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ORIGINAL ARTICLE

Machining fixture layout design using ant colony algorithm based continuous optimization method

K. P. Padmanaban · K. P. Arulshri · G. Prabhakaran

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Abstract In any machining fixture, the workpiece elastic deformation caused during machining influences the dimensional and form errors of the workpiece. Placing each locator and clamp in an optimal place can minimize the elastic deformation of the workpiece, which in turn minimizes the dimensional and form errors of the workpiece. Design of fixture configuration (layout) is a procedure to establish the workpiece–fixture contact through optimal positioning of clamping and locating elements. In this paper, an ant colony algorithm (ACA) based discrete and continuous optimization methods are applied for optimizing the machining fixture layout so that the workpiece elastic deformation is minimized. The finite element method (FEM) is used for determining the dynamic response of the workpiece caused due to machining and clamping forces. The dynamic response of the workpiece is simulated for all ACA runs. This paper proves that the ACA-based continuous fixture layout optimization method exhibits the better results than that of ACA-based discrete fixture layout optimization method.

K. P. Padmanaban (✉)
Department of Mechanical Engineering,
PSNA College of Engineering and Technology,
Dindigul, Tamilnadu, India
e-mail: padmarubhan@yahoo.co.in

K. P. Arulshri
Department of Mechanical Engineering,
KONGU Engineering College,
Perundurai,
Erode, Tamilnadu, India

G. Prabhakaran
Department of Mechanical and Industrial Engineering,
Caledonian College of Engineering,
Muscat, Sultanate of Oman
e-mail: g_prabha2006@yahoo.co.in

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1 Introduction

Fixture layout design is a major concern in the development of automated fixture design systems. The task of fixture layout design is to layout a set of locating and clamping points on workpiece surfaces such that the workpiece is accurately located and completely restrained during manufacturing operations.

Fixture layout design has received considerable attention in recent years. However, little attention has been focused on optimization of machining fixture layout under dynamic conditions of the workpiece.

Kinematic analysis-based fixture design is described in several research publications including Bausch et al [1], Menassa et al. [2] and Mani et al. [3] who have considered the rigid body models for fixture configuration and/or clamping force optimization. Few researchers have used finite element method for contact elastic models in fixture layout design. Lee et al. [4] were among the first to use the finite element method for fixture design and synthesis. Menassa et al [5] also used the finite element method to synthesize support position for plate-type workpiece. Though the finite element method is best suited for predicting the elastic deformation at any point on the workpiece surface, it has mainly been used for modeling the workpiece–fixture contact points.

A number of fixture design optimization approaches has been reported in the recent researches. King et al. [6] presented a nonlinear optimization technique to determine a statically stable fixture layout. They used a rigid body model of the fixture–workpiece system but accounting for the contact stiffness. Menassa et al. [2] used a nonlinear optimization algorithm to synthesize support positions for a plate-type workpiece with an objective of minimizing the summation of nodal displacements at specific points on the surface to be machined. In addition to this, Kashyap and DeVries [7] used the finite element analysis method to determine positions of the fixture supporting points in order to minimize normal deformations of the workpiece at the primary locating surface. DeMeter [8] presented a nonlinear optimization algorithm, but they ignored the general locator and clamp position synthesis problem. Kang [9] considered the case of a locating from a flat surface to model the geometric relation between workpiece displacement and locator displacement. It was assumed the workpiece position error resulted from manufacturing and setup errors of locators. The locator layout is finally obtained by optimizing a user-defined locating performance index. Li et al. [10] presented an optimal synthesis approach of fixture layout and clamping force that considers workpiece dynamics during machining and determined the optimal clamping force for a multiple clamp fixture subjected to quasi-static machining force. Sanchez et al. [11] calculated the contact load distribution and valid clamping regions in machining processes. They also calculated the contact load using a non-iterative means by modeling both fixture and workpiece as separate and independent FEM problems. Zhijun Li et al. [12] adopted a nested optimization strategy and proposed a methodology which was demonstrated using a vehicle side frame assembly example. Deng et al. [13] presented a model-based framework for determining the minimum required clamping force, which ensures the dynamic stability of a fixtured workpiece during machining. Tan et al. [14] described the modeling, analysis, and verification of optimal fixturing configurations by the methods of force closure, optimization and finite element modeling. The above studies have the following drawbacks: (1) either the linear or nonlinear programming methods, which often do not give global optimum solution, were used (2) the fixture layout optimization for the overall workpiece deformation was not considered, and (3) all the fixture layout optimization procedures addressed in the above studies start with an initial feasible layout.

Then, heuristic approaches have been used as alternatives to obtain reasonably good solutions within acceptable time limits. Lai et al. [15] set up an analysis model that treated locator and clamps as the same fixture layout elements for the flexible part deformation. Hamedi [16] discussed a hybrid learning system that used nonlinear finite element analysis with a supportive combination of artificial neural network and genetic algorithm. Krishnakumar et al. [17] presented a genetic algorithm-based discrete fixture layout optimization method to minimize the deformation of the workpiece under static conditions. Krishnakumar et al. [18] presented an iterative algorithm that minimized the workpiece elastic deformation for the entire cutting process by alternatively varying the fixture layout and clamping force. Qin et al. [19] developed a mathematical approach to analysis and optimal design of a fixture locating scheme.

Prabhaharan et al. [20] used a genetic algorithm-based discrete fixture layout optimization method. There are two main drawbacks of the discrete fixture layout optimization method: (1) only the nodal points are used as design variable, and hence, the optimum values are found only on these nodal points and (2) the number of GA generations is very low. Vallapuzha et al. [21] presented a new GA-based optimization method that uses spatial coordinates to represent the locations of fixture elements. They optimized only the locator's position but ignored the clamp's position. Kaya [22] used a genetic algorithm-based continuous fixture layout optimization method, but the dynamic effects of the workpiece were not considered. Marcelin [23] optimized the support positions in machining of mechanical part using GAs.

There are currently a lot of ongoing activities in the scientific community to extend/apply ant-based algorithms to many different discrete optimization problems. Recent applications cover problems like vehicle routing, sequential ordering, graph coloring, routing in communications networks so on. Dorigo et al. [24] proposed an ant colony algorithm and applied it to the traveling salesman problem. They also compared the solutions of ACA and shown to be better than other heuristic approaches like GA, evolutionary programming (EP), simulated annealing (SA), and a combination of GA and SA. Patrick [25] and Jeyaraman [26] have already proved that an ant colony algorithm is a useful technique in solving optimization problems in engineering applications. Prabhaharan et al. [27] used an ant colony system approach for the cellular manufacturing systems and compared the ACA results with GA results and proved that the performance of ACA was better than that of GA. Prabhaharan et al. [28] used an ant colony system as an optimization tool for minimizing the critical dimension deviation and allocating the cost based optimal tolerances. Prabhaharan et al [29] and Padmanaban et al [30] used an ant colony algorithm-based discrete optimization method and optimized the fixture layout under static conditions and dynamic conditions, respectively. They also proved that the performance of ant colony algorithm in the fixture layout optimization method was better than the performance of genetic algorithm.

In this paper, an ant colony algorithm-based continuous optimization method is applied for the machining fixture layout optimization problem with an objective of minimizing the workpiece elastic deformation caused during machining. Main contributions of this paper are summarized as follows: (1) FEM is used to model the workpiece elastic deformation caused due to harmonic load, (2) ACA code integrated with a finite element solver has been developed using MATLAB, (3) The dynamic effects of the workpiece are considered, since periodic forces often characterize machining process, (4) Dynamic response of the workpiece caused due to clamping and machining forces is simulated, (5) the ant colony algorithm-based continuous optimization method is employed and its performance is compared with that of ACA based discrete fixture layout optimization method, and (6) the maximum allowable workpiece elastic deformation is defined for the workpiece geometry and the fixture layout is optimized.

2 Fixture layout optimization method

The fixture layout optimization method provides an optimal fixture layout for minimizing workpiece elastic deformation. In the fixture layout optimization method, the number of design variables is the number of fixturing elements. The minimum number of fixturing elements required to constrain the two- and three-dimensional workpiece geometries is five (three locators and two clamps) and nine (six locators and three clamps), respectively.

The fixture layout optimization method can be classified as (1) discrete fixture layout optimization method and (2) continuous fixture layout optimization method. The discrete fixture layout optimization method finds an optimal position of each fixturing element for the node system (number of nodes defined along X and Y axes) defined on the workpiece geometry, whereas the continuous fixture layout optimization method finds an optimal position of each fixturing element for the range of distance defined for the each fixturing element. In this research work, the ant colony algorithm is applied for the discrete and continuous fixture layout optimization methods.

3 Finite element method

Finite element method is a powerful tool for determining the deformation at any point on the workpiece surface. In this research work, the finite element method is used for determining the dynamic response of the workpiece caused due to machining and clamping forces. The assumptions made for the finite element formulation are as follows:

- Work piece is an elastic body, whereas the fixturing elements are the rigid bodies;
- Number of degrees of freedom per node is two;
- External loads are the inplane loads;
- Machining force is an impulse force;
- Workpiece is subjected to only plane stress;
- Response of the workpiece under the dynamic conditions is considered along the plane only;
- Consistent mass system is considered in the mass matrix derivation.

3.1 Finite element formulation of workpiece elastic deformation

The global equation of motion [30] obtained by assembling the elemental equations is written as

$$[\mathrm{m}]\ \ddot{\vec{Q}} + [\mathrm{k}]\ \vec{Q} = \vec{F} \tag{1}$$

where, [m] is global mass matrix, $\ddot{\vec{Q}}$ is global nodal acceleration vector, [k] is global stiffness matrix, $\vec{Q}$ is global nodal displacement vector, and $\vec{F}$ is global load vector.

3.2 Mode superposition method (modal analysis)

In modal analysis, the mode superposition method is used to reduce the coupled equations of motion in the physical coordinates into a set of uncoupled equations of motion in modal coordinates. The set of coupled equations of motion of a multidegree of freedom system under external forces is reduced to a set of uncoupled equations of motion [30] as

$$\ddot{\vec{\eta}}(t) + \left[\omega^2\right]\ \vec{\eta}(t) = \vec{G}(t) \tag{2}$$

3.3 Solution procedure for uncoupled equations of motion

The equation of motion in terms of generalized coordinates is

$$\ddot{\vec{\eta}}(t) + \left[\omega^2\right]\ \vec{\eta}(t) = \vec{G}(t)$$

where $\vec{G}(t) = [\mathrm{X}]^{\mathrm{T}}\vec{F}(t) = \begin{Bmatrix} G_1(t) \\ G_2(t) \\ \cdot \\ \cdot \\ G_n(t) \end{Bmatrix}$ and

where, $[\mathrm{X}]^T$=modal matrix and

$$\vec{\eta}(t) = \begin{Bmatrix} \eta_1(t) \\ \eta_2(t) \\ \cdot \\ \cdot \\ \eta_n(t) \end{Bmatrix}$$

the boundary conditions used in the above differential equation are,

$$\vec{Q}(t=0) = \vec{0} = [X]\ \vec{\eta}(0)$$
$$\dot{\vec{Q}}(t=0) = \vec{0} = [X]\ \dot{\vec{\eta}}(0)$$

so that $\vec{\eta}(0) = \vec{0}$ and $\dot{\vec{\eta}}(0) = \dot{\vec{0}}$ independent solution for $\vec{\eta}(t)$is written as

$$\vec{\eta}_i(t) = \frac{1}{\omega_i}\int_0^t G_i(\tau)\sin\omega_i(t-\tau)d\tau \quad i = 1,2,\ldots..,\mathrm{n} \tag{3}$$

where, n=number of finite element equations

The physical displacements are finally found using the following equation.

$$\vec{Q}(t) = [\mathrm{X}]^{\mathrm{T}}\vec{\eta}(t) \tag{4}$$

where, $\vec{Q}(t)$is nodal displacement vector, which gives the elastic deformation at different points on the workpiece. The flow chart shown in Fig. 1 explains the step by step procedure of built in finite element solver to determine the workpiece elastic deformation.

4 Ant colony algorithm

Colonies of social insects can exhibit an amazing variety of complex behaviors and have always captured the interest of biologists and entomologists. The study of ant colonies behavior turned out to be very fruitful, giving rise to a completely novel field of research, now known as *ant algorithms*. In ant algorithms, a colony of relatively simple agents called as ants, efficiently carries out complex tasks such as resource optimization and control. Ants deposit a chemical substance (called pheromones) or induce some other physical modifications on the environment, while carrying out their own tasks. These modifications change the way sensed by the other ants in the colony and implicitly act as a signal triggering other ants' behaviors that again generate new modifications that will simulate other ants and so on.

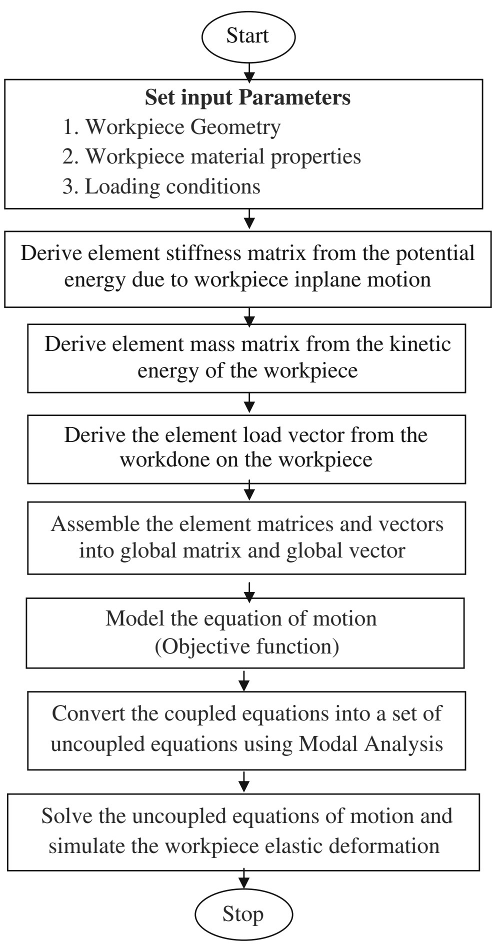

Fig. 1 Flow chart of built in finite element solver

4.1 ACA-based fixture layout optimization method

To apply the ant colony algorithm for the fixture layout optimization problems, randomly 'R' solutions are selected from different possible solutions. A critical value is fixed about which number of superior and inferior solutions are defined. Global search is carried for inferior solutions, whereas the local search is carried out for the superior solutions. In this paper, the ant colony algorithm is used in discrete and continuous fixture layout optimization methods. The various processes involved in ant colony algorithm-based fixture layout optimization method are as follows: (1) initialization, (2) global search, and (3) local search. Fig. 2 explains the distribution of ants for local search and global search.

4.1.1 Initialization

1. Twenty fixture layouts are randomly generated from the range of distance defined for each fixturing element, and their solutions are found using FEM.
2. Then, the fixture layouts are sorted according to ascending order of the solutions.
3. The solutions from 1 to12 are named as superior solutions and from 13 to 20 are named as inferior solutions.

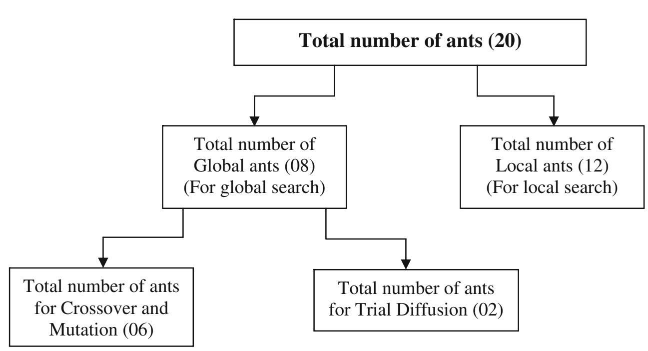

Fig. 2 Distribution of ants for local search and global search

4.1.2 Global search

The global search is done to improve the inferior solutions. This search includes crossover or random walk, mutation and trail diffusion.

Crossover or random walk In this process, the inferior solutions from 13 to 18 are replaced by the superior solutions. This process includes the following steps:

1. Replacement of each inferior solution by a superior solution is decided based on the crossover probability.
2. To replace 13th solution, a random number between 1 and 12 is generated. Then, the corresponding solution in the superior region replaces the 13th inferior solution.
3. The selected solutions in the superior region should be excluded, so that it is not selected again for replacement.
4. The above procedure is repeated up to the 18th solution.

Mutation The mutation process further improves the replaced solutions. This process includes the following steps:

1. The position of each fixturing element in the replaced 13th layout is modified by adding or subtracting with mutation step size (Δ).

The mutation step size (Δ) is obtained as $\boldsymbol{\Delta = R(1-r^{(1-T)b})}$ where, $R=(X_{jmax}-X_i)$

X_{max}	maximum range of distance defined for the fixturing elements
X_i	distance of the respective fixturing element of the respective layout.
r	random number
T	ratio of current iteration to the total no of iteration
b	a constant (obtained by trial)

2. The mutation probability (P_m) is set. Then a random number is generated between 0 and 1. If the random number generated is less than P_m, the mutation step size (Δ) is subtracted to the node number of the respective fixturing element or else it is added to the node number of the respective fixturing element. The same procedure is repeated up to the 18th solution.

Trail diffusion The trail diffusion improves the 19th and 20th solutions. This process includes the following steps.

1. Two layouts are randomly selected from the superior solutions, and they are named as parent-1 and parent-2. The position of each fixturing element in parent-1 and parent-2 is termed as XP_1 and XP_2, respectively. The new layout obtained from parent-1 and parent-2 is termed as Child. The position of each fixturing element in the Child layout is termed as XC.
2. One more random number (α) is generated between 0 and 1 for the position of each fixturing element.
3. If α is between 0 and 0.5, then the new position of each fixturing element of the new layout is obtained by

$$\mathbf{XC} = (\alpha)\mathbf{XP_1} + (\mathbf{1}-\alpha)\mathbf{XP_2}$$

4. If α is between 0.5 and 0.75, then the new position of each fixturing element of the new layout is obtained by

$$\mathbf{XC = XP_1}$$

5. If α is between 0.75 and 1, then the new position of each fixturing element of the new layout is obtained by

$$\mathbf{XC = XP_2}$$

6. The above procedure is repeated for the 20th solution also. After the crossover or random walk, mutation, and trail diffusion processes, the solutions for the modified layouts from 13th to 20th are found using FEM.

4.1.3 Local search

The local search is done to improve the superior solutions from 1 to 12. This process includes the following steps.

1. The average pheromone value is calculated by

$$P_{ave} = \frac{\sum P}{N_S}$$

where,

P	pheromone value of each solution (Initially, pheromone value is assumed to be 1.0)
N_S	number of superior solutions

2. A random number is generated between 0 and 1. If the number generated is less than average pheromone value (P_{ave}), the search is further pursued or else the ant quits, and then leaves the solution without any alteration.
3. A limiting step value L_S, which is added to the node number of the respective fixturing element when the random number generated is greater than 0.5 and subtracted to the node number of the respective fixturing element when the random number generated is less than 0.5, is calculated as follows: $\mathbf{L}_S = \mathbf{K}_1 - (\mathbf{A} \times \mathrm{K}_2)$ where, K_1 and K_2 are the values chosen such that $K_1 > K_2$.

'*A*' is the age, which is assumed to be 10 for all the solutions in the first iteration.

4. All the layouts corresponding to the superior solutions are modified by local search and solutions for the modified layouts from 1 to 12 are found using FEM.

5. The new age for each solution for the next generation is calculated as follows:

   If the current solution is less than the previous solution, the age for the new solution is calculated as follows:

   $\mathbf{A}_i = \mathbf{A}_{i-1} + 1$

   If the new solution is greater than the previous solution, the age for the new solution is calculated as follows:

   $\mathbf{A}_i = \mathbf{A}_{i-1} - 1$

   where,

   A_i is the age for the new iteration,

   $A_{i\text{-}1}$ is the age for the previous iteration.

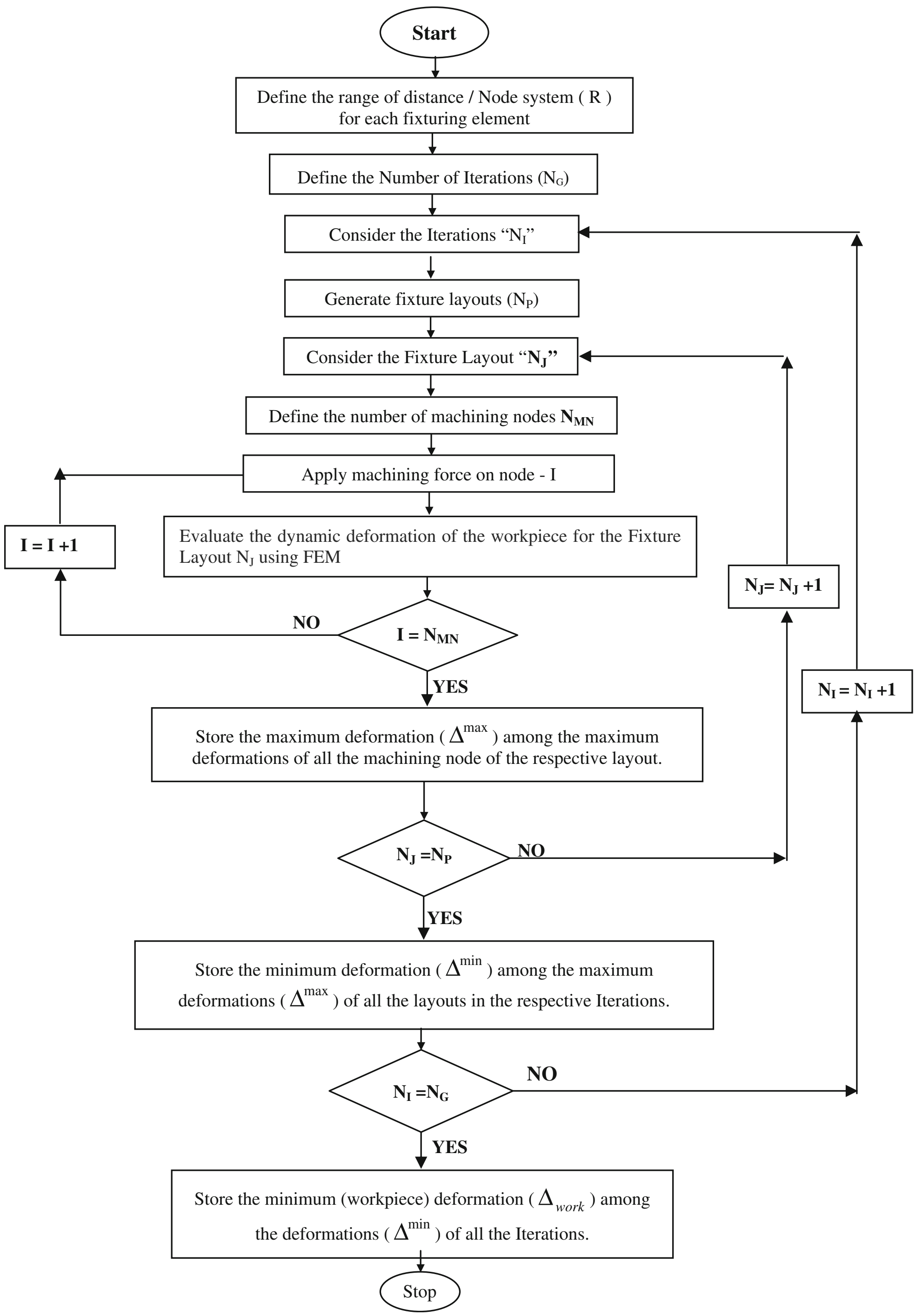

Fig. 3 Flow chart for ACA based fixture layout optimization method

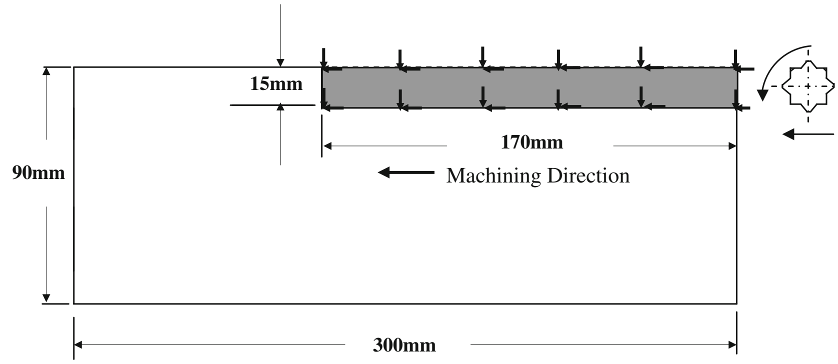

Fig. 4 Workpiece geometry

6. The new pheromone value of the ant (for each solution) in the next iteration is also calculated as follows:

$$P_i = \frac{S_i - S_{i-1}}{S_{i-1}} + P_{i-1}$$

where,

P_i is pheromone value for the new solution,
P_{i-1} is pheromone value for the old solution
S_i is the value of current solution,
S_{i-1} is the value of old solution.

The above steps, i.e., local and global searches are performed in all the iterations to improve the solutions.

5 Ant colony algorithm—convergence procedures

For each layout considered in the particular ACA iteration, the machining load is applied sequentially for all machining nodes (N_{MN}). The maximum deformation (Δ^{max}) among the maximums for each load application is found out. The same procedure is repeated for all the fixture layouts (N_P) for all the iteration. Then using ACA, the minimum deformation (Δ^{min}) among maximums for each layout is found. The same procedure is repeated for all the iterations (N_G). The algorithms converge if either the number of iterations over which no change in the objective function value is obtained, N_{chg}, or if the number of iterations, N, reaches the defined maximum number of iterations, N_G, whichever is earlier. The optimization of the fixture layout is carried out for the whole process in a single step. The different runs are performed in ACA-based continuous fixture layout optimization method until the minimum workpiece deformation corresponding to optimal layout satisfies the maximum allowable workpiece elastic deformation. The flow chart shown in Fig. 3 explains ACA-based fixture layout optimization method.

6 Problem formulation

In this paper, a milling operation is used to make a slot in the workpiece geometry. In slot milling operation, the dominant cutting force is assumed to act only along the workpiece plane. Since the cutting force is only along the plane, the response of the workpiece is assumed to be along the plane only [17], and hence, simple two-dimensional (2D) workpiece geometry is considered in this paper. In the machining fixture layout optimization problems, the design variables are the position of locators and clamps. The number of design variables, i.e., number of fixturing elements, used in each possible layout is five. First three design variables indicate the positions of the locators and next two corresponds to the clamp's position. Each design variable

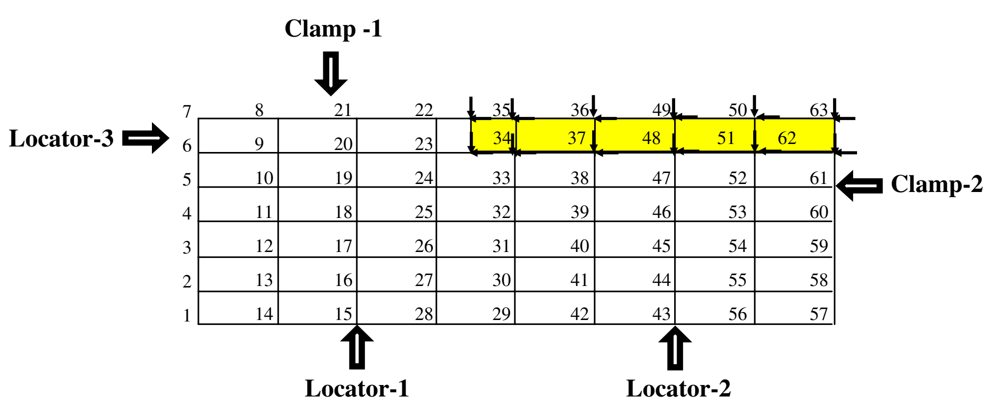

Fig. 5 Position of fixturing element using node system

is supposed to lie within a specific range. The objective function in the fixture layout optimization problems is the maximum nodal deflection of the workpiece being machined.

A case study of simple 2D workpiece geometry taken from KAYA [22] is shown in Fig. 4. The machining force of 100 and 286 N is assumed to act along the X and *Y* axes, respectively, and the clamping forces of 200 and 300 N are assumed to act at clamp-1 and clamp-2, respectively. In this case study, the fixture layout is optimized for the condition of maximum allowable workpiece deformation, which is limited to 0.3 μm. The Young's modulus, Poisson's ratio, and density of the workpiece material are 2×10^5 N/mm^2, 0.3 and 7,890 kg/m^3, respectively. The material, diameter, number of teeth, feed, and speed of the milling cutter are HSS, 100 mm, 8, 0.13 mm/tooth, and 20 m/min [31], respectively.

6.1 Discrete fixture layout optimization method

In discrete fixture layout optimization method (DFLOM), the solution region of each fixturing element is the node system. The node number of Locator-1 and Clamp-1 is defined horizontally from the left–bottom corner and left-top corner of the workpiece respectively, the node number of Locator-2 is defined horizontally from the right–bottom corner of the workpiece and the node number of Locator-3 and Clamp-2 is defined vertically from left–bottom corner and right–bottom corner of the workpiece, respectively, as shown in Fig. 5. This method finds an optimal fixture layout for the node system defined for the workpiece geometry in order to minimize the elastic deformation of the workpiece. The fixture used for constraining a simple two-dimensional workpiece geometry consists of three locators and two clamps. The boundary condition used at

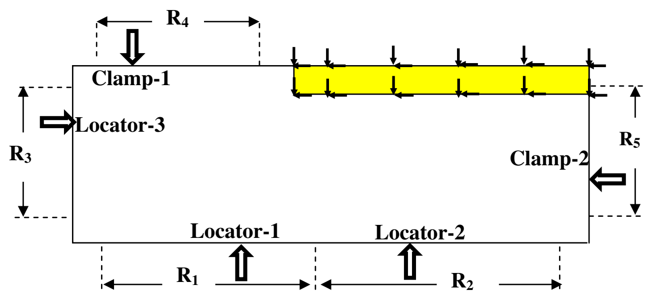

Fig. 6 Position of fixturing elements using range of distance

each locator contact point is as follows: (1) the displacement along the *Y* direction is zero at locators L_1 and L_2 and (2) the displacement along the *X* direction is zero at locator L_3. The details of machining node numbers defined for each node system is given in Table 1.

ACA-based discrete fixture layout optimization is formulated as follows:

Determine L_1, L_2, L_3, C_1, C_2
which minimize the workpiece elastic deformation
Subject to $0 < L_1 \ \& \ L_2 < N_{x1}$
$0 < L_3 \ \& \ C_2 < N_y$
$0 < C_1 < N_{X2}$

where,

L_1, L_2, and L_3	optimal node number of Locator-1, Locator-2, and Locator-3, respectively,
C_1 and C_2	optimal node number of Clamp-1 and Clamp-2, respectively.
N_{X1}	number of nodes defined along the *X*-axis for the bottom most surface of the workpiece
N_{X2},	number of nodes defined along X-axis for the top most surface of the workpiece
N_Y	number of nodes defined along the *Y*-axis

Table 1 Details of node systems and machining node numbers

Sl. no.	Node system		Number of machining nodes	Machining node numbers
	Nodes along *X*-axis	Nodes along *Y*-axis		
1	9	7	10	34, 35, 36, 37, 48, 49, 50, 51, 62, 63
2	10	7	12	34, 35, 36, 37, 48, 49, 50, 51, 62, 63, 64, 65
3	11	7	12	36, 37, 48, 49, 50, 51, 62, 63, 64, 65, 76, 77
4	12	7	14	36, 37, 48, 49, 50, 51, 62, 63, 64, 65, 76, 77, 78, 79
5	13	7	14	48, 49,50, 51, 62, 63, 64, 65, 76, 77, 78, 79, 90, 91

Table 2 Range of distance of fixturing elements

Sl. no.	Fixturing element	Range of distance of fixturing element (mm)
1	Locator 1	R_1=5–145
2	Locator 2	R_2=5–145
3	Locator 3	R_3=5–85
4	Clamp 1	R_4=5–125
5	Clamp 2	R_5=5–85

Table 3 Results of ACA-based discrete fixture layout optimization method

Sl.no	Node system		Output parameters															Least workpiece deformation (x E−1 μm)
			Optimal fixture layout															
			Optimal node numbers of					Optimal coordinate values of (mm)										
	Nodes along X-axis	Nodes along Y-axis	L_1	L_2	L_3	C_1	C_2	L_1		L_2		L_3		C_1		C_2		
								X	Y	X	Y	X	Y	X	Y	X	Y	
1	9	7	15	56	6	22	59	75	0	37.5	0	0	75	112.5	90	300	30	5.8
2	10	7	28	56	2	8	69	99.9	0	66.6	0	0	30	33.3	90	300	15	3.45
3	11	7	15	70	3	22	72	60	0	30	0	0	75	90	90	300	30	8.4
4	12	7	28	70	5	35	83	81.81	0	54.54	0	0	60	109	90	300	15	5.4
5	13	7	28	84	5	22	86	75	0	25	0	0	60	75	90	300	15	3.06

6.2 Continuous fixture layout optimization method

In continuous fixture layout optimization method (CFLOD), the solution region of each fixturing element is the range of distance in which each fixturing element is supposed to lie. The range of distance of Locator-1 and Clamp-1 is defined horizontally from the left–bottom corner and left–top corner of the workpiece, respectively, the range of distance of Locator-2 is defined horizontally from the right–bottom corner of the workpiece and the range of distance of Locator-3 and Clamp-2 is measured vertically from left–bottom corner and right–bottom corner of the workpiece, respectively, as shown in Fig. 6. This method finds an optimal fixture layout for the range of distance defined for each fixturing element in order to minimize the elastic deformation of the workpiece. The fixture used for constraining a simple two-dimensional workpiece geometry consists of three locators and two clamps. The boundary condition used at each locator contact point is as follows: (1) the displacement along the Y direction is zero at locators L_1 and L_2 and (2) the displacement along the X direction is zero at locator L_3. The range of distance defined for each fixturing element is given in Table 2.

ACA-based continuous fixture layout optimization is formulated as follows:

Determine L_1, L_2, L_3, C_1, C_2

which minimize the workpiece elastic deformation

Subject to $0 < L_1$ & $L_2 < 150$

$0 < L_3$ & $C_2 < 90$

$0 < C_1 < 130$

where,

L_1, L_2, and L_3, optimal position of Locator-1, Locator-2, and Locator-3, respectively,

C_1 and C_2, optimal position of Clamp-1 and Clamp-2, respectively.

7 Results and discussions

The results of ACA-based DFLOM, i.e., least workpiece elastic deformation, the coordinate values of optimal

Table 4 Results of ACA based continuous fixture layout optimization method

	Run	Optimal Coordinate Values of (mm)										Least Workpiece Deformation (x 10^{-1} μm)
		L_1		L_2		L_3		C_1		C_2		
		X	Y	X	Y	X	Y	X	Y	X	Y	
1	Run-1	81.1	0	26.3	0	0	36.6	43.4	90	300	11.24	2.96
2	Run-2	104.6	0	28.1	0	0	51.4	86.4	90	300	21.2	3.56
3	Run-3	86.1	0	32.4	0	0	56.6	30.1	90	300	30.4	2.98
4	Run-4	51.1	0	54.3	0	0	11.2	46.1	90	300	31.0	3.23
5	Run-5	82.0	0	30.1	0	0	36.6	45.7	90	300	11.2	2.84

position of each fixturing element, and the optimal fixture layout for each node system, are given in Table 3. The results given in Table 3 are the evident that the least workpiece elastic deformation is dependent on the node system in DFLOM. The optimal coordinate values of each fixturing element and the least workpiece elastic deformation obtained for all the runs performed in ACA-based CFLOM are given in Table 4. The simulation of workpiece

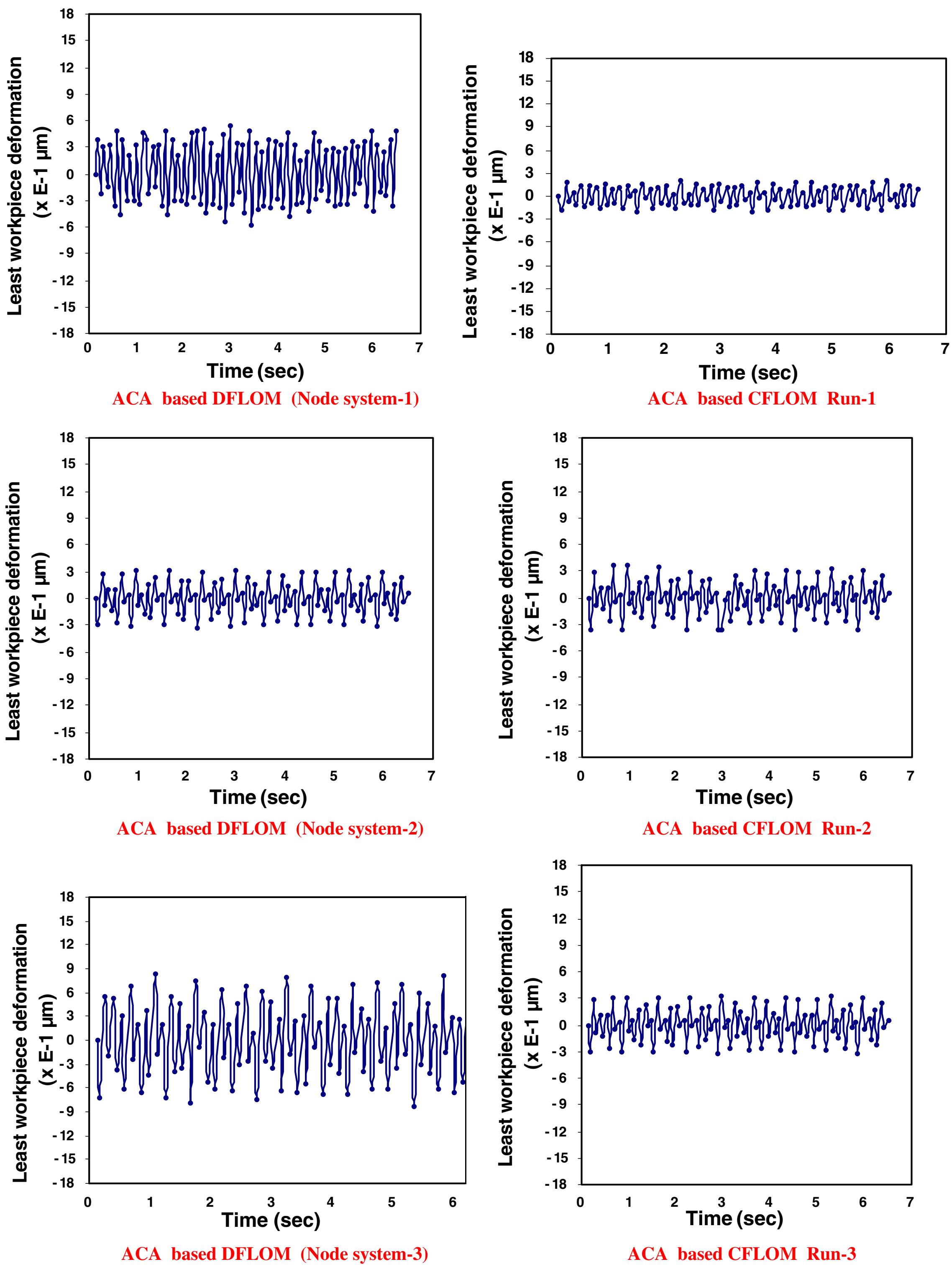

Fig. 7 Simulation of dynamic response of the workpiece

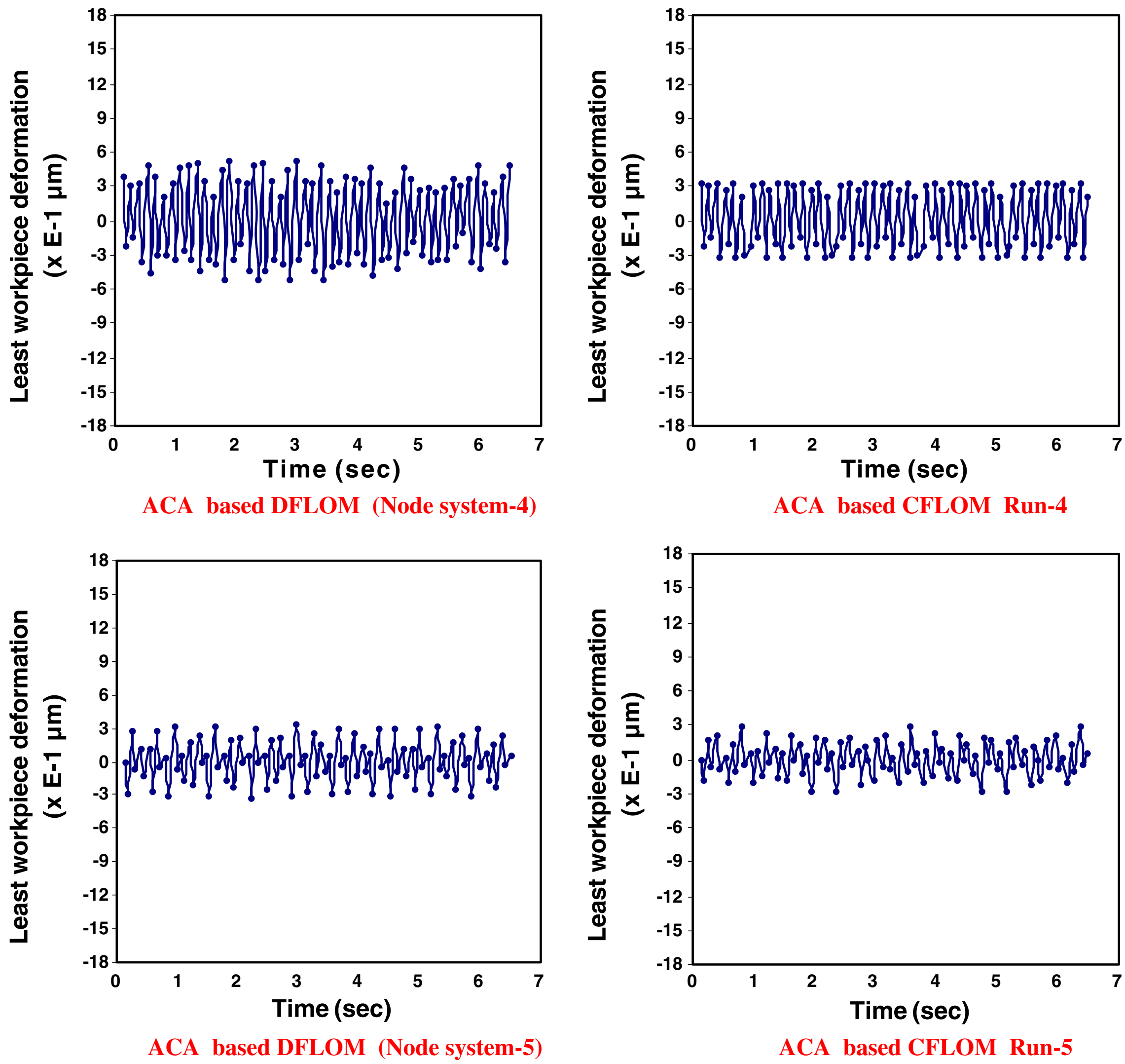

Fig. 7 (continued)

deformation obtained using ACA-based DFLOM and ACA-based CFLOM is given in Fig. 7. The comparison of least workpiece elastic deformation obtained using ACA-based DFLOM with that of obtained using ACA-based

Table 5 Comparison of results of ACA-based DFLOM and ACA-based CFLOM

Node system/run	Least workpiece deformation (*x* E−1µm)	
	ACA-based DFLOM	ACA-based CFLOM
1	5.8	2.96
2	3.45	3.56
3	8.4	2.98
4	5.4	3.23
5	3.06	2.84

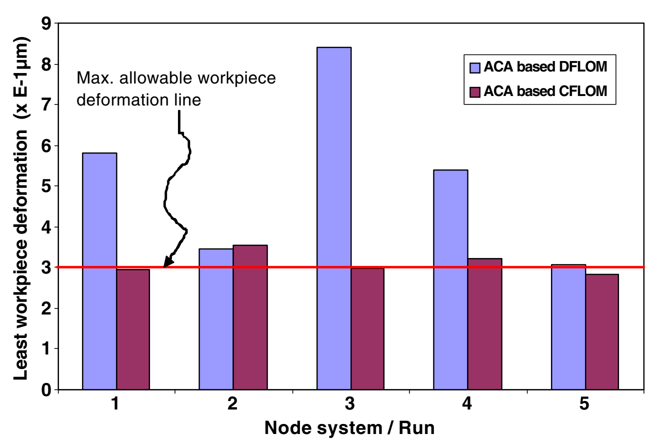

Fig. 8 Comparison of results of ACA-based DFLOM and ACA-based CFLOM

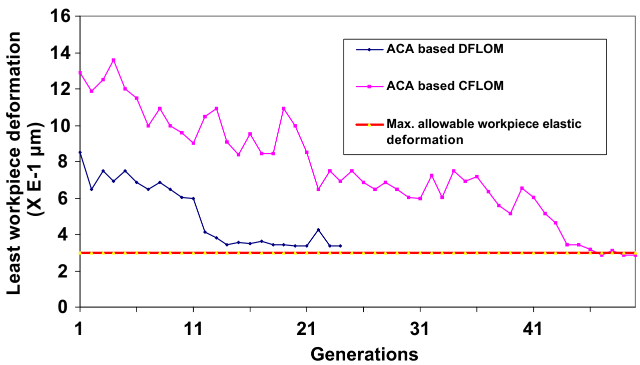

Fig. 9 Comparison of rate of convergence of ACA-based DFLOM and ACA-based CFLOM

CFLOM is given in Table 5 and shown in Fig. 8. It is obvious from Table 5, Figs. 7, 8 that the least workpiece elastic deformation obtained for all the runs performed in ACA-based CFLOM is less than that of obtained for the various node systems used in ACA based DFLOM. It is also found from Fig. 8 that (1) the least workpiece elastic deformation obtained for all the node systems used in ACA-based DFLOM does not satisfy the defined maximum allowable workpiece deformation, i.e., 0.3 µm and (2) the least workpiece elastic deformation obtained in runs 1, 3, and 5 performed in ACA-based CFLOM satisfies the defined maximum allowable workpiece deformation, i.e., 0.3 µm. The rate of convergence of ACA-based DFLOM is compared with that of ACA-based CFLOM and is shown in Fig. 9. Since the solution space (the number of possible layouts) used in DFLOM is less than that used in CFLOM, the rate of convergence is faster for the ACA-based DFLOM than that of ACA-based CFLOM.

8 Conclusion

In this paper, the fixture layout for the two-dimensional workpiece geometry was optimized with an objective of minimizing the workpiece elastic deformation using ACA-based discrete and continuous fixture layout optimization methods. The dynamic effects of the workpiece were considered. The results obtained for the case study conclude that the ant colony algorithm can be applied for the problems, in which there is no direct relation between the objective function values, and the constraints and the fixture layout optimization problem is one of such problems. Based on the solution space (the number of possible layouts) used in CFLOM, which is more than that used in DFLOM, the following conclusions are made: (1) the continuous fixture layout optimization method exhibits better results than that of discrete fixture layout optimization method and (2) the rate of convergence for ACA-based DFLOM is faster than that of ACA-based CFLOM.

Nomenclature

$\tau i(k)$	Pheromone trail
$\vec{F}$	Force vector
$\overleftrightarrow{X_j}$	Mode shape of *j*th frequency
Δ	Step size in mutation (Ant colony algorithm)
α	Random number in trail diffusion
ωj	Natural frequency of the *j*th mode
[B]	Strain–displacement matrix
[k]	Global stiffness matrix
[m]	Global mass matrix
[X]	Modal matrix
A	Area of triangular element
A_{i}	Age for the new iteration
$A_{\mathrm{i-1}}$	Age for the previous iteration
B	Breadth of workpiece geometry
L	Length of workpiece geometry
L_S	Limiting step value
m	Number of elements
n	Number of nodal displacements
N_S	Number of superior solutions
P	Pheromone value of each solution
P_{ave}	Average pheromone value
Pc	Crossover probability
P_{i}	Pheromone value for the new solution
$P_{\mathrm{i-1}}$	Pheromone value for the old solution
P_{m}	Mutation probability
R	Total no. of ants
r	Ratio of current iteration to total number of iterations
R_1, R_2, R_3, R_4, and R_5	Range of distance of Locator-1, Locator-2, Locator-3, Clamp-1, and Clamp-2
S_{i}	Value of current solution
$S_{i\text{-}1}$	Value of old solution
u and v	Displacements along x and y axes
v	Poisson's ratio
$W^{(e)}$	Work done on element
XC	Position of fixturing element of child layout
X_i	Position of *i*th fixturing element
X_{max}	Maximum value of fixturing element
XP_1 and XP_2	Position of fixturing element of parent layout-1 and parent layout-2

References

1. Bausch J, Youcef-Toumi K (1990) Kinematic methods for automated fixture reconfiguration planning IEEE International conference on robotics and automation 1396–140
2. Menassa RJ, Devries WA (1990) Design synthesis and optimization method for fixtures with compliant elements. In Proc ASME WAM Dallas TX PED 47:203–218
3. Mani M, Wilson WRD (1988) Automated design of work holding fixtures using kinematic constraint synthesis. Proceeding of the 16th North American Manufacturing Research Conference (NAMRC), 437–444.
4. Lee JD, Haynes LS (1987) Finite element analysis of flexible system. ASME J Eng Ind 109(2):134–139
5. Menassa RJ, Devries WR (1991) Optimization methods applied to selecting support positions in fixture design. ASME J Eng Ind 113:412–418
6. King LS, Hutter I (1993) Theoretical approach for generating optimal fixturing locations for prismatic work parts in automated assembly. J Manufact Syst 12(5):409–416
7. Kashyap R, DeVries WR (1999) Finite element analysis and optimization in fixture design. Struct Optim 18:193–201
8. Kang YZ, Rong YM, Yang JC (2003) Computer-aided fixture verification, Part 2—Tolerance analysis. Int J Adv Manuf Technol 21:836–884. doi:10.1007/s00170-002-1400-5
9. Demeter EC (1998) Fast support layout optimization. Int J Mach Tools Manuf 38(10):1221–1239. doi:10.1016/S0890-6955(97)00127-2
10. Li B, Melkote SN (2001) Fixture clamping force optimization and its impact on workpiece location accuracy. Int J Adv Manuf Technol 17(2):104–113. doi:10.1007/s001700170198
11. Sanchez HT, Estrems M, Faura F (2005) Fixturing analysis methods for calculating the contact load distribution and the valid clamping regions in machining processes. Int J Adv Manuf Technol 18(10):701–707 18
12. Kokkolaras Zhijun Li Michael, Izquierdo LE, Jack Hu S, Papalambros PY (2006) "Multiobjective optimization for integrated tolerance allocation and fixture layout design in multistation assembly, ASME 2006 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference September 10–13. Philadelphia, Pennsylvania, USA
13. Deng HY, Melkote SN (2006) Determination of minimum clamping forces for dynamically stable fixturing. Int J Mach Tools Manuf 46(7–8):847–857. doi:10.1016/j.ijmachtools.2005.07.040
14. Tan EYT, Kumar AS, Fuh JYH, Nee AYC (2004) Modeling, analysis and verification of optimal fixturing design. IEEE Trans Autom Sci Eng 1(2):121–132. doi:10.1109/TASE.2004.835601
15. Lai XM, Luo LJ, Lin ZQ (2004) Flexible assembly fixture layout modeling and optimization based on genetic algorithm. Chin J Mech Eng 1:89–92
16. Hamedi M (2005) Intelligent fixture design through a hybrid system of artificial neural network and genetic algorithm. Artif Intell Rev 23(3):295–311. doi:10.1007/s10462-004-7187-z
17. Krishnakumar K, Melkote SN (2000) Machining fixture layout optimization. Int J Mach Tools Manuf 40:579–598. doi:10.1016/S0890-6955(99)00072-3
18. Krishnakumar K, Satyanarayana S, Melkote SN (2002) Iterative fixture layout and clamping force optimization using the genetic algorithm. J Manuf Sci Eng 124(1):119–125. doi:10.1115/1.1414127
19. Qin GH et al (2006) Developed a mathematical approach to analysis and optimal design of a fixture locating scheme. Int J Adv Manuf Technol 29:349–359. doi:10.1007/s00170-005-2509-0
20. Prabhaharan G, Padmanaban KP, Asokan S (2006) Dynamic analysis on optimal placement of fixturing elements accounting for the effect of workpiece elasticity. Int J Manuf Sci Technol 8 (1):33–51
21. Vallapuzha S, De Meter EC, Choudhuri S, Khetan RP (2002) An investigation into the use of spatial coordinates for the genetic algorithm based solution of the fixture layout optimization problem. Int J Mach Tool Manuf 42:265–275
22. Kaya N (2005) Machining fixture locating and clamping position optimization using genetic algorithms. Int J Comput Ind 57:112–120. doi:10.1016/j.compind.2005.05.001
23. Marcelin JL (2001) Genetic search applied to selecting support positions in machining of mechanical parts. Int J Adv Manuf Technol 17:344–347. doi:10.1007/s001700170169
24. Dorigo M, Gamberdella LM (1997) Ant Colony system: a cooperative learning approach to the traveling salesman problem. IEEE Trans Evol Comput 1(1):53–66. doi:10.1109/4235.585892
25. Patrick R, Mcmullan PR (2001) An Ant Colony Optimization approach to address a JIT sequencing problem with multiple objective. Artif Intell Eng 15:309–317. doi:10.1016/S0954-1810(01)00004-8
26. Jayaraman VK, Kulkarni BD, Karale S, Shalokar P (2000) Ant Colony framework for optimal design and scheduling of batch plants. Comput Chem Eng 24:1901–1910. doi:10.1016/S0098-1354(00)00592-5
27. Prabhaharan G, Muruganandam A, Asokan P, Girish BS (2005) Machine cell formation for cellular manufacturing systems using an ant colony system approach. Int J Adv Manuf Technol 25:1013–1019. doi:10.1007/s00170-003-1927-0
28. Prabhaharan G, Asokan P, Rajendran S (2005) Sensitivity-based conceptual design and tolerance allocation using the continuous ants colony algorithm (CACO). Int J Adv Manuf Technol 25:516–526. doi:10.1007/s00170-003-1846-0
29. Prabhaharan G, Padmanaban KP (2007) Machining fixture layout optimization using FEM and evolutionary techniques. Int J Adv Manuf Technol 32(11–12):1090–1103. doi:10.1007/s00170-006-0441-6
30. Padmanaban KP, Prabhaharan G (2008) Dynamic analysis on optimal placement of fixturing elements using evolutionary techniques. Int J Prod Res 46:4177–4214. doi:10.1080/00207540601147297
31. Rao PN (2000) Manufacturing and technology (Metal Cutting and Machine tools). McGraw Hill, New York