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ORIGINAL ARTICLE

Analysis of metal cutting acoustic emissions by time series models

Federico Polito · Alberto Petri · Giorgio Pontuale · Fergal Dalton

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Abstract We analyse some acoustic emission time series obtained from a lathe machining process. Considering the dynamic evolution of the process, we apply two classes of well known stationary stochastic time series models. We apply a preliminary root mean square (RMS) transformation followed by an auto regressive moving average analysis; results thereof are mainly related to the description of the continuous part (plastic deformation) of the signal. An analysis of acoustic emission, as some previous works show, may also be performed with the scope of understanding the evolution of the ageing process that causes the degradation of the working tools. Once the importance of the discrete part of the acoustic emission signals (i.e. isolated amplitude bursts) in the ageing process is understood, we apply a stochastic analysis based on point processes' waiting times between bursts and to identify a parameter with which to characterise the wear level of the working tool. A Weibull distribution seems to adequately describe the waiting times distribution.

Keywords Time series analysis · ARMA models · Weibull distribution · Point processes · Acoustic emission

1 Introduction

When a solid body is subjected to a varying stress, acoustic waves (i.e. pressure waves propagating inside of the body), often reaching very high frequencies, are generated. Examples of this phenomenon are the noise produced by hitting a metal block with a hammer or the creaking of a wooden floor; the waves propagate inside and on the surface of the materials before dissipating in the surrounding gaseous medium. The process of generating *acoustic waves* in stressed materials is called *acoustic emission* (AE). The AE can be recorded by means of a transducer (i.e. a sensor in contact with the solid body which transforms the elastic waves into electric signals). Usually, the emission of acoustic waves may also be associated with microfractures inside the solid or, in general, a degradation of its condition. Therefore, an analysis of the AEs can reveal the level of degradation of the solid.

In particular, we note that the study of AE is developing in the field of *tool condition monitoring* (TCM), where it is important, for example, to avoid damaging machinery and to maximise productive capacity. AE analysis is an easily implemented and economical technique and it allows real-time monitoring of working tool conditions.

Other statistical analyses have been previously conducted using the same experimental setup described in this article [11, 28, 29]. Results demonstrated that acoustic emission analysis is particularly relevant for the study of the ageing of work tools and the authors elucidated methods for the elicitation and reconstruction of the pdf of the root mean square amplitude signal.

In this study, we analyse some specific high-frequency AEs by means of time series statistical-probabilistic models: auto regressive moving average (ARMA) models [3, 4] and models for point processes [3, 9]. An objective of this study is to assess their

F. Polito (✉) · A. Petri · G. Pontuale · F. Dalton
Istituto dei Sistemi Complessi,
Consiglio Nazionale delle Ricerche, Rome, Italy
e-mail: federico.polito@isc.cnr.it

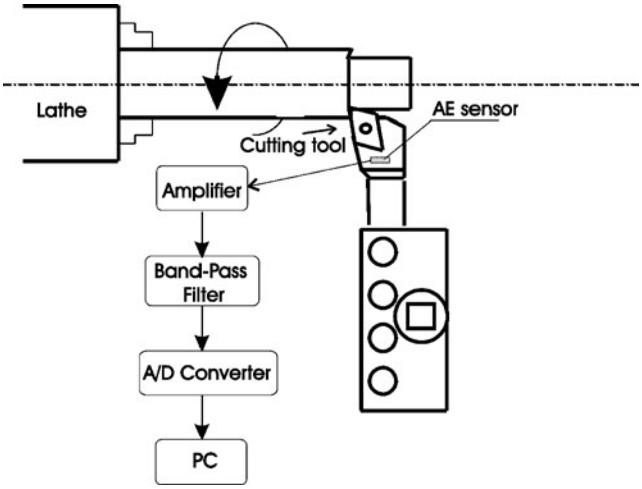

Fig. 1 The AEs were recorded by means of a transducer, preamplified, filtered, digitised and then stored

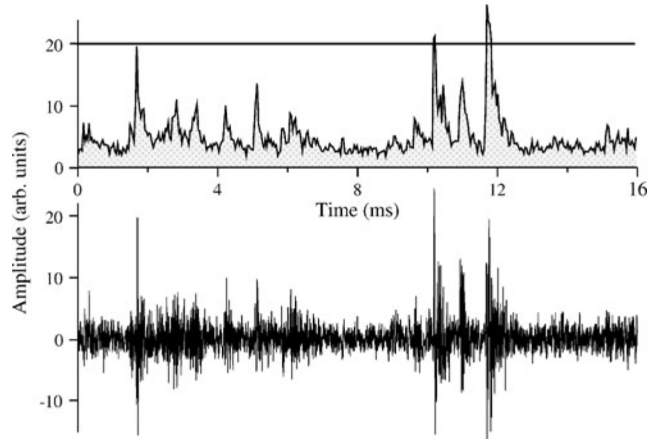

Fig. 2 A typical AE signal with its RMS transform above with a threshold at level 20

suitability for explaining AE phenomena and to establish if it might be possible to implement a tool wear monitoring algorithm.

The experimental setup (presented in Fig. 1) consisted of a mechanical lathe working on stainless steel bars and a transducer with which the resulting acoustic emissions were recorded. This signal was preamplified and filtered by a band-pass filter to isolate the frequencies of interest. The digitisation was performed by means of a digital oscilloscope with a sample frequency of $f_0 = 2.5$ MHz. The cutting speed ranged from 0.5 to 1 ms^{-1} and the cutting depth was 2 mm.

Three types of working tools were used for the analyses: new, partially and totally worn. For the new and totally worn tools, we conducted one acquisition with each of eight different tools. For the partially worn tools, only four acquisitions were conducted. For each acquisition, we recorded 15 AE time series, each composed of 40,960 consecutive points (i.e. 614,400 points for each acquisition). The time duration of a single time series is 16.38 ms. All analyses were conducted with the R environment for statistical computing ([31], http://www.R-project.org), a very powerful programming environment mainly used for statistical data analysis and modelling.

2 Data collected and preliminary analyses

When we take a close look at the collected time series that result from the experiments, it is possible to distinguish two different and superimposed parts of the signal. As we can see in the lower part of Fig. 2, there is a *continuous* part characterised by a relatively constant variability (essentially due to plastic deformation of the material) and a more interesting part composed of bursts of different and high amplitudes (usually associated with microfractures in the tool or with material splinters striking it).

The natural time scale of the cutting process and the short duration of each time series suggest that each time series will be effectively stationary; this is verified by the Dickey–Fuller test [14, par. 18.3.3].

Because of the large amount of data (due to the high sample rate necessary to capture the highest frequencies), it is necessary to operate a *data reduction* by means of a transform that has some physical meaning [17].

Definition 2.1 (Root mean square (RMS) values) Let x_s, $s \in \mathbb{N}$ be a time series taking values in $\mathbb{R}$ and T the number of samples in some interval over which the RMS y_t is to be calculated. We define

$$y_t = \sqrt{\frac{\sum_{j=1}^{T} x^2_{(t-1)T+j}}{T}}, \qquad t \in \mathbb{N}. \tag{1}$$

If a time series is of length N (multiple of T), then the RMS series $y_t \in \mathbb{R}$ has N/T points, each proportional to the acoustic energy emitted in the interval $(tT - T, tT]$. Our choice of T is motivated by examination of the AE spectral density function $\tilde{x}(f/f_0)$, which is naturally separable into lower and higher frequency regions by an almost zero density region centred at $f/f_0 \sim 5 \times 10^{-3}$. Furthermore, analysis of $\tilde{x}(f/f_0)$ on a moving window demonstrates that the low-frequency region is largely constant while the higher part depends critically on the presence or absence of bursts in the window.

As bursts are typically separated by the $O(10^4)$ samples, we conclude that the lower part of the spectrum is due to the process generating the series of bursts, whereas the higher part is due both to the dynamics of individual bursts and to the structure of the continuous part of the AE signal. In order to obtain a meaningful RMS, that is, to reduce the data to a small number of points characterising the AE noise level, we chose $T = 100$ corresponding to the spectral gap at $f/f_0 = 5 \times 10^{-3}$ (equal to a physical interval of 40 μs).

3 Results for ARMA models

We now subject this RMS series y_t to analysis to determine if some feature of the resulting model can be exploited to estimate the wear level. ARMA linear models are widely used for modelling stationary time series in general [5, 30, 32] and are very flexible with respect to real-world applications. Cohen and Berstein [7] have successfully discriminated normal and pathological patients using ARMA analyses on acoustic signals from the respiratory tract. Ho and Xie [15] have used a more generalised ARMA model to forecast failure in mechanical systems and Esquef et al. [10] have presented a review of acoustic applications of ARMA models, defining in detail a protocol for the analysis of acoustic spectral features. Some other examples may be taken from the fields of medicine [2], finance [13, 34], languages [26] and engineering [6, 17, 18, 20, 24, 25]. Previous research on machining by means of techniques related to time series analysis was conducted by Professor Wu and his team, and reported in several papers dating back to the late 1980s. Amongst others, we mention Kim et al. [19], Fassois et al. [12], Yang et al. [35], Ahn et al. [1] and Hu and Wu [16].

Although few naturally occurring processes are intrinsically linear, the aim of this section of the analysis is to understand if ARMA models are suitable for the purpose of representing AE signals, and if a linear approximation could give us enough information about the dynamical process itself.

Definition 3.1 (ARMA processes) Let us consider a real valued process $X_t, t \in \mathbb{N}$. It is called an ARMA (p, q) process (combining an AR (p) and MA (q) model) if

$$X_t = c_0 + \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q}, \quad (2)$$

where c_0 is the intercept and p and q are, respectively, the number of parameters in the autoregressive and moving average part of the process. The $p + q$ parameters $\phi_1 \cdots \phi_p, \theta_1 \cdots \theta_q$ must be chosen such that $|\phi_i| < 1, i \in \{1, \cdots, p\}$ and $|\theta_j| < 1, j \in \{1, \cdots, q\}$ to ensure stationarity and invertibility of the process. The process ϵ_t called innovations is taken to be a white noise (see, e.g. the classical text of Box et al. [4] or Brockwell and Davis [5]).

For each level of wear and for each recorded time series, we have conducted a full analysis following the Box–Jenkins iterative procedure [4]. The resulting best model is generally very simple; often, it has only three or four parameters. We can analyse the results, for example, for an RMS time series taken with a fully worn tool. At the right side of Fig. 3, we can see the autocorrelation function and the partial autocorrelation function of the series. Their shapes (the acf's decay is exponential and the pacf is zero after lag one) and an analysis of the spectral density function suggest a simple AR(1). Furthermore, if we apply the procedure

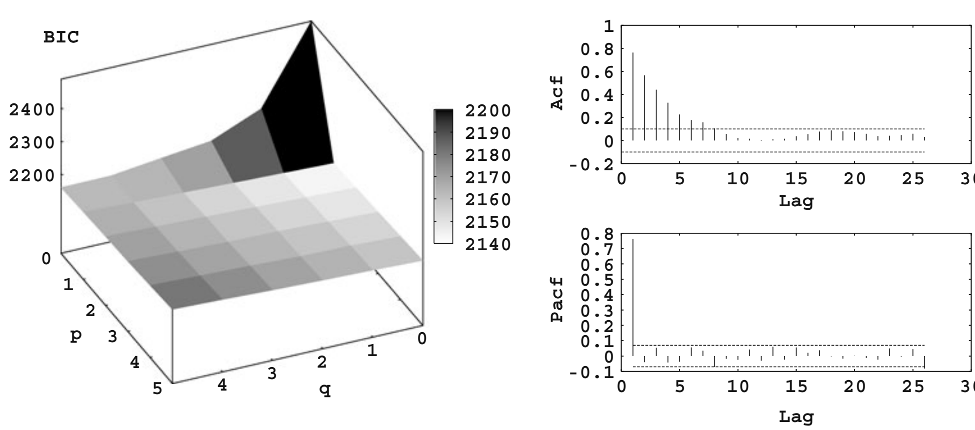

Fig. 3 BIC matrix and autocorrelation functions

in a completely automated manner [5, par. 9.3], the resulting best model is, indeed, an AR(1) model.

$$X_t = \hat{c}_0 + \hat{\phi} X_{t-1} + \hat{\epsilon}_t$$
$$= 7.5499 + 0.7632 X_{t-1} + \hat{\epsilon}_t, \quad (3)$$

where $\hat{c}_0$ is the estimated intercept, $\hat{\phi}$ is the sole estimated autoregressive parameter and $\hat{\epsilon}_t$ are the residuals (i.e. the estimated innovations). All the parameters are statistically significant.

Definition 3.2 (Bayesian information criterion (BIC)) We define the BIC for a model with $(p+q)$ parameters and N observations as

$$\mathrm{BIC}(p,q) = N \log \hat{\sigma}^2(p,q) + (p+q)\log N + - (N-p-q)\log\left(1 - \frac{p+q}{N}\right) + (p+q)\log\left(\frac{1}{p+q}\left(\frac{\hat{\sigma}_*^2}{\hat{\sigma}^2(p,q)}\right)\right), \quad (4)$$

where p is the number of parameters in the autoregressive part of the model, q is the number of parameters in the moving average part, $\hat{\sigma}^2(p,q)$ is the residuals variance calculated after having fitted an ARMA (p,q) model and $\hat{\sigma}_*^2$ is the sample variance of the observations (for details, see Priestley [30] page 375).

To better understand the behaviour of the BIC, it is useful to give the following representation also presented in Priestley [30]:

$$\mathrm{BIC}(p,q) = N \log \hat{\sigma}^2(p,q) + (p+q)\log N + o(p+q) \quad (5)$$

The BIC must thus be minimised with respect to (p,q) in order to select the model which best explains the observations with the minimum of parameters.

At the left side of Fig. 3, we can see the levelplot for the BIC matrix. For each combination of order (p,q) of the AR and MA parts, maximum likelihood estimates for the parameters are computed and BIC(p,q) is recorded on the matrix. The minimum BIC value indicates that the best model is that of Eq. 3.

Following the usual procedure to validate the model, we conduct an analysis of *whiteness* on the residuals. The autocorrelation and partial autocorrelation functions for the estimated residuals are both within the white noise confidence band for lags strictly positive, and the spectral density function is uniform. Whiteness tests in both the time and frequency domains (Ljung–Box for various lags and cumulative periodogram tests) were conducted with positive outcomes. This model therefore explains the whole linear dependence between the variables in the process.

At this point, we have a model which adequately describes the stationary part of the AE RMS signal. Furthermore, we can see that the mean of the residuals' variance decreases with increasing tool wear. However, the decrease of the residuals' variance (with respect to the wear level) is not due to the better explanation of the data by the model, but instead to the decreasing number of bursts in the time series. Therefore, ARMA models are more suitable for the description of the essentially continuous transformed signal part (the one due to plastic deformation).

4 Point processes

Taking a closer look at the residuals (Fig. 4), we note that bursts are still present, though with a smaller amplitude. Therefore, the bursts are not explained by the AR(1) model of the preceding section, and therefore not by any ARMA(p,q). This indicates a decrease in the mean burst frequency with increasing wear level. If we wish to understand the evolution of the AEs with respect to the wear level of the tool, we must take into account the dynamical and statistical process that generates the bursts.

The AE bursts are usually associated with microfractures in the work tool. They are, effectively, singular events (the exponential decay is due to the transducer response). Furthermore, the number of bursts seems to change with the wear level. Point processes are a tool widely used in modelling inherently point phenomena (see, e.g. Petri et al. [27], Lowen and Teich [22]). In this section, we consider the bursts as the outcome of a point process and try to understand the behaviour of this process as the wear

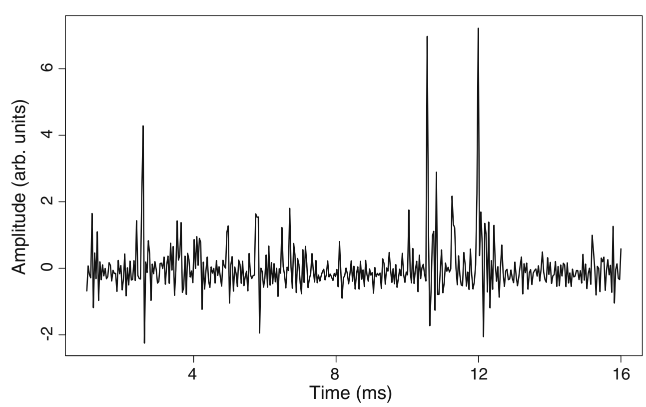

Fig. 4 A typical residuals time series after adapting an ARMA model. We note that bursts are still present

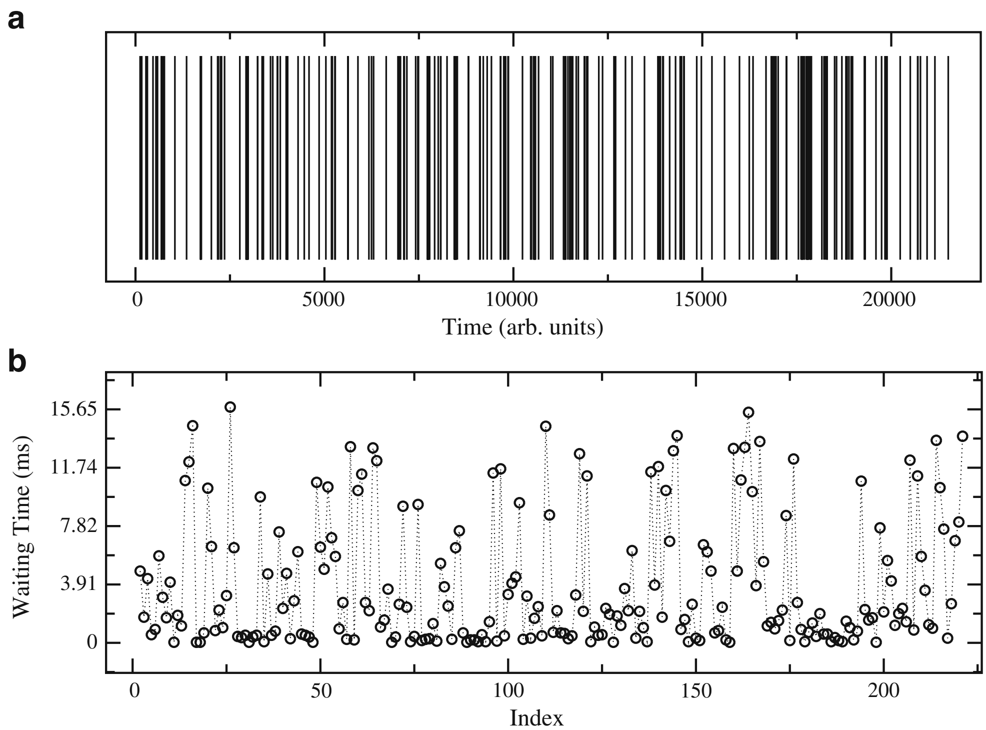

Fig. 5 **a** The sequence of burst events as a function of time. **b** The waiting time between events indexed by their order in the sequence of bursts

level increases. Figure 5 shows the overall point process for new tools and the observed waiting times between bursts. The waiting times between events are registered and presented in Fig. 5b indexed by their order in the sequence of bursts.

The identification of the events was performed by placing a threshold at various levels to obtain information about bursts of different amplitudes. In particular, we chose four levels of amplitude, 40, 50, 60 and 70. These thresholds are expressed in arbitrary units proportional to the signal's amplitude, which depends on the data collection chain. For an example, see Fig. 2, where the placement of a threshold at level 20 identifies two burst events in the RMS-transformed time series. The waiting times process is then calculated for all the thresholds considered and is, in effect, a renewal process [8]. It is possible to verify the lack of correlation (second-order dependence) in the waiting times process by calculating the spectral density function and the complete and partial autocorrelation functions.

The hypothesis of exponentially distributed waiting times (Poisson process) and Pareto (power-law) distributed waiting times (fractal point process), as in Thurner et al. [33] and Lowen and Teich [21], is addressed conducting Kolmogorov–Smirnov goodness-of-fit tests on the probability density function for all thresholds and wear levels. The tests resulted not significant, so another hypothesis for the distribution must be proposed.

As stated in Malevergne et al. [23], the Weibull distribution (also known as *stretched exponential*) can have (for typical parameters' values) both the features of an exponential distribution and a Pareto distribution:

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}}, \tag{6}$$

where β and α are, respectively, the scale and the shape parameter ($x \geq 0, \alpha, \beta > 0$).

When $\alpha = 1$, a Weibull is an exponential distribution with rate $\lambda = \frac{1}{\beta}$. This kind of distribution, therefore,

Table 1 Maximum likelihood estimates and P values of Kolmogorov–Smirnov tests

Thresh.	New			Worn			P value	
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\mu}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\mu}$	New	Worn
70	0.845	98.53	90.92	0.764	138.98	124.56	0.9907	0.3494
60	0.875	119.68	111.95	0.786	138.99	125.38	0.5507	0.2388
50	0.822	109.99	100.53	0.942	148.79	144.06	0.606	0.1154
40	0.754	83.1	74.27	0.907	138.90	132.02	0.3204	0.1055

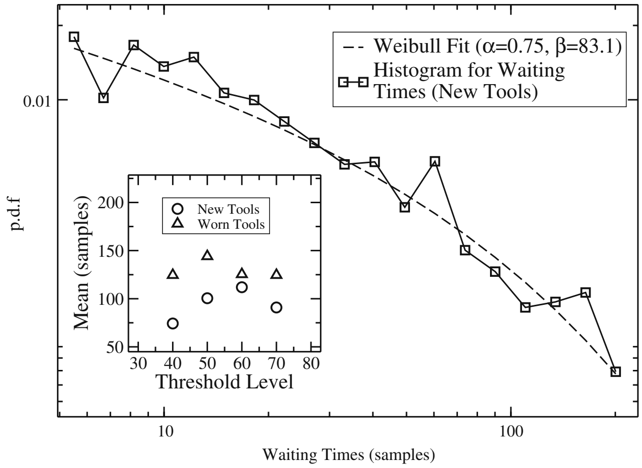

Fig. 6 Weibull fit for the pdf of the waiting times process. The mean inter-burst waiting time consistently increases with increasing tool wear

allows us to describe point processes that have waiting times following an *exponential behaviour*, but that can exhibit a heavier tail. Table 1 summarises the maximum likelihood estimates for the Weibull parameters $\hat{\alpha}$ and $\hat{\beta}$. We have calculated the estimates for each threshold and each wear level, finding the estimated mean $\hat{\mu}$, and performed a Kolmogorov–Smirnov test to see if the Weibull hypothesis can hold. As can be seen from the *P*-values in the table, the test is not significant for each level and each threshold. Table 1 contains only the estimates for the *new* and *totally worn* tools. The half-worn tool has sufficient data for this analysis only with the lowest threshold; the result is consistent with the other wear levels (i.e. $\hat{\mu} = 113.15$).

Figure 6 shows the maximum likelihood Weibull fit for the distribution of waiting times between the bursts (new tools, threshold equal to 40). The estimated parameters are $\hat{\alpha} = 0.75$ and $\hat{\beta} = 81.3$. In the inset, we note that, for all thresholds considered, the estimated mean of the waiting time increases with the tool wear level. For purposes of TCM, we could therefore monitor the mean of the estimated distribution to decide whether it is necessary to change the tool or not.

5 Conclusions

In this analysis, we have shown an application of time series models to AE signals from metal cutting processes. Before this work, few stochastic analyses were performed on this particular type of data. After transforming them by means of the RMS transform, we applied standard time series statistical techniques to explain the underlying process that generates the phenomenon under consideration. In particular, initially, we used linear ARMA models together with the well-known Box–Jenkins iterative procedure. We found that these linear models, with a small number of parameters, are suitable for the description of the linear contribution of the background part of the signal. The variance of the residuals decreases when the wear level increases, but this effect is due mainly to the decreasing number of bursts in the time series. Even though, for the purposes of TCM, we could, in principle, monitor that variance, we have obtained better results looking directly at the underlying mechanism generating the burst process.

A renewal point process with Weibull distributed waiting times seems to represent the burst process very well. In particular, for each threshold and each wear level, the Kolmogorov–Smirnov test results not significant. When the tool wear level increases, the tail of the distribution becomes heavier and the estimated mean of the waiting times process increases. The number of bursts is actually correlated with increasing tool wear; therefore, we could identify the estimated mean as the parameter that should be used to monitor the condition of the working tool (the tool will be substituted when the estimated mean becomes sufficiently high).

In conclusion, it is important to underline that, from a methodological point of view, both ARMA models and Weibull point processes are suitable for modelling the phenomenon (even though ARMA models are somewhat limited to explaining the linear contribution of the plastic deformation). What appears of importance is that the shape of the distribution of waiting times changes in the sense that the tail becomes heavier when the wear level increases. Therefore, the process creating the bursts seems to evolve in a fundamental manner.

In future work, it would certainly be very interesting to follow the evolution of the distribution shape along the whole life of the cutting tool to try to understand better the dynamical properties of the underlying process.

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