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ORIGINAL ARTICLE

# Effect of fixture compliance and cutting conditions on workpiece stability

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**Abstract** This paper computes and investigates the effect of fixture compliance and cutting conditions on workpiece stability and uses it as a basis for selecting a suitable fixture among several alternatives. We use two criteria, the minimum eigenvalue of the fixture stiffness matrix and the largest displacement of the workpiece due to the cutting forces to assess the stability of the workpiece. First, the minimum eigenvalues of the fixture stiffness matrices, for the fixtures being considered, are computed. Second, since the eigenvalues are not dependant on the cutting forces, a displacement measure, the largest displacement of the workpiece due to the cutting force, is computed for each fixture. This displacement is a function of the cutting force and the fixture compliance. The choice of fixture is often a compromise between the two criteria. We also consider the combined influence of fixture compliance and cutting conditions on workpiece stability. The results from the simple study used for illustration show that the eigenvalues remain constant under different cutting conditions whilst the largest displacement reduces by 68%, a significant reduction.

**Keywords** Fixture stiffness matrix · Fixture compliance · Contact force · Workpiece stability

## 1 Introduction

Object manipulation is the art of moving things. For example, an object can be moved from one point to another by a robot hand that is in contact with the object. The contacts serve two functions: they transmit forces and impose motion constraints on the object. The motion of the robot is constrained by the interactions at the contacts for control of the motion of the object. If the robot hand has multiple fingers, there is a coordinated manipulation of the object by the multiple fingers. Furthermore, the motion of the object may be constrained in some directions because of contact with fingers. The multi-fingered robot hand in contact with the object forms a closed loop. Hence, the fingers' effort and the contact forces and moments required for support and for moving the object are coupled. Any external force that acts on the object to bring about motion will cause a consequential readjustment of forces at the finger tips of the multi-fingered robot hand. The tips of the multi-fingered robot hand may deform in order to accommodate the object and prevent motion. In the robotics literature, the motion and force are related by the equation $\mathbf{v}=C\mathbf{f}$ [1], where $\mathbf{v}$ is the velocity of the object, $C$ is the compliance of the fingers and $\mathbf{f}$ is the applied force from the fingers. $C$ is a diagonal matrix, which means that motion is allowed only in particular directions. For example, in the compliance matrix $C = \mathrm{diag}\left[C_x, C_y, C_z\right]$, when $C_z$ is set to zero, then motion is allowed in the $x$–$y$ direction only. This analogy can be applied to fixturing so that a workpiece held in a machining fixture can be considered as a special case within the same understanding. In the case of machining fixtures, the workpiece cannot accelerate on its own but is displaced when an external force, such as the machining force, acts on the workpiece. Since displacement is the main concern here for workpieces being machined, a displacement equation will be most appropriate.

This paper examines the effect of fixture compliance and cutting conditions on workpiece stability and discusses the coordination of multiple-point contact of the fixture

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elements with the workpiece. The coordination problem is divided into a number of phases: determining the force by multiple fixture elements, determining the contact and fixture stiffness and fixture compliance and determining the workpiece displacement. The force is used for maintaining equilibrium and for generating the restoring force. Frictional constraints are incorporated in the force equation.

When a workpiece is subjected to external machining forces, the workpiece is stable when the displacements of the workpiece are at a minimum. In the stability analysis reported in this study, we are concerned with how externally applied forces affect the workpiece displacement due to deformation of contact points between the workpiece and locators/clamps. We need to find the minimum rigid body displacement of the workpiece when the external force is applied to the workpiece. This can be solved by solving the eigenvalue problem $\mathbf{k}z=\lambda z$. $\mathbf{k}$ is the fixture stiffness matrix. The minimum eigenvalue $\lambda$ represents the minimum displacement at the contact points. $z$ is the position vector associated with the minimum displacement. The effect of cutting condition on workpiece displacement within the fixture was also investigated.

Shawki and Abdel [2–4] investigated and showed through experiments that locators and clamps elastically deform at their contact points. For this reason, there is a need to consider compliant contacts. Contact compliance in fixturing may be due to the elasticity of either the contact points of the fixture elements or the workpiece or both. In this paper, we consider fixture element position and contact forces of the fixture elements. When held by the fixture elements, the workpiece is assumed to be deformable at the points of contact with the fixture elements (locators/clamps). The reaction forces at the contact points are restoring forces that depend linearly on deformation; hence, reaction forces are functions of the contact point deformation. In this work, we take a different approach from that of Howard and Kumar [5] to study the stability of the grasped object. In their work, they assumed that all normal forces are known, the fingers are fixed but the workpiece changes position making the contact points change and no friction forces are considered. In this investigation, the contacts between the workpiece and locators and between the workpiece and clamps are modelled as elastic contact with friction. The workpiece and locators/clamps are assumed to be rigid bodies. The contact points do not change locations due to the deformations at the contact points.

The question that needs to be answered is: What effects do fixture compliance and cutting conditions have on workpiece stability? Workpiece stability is compromised when sliding on or revolving about the main axes occurs. The main objective in this research is to develop tools that can be used to answer this question. To do this, the following tasks will be accomplished: Investigate the effect of fixture compliance on workpiece stability and propose a quantitative measure for defining a fixtured workpiece's stability. Select a typical fixture for a machining operation and investigate the combined effect of fixture compliance and cutting conditions on this workpiece's stability.

## 2 Literature review

Workpiece stability is an essential issue in fixture design. Workpiece stability is affected by a number of factors; notably among them are clamping sequence, clamping forces, fixture layout and fixture deformation. A number of research efforts have been focused in this area. According to [6], though the rational selection of the clamping sequence is important for control of workpiece displacement in fixture design, no clamping sequence will ensure simultaneous minimization of all types of deviations, making it difficult to propose any preferred clamping sequence. The same work showed that, even with the same fixturing forces, the force distribution in fixtures may vary significantly with different application sequences of clamping forces. It has, however, been shown that the placement of clamps in relation to locators may have a larger influence on accuracy than absolute placement of the locators themselves [7]. DeMeter [8, 9] applied restraint analysis to a fixture with frictionless or frictional surface contacts and proposed a linear model for predicting the impact of locator and clamp placement on workpiece displacement throughout machining operations. This model can be used to evaluate whether or not a fixture provides total restraint. The machined workpiece was treated as a rigid body. According to [10], the proper sequences of placing the clamps can actually relax the stringent requirement in the positioning accuracy of the clamps whilst assuring the stability of the workpiece during the clamping process. [11, 12] also analysed the clamping sequence from the viewpoint of minimising contact forces under the assumption of a rigid workpiece–fixture system. Researchers in [13] used an analytical model to capture the effect of clamping sequence on the workpiece accuracy. In their work, they assumed that multiple clamps have to be applied simultaneously when applied to the same part face. Researchers in [14] used the FEM to simulate the effect of clamping sequence upon the machined surface position of the workpiece. In fact, all these works never investigated the effect of cutting conditions on workpiece stability.

Various clamping force analysis schemes have been used in analysing the required minimum clamping force. The authors in [15] proposed a method to analyse the minimum clamping forces. It was derived from the correlation between clamping moment and cutting force. This method increases the search efficiency by pruning inadequate search directions. In their work, several stability criteria were set up after theoretical derivation. This work, however, is only useful for force analysis with only one clamping plane. Employing linear springs to approximate the stiffness characteristics of contact between workpiece and fixture, researchers in [16] were able to minimise workpiece location error by optimising clamping force. Li and Melkote [17] presented an algorithm based on the contact elasticity method for determining the optimum clamping forces for a multiple clamp fixture–workpiece system subjected to quasi-static loads. The method seeks to minimise the impact of workpiece motion due to clamping and machining loads on the part location accuracy by systematically optimising the clamping forces. A contact mechanics model is used to determine a set of contact forces and displacements, which are then used for the clamping force optimization. Researchers in [18] investigated the stability of a workpiece in an automated fixture design (AFD) environment. Their work reported the development of a computational methodology for quantitatively analysing the workpiece's stability. Expressing the virtual disturbance, which has the same tendency as those fixturing and machining forces to destabilise the workpiece as a wrench in the screw coordinates, they characterise the stability of the workpiece based on its capability to overcome the virtual disturbance. The stability characteristics for each fixturing configuration (i.e., positions for fixturing components of supporters, locators and clamps) itself are obtained from the stability analysis and used in the re-design process to determine the better design of fixturing configuration. They, however, ignored the effects of frictional forces.

Other researchers [7, 14] used time-varying cutting loads to determine the clamping force required. It has been observed that a loss of contact between the workpiece surface and a locator is an indication of the failure of total restraint [14]. The conclusion is that the clamping force required for maintaining stability is the minimum value that prevents separation. In their analysis, [7] ignored the friction forces. This simplification leads to higher than necessary fixturing forces. Considering friction in their analysis, the authors in [19] constructed the limit surface in force/moment space as a convenient formalism to check the stability of the workpiece and to specify clamping forces by searching in the infinite clamping plane. The method, however, lacks theoretical sufficiency and is rather inefficient. When applied to 3D analysis that is usually required for most fixturing configurations, the method becomes too complex and time-consuming to be applied. A systematic approach was used [20] in verification and optimization of the fixturing scheme. They assumed the workpiece fixture as being perfectly rigid bodies in frictional contact and predicted the clamping forces required to maintain the workpiece stability. [21, 22] proposed a computational geometry approach to optimum clamping synthesis of machining fixtures. [23] designed a force control clamping system using feedback control, and it provides an effective means for a variable clamping force system. [24] reported a minimum clamping force algorithm for machining fixtures.

Research activity of workpiece stability in fixturing for a workpiece–fixture system has been generally focused on force analysis, fixture layout and clamping stability. Three problems with these methods are that (1) fixture layout is unable to adapt to deformations that might occur at the contact interface between the workpiece and the fixture elements, (2) clamping stability considers only the clamps as the external forces, ignoring the fact that the cutting forces may also induce displacements due to contact deformation and (3) force analysis only provides a means of assessing the state of equilibrium of the workpiece–fixture system. Given that deformation occurs at the contact interface between the workpiece and the fixture elements, it is important to investigate its effect on workpiece stability. The second issue that will be of importance to investigate is the effect of cutting conditions on workpiece stability. [2–4] were the first to investigate fixture rigidity on dimensional accuracy and showed through experiments that locators and clamps elastically deform at their contact points. Their work reported that the rigidity of a fixture is a prime cause of dimensional inaccuracy and, therefore, the most important aspect of fixture design. Their work reported the lack of the study of fixture rigidity. Analytical methods for determining the stiffness of the fixture locators for a given stiffness requirement of the workpiece at the machining surfaces have also been used [25]. Meanwhile, a kinetic fixture model [26], in which a fixture stiffness matrix is derived to link the external loads with the workpiece displacement, has been developed. They define workpiece stability as no slippage between any locating point and the workpiece surface and depend purely on the application of the clamping force. This criterion was based on the concept of the friction cone and the definition of a stability index, which depends on whether the contact force is within the friction cone. The friction cone was defined as the maximum friction limitation. If the contact force falls outside the friction cone, it is unstable. If the contact force is on the friction cone, it is marginally stable. If the contact force is inside the friction cone, it is stable. These works showed the link between the clamping forces and the stiffness of the fixture elements but did not investigate the effect of fixture compliance on workpiece stability.

Clearly, from the above review, there are only a few recent works that consider contact compliance (i.e., contact deformation). Some of them model the contact as a rigid frictional point contact ignoring any deformation; others consider finite element deformation models; others consider only the fixture deformation, ignoring deformation on the workpiece where it has contact with the fixture elements. From the fixturing literature, there are no metrics that can be used to quantify workpiece stability, nor do they provide any means of selecting from among existing fixtures the best fixture that can provide what is required. In the robotics literature, various quality (metrics) measures are available for assessing the stability of a grasped object by robot fingers. Some of the notable approaches are based on assessing the static stability margin in a given contact arrangement. This approach was pioneered by [27], and subsequently extended in [28–33]. Researchers in [5] derived conditions for fixture/grasp static stability based on local curvature, reaction forces, composite stiffness, contact position and orientation. Researchers in [34] developed frame-independent metrics for grasp stability based on the generalised eigen-decomposition of the stiffness matrix. However, all of these quality measures assume perfectly rigid bodies which do not experience any contact deformation in response to applied loads. In addition, characterization of the workpiece stability as a function of the number and position of the fixture elements is lacking. They also did not investigate cutting conditions on workpiece stability.

Characterising stability in terms of the worst-case workpiece displacement and the minimum eigenvalue of the fixture stiffness matrix will be a good way to assess workpiece stability. Since fixture configuration affects workpiece stability, it is important that any model developed be frame invariant and must employ any number of contacts.

## 3 Methodology and system description

### 3.1 Overview of the methodology

Workpiece stability is affected by a number of factors; notably among them are clamping sequence, clamping forces, fixture layout and fixture deformation. Workpiece stability is associated with workpiece displacement whilst it is machined. The stability is compromised when sliding on or revolving about the main axes occurs. This problem of displacement may be the result of fixture deformation (fixture compliance) as a result of the application of the machining forces. To investigate the effect of fixture compliance on workpiece stability, a force analysis routine is first used to derive the equations that govern the static behaviour of the workpiece and fixture elements. Frictional constraints are incorporated in the force equation. Second compliance models are developed to determine the fixture stiffness matrix. Thirdly, a relationship between the cutting forces and the workpiece displacement is established.

The workpiece stability is determined by the minimum eigenvalue of the fixture stiffness matrix in response to all external forces. The stability of the workpiece is further characterised by the maximum displacement due to the cutting forces. Results show that a compromise must sometimes be made when using these two criteria to choose the best fixture. The effect of cutting condition on workpiece displacement within the fixture is also investigated.

### 3.2 System description

In order to simplify the analysis, we assume that (1) the workpiece is of a rectangular shape and the fixture elements have already established contact with the workpiece. (2) The fixture element tips maintain contact with the workpiece. (3) The fixture elements are modelled as rigid bodies but deformable at their points of contact with the workpiece. The workpiece is also considered as a rigid body but deforms at the points of contact with the fixture elements tips. (4) Every fixture element tip makes a frictional contact with the workpiece. (5) After deformation, the corresponding surface points on the workpiece and locator/clamp surface are coincident within the contact surface.

Let GCS be the global coordinate system considered as the reference frame $O$, CCS contact coordinate system which is the reference frame at the contact point between the workpiece and fixture elements, FCS the fixture coordinate system and WCS the workpiece coordinate system defined at the centre of the workpiece. Fixture elements are the locators and clamps.

## 4 Contact force and wrench

Let the components of the contact force at the $i$th locator be ${}^{i}\mathbf{F}_l = ({}^{i}f_{la}, {}^{i}f_{lb}, {}^{i}f_l)$ and that at the $i$th clamp be ${}^{i}\mathbf{F}_c = ({}^{i}f_{ca}, {}^{i}f_{cb}, {}^{i}f_c)$. Let the components of the position vector of the contact points at the $i$th locator and clamp be ${}^{i}\mathbf{r}_l = \left({}^{i}r_l^x, {}^{i}r_l^y, {}^{i}r_l^z\right)$ and ${}^{i}\mathbf{r}_c = \left({}^{i}r_c^x, {}^{i}r_c^y, {}^{i}r_c^z\right)$, respectively. Here, ${}^{i}f_l$, ${}^{i}f_c$ are the normal forces and ${}^{i}f_{la}$, ${}^{i}f_{ca}$ and ${}^{i}f_{lb}$, ${}^{i}f_{cb}$ are orthogonal tangential forces at the $i$th locator and clamp, respectively. ${}^{i}r_l^z$, ${}^{i}r_c^z$ are the distances between the $i$th locator and clamp and the reference point $O$ along the $z$ axis, respectively, ${}^{i}r_l^y$, ${}^{i}r_c^y$ are the distances between the $i$th locator and clamp and the reference point $O$ along the $y$ axis, respectively, and ${}^{i}r_l^x$, ${}^{i}r_c^x$ are the distances between the $i$th locator and clamp and the reference point $O$ along the $x$ axis, respectively. Let $W_{\mathrm{g}}$ be the weight of the workpiece.

Consider the body shown in Fig. 1. Point $i$ is a representative locator. We now find the sum of the forces

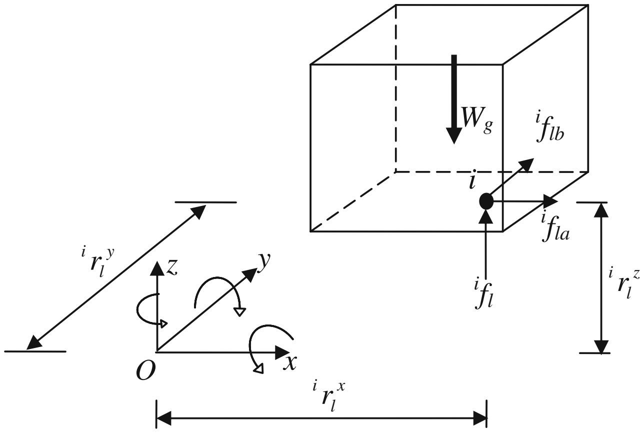


**Fig. 1** Contact forces and wrench

in the $x$–$y$–$z$ directions and the moments about $O$ an arbitrary reference frame. This is given as

$$\begin{cases} {}^iF_{lx} = {}^if_{la} \\ {}^iF_{ly} = {}^if_{lb} \\ {}^iF_{lz} = {}^if_l \\ {}^iM_{lx} = {}^if_l{}^ir_l^y - f_{lb}{}^ir_l^z \\ {}^iM_{ly} = {}^if_{la}{}^ir_l^z - {}^if_l{}^ir_l^x \\ {}^iM_{lz} = {}^if_{lb}{}^ir_l^x - {}^if_{la}{}^ir_l^y \end{cases} \tag{1}$$

This is called the wrench and can be written as

$$\begin{aligned} {}^i\mathbf{W}_l = \begin{bmatrix} {}^iF_{lx} \\ {}^iF_{ly} \\ {}^iF_{lz} \\ {}^iM_{lx} \\ {}^iM_{ly} \\ {}^iM_{lz} \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -{}^ir_l^z & {}^ir_l^y \\ {}^ir_l^z & 0 & -{}^ir_l^x \\ -{}^ir_l^y & {}^ir_l^x & 0 \end{bmatrix} \begin{bmatrix} {}^if_{la} \\ {}^if_{lb} \\ {}^if_l \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I} \\ {}^i\mathbf{R}_l \end{bmatrix} {}^i\mathbf{F}_l = {}^i\mathbf{G}_l{}^i\mathbf{F}_l \end{aligned} \tag{2}$$

Here, ${}^i\mathbf{R}_l = \begin{pmatrix} 0 & -{}^ir_l^z & {}^ir_l^y \\ {}^ir_l^z & 0 & -{}^ir_l^x \\ -{}^ir_l^y & {}^ir_l^x & 0 \end{pmatrix}$, ${}^i\mathbf{F}_l = \begin{bmatrix} {}^if_{la} \\ {}^if_{lb} \\ {}^if_l \end{bmatrix}$ and ${}^i\mathbf{G}_l = \begin{bmatrix} \mathbf{I} \\ {}^i\mathbf{R}_l \end{bmatrix}$. Now assume that we have $m$ locators and $n$ clamps. Then, we have to combine all the wrenches at the locators and the clamps to get the contact wrench $\mathbf{W}$

$$\begin{aligned} \mathbf{W}_C &= \sum_1^m {}^i\mathbf{W}_l + \sum_1^n {}^i\mathbf{W}_c \\ &= \begin{bmatrix} \mathbf{I} & \cdots & \mathbf{I} & \mathbf{I} & \cdots & \mathbf{I} \\ {}^1\mathbf{R}_l & \cdots & {}^m\mathbf{R}_l & {}^1\mathbf{R}_c & \cdots & {}^n\mathbf{R}_c \end{bmatrix} \begin{bmatrix} {}^1\mathbf{F}_l \\ \vdots \\ {}^m\mathbf{F}_l \\ {}^1\mathbf{F}_l \\ \vdots \\ {}^n\mathbf{F}_c \end{bmatrix} = \mathbf{G}_T\mathbf{F}_C \end{aligned} \tag{3}$$

We shall call $\mathbf{G}_T = [\mathbf{G}_l \quad \mathbf{G}_c] = \begin{bmatrix} \mathbf{I} & \cdots & \mathbf{I} & \mathbf{I} & \cdots & \mathbf{I} \\ {}^1\mathbf{R}_l & \cdots & {}^m\mathbf{R}_l & {}^1\mathbf{R}_c & \cdots & {}^n\mathbf{R}_c \end{bmatrix}$ the grasp matrix of the system and $\mathbf{F}_C = \left[{}^1\mathbf{F}_l \quad \cdots \quad {}^m\mathbf{F}_l \quad {}^1\mathbf{F}_l \quad \cdots \quad {}^n\mathbf{F}_c\right]^T$ the contact force.

We need to consider the gravity wrench, $\mathbf{W}_g$, due to the gravity force. Let the components of the gravity force, position and moment, respectively, be $\mathbf{F}_g = (0, 0, -W_g)$, $r_g = (r_{gx}, r_{gy}, r_{gz})$ and $\mathbf{M}_g = (-W_g r_{gy}, W_g r_{gx}, 0)$. Then, the sum of the forces and moments in the $x$–$y$–$z$ directions due to gravity are: $\begin{cases} F_{gx} = 0 \\ F_{gy} = 0 \\ F_{gz} = -W_g \\ M_{gx} = -W_g r_{gy} \\ M_{gy} = W_g r_{gr} \\ M_{gz} = 0 \end{cases}$, where $W_g = m_w g$ and $m_w$ is the mass of the workpiece. The gravity wrench can be written as:

$$\begin{aligned} \mathbf{W}_g = \begin{bmatrix} \mathbf{F}_g \\ \mathbf{M}_g \end{bmatrix} = \begin{bmatrix} F_{gx} \\ F_{gy} \\ F_{gz} \\ M_{gx} \\ M_{gy} \\ M_{gz} \end{bmatrix} &= -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & r_{gy} \\ 0 & 0 & -r_{gx} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ W_g \end{bmatrix} \\ &= -\mathbf{G}_g\mathbf{F}_g \end{aligned} \tag{4}$$

Equation 4 means that $\mathbf{W}_g$ is considered as an external wrench. Now, consider a machining wrench $\mathbf{W}_{cut}$ applied to the workpiece for machining purposes. If we consider the sum of the machining and gravity wrenches as external wrenches, represented by $\mathbf{F}_e = \begin{bmatrix} f_e \\ M_e \end{bmatrix}$, then the equilibrium equation for the whole system can be written as

$$\mathbf{F}_e + \mathbf{G}_T\mathbf{F}_C = 0, \tag{5}$$

where $\mathbf{F}_e = \begin{bmatrix} f_e \\ M_e \end{bmatrix} = \mathbf{W}_{cut} + \mathbf{W}_g$.

## 5 Stiffness and compliance models

We assume the fixture to be linear elastic at the contacts, i.e. forces are proportional to their displacements. Let the $i$th contact point stiffness between the locator and clamp in the $x$–$y$–$z$ directions be, respectively, $({}^ik_{C_{la}}, {}^ik_{C_{lb}}, {}^ik_{C_l})$ and $({}^ik_{C_{ca}}, {}^ik_{C_{cb}}, {}^ik_{C_c})$. This contact point displaces when the workpiece displaces. Let the components of the displacements at the tips of the locators and clamps be represented, respectively, as ${}^i\mathbf{d}_l = ({}^id_{C_{la}}, {}^id_{C_{lb}}, {}^id_{C_l})$ and ${}^i\mathbf{d}_c = ({}^id_{C_{ca}},$

${}^{i}d_{C_{cb}}, {}^{i}d_{C_c}$). Let $\mathbf{d}_E = \left[d_{Ex}, d_{Ey}, d_{Ez}, \theta_{Ex}, \theta_{Ey}, \theta_{Ez}\right]^T$ or $\mathbf{d}_E = [\mathbf{x}\,, \theta]^T$ be a six-dimensional vector representing the workpiece displacement (translational and rotational). The contact force at the i*th* contact point will then be

$$\begin{aligned} {}^{i}f_{C_{la}} &= -\,{}^{i}k_{C_{la}}{}^{i}d_{C_{la}} \\ {}^{i}f_{C_{lb}} &= -\,{}^{i}k_{C_{lb}}{}^{i}d_{C_{lb}} \\ {}^{i}f_{C_{l}} &= -\,{}^{i}k_{C_{l}}{}^{i}d_{C_{l}} \end{aligned} \tag{6}$$

or

$${}^{i}\mathbf{F}_l = \begin{bmatrix} {}^{i}f_{C_{la}} \\ {}^{i}f_{C_{lb}} \\ {}^{i}f_{C_l} \end{bmatrix} = - \begin{bmatrix} {}^{i}k_{C_{la}} & 0 & 0 \\ 0 & {}^{i}k_{C_{lb}} & 0 \\ 0 & 0 & {}^{i}k_{C_l} \end{bmatrix} \begin{bmatrix} {}^{i}d_{C_{la}} \\ {}^{i}d_{C_{lb}} \\ {}^{i}d_{C_l} \end{bmatrix} = -{}^{i}\mathbf{k}_{C_l}{}^{i}\mathbf{d}_l, \tag{7}$$

where ${}^{i}\mathbf{k}_{C_l} = \begin{bmatrix} {}^{i}k_{C_{la}} & 0 & 0 \\ 0 & {}^{i}k_{C_{lb}} & 0 \\ 0 & 0 & {}^{i}k_{C_l} \end{bmatrix}$ and ${}^{i}\mathbf{d}_l = \begin{bmatrix} {}^{i}d_{C_{la}} \\ {}^{i}d_{C_{lb}} \\ {}^{i}d_{C_l} \end{bmatrix}$ are the *i*th contact stiffness and displacement, respectively. We can combine all the contact forces to define an *n*-column matrix of $\mathbf{F}_C$ so that

$$\mathbf{F}_C = \begin{bmatrix} {}^{1}\mathbf{F}_l \\ \vdots \\ {}^{m}\mathbf{F}_l \\ {}^{1}\mathbf{F}_c \\ \vdots \\ {}^{n}\mathbf{F}_c \end{bmatrix} = - \begin{bmatrix} {}^{1}\mathbf{k}_{C_l} & & & & & \mathbf{O} \\ & \ddots & & & & \\ & & {}^{m}\mathbf{k}_{C_l} & & & \\ & & & {}^{1}\mathbf{k}_{C_c} & & \\ & & & & \ddots & \\ \mathbf{O} & & & & & {}^{n}\mathbf{k}_{C_c} \end{bmatrix} \begin{bmatrix} {}^{1}\mathbf{d}_l \\ \vdots \\ {}^{m}\mathbf{d}_l \\ {}^{1}\mathbf{d}_c \\ \vdots \\ {}^{n}\mathbf{d}_c \end{bmatrix} = -\mathbf{k}_C\mathbf{d}_C. \tag{8}$$

Here, $\mathbf{k}_c$ and $\mathbf{d}_c$ are the contact stiffness and displacement, respectively. The stiffness of the contact point determines the strength of the fixture element and the positioning accuracy of the workpiece in the presence of a disturbing force. Also, the stiffness is an important control variable, which allows the fixture to accommodate contact forces with acceptable displacements. In effect, the workpiece will displace by an amount which depends on the stiffness of the fixture elements and the applied force.

Let us assume that the elastic deformations at the contact points are so small that the grasp matrix does not change under the action of the external wrench, $\mathbf{F}_e$. In that case, Eq. 3 can be written as

$$\mathbf{F}_e = -\mathbf{G}_T\mathbf{F}_C \tag{9}$$

We need to find the rigid body displacement of the workpiece caused by the application of the external force in relation to the fixture elements. We assume small displacements and define the *i*th locating point displacement on the workpiece surface with respect to the workpiece mass centre. When the workpiece displaces, its centre of mass translates and rotates. The workpiece displacement $\mathbf{d}_E$ is measured by the displacement of its mass centre. The displacement of the *i*th locating point in relation to the workpiece mass centre is

$$\mathbf{d}_i = \mathbf{x} + \theta \times \mathbf{r}_i = \left[\mathbf{I} \quad \mathbf{R}^T\right]\mathbf{d}_E = \mathbf{G}^T\mathbf{d}_E. \tag{10}$$

Here, it is assumed that the reference frame of the centre of mass of the workpiece is coincident with the reference frame *O*. The displacement of each contact point can now be defined in relation to the workpiece mass centre, as shown in Eq. 10, so that, for the *i*th locator,

$${}^{i}\mathbf{d}_l = {}^{i}\mathbf{G}_l^T\mathbf{d}_E \tag{11}$$

For each contact point, we substitute the displacement into Eq. 8 for the column matrix of $\mathbf{d}_C$ to get

$$\mathbf{d}_C = \begin{Bmatrix} {}^{1}\mathbf{d}_l = {}^{1}\mathbf{G}_l^T\mathbf{d}_E \\ \vdots \\ {}^{m}\mathbf{d}_l = {}^{m}\mathbf{G}_l^T\mathbf{d}_E \\ {}^{1}\mathbf{d}_c = {}^{1}\mathbf{G}_c^T\mathbf{d}_E \\ \vdots \\ {}^{n}\mathbf{d}_c = {}^{n}\mathbf{G}_c^T\mathbf{d}_E \end{Bmatrix} = \mathbf{G}_T^T\mathbf{d}_E \tag{12}$$

Substituting Eqs. 12 and 8 into Eq. 9, we obtain

$$\mathbf{F}_e = \mathbf{G}_T\mathbf{k}_C\mathbf{G}_T^T\mathbf{d}_E \tag{13}$$

The term $\mathbf{k} = \mathbf{G}_T\mathbf{k}_C\mathbf{G}_T^T$ in Eq. 13 is called the *fixture stiffness matrix* of the fixture. This allows us to express the fixture stiffness in terms of the contact wrenches and the workpiece displacement. The inverse of the fixture stiffness matrix is called the *compliance matrix* **C**. Equation 13 shows that the contact stiffness is dependent on the fixture stiffness and contact positions of the fixture elements. The fixture stiffness matrix is associated with a fixed-coordinate origin, chosen arbitrarily (point *O*). We shall call this point the compliance centre. It is the central point for the whole system. Each of the fixture elements in contact with the workpiece is coupled to this axis. The fixture stiffness matrix, **k**, allows the fixture stiffness and the compliance centre to be arbitrarily chosen. This is important in fixture design, because sufficient rigidity needed for the fixture can be provided by choosing appropriate positions for the fixture elements. For a system in which only the clamping forces act as external forces, the clamping forces can be computed as: $\mathbf{F}_c = \mathbf{G}_l\mathbf{k}_l\mathbf{G}_l^T\mathbf{d}_E$.

## 6 Local elastic deformation of contact point due to contact forces

Machining forces on the workpiece could cause local elastic deformations at the points of contact between the locators and clamps, resulting in workpiece displacement. This is known as contact deformation, and contact stiffness plays a major role in such a deformation. The contact reaction forces between the surfaces of the workpiece and fixture elements are a direct consequence of the clamping and cutting forces imparted to the workpiece in addition to the weight of the workpiece. The contact deformation during clamping and machining is a function of the material properties, contact geometry of the workpiece/clamp, contact forces.

Consider two contacting bodies A and B (Fig. 2), which are both linearly elastic at the contact region ${}^{i}C$. Let body A be the locator/clamp and body B be the workpiece. It has been investigated [2–4] and shown through experiments that the workpiece–fixture system deforms elastically at the contact points. Therefore, the contact between A and B is modelled with linear springs. The stiffness of the workpiece–fixture contact can then be modelled using the linear springs as shown in Fig. 2. Let the locator have normal stiffness ${}^{i}k_f$, in the normal $z$ direction and tangential stiffnesses be ${}^{i}k_{fa}$ and ${}^{i}k_{fb}$ in the tangential $x$ and $y$ directions, respectively, at the contact region. Similarly, the normal and tangential stiffnesses of the workpiece are ${}^{i}k_w$, ${}^{i}k_{wa}$ and ${}^{i}k_{wb}$, respectively, at the contact region. The contact stiffness is defined the same way as what has been used to determine the equivalent stiffness of a group of serially connected linear springs.

The composite stiffness at each contact point can be modelled as the summation in series of the individual stiffnesses of the workpiece and locator/clamp as:

$$\frac{1}{{}^{i}k_{C_l}} = \left[\frac{1}{{}^{i}k_f} + \frac{1}{{}^{i}k_w}\right] \tag{14}$$

as the normal contact stiffness at the $i$th contact point between the locator and workpiece and

$$\frac{1}{{}^{i}k_{C_{la}}} = \left[\frac{1}{{}^{i}k_{fa}} + \frac{1}{{}^{i}k_{wa}}\right] \tag{15}$$

as the tangential contact stiffness at the $i$th contact point between the locator and workpiece. The expressions on the right-hand sides of Eqs. 14 and 15 are only first estimates.

Subscripts $f$ and $w$ refer purely to the locator/clamp and workpiece, respectively; (${}^{i}k_f$, ${}^{i}k_w$) and (${}^{i}k_{fa}$, ${}^{i}k_{wa}$) are the normal and tangential stiffnesses of the locator/clamp and workpiece, respectively. Substituting Eqs. 14 and 15 into Eq. 6 will give the $i$th contact force. Also, it is worth noting that, for all tangential directions with subscript $a$, the tangential directions with subscript $b$ are similarly written. Thus, the contact stiffnesses are related to the material properties of the fixture elements and the workpiece at contact point $C$.

### 6.1 Estimation of normal and tangential contact stiffnesses

A semi-empirical method for determination of normal contact deformation has been provided in [3]. The equation is adopted with a little modification. The equivalent normal contact stiffness can then be expressed as:

$${}^{i}k_{C_l} = \left(\left[0.77\left(\frac{1}{{}^{i}E_f} + \frac{1}{{}^{i}E_w}\right)^{\frac{2}{3}} \cdot \frac{1}{\rho^{1/3}}\right]k\right)^{-1},$$

where $E_f$ and $E_w$ are the moduli of elasticity of locator and workpiece materials, respectively. $k = \frac{P^{2/3}}{P}$ and $P$ is the normal contact force. The normal stiffness is determined as $1.06 \times 10^5$ N/mm for steel on steel and $0.69 \times 10^5$ N/mm for steel on aluminium, for an average value of $k = 0.0714$ with $P$ ranging from 0 to 5,000 N. The radius of curvature $\rho$ is taken to be 20 mm [3]. The tangential stiffness is estimated to be about one-third of the normal stiffness value on the fact that the friction capacity equals the normal load times the coefficient of friction (0.3).

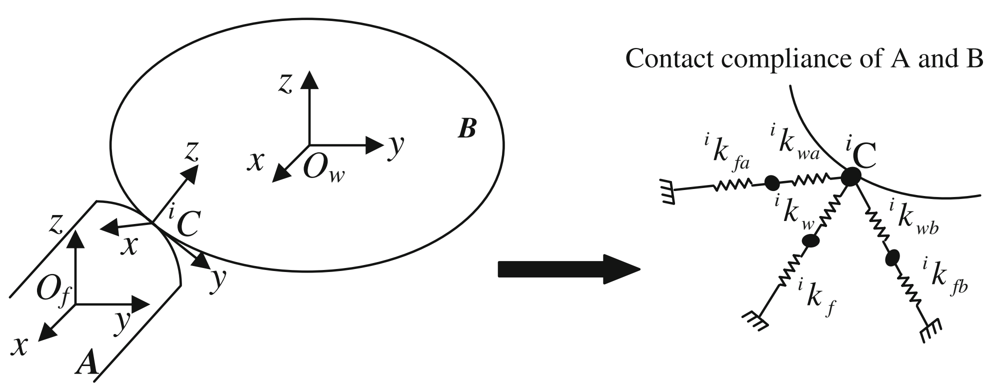


**Fig. 2** Locator/clamp in contact with workpiece

## 7 Stability analysis

When a workpiece is fixtured, all the fixture elements act together. The overall fixture stiffness matrix as given in Eq. 13 can now be used in Eq. 16 below. A machining force applied to the workpiece moves the workpiece to a new position. The fixture resists attempts by this force to stretch or compress it or move it rigidly. Thus, the restoring force tends to bring the workpiece back to its equilibrium state. The restoring force is generated at the tips of the fixture elements in response to small displacements of the workpiece from its originally stable (equilibrium) position after clamp actuation. The relationship between the cutting forces and moments, $\mathbf{F}_e$, and the workpiece rigid body displacement, $\mathbf{d}_E$, in the GCS is given by:

$$\mathbf{F}_e = \mathbf{k}\mathbf{d}_E, \tag{16}$$

where $\mathbf{k}$ is the effective stiffness matrix of the workpiece–fixture system. From Eq. 13, it is clear that the fixture stiffness matrix $\mathbf{k}$ is determined by the positions of the locators/clamps. The stiffness matrix $\mathbf{k}$ can be changed by using different materials or by changing the contact positions of the fixture elements. The contact positions of the fixture elements are defined in a coordinate system. This stiffness matrix is different from that of robotics in that there is no readjustment of the stiffness during fixturing. Another difference between robotics and fixtures lies in the fact that all of the robot fingers are active and apply forces, whereas all of the fixture locators are passive elements and only clamps can be considered to be active. Third, in fixtures, the contact locations of the fixture elements are fixed and predetermined. Fourth, the contact point positions change with workpiece displacement in robot finger grasping. In fixtures, contact point positions do not change with workpiece displacement. Fifth, in robotics, each finger's stiffness can be varied independent of other fingers to control the contact force. This is not possible with fixtures. In order to have a small value of $\mathbf{d}_E$, a high $\mathbf{k}$ is necessary for fixed $\mathbf{F}_e$. On the other hand, a lower $\mathbf{F}_e$ will reduce workpiece displacement.

The fixture stiffness matrix $\mathbf{k}$ represents a measure of the restoring forces along each contact axis of motion after the fixtured workpiece is displaced from its equilibrium position. So that the stability of the workpiece can be measured using the stiffness matrix. Equation 16 gives the relationship between the fixture stiffness matrix and the workpiece displacement. However, in a situation where one must select among several alternative fixtures, it will be prudent to select the system that gives the least of the smallest eigenvalues of all the alternatives.

The stiffness matrix of the fixture system must be invertible. If it is not invertible, then the system is not stable. This means that the fixture elements are free to move or deflect without deforming. If fixture elements move in this way, then the applied cutting forces can produce infinite or undetermined displacements. The fixture structure has an invertible stiffness matrix if and only if $\det|\mathbf{k}| \neq 0$. The determinant of the stiffness matrix can be calculated as the product of its eigenvalues, i.e., $\det|\mathbf{k}| = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 \cdot \lambda_5 \cdot \lambda_6$. For a stable workpiece, the larger the determinant is, the greater ability the fixture has to withstand any disturbance force–moment pair on the workpiece. If any of the eigenvalues are equal to or less than zero, the structure is not stable.

It can be seen from Eq. 16 that the fixture stiffness matrix varies with the fixture element positions (i.e., positions of locators/clamps). The applied cutting force displaces the workpiece by an amount which depends on the stiffness of a fixture. In fixturing application, it is important that the displacement of the workpiece is as small as possible to achieve quality in machining. In this analysis, it is assumed that the measure of stability depends on the eigenvalues of fixture stiffness matrix. The eigenvalues from this analysis cannot be compared directly to the eigenvalues computed using a different frame of reference. The reason for this is that the terms of the fixture stiffness matrix are in millimetres. Thus, the eigenvectors of the two analyses will be different from each other due to a rigid body translation or rotation. Since the eigenvectors are different, their eigenvalues will also be different. [34] presents a method for comparing solutions to problems where different reference frames are used. They suggest the measure be normalised by multiplying the stiffness matrix by the inverse matrix of the mass distribution of the workpiece. This way, the units will be the same and a change in reference frame or units will not affect the measure. Assuming $S_w$ to be the stability measure,

$$S_w = \min\left\{\text{eigenvalue}\left(\mathbf{M}_w^{-1}\mathbf{k}\right)\right\} = \min\{\lambda_1, \ldots..\lambda_6\}, \tag{17}$$

where $\mathbf{M}_w$ is a non-singular matrix, which depends on the mass distribution of the workpiece. In the absence of information about the mass distribution of the workpiece, it can be assumed that the mass distribution $\mathbf{M}_w$=$\mathbf{I}$, where $\mathbf{I}$ is a unit matrix [34].

Since the eigenvalues are not dependent on the cutting forces, an additional consideration must also be used when selecting the fixture. From Eq. 16, the workpiece displacement due to the cutting forces is defined as $\mathbf{d}_E$= $\mathbf{CF}_e$. From this relationship, we note that the displacement of the workpiece is a function of the cutting forces and the compliance of the fixture. To select from the available fixtures, a measure can be defined for each fixture from Eq. 16. The measure is the largest displacement of the

**Table 1** Locator and clamp positions of three different fixture arrangements

| | Fixture 1 | | | Fixture 2 | | | Fixture 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | *x* | *y* | *z* | *x* | *y* | *z* | *x* | *y* | *z* |
| Locator 1 | 60 | 70 | 0 | 60 | 70 | 0 | 10 | 10 | 0 |
| Locator 2 | 110 | 10 | 0 | 110 | 10 | 0 | 10 | 70 | 0 |
| Locator 3 | 10 | 10 | 0 | 10 | 10 | 0 | 110 | 40 | 0 |
| Locator 4 | 110 | 80 | 18 | 0 | 70 | 18 | 110 | 0 | 18 |
| Locator 5 | 10 | 80 | 18 | 0 | 10 | 18 | 10 | 0 | 18 |
| Locator 6 | 0 | 40 | 18 | 0 | 60 | 18 | 0 | 40 | 18 |
| Clamp 1 | 120 | 40 | 18 | 120 | 40 | 18 | 120 | 40 | 18 |
| Clamp 2 | 60 | 0 | 18 | 60 | 0 | 18 | 60 | 80 | 18 |

Note: All dimensions are in millimetres

workpiece due to the cutting force. Therefore, we define a displacement measure as,

$$Q_{dis} = \|\mathbf{d}_E\| = \|\mathbf{C}\mathbf{F}_e\| = \sqrt{\mathbf{d}_E{}^T\mathbf{d}_E} = \sqrt{\mathbf{F}_e^T\mathbf{C}^T\mathbf{C}\mathbf{F}_e}$$
$$= \sqrt{\mathbf{F}_e^T\mathbf{C}^2\mathbf{F}_e} \qquad (18)$$

where $\|\bullet\|$ is the root mean square (rms) norm. A given tolerance $\delta$ for the workpiece is satisfied when $Q_{dis} \leq \delta$. Also, since workpiece displacements are to be as small as possible, the fixture that gives the minimum $Q_{dis}$ among the fixtures is the best fixture.

## 8 Examples

In all the cases in these examples, a coefficient of friction of 0.62 is assumed and a workpiece of dimension 120 × 80 × 30 mm is used. Let us assume that a 76.20-mm-diameter plain milling cutter with ten teeth is used to slab mill a workpiece with a depth of cut of 9.525 mm. A 3–2–1 locating scheme in which the workpiece is supported by six spherically tipped locators, each having a contact radius of $a$ = 5 mm, was used. The fixture elements are made of steel, having an elastic modulus of $E$ = 200 GPa and Poisson ratio ν = 0.296.

**Table 3** Comparison of displacement and stability criteria for the three fixtures (aluminium)

| Material | Criteria | Fixture 1 | Fixture 2 | Fixture 3 |
|---|---|---|---|---|
| Aluminium workpiece | Minimum eigenvalue $S_w \times 10^5$ (N/mm) | 0.60 | 0.78 | 0.78 |
| | Largest displacement $Q_{dis}$ ($\mu$m) equation (18) | 9.7 | 8.3 | 8.1 |

The first workpiece is a low carbon steel and Brinell hardness number of 125 BHN. The workpiece is clamped in a steel fabricated fixture. The cutting speed $V_c$ is 18.288 m/min and the feed rate is 0.0508 mm/tooth. Using [35], the cutting force and torque are calculated to be 2,734 N and 103.85 Nm, respectively. The tangential and radial components of the cutting force are represented by $F_t$ and $F_R$, which can be split into the force components $F_x$ and $F_z$ in the Cartesian coordinate system. The magnitudes of these force components can be calculated from the magnitude of the cutting force $F_c$ by using the relationships ($F_t$=$F_c$, $F_R$ = 0.3, $F_c$ = 820.2 N) recommended in [36]. Therefore, the magnitudes of the forces are calculated as follows: $F_x$ = 2592.98 N and $F_z$ = 1193.12 N. A second workpiece is made of aluminium with a Brinell hardness of 120 BHN and an elastic modulus of $E$ = 70 GPa, Poisson ration ν = 0.334 is machined under the same cutting conditions as the previous workpiece.

A third workpiece made of aluminium with a Brinell hardness of 120 BHN. The cutting tool speed is 152.4 m/min. All other conditions remain the same. In this case, the cutting force and torque are calculated to be 882 N and 33.5 Nm, respectively. The force components are, respectively, $F_x$ = 836.51 N and $F_z$ = 384.91 N. Table 1 shows the locator and clamp positions for three different fixturing arrangements from which to select. This is used to simulate the different fixturing arrangements. All values are in millimetres.

The first part of this example is to demonstrate how to select a fixture for a rectangular workpiece under milling operations for different workpiece materials. This example demonstrates how the stability criteria and minimum displacement (Eqs. 17 and 18) are used to select the best fixture. Table 2 shows the comparison of these two stability

**Table 2** Comparison of displacement and stability criteria for the three fixtures (same cutting conditions)

| Material | Criteria | Fixture 1 | Fixture 2 | Fixture 3 |
|---|---|---|---|---|
| Steel workpiece | Minimum eigenvalue $S_w \times 10^5$ (N/mm) | 0.92 | 1.20 | 1.10 |
| | Largest displacement $Q_{dis}$ (μm) equation (18) | 19.5 | 16.8 | 16.4 |
| Aluminium workpiece | Minimum eigenvalue $S_w \times 10^5$ (N/mm) | 0.60 | 0.78 | 0.78 |
| | Largest displacement $Q_{dis}$ (μm) equation (18) | 30.0 | 25.7 | 25.2 |

indices for three different fixture arrangements. The cutting force for this analysis was 2,734 N.

From the results in Table 2, we easily notice that, for the steel workpiece, fixture 1 has the minimum eigenvalue. This means that this arrangement of locator and clamp positions is best among the three fixtures. However, it has the highest $Q_{dis}$. This means that the workpiece may have higher dimensional errors than fixtures 2 and 3. Next is fixture 2 with minimum eigenvalue and $Q_{dis}$ higher than those of fixture 3. Judging from the values obtained for fixtures 1 and 2, a compromise choice may be appropriate here. The best compromise fixture arrangement to choose will therefore be that of fixture 3.

The minimum eigenvalues for the aluminium workpiece for both fixtures 2 and 3 are greater than that of fixture 1. However, the largest displacement $Q_{dis}$ for fixture 1 is greater than that for fixtures 2 and 3. As a compromise, fixture 3 may be well suitable for the operation.

Having done this, we now consider the combined influence of fixture compliance and cutting conditions on workpiece stability for the three fixtures. Different cutting conditions are used. The cutting force and cutting speed for this case are 882 N and 152.4 m/min. The material of the workpiece is aluminium. The results are tabulated in Table 3.

By comparing tabulated results from Tables 2 and 3 for the aluminium workpiece, it can be seen that the eigenvalues remained the same when the cutting conditions changed. However, the displacements for all three fixture arrangements were reduced by 68% each, a significant reduction. It can be concluded that cutting conditions have significant effect on workpiece displacement. The possibility of achieving higher dimensional accuracy is shown by three fixturing arrangements when the cutting conditions changed. However, fixture three shows the smallest largest displacement and may be the compromise fixture.

## 9 Conclusions

In this paper, we have computed and analysed fixture compliance and cutting conditions on workpiece stability and used it as a basis for selecting a suitable fixture among several alternatives. Two considerations for selecting a fixture based on workpiece stability were developed. In the first, the minimum eigenvalues of the fixture stiffness matrices, for the fixtures being considered, are computed. The minimum eigenvalues represent the minimum displacements at the contact points (locators and clamps). The fixture having the smallest value for the minimum eigenvalue is the best choice based on this consideration. Since the eigenvalues are not dependant on the cutting forces, an additional consideration must also be used when selecting a fixture. The displacement of the workpiece is a function of the cutting force and the compliance of the fixture. To select from the available fixtures, a displacement measure, the largest displacement of the workpiece due to the cutting force, is computed for each fixture. Since workpiece displacements are to be as small as possible, the fixture that gives the minimum of the largest displacement of each of the fixtures is the best fixture based on this consideration. The choice of the fixture to use is often a compromise between the two considerations. We also consider the combined influence of fixture compliance and cutting conditions on workpiece stability. The results from the simple study used for illustration show that the eigenvalues remains constant whilst the largest displacement reduces by 68%, a significant reduction, for all the fixturing arrangements. This shows that cutting conditions have a significant effect on workpiece stability.

These results are very interesting and require further investigation. This includes different workpiece materials, depth of cut, workpiece size, tool type and an experimental verification. The effects of other cutting conditions need to be investigated. The results will be reported in a separate paper.

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