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ORIGINAL ARTICLE

Optimization of supply chains for single-vendor–multibuyer consignment stock policy with genetic algorithm

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Abstract In this article, we have developed four consignment stock inventory models of supply chain. The lead time is assumed to be dependent because, at the time of contract with the manufacturer, the retailer may intend to reduce the lead time for which the retailers pay an additional cost. The lead time of consignment stock strategy has been controlled to minimize joint total expected cost and simultaneously optimize other decision variables such as quantity transported, lead time, number of transport operations, and delay deliveries under stochastic environment so as to gain the competitive advantage in the business strategy. Numerical examples and sensitivity analysis are presented to illustrate the solution procedure.

Keywords Consignment stock · Single-vendor–multibuyer model · Supply chain · Genetic algorithm · Controllable lead time and crashing cost

1 Introduction

In today's globalized economy, business is looking for ways to optimize the supply chain (SC) network by means of integration and cooperation of network echelons. Inventory is one of the most widely discussed areas for improving SC efficiency. Wal-Mart and Procter & Gamble popularized it in the late 1980s. Since the holding of inventories in an SC can cost anywhere between 20% and 40% of product value, effective management of inventory is hence critical in SC operations [1]. In this environment, SC management (SCM) has become an effective business tool to reduce echelon inventory cost. Houlihan [2] is credited to be the first person for coining the term SC with insight concepts with a strong case for viewing the SC as a strategy for global business decisions. The SCM is generally viewed as a strategy for integrated network business that works together to source, produce, and ultimately distribute products and services to the customer at the right quantities, right place, and right time. Each echelon of SC performs independent business with integrated information sharing among all echelons, and it holds some inventories which may be unavoidable due to existing uncertainty in the business. In the area of inventory, one of the effective industrial approaches that is quickly gaining ground is the consignment stock (CS) inventory model. The CS of vendor-managed inventory in which vendor stocks his finished products in buyer's warehouse. The vendor will guarantee for the quantity stored in the buyer's warehouse that will be kept between minimum (s) and maximum (S) levels with supporting shortages in stochastic demand and lead time. For single-vendor–single-buyer case, the demand rate is assumed to be consistent, but this may be the reverse in the case of multiple buyers wherein scope of demand and lead time variation are quite evident.

The most radical application of CS approach leads to suppression of vendor inventory, as this party will use the buyer's warehouse to stock his finished products. CS with single-vendor–multibuyer model is viewed as a classification of divergent SC with end to multiend case. CS is a combination of push and pull system. The vendor adopts the push system whereas the buyer adopts the pull system. The change of ownership commences during the pull system.

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It is found in literature that a little research has been done in CS. The basic fundamental of CS is explained in Braglia and Zavanella [3], Valentini and Zavanella [4], Simone and Grubbström [5], and Srinivas and Rao [6]. Some of the terms are incorporated into CS inventory programs in which payment for supplier inventory is not released until goods have been sold at the customer location. CS policy is conveniently adopted in small size and less costly items. Typically, it is best suited for automobile components [5], fashion products, pharmaceutical, electronic, fast-moving consumer goods, retail items of super, and hypermarkets.

This paper is structured in six sections: Section 2 describes the literature review and the work done in the area of joint total expected cost (JTEC) of single vendor–single buyer and single vendor–multibuyers with controllable lead times, vendor-managed inventory, CS, and also the Hills inventory model. Section 3 will briefly present the genetic algorithm (GA) method used for single-vendor–multibuyer CS policy. In Section 4, analytical models of single-vendor–multibuyer inventory models of different CS models like CS policy without delay deliveries, CS with delay deliveries, CS with information sharing and with delay model, and CS with controllable lead time model are discussed. Section 5 gives the illustrative example and results, and Section 6 gives the conclusions and future studies.

2 Literature review

The general buying and payment mode includes various strategies [7], among them electronic fund transfer (EFT) and CS inventory are important. The EFT or e-cash facilitates consignment inventory programs with electronic payment on consumption initiated at the point of sale. In the CS inventory, one of the main objectives in buying and payment is to negotiate the most favorable payment terms. Some of the terms are incorporated into CS inventory programs in which payment for supplier inventory is not released until goods have been sold at the customer location. Corbett [8] is credited to be the first person to give about the fundamentals of CS policy whereas Valentini and Zavanella [4] presented an industrial case and performance analysis of CS policy for a single vendor–single buyer. Braglia and Zavanella [3] presented an analytical modeling approach which concerns the deterministic single vendor–single buyer, allowing the analyst to identify the optimal inventory level and shipment policy for minimizing total costs. Wee and Yang [9] developed an optimal pricing and replenishment strategies in a lean and agile SC-integrated system for single vendor–multibuyers. Piplani and Viswanathan [10] discussed supplier-owned inventory (SOI) which possesses the concepts of CS. They evaluated performance of policy and concluded that SOI arrangement is always beneficial for the SC as a whole. He could show that SOI would be beneficial to the buyer with assuming that they continue to pay the same price to suppliers, but he could not discuss its impact on suppliers and JTEC as a whole. Simone and Grubbström [5] extended the discussion by Braglia and Zavanella [3] by giving the explicit analytical expression of ordering quantity, number of shipments, and delay deliveries in two cases: vendor stock holding cost is higher to that of buyer stock holding cost (no delay), and vendor stock holding cost is less to that of buyer stock holding cost (maximum delay). In practical application of CS model, there will always be vendor stock holding cost < buyer stock holding cost because of the downstream movement of the product.

Pan and Yang [11] were credited for minimizing the joint total economic cost of vendor's and buyer's inventory model with controllable lead time which is a decision variable; however, shortages are not allowed in their paper. Ryu and Lee [12] analyzed the effect of investment strategies to control lead times. Liao and Shyu [13] decomposed lead time into n components each having a different crashing cost for reduced lead time. The lead time is the only decision variable in their model. They assume that the order quantity is predetermined. Ben-Daya and Raouf [14] considered both lead time and order quantity as decision variables. Their model uses different representations of the relationship between lead time crashing cost and lead time. Ouyang et al. [15] discussed integrated vendor–buyer model with stochastic demand to integrate production inventory model. The shortages are permitted, and it is assumed that the lead time is controllable with added cost so as to optimize ordering quantity. The lead time crashing component can be more than three components. It is in the interests of both parties involved to reduce the lead time as much as economically possible by a technique such as work study [16].

Most of the published papers has assumed a deterministic environment. When demand during the cycle time is not deterministic but stochastic, the system lead times become an important issue, and its control leads to some quantitative benefits. The system lead time [17] consists of order presentation, order transit, supplier lead time, delivery lead time, and setup time. Lead time crashing facilitates lower lead time and enables quick response and production line structuring. It also reduces inventories in SC and improves the coordination between different stages of the network. Whereas, in general problems, whenever the lead time reduces for either larger/or smaller demands for immediate delivery, companies may face stock out problems, but in the method proposed the stock out is eliminated/or minimized. Persona et al. [18] proposed an analytical model able to take into account the effects of obsolescence in an SC-based CS model. They used deterministic single-vendor–single-buyer CS model as a basis to develop the model. Results showed that the presence of obsolescence reduces the optimal inventory level, specifically for short-life components. Wee et al. [19] examined the total cost function of a single-vendor–multibuyer for deteriorating item inventory models and proposed a model to overcome the flaws existing in Yang and Wee [20] model. Yang et al. [21] developed a genetic-algorithm-based return policy for unsold items to enable the buyer to develop a supplier selection and replenishment policy.

Recently, Srinivas and Rao [6] extended and analyzed the models proposed by Braglia and Zavanella [3] and Ouyang et al. [15] for single-vendor–single-buyer inventory models with emphasis on crashing lead time. Their model suggests that CS with stochastic lead time reduction yields less JTEC. The literature review paper of Aytug et al. [22] and Chaudhury and Luo [23] reveals that no approach attempt has been made to develop a heuristic method such as GA to determine inventory levels in SC echelons. Daniel and Rajendran [24] studied GA, enumeration, and random search procedure methods to single-product serial SC operating with a base stock periodic review system to optimize the base stock inventory levels in SC so as to minimize total SC cost, comprising holding and shortage costs at all installations in SC. They found that the solution generated by proposed GA is not significantly different from the optimal solution yielded by complete enumeration, but it is significantly good for deterministic replenishment lead times and other with random replenishment lead times. They did not check for multibuyer stochastic demand and lead time models.

This paper addresses the problem of CS in SC to minimize JTEC for a single-product, single-vendor–multi-buyer model. It is an extension of the study of Srinivas and Rao [6]. To simplify the analysis, we have assumed that there is only one entity per tier. The authors tested both enumeration technique as well as GA. The former took more CPU time (more than a couple of hours) for more than three buyers with five process variables, and with the latter method, the results were yielded in less than 20 s of CPU for all models. The running time of enumeration technique grows exponentially [25] while increasing the number of variables. Hence, we have restricted it to GA method and applied this mode up to ten buyers.

2.1 Summary of necessary notations used in this paper

s	batch setup cost (\$; vendor)
A_t	order emission cost (\$; buyer)
h_v	vendor stock holding cost (\$) per unit per unit time
h_b	buyer stock holding cost (\$) per unit per unit time
p	vendor production rate (continuous)
D	demand rate in units per unit time seen by the buyer (continuous)
σ	standard deviation of demand for each buyer per unit time
π	unit back order cost (\$) for the buyer
L_g	length of the lead time component, g=1 to 4
C_L	lead time crashing cost (\$) per cycle
k^1	delay deliveries (<n)
ϕ	normal probability density function
$\Phi_{(z)}$	cumulative distribution function
n	number of transport operations/production batch
m	delayed deliveries shifted to different buyers ($\leq k^1$)
q	shifted delayed deliveries among ith buyer to jth different buyer, $\sum q_{ij} = m$ and $i \neq j$
y	buyer range
i	buyer, i=1, 2, …. y (maximum buyer size is ten)
z	safety factor
c	cycle time

2.2 Assumptions used to develop the proposed models

1. Single-product (one setup for vendor) flow with continuous review inventory replenishment system over an infinite horizon for single vendor–multiple buyers
2. Buyer and vendor carrying cost is independent of quantity transported but proportional to the holding time
3. Demand rate and the delivery lead time for each buyer are continuous variables with known stationary probability distributions, and demand is normally distributed
4. Shortages during the lead time is permitted on the basis of fixed cost
5. If demand exceeds on hand inventory, it is considered as shortage
6. $p \geq \sum D$ and $n_i \geq 1 \; \forall \; i$

3 Genetic algorithm: an introduction

We propose a GA approach to optimize the CS-based inventory models JTEC in SC. This study attempts to perform both performance analysis and optimization of various inventory policy settings. GA is a class of evolutionary algorithms that utilize the theories of evolution and natural selection. GA begins with a population of randomly generated strings that represent the problems' possible solutions. Thereafter, each of these strings is evaluated to find its fitness. The initial population is subjected to genetic evolution to procreate the next generation of candidate solutions [26]. The members of the population are processed by GA operators such as reproduction, crossover, and mutation to create the progenies for the next generation of candidate solutions. The progenies are then evaluated and tested for termination until a satisfactory solution (based on the acceptability or search stoppage criterion) already at hand is found, and the search is stopped.

3.1 Working mechanism of GA

1. Encode the initial chromosome
2. Initialize a set of feasible solutions randomly (i.e., initialize a population of chromosomes)
3. Compute fitness value $f_t = \frac{1}{1+\mathrm{JTEC}_{\left(n,k^1,m,q,c,L_g,\Phi(z)\right)}} \in$ chromosome $\forall$ population.
4. Select chromosomes for reproduction by making use of the roulette wheel selection procedure and fitness function value
5. Apply crossover and mutation on the selected chromosomes to produce new chromosomes
6. Compute the fitness value if the stopping condition is reached (≤500 generations), return the best solution; if not, go to step 4

GA works on a population or collection of solutions to the given problem. Each individual in the population is called chromosome. Designing chromosome is a very important step in GA, which contains decision variables that are to be optimized.

The chromosome structures for various models are summarized below.

Basic CS model	$(c,\ n,\ \Phi_{(z)})$
CS with delay model	$(c,\ n,\ k^1, \Phi_{(z)})$
CS with information-sharing and delay model	$(c,\ n,\ k^1,\ m,\ q,\ \Phi_{(z)})$
CS-LT model	$(c,\ n,\ \Phi_{(z)})$

Binary coding is used for c and "$\Phi_{(z)}$," for converting into binary coding, first multiplied with 1,000 to remove decimal point and then converted to binary again after crossover and mutation and again divided by 1,000, whereas integer encoding is used for (n, k^1, m, q). The population size is fixed as 150; crossover rate and mutation rate for the proposed GA are fixed by conducting a pilot study with different combinations of probability of crossover (pc) from 0.7 to 0.8 and probability of mutation (pm) of 0.05 with respect to four different CS policies. The number of generations is fixed as 500. Crossover is known as the recombination and exchanges information among the strings present in the mating pool and creates new strings. In crossover, two strings are picked from the mating pool, and some portions of these strings are exchanged between them, attempting to produce new strings of superior fitness by effecting large changes in a string to jump in search of the optimum in the solution space.

An example of chromosome for one-vendor–four-buyer CS with delay model is given below.

Chromosome encoding:

	c	n_1	n_2	n_3	n_4	k_1^1	k_2^1	k_3^1	k_4^1	$\Phi(z)$
1st parent chromosome	0.112	5	6	4	2	3	5	1	1	0.980
2nd parent chromosome	0.122	8	4	5	6	7	2	2	2	0.862

Crossover:

Parent strings before crossover

binary coding			*integer coding*			*binary coding*		
1 1	1 0 0	0 0	5 6 4	2 3 5 1	1	1	1 1 1 0 1 0 1	0 0
1 1	1 1 0	1 0	8 4 5	6 7 2 2	2	1	1 0 1 0 1 1 1	1 0

Offsprings after two point crossover operator

1 1	1 1 0	0 0	5 6 4	6 7 2 2	1	1	1 0 1 0 1 1 1	0 0
1 1	1 0 0	1 0	8 4 5	2 3 5 1	2	1	1 1 1 0 1 0 1	1 0

After decoding:

	c	n_1	n_2	n_3	n_4	k_1^1	k_2^1	k_3^1	k_4^1	$\Phi(z)$
1st child chromosome	0.120	$\boxed{5}$	6	4	6	$\boxed{7}$	2	2	1	0.860
2nd child chromosome	0.114	8	$\boxed{4}$	5	2	3	$\boxed{5}$	1	2	0.982

Variables in the offspring after crossover may cross the permissible independent boundary range. It is found from the first offspring (see the rounded value) after decoding, n_1=5, $k_1^1 = 7$ and in the second offspring, n_2=4, $k_2^1 = 5$. But the constraint is $n_1 > k_1^1$ and $n_2 > k_2^1$. As the constraints are violated in both the offsprings, repair function is hence used to correct these defective chromosomes.

Repair function In the first offspring k_1^1, value is replaced by the corresponding k_1^1 value of the second offspring. If k_1^1 value in the second offspring is also greater than n_1, then randomly substitute k_1^1 with a value less than n_1.

Offsprings after repair:

	c	n_1	n_2	n_3	n_4	k_1^1	k_2^1	k_3^1	k_4^1	$\Phi(z)$
1st child chromosome	0.120	5	6	4	6	3	2	2	1	0.860
2nd child chromosome	0.114	8	4	5	2	3	2	1	2	0.982

Mutation The need for local search around a current solution also exists and is accomplished by mutation. Mutation is additionally aimed to maintain diversity in the population. Mutation creates a new solution in the neighborhood of a current solution by introducing a small change in some aspect of the current solution and helps to ensure that no point in the search space has a zero probability of being examined. For binary coding, normal swap mutation operator is used. All bits in binary number is mutated with pm=0.05; for this purpose, a uniform random number is generated between 0 and 1; if number is less than pm, then that bit is changed from 0 to 1 or vice versa. For integer, coding genes in parent population are mutated with pm=0.05, with sampling a uniform random number, u. If $u \le$ pm, then the value of the corresponding gene is altered as given below:

$$S_{\text{new}} = S_{\text{old}}\,(1 - x) + 2\,x\;u\;S_{\text{old}} \tag{1}$$

Where S_{new} is new gene after mutation; S_{old} is gene before mutation; u is a uniform random number between 0 and 1 and x denotes the fraction of S_{old}. In this study, x is set to 0.2 after experimentation. It is to be noted that if the computed S_{new} takes a noninteger value, then it is to be rounded off to the nearest integer. After mutation, if any damaged genes exist, the same repair function as discussed in crossover is used to repair the damaged genes. After crossover and mutation, the new population is called a child population. We have now N chromosomes in the initial population and N chromosomes in the parent population. The best N chromosomes, among available 2 N chromosomes in the initial and parent population put together, with respect to JTEC are chosen for entry into the parent population as the surviving chromosomes for the next generation.

3.2 Optimum GA parameters

Population size, number of generations, pc, and pm are the GA parameters. A large population size means a better exploration of the search space, while a large number of generations allows for better exploitation of the promising solutions found. Generally, the larger these parameters are, the better the algorithm will perform, but at the expense of longer run times because more fitness evaluations will be involved. Population size is fixed as 150, and the number of generations is fixed as 500 after experimentation. pc varied from 0.5 to 1 with step of 0.1, and optimum value was found at 0.7. pm is varied from 0.05 to 0.15 in steps of 0.01 and finally fixed at 0.05 as it is giving minimum total cost.

4 Vendor–buyer inventory models

4.1 Consignment stock model

In this model, vendor use buyer's warehouse for keeping the goods produced by the vendor without changing the ownership. To fulfill this concept, the vendor should be close to the buyer production line. This creates a condition of shared benefit, neither the vendor nor the buyer will benefit until the product is sold to an end user. This shared-risk benefit condition will often be enough to convince the buyer to stock the products. The key benefit to the buyer should be obvious that the buyer does not have to tie up capital $h_{\text{b, finance}}$ (buyer financial capital). This does not mean that there is no inventory carrying costs for the buyer; he does still incur costs $h_{\text{b, stock}}$ (buyer inventory stock carrying cost) related to storing and managing the inventory, i.e., both parties incur holding cost, depending on different rates and the length of time for which materials have been stocked in SC. Finally, the buyer sees a lower inventory cost per unit, i.e., only $h_{\text{b, stock}}$ instead of the entire $h_{\text{b, stock}} + h_{\text{b, finance}}$. The vendor will have setup cost and holding cost whereas the buyer will have order emission cost and holding cost.

The expected shortage quantity at the end of the cycle is given by

$$E(X) = \int_R^\infty (X - R)dF(X) = \sigma\sqrt{L}\Psi(z), \quad \forall X > R$$

Considering the safety factor z as a decision variable instead of reorder point R where $\Psi(z) = \phi(z) - z[1 - \Phi(z)]$, and $\varphi(z)$ is the standard normal probability density function $\phi(z) = 0.399e^{-\frac{z^2}{2}}$, and Φ(z) is the cumulative distribution function.

The average total cost for CS model is:

$$T_{\mathrm{C}}^{\mathrm{CS}} = \text{vendor set up cost} + \text{average vendor holding cost} + \text{buyer ordering cost} + \text{average buyer holding cost} + \text{safety stock cost} + \text{shortage cost}$$

$$T_C^{\mathrm{CS}} = \left(\frac{s}{c}\right) + h_{\mathrm{v}}\frac{c}{2p}\left(\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right) + \frac{1}{c}\left(\sum_{i=1}^{y} n_i A_{ti}\right) + \sum_{i=1}^{y}\left(\frac{h_{\mathrm{b}i}}{2}\left\{D_i c - (n_i-1)D_i\left[\frac{D_i c}{n_i p} + \sum_{i\neq j}\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right) + \left(\sum_{i=1}^{y}\left(h_{\mathrm{b}i} z\sigma_i\sqrt{L_g}\right)\right) + \left(\frac{1}{c}\sum_{i=1}^{y}\pi_i\sigma_i\sqrt{L_g}\Psi(z)\right) \tag{2}$$

Equation 2 is modified as

$$T_{\mathrm{C}}^{\mathrm{CS}} = \frac{1}{c}\left(s + \sum_{i=1}^{y} n_i A_{ti}\right) + h_{\mathrm{v}}\frac{c}{2p}\left(\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right) + \sum_{i=1}^{y}\left(\frac{h_{bi}}{2}\left\{D_i c - (n_i-1)D_i\left[\frac{D_i c}{n_i p} + \sum_{i\neq j}\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right) + \left(\sum_{i=1}^{y}\left(h_{bi} z\sigma_i\sqrt{L_g}\right)\right) + \left(\frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\Psi(z)\right)\right) \tag{3}$$

Equation 3 is further modified as

$$T_{\mathrm{C}}^{\mathrm{CS}} = \frac{1}{c}G + Hc + \sum_{i=1}^{y}\left(h_{bi} z\sigma_i\sqrt{L_g}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\Psi(z)\right) \tag{4}$$

where $G = s + \sum_{i=1}^{y}(n_i A_{ti})$

$$H = h_{\mathrm{v}}\frac{1}{2p}\left[\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{h_{bi}}{2}\left\{D_i - (n_i-1)D_i\left[\frac{D_i}{n_i p} + \sum_{i\neq j}\frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right]\right\}\right)$$

Taking the minimum cost for optimum values of (c, n, and z) and the derivative of joint total cost in Eq. 4 *w.r.t* 'c' and equating it to zero gives

$$0 = -\frac{1}{c^2}G + H + 0 - \frac{1}{c^2}\sum_{i=1}^{y}\left\{\pi_i\sigma_i\Psi(z)\sqrt{L_g}\right\}$$

$$\frac{1}{c^2}\left[G + \sum_{i=1}^{y}\left(\pi_i\sigma_i\Psi(z)\sqrt{L_g}\right)\right] = H$$

$$c = \sqrt{\frac{G + \sum_{i=1}^{y}\left(\pi_i\sigma_i\Psi(z)\sqrt{L_g}\right)}{H}}$$

After substituting the value of c and Ψ (z) in Eq. 4,

$$T_c^{CS} = \frac{1}{c}G + cH + \sum_{i=1}^{y} zh_{bi}\sigma_i\sqrt{L_g} + \frac{1}{c}\sum_{i=1}^{y}\left[\pi_i\sigma_i\sqrt{L_g}\{\phi(z) - z(1-\Phi(z)\}\right]$$
$$T_c^{CS} = \frac{1}{c}G + cH + \sum_{i=1}^{y} zh_{bi}\sigma_i\sqrt{L_g} + \frac{1}{c}\sum_{i=1}^{y}\left[\pi_i\sigma_i\sqrt{L_g}\phi(z) - z\pi_i\sigma_i\sqrt{L_g} + z\pi_i\sigma_i\sqrt{L_g}\Phi(z)\right]$$

Taking partial derivative of Eq. 4 with respect to z and equating it to zero gives

$$0 = 0 + 0 + \sum_{i=1}^{y} h_{bi}\sigma_i\sqrt{L_g} + \frac{1}{c}\sum_{i=1}^{y}\left[0 - \pi_i\sigma_i\sqrt{L_g} + \pi_i\sigma_i\sqrt{L_g}\Phi(z)\right]$$
$$0 = \sum_{i=1}^{y}\left(h_{bi}\sigma_i\sqrt{L_g}\right) - \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\Phi(z)\right)$$
$$\frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\Phi(z)\right) = \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\right) - \sum_{i=1}^{y}\left(h_{bi}\sigma_i\sqrt{L_g}\right)$$
$$\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\Phi(z)\right) = \sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\right) - c\sum_{i=1}^{y}\left(h_{bi}\sigma_i\sqrt{L_g}\right)$$
$$\Phi(z) = \frac{\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\right) - c\sum_{i=1}^{y}\left(h_{bi}\sigma_i\sqrt{L_g}\right)}{\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\right)}$$
$$\Phi(z) = 1 - \frac{c\left[\sum_{i=1}^{y}\left(h_{bi}\sigma_i\sqrt{L_g}\right)\right]}{\left[\sum_{i=1}^{y}\pi_i\sigma_i\sqrt{L_g}\right]}$$

For values of (c, z, n), the minimum JTEC is

$$T_c^{CS} = \frac{G}{\sqrt{\frac{G+\sum_{i=1}^{y}\left(\pi_i\sigma_i\Psi(z)\sqrt{L_g}\right)}{H}}} + H\sqrt{\frac{G+\sum_{i=1}^{y}\pi_i\sigma_i\Psi(z)\sqrt{L_g}}{H}} + \sum_{i=1}^{y}\left(h_{bi}z\sigma_i\sqrt{L_g}\right) + \frac{\sum_{i=1}^{y}\left(\pi_i\sigma_i\Psi(z)\sqrt{L_g}\right)}{\sqrt{\frac{G+\sum_{i=1}^{y}\left(\pi_i\sigma_i\Psi(z)\sqrt{L_g}\right)}{H}}} = \frac{G}{\sqrt{\frac{G+M_1}{H}}} + H\sqrt{\frac{G+M_1}{H}} + M_2 + \frac{M_1}{\sqrt{\frac{G+M_1}{H}}}$$

where $M_1 = \sum_{i=1}^{y}\left(\pi_i\sigma_i\Psi(z)\sqrt{L_g}\right)$ and $M_2 = \sum_{i=1}^{y}\left(h_{bi}z\sigma_i\sqrt{L_g}\right)$

$$= \sqrt{\frac{H}{G+M_1}}[G+M_1] + H\sqrt{\frac{G+M_1}{H}} + M_2$$
$$= \sqrt{\frac{H}{G+M_1}}[G+M_1] + \sqrt{H}\sqrt{G+M_1} + M_2$$
$$= \sqrt{H}\sqrt{G+M_1} + \sqrt{H}\sqrt{G+M_1} + M_2$$
$$= 2\sqrt{H[G+M_1]} + M_2$$

Substituting M_1 and M_2 values in the above equation gives

$$T_c^{CS} = 2\sqrt{H\left(G + \sum_{i=1}^{y}\pi_i\sigma_i\psi(z)\sqrt{L_g}\right)} + \sum_{i=1}^{y}\left(h_{bi}z\sigma_i\sqrt{L_g}\right) \quad (5)$$

Maximum level of inventory (I) for buyer i is

$$I^{i}_{\max.} = \left\{ D_i c - (n_i - 1) D_i \left[\frac{D_i c}{n_i p} + \sum_{i \neq j} \frac{D_j c}{n_j p} \left(\frac{n_j}{n_i} \right) \right] \right\} + z\sigma_i \sqrt{L_g} \tag{6}$$

4.2 Consignment stock with delay delivery (CS-k^1)

The CS model is not suitable for limited/small periods because the maximum level of buyer's inventory may reach even for limited periods. Hence, CS model with delayed delivery period (CS-k^1) is preferred for limited periods. In CS-k^1 model, the last delivery is delayed until there is no longer an increase in the maximum level already reached. That means, we have to delay the delivery always whenever maximum level inventory stock is reached. The average joint total cost in this model is:

$$T_{\mathrm{C}}^{\mathrm{CS}-k^1} = \frac{1}{c} G + c\,E + \sum_{i=1}^{y} \left(h_{bi} z \sigma_i \sqrt{L_g} \right) + \frac{1}{c} \sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_g} \Psi(z) \right) \tag{7}$$

where

$$E = h_{\mathrm{v}} \left\{ \frac{1}{2p} \left[\sum_{i=1}^{y} \frac{D_i^2}{n_i} \right] + \sum_{i=1}^{y} \left(\frac{D_i}{n_i} \frac{(p - D_i)}{n_i p} \frac{(k_i^1 + 1)}{2} k_i^1 \right) \right\} + \sum_{i=1}^{y} \left\{ \frac{h_{bi}}{2} \left\{ (n_i - k_i^1) \frac{D_i}{n_i} - (n_i - k_i^1 - 1) D_i \left[\frac{D_i}{n_i p} + \sum_{i \neq j} \left(\frac{D_j}{n_j p} \left(\frac{n_j}{n_i} \right) \right) \right] \right\} \right\}$$

the minimum cost for optimum values of (c, n, k^1, and z) will be

$$T_{\mathrm{C}}^{\mathrm{CS}-k^1} = 2 \sqrt{ \left(G + \sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_g} \Psi(z) \right) \right) E } + \sum_{i=1}^{y} \left(h_{bi} z \sigma_i \sqrt{L_g} \right) \tag{8}$$

Maximum inventory level for buyer is

$$I^{i}_{\max.} = \left\{ (n_i - k_i^1) \frac{D_i c}{n_i} - (n_i - k_i^1 - 1) D_i \left[\frac{D_i c}{n_i p} + \sum_{i \neq j} \left(\frac{D_j c}{n_j p} \left(\frac{n_j}{n_i} \right) \right) \right] \right\} + z\sigma_i \sqrt{L_g} \tag{9}$$

Equation 9 ensures that not less than a single delay has been delayed. When $k_i^1 = 0$, Eq. 9 becomes the maximum level of buyer's stock in basic CS model Eq. 6, and when $k^1 = (n-1)$, Eq. 9 matches with maximum level of buyer's stock of Hill [27] model in which maximum buyer inventory is equal to nq, where q is the quantity transported per delivery. The delay delivery strategy is much explained in Simone and Grubbström [5]. They also provided a quick method for calculating the optimal total number of deliveries and number of deliveries to be delayed.

4.3 Consignment stock with information sharing and with delay (CS-IS-k^1)

Goyal [28] is credited to be the first person to describe integrated models of single vendor–single buyer. Goyal [29] proposed a joint expected lot size model to minimize total relevant costs, which is compared with total costs incurred if vendor and buyer act independently. Banerjee [30] generalized Goyal's [29] model by assuming that vendor with finite rate produces for a buyer on a lot-for-lot basis under deterministic conditions. Goyal [31] generalized Banerjee's [30] model by relaxing the assumption of the lot-for-lot policy of the vendor. In an integrated inventory model, one partner's gain exceeds the other partner's loss. Therefore, the net benefit can be shared in some equitable fashion [32]. They also summarized the literature on integrated vendor–buyer models up to 1989. Yang et al. [33] proposed an integration of vendor–buyer inventory approach based on just-in-time concept to the success of SCM by minimizing the joint inventory cost.

CS with partial information-sharing model includes information of demand, shipments, and inventory. It is known that partial information sharing benefits the vendor more compared with buyer due to reduction in vendor inventory and also due to adjusted shipments between buyers; otherwise, the vendor may have to keep (see Table 2 in Section 5). In this view, SC is constructed in such a way that if buyer does not need a particular scheduled delivery lot, the vendor finds an alternate buyer in the SC network. To fulfill this, the vendor adjusts the exact delivery quantity required to the different alternate buyers, i.e., the shifted quantity should be equal to scheduled quantity of different alternate buyers.

The average total cost in this model is

$$T_{\mathrm{C}}^{\mathrm{IS}} = \tfrac{1}{c} G + Uc + \sum_{i=1}^{y} \left(h_{bi} z \sigma_i \sqrt{L_g} \right) + \tfrac{1}{c} \sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_g} \Psi(z) \right) \tag{10}$$

where

$$U = h_{\mathrm{v}}\left\{\frac{1}{2p}\left[\sum_{i=1}^{y}\frac{D_i^2}{n_i}\right] + \sum_{i=1}^{y}\left(\frac{D_i}{n_i}\frac{(p-D_i)}{n_i p}\frac{\left(k_i^1 - m_i + 1\right)}{2}\left(k_i^1 - m_i\right)\right)\right\}$$
$$+\sum_{i=1}^{y}\left\{\frac{h_{\mathrm{b}i}}{2}\left\{\left(n_i - k_i^1 + \sum_{i\neq j} q_{ij}\right)\frac{D_i}{n_i} - \left(n_i - k_i^1 - 1 + \sum_{i\neq j} q_{ij}\right)D_i\left[\frac{D_i}{n_i p} + \sum_{i\neq j}\left(\frac{D_j}{n_j p}\left(\frac{n_j}{n_i}\right)\right)\right]\right\}\right\}$$

From Eq. 10, the minimum total cost for optimum values (c, n, k^1, m) is calculated as

$$T_{\mathrm{C}}^{\mathrm{IS}} = 2\sqrt{\left(G + \sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\Psi(z)\right)\right)U} + \sum_{i=1}^{y}\left(h_{bi}z\sigma_i\sqrt{L_g}\right) \tag{11}$$

Maximum level of inventory for buyer i is

$$I_{\max.}^{i} = \left\{\left(n_i - k_i^1 + \sum_{i\neq j} q_{ij}\right)\frac{D_i c}{n_i} - \left(n_i - k_i^1 - 1 + \sum_{i\neq j} q_{ij}\right)D_i\left[\frac{D_i c}{n_i p} + \sum_{i\neq j}\left(\frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right)\right)\right]\right\} + z\sigma_i\sqrt{L_g} \tag{12}$$

4.4 Consignment stock with controllable lead time (CS-LT) model

In this model, the vendor will negotiate with a buyer closely to reduce lead time as much as possible down to a point where it is acceptable to the buyer with his stable production and delivery schedule. The inventory is reviewed continuously, and shortages are allowed with full backorder. It should be noted that the delivery lead time is null; however, the batch is to be produced, so that there exists a system lead time other than zero. Adding an additional cost the system lead time can be controlled. Thus, the system lead time is crashed one at a time, starting from first independent component because it has minimum unit crashing cost per unit time and then the second independent component and so on. It is clear that, when lead time is reduced, its corresponding handling cost for that time is reduced. The length of lead time ensures the order transit arrival even though lead time is crashed and shortages if any are permitted and backordered. Since lead time is a decision variable in this model, the extra costs incurred by the vendor will be fully transferred to the buyer if shortened lead time is requested, which can be viewed as an investment. The total lead time crashing cost per cycle is

$$C_L = c_u\left(L_{g-1} - L\right) + \sum_{w=1}^{4} c_w(b_w - a_w) \tag{13}$$

$$L_g = L_o - \sum_{w=1}^{4} c_w(b_w - a_w) \tag{14}$$

where $L_0 = \sum_{w=1}^{4} b_w$, and L_g is the length of the lead time with components g=1, 2, 3, 4, which are to be crashed to minimum duration and $L \in [L_g, L_{g-1}]$ for g^{th} component has a normal duration b_w and minimum duration a_w and crashing cost per unit time c_u, such that $c_1 \leq c_2 \leq \ldots \leq c_u$ (Table 1).

$$T_{\mathrm{C}}^{\mathrm{CS-LT}} = \frac{1}{c}G + cV + \sum_{i=1}^{y}\left(h_{\mathrm{b}i}z\sigma_i\sqrt{L_g}\right) + \frac{1}{c}\sum_{i=1}^{y}\left(\pi_i\sigma_i\sqrt{L_g}\Psi(z)\right) + \frac{1}{c}\sum_{i=1}^{y} C_{L_g} \tag{15}$$

Table 1 Lead time crashing cost

Lead time component	Leading time (days)	(b_w – a_w) days	Unit crashing cost c_u ($/day)	Total crashing cost C_L ($)
1	14	0	0	0
2	10.5	3.5	0.4	1.4
3	7	3.5	1.2	5.6
4	5.25	1.75	5.0	14.35

where $G = s + \sum_{i=1}^{y} n_i A_{ti}$ and

$$V = h_{\mathrm{v}} \frac{1}{2p} \left[\sum_{i=1}^{y} \frac{D_i^2}{n_i} \right] + \sum_{i=1}^{y} \left(\frac{h_{\mathrm{b}i}}{2} \left\{ D_i - (n_i - 1) D_i \left[\frac{D_i}{n_i p} + \sum_{i \neq j} \frac{D_j}{n_j p} \left(\frac{n_j}{n_i} \right) \right] \right\} \right)$$

$$c = \sqrt{\frac{G + \sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_g} \Psi(z) \right) + \sum_{i=1}^{y} C_{L_g}}{V}} \tag{16}$$

$$\Phi(z) = 1 - \frac{c \left[\sum_{i=1}^{y} \left(h_{bi} \sigma_i \sqrt{L_g} \right) \right]}{\left[\sum_{i=1}^{y} \left(\pi_i \sigma_i \sqrt{L_g} \right) \right]} \tag{17}$$

Table 2 Summary of results up to ten buyers with single vendor

Variable	Model	Buyer size								
		2	3	4	5	6	7	8	9	10
JTEC ($)	CS	3,440	4,183	5,004	6,006	6,472	7,231	8,094	8,722	9,381
	CS-k^1	3,294	4,096	4,979	6,004	6,472	7,291	8,090	8,722	8,241
	CS-IS-k^1	3,120	3,829	4,731	5,931	5,885	7,106	8,090	8,720	9,238
	CS-LT	3,280	4,038	4,824	5,785	6,227	6,940	7,766	8,357	8,852
Total buyer max. stock	CS	651	798	962	1,160	1,249	1,403	1,580	1,475	1,785
	CS-k^1	500	626	807	1,160	1,249	1,403	1,580	1,470	1,806
	CS-IS-k^1	539	699	885	1,144	1,440	1,358	1,575	1,470	1,800
	CS-LT	610	770	928	1,113	1,165	1,354	1,510	1,631	1,728
Total buyer min. stock	CS	34	50	63	76	85	102	115	126	138
	CS-k^1	34	50	60	64	83	100	110	125	136
	CS-IS-k^1	30	47	60	76	79	88	94	99	116
	CS-LT	17	26	32	38	43	55	62	70	73
Number of shipments	CS	5	6	7	7	7	7	8	9	10
	CS-k^1	6	9	8	7	7	8	8	8	10
	CS-IS-k^1	9	8	10	8	12	13	8	11	11
	CS-LT	4	6	7	7	7	7	8	9	10
Delay deliveries	CS-k^1	2	3	3	3	3	3	4	3	2
	CS-IS-k^1	4	3	3	3	6	4	4	4	5
q_{ij}	CS-IS-k^1	2	2	2	2	5	3	3	3	5

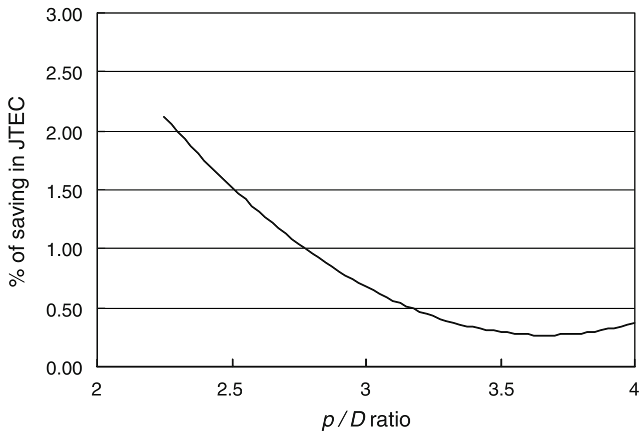

Fig. 1 Effect of (p/D) ratio on the percent of savings in JTEC

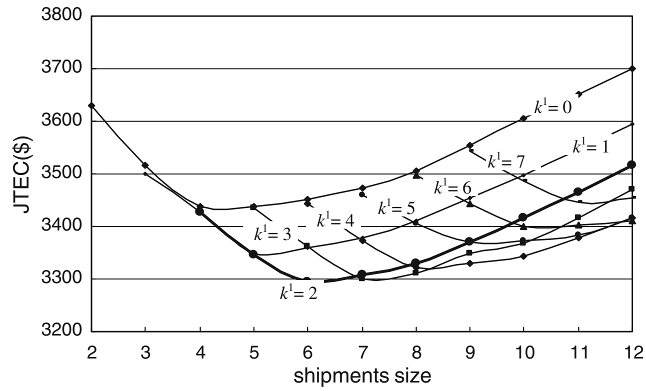

Fig. 3 JTEC for CS-k^1 model with delay deliveries varying from zero to seven and with different shipments

Minimum cost for optimum values of (c, n, L, z) is

$$T_{\mathrm{C}}^{\mathrm{CS-LT}} = 2\sqrt{\left(G + \sum_{i=1}^{y}\left(\pi_i \sigma_i \sqrt{L_g}\Psi(z)\right) + \sum_{i=1}^{y} C_{L_g}\right)V} + \sum_{i=1}^{y}\left(h_{bi} z \sigma_i \sqrt{L_g}\right) \tag{18}$$

Maximum level of inventory for buyer i is

$$I_{\max.}^{i} = \left\{ D_i c - (n_i - 1) D_i \left[\frac{D_i c}{n_i p} + \sum_{i \neq j} \frac{D_j c}{n_j p}\left(\frac{n_j}{n_i}\right) \right] \right\} + z\sigma_i\sqrt{L_g} \tag{19}$$

4.5 Algorithm to consignment stock model

In this section, an iterative algorithm (single vendor–two buyers) including the crashing expenses is presented to find minimum JTEC with optimal decision variables.

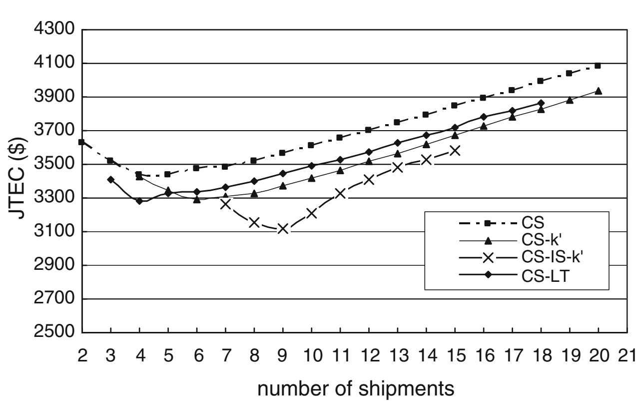

Fig. 2 Total system cost for different CS models and shipment size

Step 1: Set transport operations for first buyer n_1=1 and for second buyer n_2=1.

Step 2: For two buyers with each n_1 and n_2 and for all lead time components, perform steps 1 to 5.

1. Start with z_{i1}=0 using $\Psi(z) = \phi(z) - z[1 - \Phi(z)] \Rightarrow \Psi(z_{i1})$=0.39894 and $\forall\ z_{i1}=0 \Rightarrow \Phi z=0.5$ (from normal distribution table).
2. Substitute $\Psi(z)$ into Eq. 16 to evaluate c.
3. Using c, determine $\Phi(z)$from Eq. 17, then find z for the next iteration by checking the standard normal table and hence $\Psi(z)$ for the next iteration.

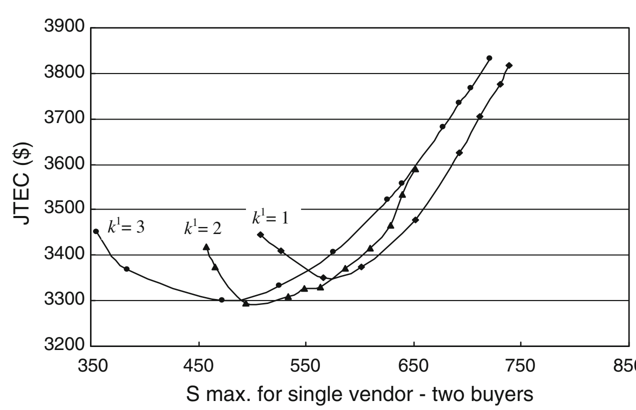

Fig. 4 JTEC for CS-k^1 model with delay deliveries varying from zero to three and with max. stock level of buyers

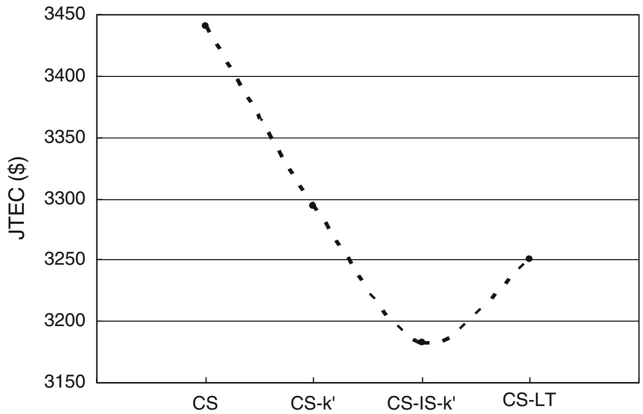

Fig. 5 JTEC of single vendor–two buyers for different strategies

4. Repeat 2 to 3 until no change occurs in the values of c and z.
5. Find corresponding minimum $\mathrm{JTEC}\left(c,n_2,n_1,L_{g,1},L_{g,2},z\right)=\mathrm{JTEC}\left(c^*,n_2,n_1,L_{g,1},L_{g,2},z^*\right)$

Step 3: For both the buyers with each $L_{g,i}$, i=1, 2, repeat steps 4 to 11 to get $\mathrm{JTEC}\left(c^*,n_2^*,n_1^*,L_{g,1}^*,L_{g,2}^*,z^*\right)$.

Step 4: If $\mathrm{JTEC}\left(c^*,n_2,n_1,L_{g,1},L_{g,2}^*,z^*\right)\leq\mathrm{JTEC}\left(c_{L_{1-1}}^*,n_{2(L_{1-1})},n_{1(L_{1-1})},L_{g,(1-1)},L_{g,2(L_{1-1})}^*,z_{L_{1-1}}^*\right)$, then go to step 3; otherwise, go to step 5.

Step 5: Set $\mathrm{JTEC}\left(c^*,n_2,n_1,L_{g,1}^*,L_{g,2}^*,z^*\right)=\mathrm{JTEC}\left(c_{L_{1-1}}^*,n_{2(L_{1-1})},n_{1(L_{1-1})},L_{g,(1-1)},L_{g,2(L_{1-1})}^*,z_{L_{1-1}}^*\right)$

Step 6: Set n_2=2; repeat steps 2 to 5 to get JTEC (c^*, n_2, n_1, $L^*_{g,1}$, $L^*_{g,2}$, z^*).

Step 7: If $\mathrm{JTEC}\left(c^*,n_2,n_1,L_{g,1}^*,L_{g,2}^*,z^*\right)\leq\mathrm{JTEC}\left(c_{n_{2-1}}^*,n_{2(n_{2-1})},n_{1(n_{2-1})},L_{g,1(n_{2-1})}^*,L_{g,2(n_{2-1})}^*,z_{n_{2-1}}^*\right)$, then go to step 6, otherwise go to step 8.

Step 8: Set $\mathrm{JTEC}(c^*,n_2^*,n_1,L_{g,1}^*,L_{g,2}^*,z^*)=\mathrm{JTEC}(c_{n_{2-1}}^*,n_{2(n_{2-1})},n_{1(n_{2-1})},L_{g,1(n_{2-1})}^*,L_{g,2(n_{2-1})}^*,z_{n_{2-1}}^*)$

Step 9: Set n_1=2; repeat steps 2 to 7 to get JTEC (c^*, n_2^*, n_1, $L^*_{g,1}$, $L^*_{g,2}$, z^*).

Step 10: If $\mathrm{JTEC}(c^*,n_2^*,n_1,L_{g,1}^*,L_{g,2}^*,z^*)\leq\mathrm{JTEC}(c_{n_{1-1}}^*,n_{2(n_{1-1})},n_{1(n_{1-1})},L_{g,1(n_{1-1})}^*,L_{g,2(n_{1-1})}^*,z_{n_{1-1}}^*)$, then go to step 9; otherwise, go to step 11.

Step 11: If $\mathrm{JTEC}(c_{n_{1-1}}^*,n_{2(n_{1-1})}^*,n_{1(n_{1-1})},L_{g,1(n_{1-1})}^*,L_{g,2(n_{1-1})}^*,z_{(n_{1-1})}^*)=\mathrm{JTEC}(c^*,n_2^*,n_1^*,L_{g,1}^*,L_{g,2}^*,z^*)$, then c^*, n^*_2, n^*_1, $L^*_{g,1}$, $L^*_{g,2}$, z^* are the optimal variables.

5 Illustrative example

The input values to all the models discussed refer to Ben-Daya and Raouf [14], Braglia and Zavanella [3], Ouyang et al. [15], and Srinivas and Rao [6]: h_v=\$4per unit per year, h_b=\$5per unit per year, $D_{i=1,\,2}$ (units per year, for two buyers)= 1,000, 1,300, $p/\Sigma D_i$=3.2, $\sigma_{1,2}$=44.72, 50, s=\$400 per setup, $A_{t,\ i=1,2}$=\$25 per order, π=\$50 per unit. Later extended to ten buyers with $D_{3,4,\ldots10}$=800, 1,000, 1,500, 600, 1,200, 1,500, 1,000, 800 and $\sigma_{3,4,\ldots10}$=35.7, 30, 30, 20, 30, 30, 30, 20. The summary of brief results was given in Table 2.

It is found that number the of shipments in CS-IS-k^1 is more due to partial information sharing, whereas it is almost equal in case of CS, CS-k^1, and CS-LT. The JTEC cost in CS-LT is much lesser compared with CS and CS-k^1 due to considerable reduction in buyer total cost (Table 2) because of low inventory. The cost savings with CS-k^1 and CS-IS-k^1 policies decrease as uncertainty in demand and lead time increases, whereas for CS-LT model it increases as uncertainty in demand and lead time increases (Fig. 5). Therefore, when uncertainty in demand and lead time is more, one should prefer CS-LT policy as it lowers the lead time. Buyer maximum stock level and minimum stock in the case of CS-LT are always low (Table 2). The highest difference is for CS, then CS-IS-k^1 and CS-k^1. The difference in the case of CS-k^1 and CS-IS-k^1 is controlled due to delay and information sharing. Srinivas and Rao [6] developed the CS-LT policy for single vendor–single buyer which terminates the iterative analysis algorithm for minimum JTEC of \$6,335 with two components of lead time reduction with an aggregate lead time of (6+6+16) 28 days for a set of given input values. For the same input values (single vendor–single buyer), Ouyang et al. [15] got a controlled lead time of 28 days and JTEC of \$6,660.4, but

Table 3 Comparison of different strategies

Variable	Hill [27]	Braglia and Zavanella [3] (single vendor–single buyer)			The proposed models (single vendor–two buyers)			
		CS-2	CS-1	CS	CS-k^1	CS	CS-IS-k^1	CS-LT
Max. level of buyer stock	110	164	267	376	500	651	539	610
Number of shipments	5	3	3	4	6	5	9	4
Delay deliveries	–	2	1	–	2	–	4	–
Total cost (\$)	1,903	1,929	2,003	2,035	3,294	3,440	3,120	3,280

in the case of CS-LT single vendor–multibuyer, the lead time reduces up to the minimum.

The results show that, by having more buyers in the SC, the projected single-buyer total cost saving is increasing with considerable amount for CS-IS-k^1 and CS-LT (Table 2). The lowest projected cost for a set of given input data in the case of single vendor–single buyer CS-LT model is \$836 while ten buyers exist in SC network. Up to buyer size six, the CS-IS-k^1 gives the lowest cost compared to CS-LT, but when buyer size further increases, the CS-LT yields the lowest joint total cost. For basic CS model in the case of Braglia and Zavanella [3], the JTEC is \$2,035 for single vendor–single buyer, whereas the proposed model of single-vendor–two-buyer JTEC is \$3,440, and its projected single-vendor–single-buyer cost is \$315 less compared to Braglia and Zavanella [3]. The difference is due to reduction in shipments and reduction in the buyer's total carrying cost.

The results given through Figs. 1, 2, 3, 4, and 5 are referring to single-vendor–two-buyer model. If the total demand rate is closer to the production rate, greater saving can be obtained. In other words, by gradually declining the ratio of the production rate to the demand rate, the percentage of JTEC saving is increased. In contrast, by inclining the value of (p/D), the saving is decreased. However, it does not mean that the saving diminishes to zero as (p/D) becomes significantly high, as shown in Fig. 1. The buyer maximum stock level with minimum JTEC ranges from 500 to 650. The total cost in the case of CS-LT is \$3,280 (two buyers) with buyer maximum level of 610 (two buyers; Tables 2 and 3). For more than six buyers, CS-LT models yield less JTEC (Table 2). There is a close range for CS-k^1 and CS-IS-k^1, but for basic CS, the minimum total cost occurs at buyer's maximum level (Table 2). From the fundamentals of CS policy, the vendor always prefers to have maximum stock level at the buyer's warehouse. Figure 2 gives joint total system cost in the case of two buyers while increasing the shipment size. The minimum JTEC is n=4 in the case of CS-LT model, whereas for CS, CS-k^1, and CS-IS-k^1, it is 5, 6, and 9 (Fig. 3). CS-k^1=2 gives the lowest JTEC at n=6. CS-$k^1 \in$ CS $\forall$ k^1=0 always produces a maximum cost, if it adopted basic CS model (Fig. 3). For k^1=2, a low buyer and vendor inventory cost for all the ranges of maximum buyer inventory levels (Fig. 4) are found.

6 Conclusions and future scope

Four types of models are developed, basic CS, CS with delay, CS with information sharing and delay delivery, and CS-LT model. The CS inventory management policy with information sharing and delay delivery and controllable lead time has proven to be suitable for facing new SCM challenges with stochastic demand for single vendor–multibuyers, and after buyer size is increased, controllable lead time is suitable. The proposed models not only can make tradeoffs but can also enable decision makers to deal with inconsistent judgments systematically. The controllable lead time model total cost depends on lead time components and crashing cost components.

Future studies have to be made in the area of CS for multivendor–multibuyer with multiple products and can be extended to multiechelons. The studies can be extended to zero value at the end of predefined lifetime of products. The implementation of radiofrequency identification can also be studied, which can have significant potential in delivering business benefits.

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