. However, the computer aided engineering (CAE) simulation requires the mold designer to run the simulation, perform the design evaluation, and redesign based on experience, until a satisfactory design is obtained. This manual design process does not guarantee the optimal design solution and so has led to increasing interest in the utilization of design optimization techniques in the mold design procedure.

Several studies reported have investigated the optimization of the injection molding process. Pandelidis and Zhou [1] presented the optimization of gate location using the combined scheme of a simulated annealing and hill-climbing method. The quality of a gating design was presented as an additive function of a temperature differential term and an overpack term, with appropriate weighting term. Young [2] developed a method of gate location optimization based on the minimization of the mold filling pressure, uneven filling pattern, and temperature difference during the mold filling process. Ye et al. [3] developed a scheme to optimize the part quality in injection molding. A mathematical definition of part warpage is presented, and simulated annealing method is used to search for optimum process condition. Chang et al. [4] combined the usage of Taguchi approach and CAE flow simulation software for optimal design of injection molding process parameters. Irani et al. [5] developed a system that automates the process of gate design. The gate design is performed in two stages, global search followed by a local search. During the global search, the candidate gating plans are generated using feature connectivity information. These gating plans are evaluated and redesigned in iterative until the best cavity inlet conditions for each plan is obtained. Then, the best in the trial set is perturbed locally in a search for a better gating plan. The limitation of this system is that the features used are very simple geometry. Zhai et al. [6–9] has investigated the objective function and search scheme

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of gate location optimization and some good results had been achieved. Lee and Kim [10] proposed a method adopting the natural human capability of sense instead of utilizing computers for interpretation and reasoning about complex part geometry to choose the initial gate locations. The methodology comprises a two-stage process: choosing the best among a set of gate locations generated based on a human designer's intuition and locally searching for the better gate location. Gao and Wang [11] used Kriging model for injection molding optimization to minimize the part warpage. Deng et al. [12] presented a particle swarm optimization algorithm for the optimization of multiclass design variables such as the part thickness, process parameters (melt temperature, mold temperature, and injection time), and gate location. Lam and Seow [13, 14] and Jin and Lam [15, 16] developed the flow path concept for cavity balancing. An automatic flow path generation routine was developed, and the flow front within a cavity was altered by varying the wall thickness of the part along the flow path to achieve balanced flow. Lam et al. [17] also developed an automated routine to handle design constraints in automated gating synthesis, taking advantages of functionality of both computer aided design (CAD) and CAE systems.

The related studies previously described did not combine numerical simulation, sensitivity analysis, and numerical optimization to design injection mold. In this study, optimization of gate location in the mold design is addressed based on the results of mold filling simulations with finite difference approximation being used to evaluate the sensitivity and the sequential linear programming (SLP) algorithm implemented as the optimization algorithm. During the optimization process, an objection function depending on the time difference for the extremities of the mold to be filled is evaluated. Through minimizing of the objective function, a uniform pattern of mold filling can be achieved.

2 Filling simulation

Since molten polymer flows during processing operations with low Reynolds (usually Re $<10^{-2}$) and the thickness of injection molded parts is usually very small, the generalized Hele–Shaw approximation is used to model the filling process. After introducing appropriate simplifying assumptions, the equations governing the injection filling process can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$0=\frac{\partial}{\partial z}\left(\eta\frac{\partial u}{\partial z}\right)-\frac{\partial p}{\partial x} \tag{2}$$

$$0=\frac{\partial}{\partial z}\left(\eta\frac{\partial v}{\partial z}\right)-\frac{\partial p}{\partial y} \tag{3}$$

$$\rho Cp\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\eta\dot{\gamma}^2+k\frac{\partial^2 T}{\partial z^2} \tag{4}$$

where x, y are the planar coordinate and z is the gap-wise coordinate, t is time, (u, v) are the velocity components in the (x, y) directions, respectively. T is the temperature, p is the pressure, η is the shear viscosity, $\dot{\gamma}$ is the shear rate, and ρ is the density with C_p being the specific heat, and k the thermal conductivity.

The boundary conditions are adopted as follows. (a) The normal velocity is zero along the periphery of the cavity, which implies that $\nabla p=0$. (b) The flow rate at the injection node is given. (c) $p=0$ in the empty area. (d) The melt front temperature is assumed to be uniform and is set equal to the temperature in the hot core region behind the advancing front. (e) The inlet temperature is assumed to be uniform across the gap and taken as the injection melt temperature. (f) A constant temperature is given along the upper and lower walls.

A hybrid finite element and finite difference numerical scheme is used to solve the previously mentioned governing equation to get the temperature and pressure field. A control volume method is used to calculate the filling area evolution and filling factors associated with each cell are employed to represent the filling status. At time t the polymer filling factor α is known in each control volume ($\alpha=1$ in totally filled ones; $\alpha=0$ in totally empty ones; and $0<\alpha<1$ in partially filled ones). A mass balance is written in each control volume leading to new values of the filling factor at time $t+\Delta t$. Then the filling area at time $t+\Delta t$ can be determined according to the calculated filling factor of each cell. Pressure field, filling factors, and temperature field are computed in iterative until the end of the filling stage. The time of polymer melt reached in each node in the finite element model can be determined from such process.

3 Definition of optimization problem

Determination of gate location is one of the important issues in the mold design. A gate location will affect the filling time and temperature and pressure distributions by altering the balance and direction of the polymer flow. Proper arrangement of the inlet gate can reduce the required pressure, whereas, an improper choice of gate location always causes overpacking, high shear stress, poor weld lines, and excessive warpage among other things. Therefore, the placement of a gate in an injection mold is one of the most important variables of the total mold design, and it is necessary to search for an optimum gate location to improve part quality. In this context, the optimization of gate location is studied. A gate is modeled as point sources, and coordinates of the gate location are selected as design variables.

To apply optimization theory to the injection molding process, quantitative measure of the part quality first need to be developed since the ultimate goal in optimizing the injection molding design is to improve part quality. The part quality can be described with many end product properties such as mechanical, thermal, electrical, optical, or geometrical properties. There are two types of part quality measures: direct and indirect method. Direct method can determine the measurable quantities that characterize a product. In contrast, an indirect measure of quality is a quantity that is correlated but does not produce a direct estimate of that quality.

The indirect quality measures used in this paper are those related to warpage. Part warpage, a dimensional distortion that causes structural unfitness and esthetic problems, is one of the critical quality issues for injection molded parts. When the molded part does not satisfy a dimensional tolerance, it is useless as the final product. A major cause of part warpage is the residual stresses induced by unbalanced filling. When such residual stress has no chance to relax, the plastic parts will gradually warp upon ejection as time passes. For this reason, achieving balanced flow is the objective of the optimization scheme in this study.

Lam et al. [13, 14] have developed the flow path concept for cavity balancing. For plastic injection molding, flow path is defined as the path traced by a melt particle when it is first injected into a cavity until the mold cavity has been completely filled. It may be visualized as the trajectory from the injection gate to the extremities of the cavity. An automatic flow path generation routine was developed [15, 16]. For the part with uniform thickness, balanced flow is achieved if all flow paths are of equal length. However, equal flow path length cannot be achieved practically. Instead, the variation between the lengths of the flow path is adopted as a measure of the uniformity of fill. The lesser the variation between the lengths of the flow paths indicates that the more balanced is the flow. Thus, Lam and Jin [16] used the standard deviation of the flow path lengths as the objective function for gate location optimization to achieve a balanced flow. However, for the part with nonuniform thickness, the filling time for each element will vary even though the other conditions are the same for all elements. This means that the length of flow path is no longer proportional to the filling time, and the standard deviation of flow lengths cannot be used as the measurement of the uniformity of the fill pattern. It is, therefore, better to employ the standard deviation of filling times for all boundary nodes directly [16]. Later, Zhai et al. [8] employ injection pressure as a proxy to a balanced flow. The injection pressure for a design with a nonuniform flow pattern will be higher than that for a uniform flow pattern, as an unbalanced flow will lead to overpacking and thus, higher injection pressure. Therefore, uniform flow pattern can be achieved through minimizing injection pressure under constant injection rate.

Although the standard deviation of filling time describes the overall variation of the filling time and thus, uniformity of fill, it does not directly reflect the difference between the maximum and minimum boundary filling time. The difference between the maximum and minimum boundary filling time could also be employed to reflect the uniformity of fill and is used as an objective function in this paper. Thus, the gate location optimization based on the difference between the maximum and minimum boundary filling time can be stated as: Minimize

$$F(X) = \left(t_{\mathrm{BoundaryNode}}\right)_{\max} - \left(t_{\mathrm{BoundaryNode}}\right)_{\min} \tag{6}$$

Subject to

$$\begin{array}{l} G(X) \leq G_{\mathrm{clamp}} \\ X \in \Omega \end{array} \tag{7}$$

where F(X) is the objective function, $X=[x, y, z]$, x, y, z are the coordinates of the corresponding gate, Ω is the feasible search space, G is the calculated clamp force, and G_{clamp} is the maximum clamp force. The constraint function limits the clamp force below a reference value. $t_{\mathrm{BoundaryNode}}$ is the time when the polymer melt reaches a boundary node in a finite element model, and the subscript max and min refer to the maximum and minimum time of polymer melt reaching the boundary nodes, respectively. In a mold filling simulation, the time of resin flow reaching each boundary node can be determined by the control volume finite

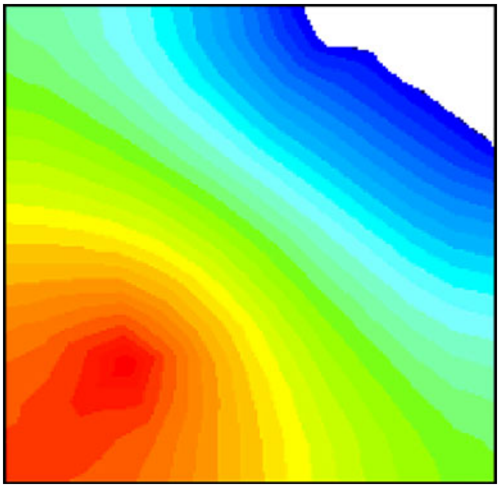

Fig. 1 Melt front advancement of initial design (at the time, t=0.619 s)

Table 1 Optimization results of the rectangular plate

		Initial value	Optimum value			Expected value
			SDT	PF	DT	
Coordinates of gate location	X(m)	0.050	0.100	0.099	0.099	0.100
	Y(m)	0.050	0.099	0.099	0.100	0.100
Warpage (mm)		0.76	0.68	0.68	0.68	0.68

element method. If the mold is filled evenly, the time difference for the boundary to be filled is minimized.

4 Optimization methodology

In this study, numerical optimization is combined with numerical simulation to search for the optimum gate location.

The optimization technique is primarily classifying into two search methods: the direct search method and the gradient-based method. The former uses only the value of the objective function to reach the minimum or maximum; the latter uses the gradients of the objective and constraint functions. The latter is generally considered superior to the direct method in its efficiency and effectiveness for most functional optimization problems. For this reason, the gradient-based optimization algorithm is used to search for the optimum gate location.

There are different ways of evaluating design sensitivities such as the direct differentiation method, the adjoint method, and the finite difference method. Finite difference approximation is the simplest technique for calculating derivatives of response with respect to a design variable. Though this technique is often computationally expensive, it is easy to implement and very popular. In this study, as the objective functions are normally highly nonlinear and implicit functions of design variable, we opt to use the first-order forward finite difference approximation to evaluate the design sensitivity.

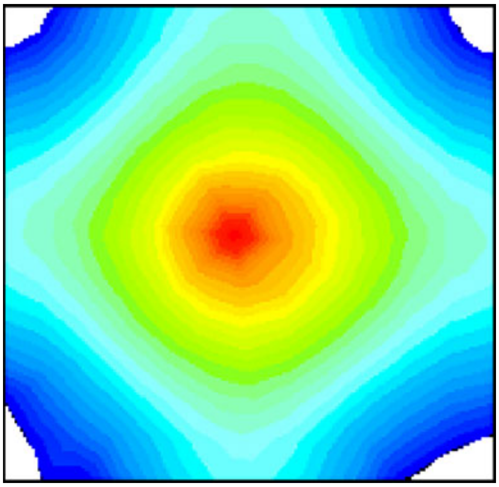

Fig. 2 Melt front advancement of optimum design (at the time, t=0.634 s)

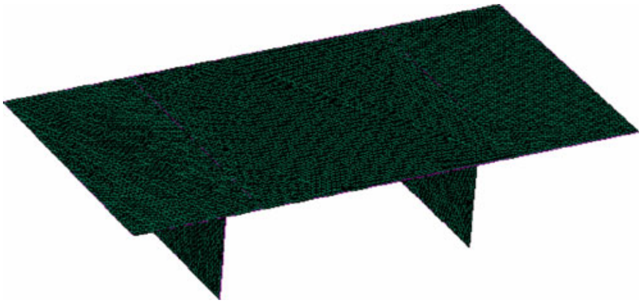

Fig. 3 Schematic diagram of a toy table

The gradient of the objective function is approximated with the forward finite difference method and can be expressed as follows:

$$\partial f/\partial x_i|_{x=x_0} = (f(x_0 + \Delta x_i) - f(x_0))/\Delta x_i \qquad (8)$$

where i=1 to n number of design variables.It should be noted that the finite difference method used for sensitivity analysis, although simple, suffers from two major drawbacks. Firstly, the accuracy of the approximation in the abovementioned equation depends on the magnitude of the perturbation, Δx_i. If Δx_i is too small, then round-off errors will be significant, and if Δx_i is too large, then truncation errors will degenerate the accuracy. Secondly, the use of finite difference method for the sensitivity analysis is expensive because for finding each new design, the finite element analysis program must run n+1 times to complete the sensitivity loop. This becomes especially more crucial when the number of design variables increase. The use of a more efficient sensitivity analysis method is currently under investigation.

In this study, SLP is used to search for the optimum gate locations. Consider an optimization problem of the form

$$\text{Minimize} f(\mathbf{x}) \qquad (9)$$

$$\text{Subjectto}\, g_{\mathrm{j}}(\mathbf{x}) \geq 0, \mathrm{j} = 1, ..., n_{\mathrm{g}} \qquad (10)$$

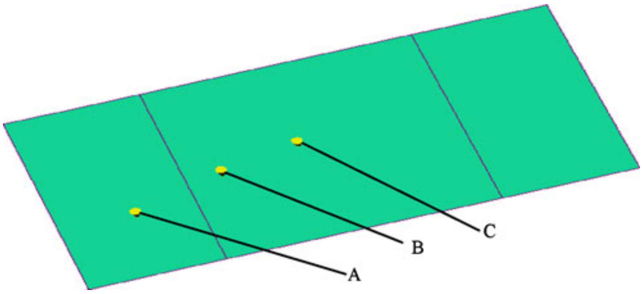

Fig. 4 Initial and optimum gate locations

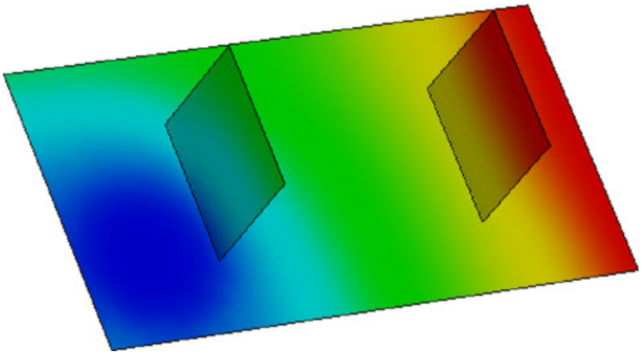

Fig. 5 Melt front advancement of initial design

where n_g is the number of the constraints. The SLP approach starts with a trial design $\mathbf{x}_0$ and replaces the objective function and constraints by linear approximations obtained from a Taylor series expansion about $\mathbf{x}_0$Minimize

$$f(x_0) + \sum_{i=1}^{n} (x_i - x_{0i})(\partial f/\partial x_i)_{x_0}$$

Subject to

$$g_j(\mathbf{x}_0) + \sum_{i=1}^{n} (x_i - x_{0i})\left(\partial g_j/\partial x_i\right)_{\mathbf{x}_0} \geq 0, j = 1, \ldots, n_g\, a_{li}$$

$$\leq x_i - x_{0i} \leq a_{ui}$$

whereas a_{li} and a_{li} are the lower and upper bounds, respectively, while on the allowed change in $\mathbf{x}_\mathrm{i}$, n is the number of the design variable.

Because of the approximation involved, it is rare that the final design of the linearized problem, $\mathbf{x}_\mathrm{L}$, is acceptably close to the optimum design. However, if the move limits are small enough to guarantee a good approximation within these move limits, $\mathbf{x}_\mathrm{L}$ will be close to the optimum than $\mathbf{x}_0$. We can, therefore, replace $\mathbf{x}_0$ by $\mathbf{x}_\mathrm{L}$ and repeat the linear optimization with Eq. (9) and (10) linearized about the new starting point.

5 Mesh adjustment

Because the coordinates of the gates are chosen as design variables and the gate locations are defined as continuous function of design variables to facilitate usage of gradient-based optimization algorithms, the gate locations obtained during the optimization process may not coincide with the nodes of the preassigned finite element mesh. The mesh should be adjusted accordingly.

Table 2 Optimized results of the rectangular plate

		Initial value	Optimum value			Expected value
			SDT	PF	DT	
Coordinates of gate location	X(m)	0.025	0.061	0.101	0.100	0.100
	Y(m)	0.030	0.045	0.051	0.049	0.050
Warpage (mm)		0.59	0.56	0.53	0.53	0.53

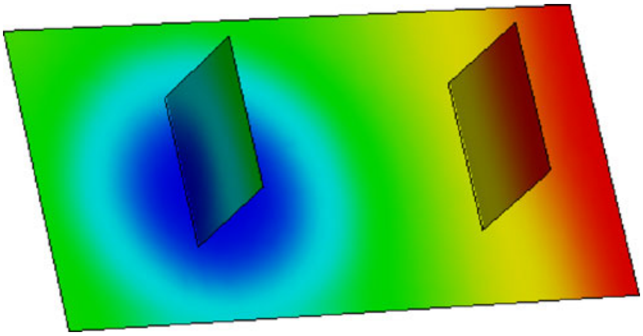

Fig. 6 Melt front advancement of optimum design with gate located at *B*

The gates are modeled as point sources in the interior of the cavity via concentrated nodal load during simulation. The pressure field at a point source is singular, so the error in the computed pressure solution depends on the geometry of the mesh connected to the gate node. In our analysis, the geometry of the finite element connected to the gate node is fixed throughout the design optimization to neutralize the effects of the errors introduced by the point source in such a process. The preassigned node nearest the gate location is moved to the gate location, and the adjacent nodes are moved in such way that the geometry of the elements connected to the gate node remains unchanged; distortion of the mesh near the gate is avoided. This mesh adjustment scheme avoids the regeneration of mesh and reduces the design effects on the singularity errors.

6 Examples

The performance of the proposed gate location optimization method is demonstrated by two examples. For both studies, PS (Styron 685, Dow Chemical), tool steel, and water are

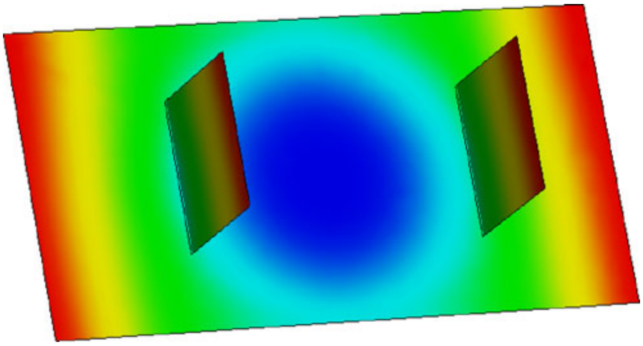

Fig. 7 Melt front advancement of optimum design with gate located at *C*

employed as the polymer material, mold material, and coolant, respectively.

6.1 Example 1

A square plate is chosen as the first example for verification purposes since the optimal gate location is known to be at the center of the plate. The part used in this example has dimensions of 0.2 m×0.2 m×0.002 m. The finite element modeling of the part creates 155 nodes and 266 triangular elements. The four mesh nodes corresponding to the corner of the plate are chosen as boundary nodes.

The initial gate (0.050, 0.050) is located at the lower left hand corner of the plate. The resin flow of this gate design is unbalanced, and polymer melt reaches the boundary of the part at different times (see Fig. 1). The part exhibits excessive warpage of 0.76 mm due to overpacking caused by unbalanced flow. SLP is employed to search for the optimal gate location, with standard deviation of filling time (SDT), pressure at end of fill (PF), and difference of the boundary filling time (DT) chosen as an objective function, respectively. The coordinates of the optimum gate location for different objective functions are shown in Table 1.

Table 1 indicates that the optimum gate locations obtained by the three objective functions are all very close to the expected location (0.100, 0.100), i.e., the center of the part with an error of about 0.001 m. The three objective functions are all effective for this example. Figure 2 shows the melt front advancement for design with gate located at (0.100, 0.100). It can be seen that melt reaches the four corners almost simultaneously. The uniformity of flow pattern leads to smaller warpage of about 0.68 mm, approximately 11% lower than that for the initial design (0.76).

6.2 Example 2

The second example considers a toy table. It has dimensions of 200×100×50 mm (Fig. 3). The finite element model of the part consists of 1,225 nodes and 2,288 triangular elements.

The initial gate is located at point A (0.025, 0.030) in Fig. 4. Melt advancement for the design with gate located at point A is shown in Fig. 5. It can be seen that the resin flow of this gate design is unbalanced and the molded part exhibits warpage of 0.59 mm. The proposed SLP is used to search for the optimal gate location with standard deviation of filling time (SDT), pressure at end of fill (PF), and difference of the boundary filling time (DT) employed as objective function, respectively. The optimum gate locations obtained by the different objective functions are shown in Table 2. From Table 2, it can be observed that different objective functions result in different optimal gate locations. The optimum gate obtained with SDT as objective function is approximately located at (0.061, 0.045), which is labeled as point B in Fig. 4; whereas, the optimal gate location for objective function of PF is very close to that for objective function of DT, which is near (0.100, 0.050) shown as point C in Fig. 4. Melt advancement for designs with gate located at points B and C is shown in Figs. 6 and 7, respectively. It can be observed that the fill pattern of the optimum design for the objective function of SDT is not as uniform as that achieved by employing the objective function of PF or DT. This is reflected by the slightly higher warpage for the objective function of SDT (0.56 mm) as compared with that of PF and DT (0.53 mm).

7 Conclusion

This paper is concerned with the gate location optimization to achieve a balanced flow for injection molding. The numerical simulation for injection mold filling process is combined with sequential linear programming to find the optimum gate by minimizing the difference of the maximum and minimum times of boundary filling. This optimization algorithm is as efficient as that adopting injection pressure as an objective function, but it can obtain better results when compared with adopting the objective function of standard deviation of filling time.

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