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ORIGINAL ARTICLE

Kinematics of a five-degrees-of-freedom parallel manipulator using screw theory

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Abstract In this work, the kinematic analysis of a five-degrees-of-freedom decoupled parallel manipulator is approached by means of the theory of screws. The architecture of the parallel manipulator under study is such that the translational motion of the moving platform, with respect to the fixed platform, is controlled by means of a central limb provided with two active prismatic joints while its rotational motion is controlled by means of a three-degrees-of-freedom spherical parallel manipulator. The forward position analysis is presented in semiclosed form solution applying recursively the Sylvester dialytic elimination method. On the other hand, the velocity and acceleration analyses are carried out using the theory of screws. Simple and compact expressions to compute the velocity state and the reduced acceleration state of the moving platform, with respect to the fixed platform, are easily derived in this contribution by taking advantage of the Klein form of the Lie algebra $se(3)$. Finally, only few and slight modifications to the proposed method of kinematic analyses are required in order to approach the position, velocity, and acceleration analyses of parallel manipulators with similar topologies.

Keywords Parallel manipulator · Decoupled motion · Screw theory · Kinematics

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1 Introduction

Due to their compact topology, parallel manipulators are more accurate and stiffer than their serial counterparts. Owing to these merits, over the past few decades, a growing interest devoted to the development of parallel kinematic machines, with heavy payload capacities, has been reported day to day in the literature. In fact, among flight simulators, which seems to be the first transcendental application [1], parallel manipulators have found interesting applications such as walking machines, pointing devices, machine tools, micromanipulators, and so on. On the other hand, the limitations of parallel manipulators with identical limbs cannot be ignored. Consider, for instance, that, in this kind of mechanism, the rotation and position capabilities of the moving platform are highly coupled, resulting in the forward finite kinematics, and their control and calibration, being rather complicated, in addition to a limited and complex-shaped workspace.

The dexterity and workspace of a parallel manipulator can be improved by assembling a serial manipulator to its moving platform, a natural and evident possibility. For example, a spherical wrist can be connected at the moving platform of a Tricept, yielding a six-degrees-of-freedom (dof) spatial mechanism [2]; other combinations were reported in [3, 4]. Furthermore, taking into account that, if the spherical wrist is an open chain, then, in order to improve the stiffness, the serial manipulator can be replaced by a parallel manipulator, producing a series-parallel manipulator; see, for instance, [5–8]. Another possibility consists of transforming the motion of the moving platform, with respect to the fixed platform, in decoupled motions; thus, the translational and rotational motions of the moving platform can be computed separately; this option was investigated in Hunt and Primrose [9] for in-fully parallel manipulators and was applied by Zlatanov et al. [10] in the design of a six-dof, three-legged parallel manipulator. Jin et al. [11] proposed a versatile, three-legged parallel manipulator, with six dof, in which, by choosing the scheme of actuation, it is possible to generate decoupled and hybrid motions over the moving platform. Taking into account that the forward position analysis of parallel manipulators with decoupled motions can be easily derived in closed or semiclosed-form solution, Gallardo-Alvarado et al. [12] introduced a family of nonoverconstrained redundantly actuated parallel manipulators. Furthermore, as is shown in [13–15], closed-form solutions for solving the kinematics of parallel manipulators with fewer than six dof, known as defective or limited-dof parallel manipulators, over the moving platform can be easily derived using simple geometric procedures.

Limited-dof parallel manipulators, after the success of the robot Delta [16], have been finally recognized, in both industrial and academic communities, as interesting inventions with potential applications in many areas such as machine tools [17–22], hyperredundant manipulators [23], haptic devices based on parallel mechanisms [24, 25], microrobotics [26–29], and so on. In that way, five-dof parallel manipulators represent an excellent option for the development of multiaxis machine tools that will possibly replace, surely, the traditional Cartesian machine tools; for details about interesting prototypes based on parallel kinematic devices, the reader is invited to visit the web site http://www.parallemic.org/ hosted by Dr. Ilian Bonev. On the other hand, following that fashion, it is notorious how most researchers are focused on the study of the so-called two rotations + three translations parallel mechanisms, 2R3T for brevity. Certainly, the restriction of one rotation, usually normal to the moving platform, is the main attraction of such a trend. However, interesting applications using 3R2T parallel mechanisms, such as the kinematic simulation of the human spinal column, recently attracted the attention of some kinematicians [30–32]. In this work, the position, velocity, and acceleration analyses of 3R2T limited-dof parallel manipulators are approached by means of simple geometric procedures and the theory of screws.

2 Description of the parallel manipulator

The parallel manipulator under study, see Fig. 1, consists of a moving platform and a fixed platform connected to each other by means of four active extendible limbs, and since this mechanism does not have passive legs, it offers the two main advantages of a parallel manipulator: accuracy and stiffness. At once, the moving platform is connected at the limbs by means of four distinct spherical joints, whereas the fixed platform is connected at the limbs by means of a central prismatic joint and three distinct universal joints. The position of the moving platform is controlled using a central limb, which is a simple prismatic + prismatic + spherical (PPS-type) kinematic chain, while its orientation is controlled by means of a spherical parallel manipulator composed of three universal + prismatic + spherical (UPS-type) kinematic chains. According to a revised version of the Kutzbach–Grübler formula, the mechanism under study possesses five dof, and therefore, the prismatic pairs have the privilege to be selected as the active joints of the parallel manipulator.

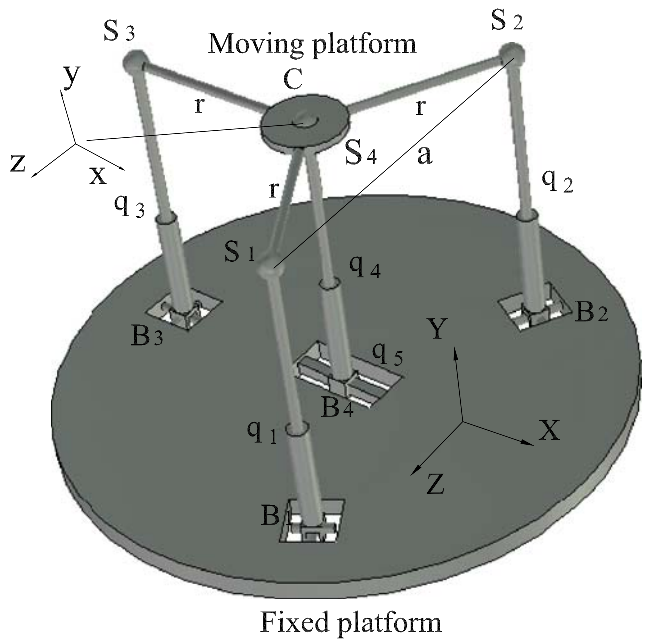


Fig. 1 The parallel manipulator under study

The proposed topology is so simple that the moving platform has the ability to perform independent translational and rotational displacements. Furthermore, it is interesting to mention that, due to the spherical joints attached at the moving platform, the parallel manipulator can be considered as a nonoverconstrained parallel manipulator and, therefore, does not require additional conditions of manufacture.

3 Finite kinematics of the parallel manipulator

In this section, the inverse and forward position analyses of the mechanism under study are presented. The inverse position analysis consists of finding the length of each limb of the parallel manipulator given the pose, position, and orientation of the moving platform with respect to the fixed platform.

Clearly, given the pose of the moving platform, the coordinates of the centers of the spherical joints $\mathbf{S}_i = (X_i, Y_i, Z_i)$ $i = 1, 2, 3$ and the center $\mathbf{S}_4 = (X_4, Y_4, Z_4)$ of the moving platform are easily computed using the global reference frame XYZ, see Fig. 1, as follows

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{R} & \tau \\ 0_{1\times 3} & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix} \quad i = 1, \ldots, 4, \tag{1}$$

where R is the rotation matrix, τ is the position vector of the origin of the moving reference frame xyz with respect to the origin of the global reference frame XYZ, and $\mathbf{s}_i = (x_i, y_i, z_i)$ $i = 1, \ldots, 4$ are the coordinates of the centers of the spherical joints expressed in the moving reference frame xyz. Furthermore, once the points $\mathbf{S}_i$ are computed, it follows that

$$q_i^2 = (\mathbf{S}_i - \mathbf{B}_i) \cdot (\mathbf{S}_i - \mathbf{B}_i) \quad i = 1, 2, 3, \tag{2}$$

where $\mathbf{B}_i$ $i = 1, 2, 3$ are the nominal positions of the universal joints.

Finally, the generalized coordinates q_4 and q_5 are obtained as follows:

$$q_4 = \mathbf{S}_4 \cdot \hat{j} \tag{3}$$

and

$$q_5 = (\mathbf{S}_4 - \mathbf{B}_4) \cdot \hat{i}, \tag{4}$$

where $\mathbf{B}_4$ is the origin of the global reference frame XYZ and $\hat{i}$ and $\hat{j}$ are unit vectors along the X and Y axes, respectively.

On the other hand, the forward position analysis, a more challenging task, consists of finding the pose of the moving platform, with respect to the fixed platform, of the parallel manipulator given a set $\{q_1, q_2, \ldots, q_5\}$ of generalized coordinates that are indicated in Fig. 1. The forward position analysis is approached using the well-known Sylvester dialytic elimination method, and it is included here only for the sake of completeness; for a detailed explanation of kinematic applications of such a method, the reader is referred to [33].

According to the reference frame XYZ, the center of the moving platform, point $\mathbf{S}_4$, is obtained directly using only the generalized coordinates q_4 and q_5 as follows:

$$\mathbf{S}_4 = q_5\hat{i} + q_4\hat{j}. \tag{5}$$

Once the center of the moving platform is determined, in what follows, the coordinates of the spherical joints of the spherical parallel manipulator will be computed.

Three linear equations emerge immediately taking into account that

$$\mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 - 3\mathbf{S}_4 = \mathbf{0} \tag{6}$$

On the other hand, the length of the limbs of the spherical parallel manipulator are restricted to

$$(\mathbf{S}_i - \mathbf{B}_i) \cdot (\mathbf{S}_i - \mathbf{B}_i) - q_i^2 = 0 \quad i = 1, 2, 3 \tag{7}$$

Furthermore, the radius r of the moving platform brings the following constraints:

$$(\mathbf{S}_i - \mathbf{S}_4) \cdot (\mathbf{S}_i - \mathbf{S}_4) - r^2 = 0 \quad i = 1, 2, 3 \tag{8}$$

Thus, from expressions 7 and 8, it follows that

$$(\mathbf{S}_i - \mathbf{B}_i) \cdot (\mathbf{S}_i - \mathbf{B}_i) - q_i^2 - (\mathbf{S}_i - \mathbf{S}_4) \cdot (\mathbf{S}_i - \mathbf{S}_4) + r^2 = 0 \tag{9}$$

Equations 6 and 9 represent a linear system of six equations in nine unknowns. This linear system is solved in terms of the variables X_1, Y_2, and Z_3. Furthermore, three compatibility equations can be written as follows:

$$\left.\begin{array}{l} (\mathbf{S}_1 - \mathbf{S}_2) \cdot (\mathbf{S}_1 - \mathbf{S}_2) - \mathrm{a}^2 = 0 \\ (\mathbf{S}_2 - \mathbf{S}_3) \cdot (\mathbf{S}_2 - \mathbf{S}_3) - \mathrm{a}^2 = 0 \\ (\mathbf{S}_1 - \mathbf{S}_3) \cdot (\mathbf{S}_1 - \mathbf{S}_3) - \mathrm{a}^2 = 0 \end{array}\right\} \tag{10}$$

where a is the length of the side of the equilateral triangle formed over the moving platform by the three spherical joints of the spherical parallel manipulator.

Finally, after a few computations, a higher nonlinear system of three equations in the unknowns X_1, Y_2, and Z_3 is obtained as follows:

$$\left.\begin{array}{lr} K_1' Y_2^2 + K_2' Z_3^2 + K_3' Y_2^2 Z_3 + K_4' Y_2 Z_3^2 & \\ \quad + K_5' Y_2 Z_3 + K_6' Y_2 + K_7' Z_3 + K_8' = 0 & \text{(11a)} \\ K_1'' X_1^2 + K_2'' Z_3^2 + K_3'' X_1^2 Z_3 + K_4'' X_1 Z_3^2 & \\ \quad + K_5'' Y_2 Z_3 + K_6'' X_1 + K_7'' Z_3 + K_8'' = 0 & \text{(11b)} \\ K_1''' X_1^2 + K_2''' Y_2^2 + K_3''' X_1^2 Y_2 + K_4''' X_1 Y_2^2 & \\ \quad + K_5''' X_1 Y_2 + K_6''' X_1 + K_7''' Y_2 + K_8''' = 0. & \text{(11c)} \end{array}\right\} \tag{11}$$

where K_*', K_*'', and K_*''' are coefficients that are computed according to the parameters and generalized coordinates of the spherical parallel manipulator. The solution of this type of equation is well-known, see for instance [33–35], and it is recalled here only for the sake of completeness. In what follows, the nonlinear system of Eq. 11 is reduced systematically into a 16th-polynomial expression in the unknown X_1.

With the purpose to eliminate Z_3, Eqs. 11a and 11b are rewritten, respectively, as follows:

$$\mathrm{p}_1 Z_3^2 + \mathrm{p}_2 Z_3 + \mathrm{p}_3 = 0 \tag{12}$$

and

$$\mathrm{p}_4 Z_3^2 + \mathrm{p}_5 Z_3 + \mathrm{p}_6 = 0, \tag{13}$$

where $\mathrm{p}_i\, i = 1, 2, 3$ are second-order polynomials in Y_2 while $\mathrm{p}_i\, i = 4, 5, 6$ are second-order polynomials in X_1.

Multiplying Eqs. 12 and 13, respectively, by p_4 and p_1, and subtracting, it follows that

$$(\mathrm{p}_1\mathrm{p}_5 - \mathrm{p}_2\mathrm{p}_4) Z_3 + \mathrm{p}_1\mathrm{p}_6 - \mathrm{p}_3\mathrm{p}_4 = 0 \tag{14}$$

Similarly, multiplying Eq. 12 by p_6 and Eq. 13 by p_3, and subtracting, one obtains

$$(\mathrm{p}_3\mathrm{p}_4 - \mathrm{p}_1\mathrm{p}_6) Z_3 + \mathrm{p}_3\mathrm{p}_5 - \mathrm{p}_2\mathrm{p}_6 = 0 \tag{15}$$

Considering Eqs. 14 and 15 as two linear homogeneous equations in the unknowns Z_3 and 1, then, casting into a matrix form such equations, it is possible to write

$$\mathrm{M}_1 \begin{bmatrix} Z_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{16}$$

where

$$\mathrm{M}_1 = \begin{bmatrix} \mathrm{p}_1\mathrm{p}_5 - \mathrm{p}_2\mathrm{p}_4 & \mathrm{p}_1\mathrm{p}_6 - \mathrm{p}_3\mathrm{p}_4 \\ \mathrm{p}_3\mathrm{p}_4 - \mathrm{p}_1\mathrm{p}_6 & \mathrm{p}_3\mathrm{p}_5 - \mathrm{p}_2\mathrm{p}_6 \end{bmatrix}$$

Thus, in order to avoid arbitrary solutions, one eliminant is obtained when $\det \mathrm{M}_1 = 0$, canceling the variable Z_3. Indeed,

$$\mathrm{p}_7 Y_2^4 + \mathrm{p}_8 Y_2^3 + \mathrm{p}_9 Y_2^2 + \mathrm{p}_{10} Y_2 + \mathrm{p}_{11} = 0, \tag{17}$$

where $\mathrm{p}_i\, i = 7, 8, \ldots, 11$ are fourth-order polynomials in X_1.

With the purpose to eliminate the unknown Y_2, Eq. 11c is rewritten as follows:

$$\mathrm{p}_{12} Y_2^2 + \mathrm{p}_{13} Y_2 + \mathrm{p}_{14} = 0, \tag{18}$$

where $\mathrm{p}_i\, i = 12, 13, 14$ are second-order polynomials in X_1.

Multiplying Eq. 17 by p_{12} and Eq. 18 by $\mathrm{p}_7 Y_2^2$, the subtraction of the resulting equations yields

$$(\mathrm{p}_{13}\mathrm{p}_7 - \mathrm{p}_{12}\mathrm{p}_8) Y_2^3 + (\mathrm{p}_{14}\mathrm{p}_7 - \mathrm{p}_{12}\mathrm{p}_9) Y_2^2 - \mathrm{p}_{12}\mathrm{p}_{10} Y_2 - \mathrm{p}_{12}\mathrm{p}_{11} = 0 \tag{19}$$

Moreover, multiplying Eq. 18 by Y_2:

$$\mathrm{p}_{12} Y_2^3 + \mathrm{p}_{13} Y_2^2 + \mathrm{p}_{14} Y_2 = 0 \tag{20}$$

Furthermore, multiplying Eq. 17 by $\mathrm{p}_{12} Y_2 + \mathrm{p}_{13}$ and Eq. 18 by $\mathrm{p}_7 Y_2^3 + \mathrm{p}_8 Y_2^3$, and after subtracting, one obtains

$$(\mathrm{p}_{12}\mathrm{p}_9 - \mathrm{p}_7\mathrm{p}_{14}) Y_2^3 + (\mathrm{p}_{12}\mathrm{p}_{10} + \mathrm{p}_{13}\mathrm{p}_9 - \mathrm{p}_8\mathrm{p}_{14}) Y_2^2 + (\mathrm{p}_{12}\mathrm{p}_{11} + \mathrm{p}_{13}\mathrm{p}_{10}) Y_2 + \mathrm{p}_{13}\mathrm{p}_{11} = 0 \tag{21}$$

Equations 18–21 can be considered as a homogeneous linear system of four equations in the unknowns Y_2^3, Y_2^2, Y_2, and 1, that are casting in a matrix form as follows:

$$\mathrm{M}_2 \begin{bmatrix} Y_2^3 \\ Y_2^2 \\ Y_2^1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{22}$$

where M_2 is a square matrix given by

$$\mathrm{M}_2 = [\mathrm{m}_1 \quad \mathrm{m}_2 \quad \mathrm{m}_3 \quad \mathrm{m}_4] \tag{23}$$

with columns

$$\mathrm{m}_1 = \begin{bmatrix} 0 \\ \mathrm{p}_{13}\mathrm{p}_7 - \mathrm{p}_{12}\mathrm{p}_8 \\ \mathrm{p}_{12} \\ \mathrm{p}_{12}\mathrm{p}_9 - \mathrm{p}_7\mathrm{p}_{14} \end{bmatrix}$$

$$\mathrm{m}_2 = \begin{bmatrix} \mathrm{p}_{12} \\ \mathrm{p}_{14}\mathrm{p}_7 - \mathrm{p}_{12}\mathrm{p}_9 \\ \mathrm{p}_{13} \\ \mathrm{p}_{12}\mathrm{p}_{10} + \mathrm{p}_{13}\mathrm{p}_9 - \mathrm{p}_8\mathrm{p}_{14} \end{bmatrix}$$

$$\mathrm{m}_3 = \begin{bmatrix} \mathrm{p}_{13} \\ -\mathrm{p}_{12}\mathrm{p}_{10} \\ \mathrm{p}_{14} \\ \mathrm{p}_{12}\mathrm{p}_{11} + \mathrm{p}_{13}\mathrm{p}_{10} \end{bmatrix}$$

$$\mathrm{m}_4 = \begin{bmatrix} \mathrm{p}_{14} \\ -\mathrm{p}_{12}\mathrm{p}_{11} \\ 0 \\ \mathrm{p}_{13}\mathrm{p}_{11} \end{bmatrix}$$

Henceforth, a 16th univariate polynomial expression in the unknown X_1 is obtained taking into account that, in order to avoid arbitrary solutions, necessarily, $\det \mathrm{M}_2 = 0$. Moreover, Y_2 and Z_3 are determined solving the quadratic equation Eq. 11. Furthermore, the remaining coordinates are computed using Eqs. 6 and 9.

Once the coordinates of the centers of the spherical joints are computed, the pose of the moving platform, with respect to the fixed platform, is determined according to the resulting transformation matrix T:

$$\mathrm{T} = \begin{bmatrix} \mathrm{R} & \mathbf{S}_4 \\ 0_{1\times 3} & 1 \end{bmatrix}. \tag{24}$$

A simple method to compute the rotation matrix R based upon the coordinates of three distinct points of a rigid body is reported in Gallardo-Alvarado et al. [36].

Finally, it is prudent to mention that the Sylvester dialytic elimination method in general is not free of spurious solutions; thus, the correctness of the obtained solutions must be verified, substituting it into Eqs. 6–8. Furthermore, the determination of the actual configuration of the parallel manipulator, a necessary step to approach the infinitesimal kinematics of the mechanism, can be achieved using sensors.

4 Infinitesimal kinematics of the parallel manipulator

In this section, the velocity and acceleration analyses of the parallel manipulator are carried out by means of the theory of screws, which is isomorphic to the Lie algebra *se*(3). The benefits of using this mathematical resource in analyzing parallel manipulators are innegable; see, for instance, [37–43]. In this work, two fundamental operations of the Lie algebra play a central role, the Lie product and the Klein form.

Let $\$_1 = (\hat{s}_1, \mathbf{s}_{O1})$ and $\$_2 = (\hat{s}_2, \mathbf{s}_{O2})$ be two elements of the Lie algebra *se*(3). The Lie product, $[*\quad *]$, is defined as follows:

$$[\$_1 \quad \$_2] = \begin{bmatrix} \hat{s}_1 \times \hat{s}_2 \\ \hat{s}_1 \times \mathbf{s}_{O2} - \hat{s}_2 \times \mathbf{s}_{O1} \end{bmatrix} \tag{25}$$

whereas the Klein form, $\{*; *\}$, is defined as follows:

$$\{\$_1; \$_2\} = \hat{s}_1 \cdot \mathbf{s}_{O2} + \hat{s}_2 \cdot \mathbf{s}_{O1}. \tag{26}$$

It is said that the screws $\$_1$ and $\$_2$ are reciprocal if $\{\$_1; \$_2\} = 0$; for instance, this condition occurs when

1. The primal parts of the screws, unit vectors $\hat{s}_1$ and $\hat{s}_2$, are concurrent
2. The vectors $\hat{s}_1$ and $\hat{s}_2$ are orthogonal, respectively, to the vectors $\mathbf{s}_{O2}$ and $\mathbf{s}_{O1}$

4.1 Velocity analysis

Let $\mathbf{V}_C = [\omega, \mathbf{v}_C]^{\mathrm{T}}$ be the velocity state of the moving platform, with respect to the fixed platform, where the primal part, $\mathrm{P}(\mathbf{V}_C) = \omega$, is the angular velocity of the moving platform and the dual part, $\mathrm{D}(\mathbf{V}_C) = \mathbf{v}_C$, is the linear velocity of its center $\mathbf{S}_4$. Then, the velocity state $\mathbf{V}_C$ can be written in screw form; the corresponding infinitesimal screws are depicted in Fig. 2, through any of the UPS-type limbs, as follows:

$$\mathrm{J}_i \Omega_i = \mathbf{V}_C \quad i = 1, 2, 3 \tag{27}$$

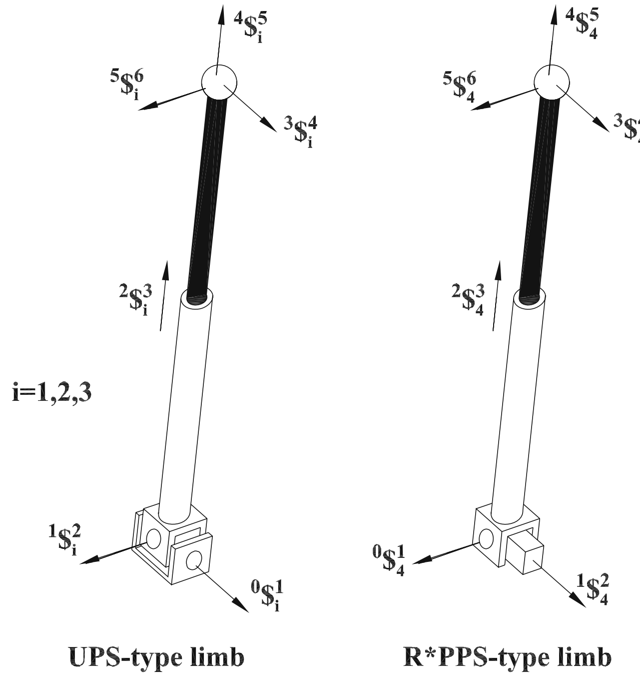


Fig. 2 The infinitesimal screws of the connector limbs of the parallel manipulator

where

$\mathrm{J}_i = \left[{}^0\$_i^1, {}^1\$_i^2, {}^2\$_i^3, {}^3\$_i^4, {}^4\$_i^5, {}^5\$_i^6\right]$ is the i-th Jacobian of the i-th limb, in which ${}^k\$_i^{k+1}$ denotes the infinitesimal screw between two consecutive bodies k and $k+1$ of the i-th limb

$\Omega_i = \left[{}_0\omega_1^i, {}_1\omega_2^i, {}_2\omega_3^i, {}_3\omega_4^i, {}_4\omega_5^i, {}_5\omega_6^i\right]^{\mathrm{T}}$ is a matrix containing the joint velocity rates in which ${}_k\omega_{k+1}^i$ denotes the relative velocity rate, associated to the corresponding screw, of two consecutive bodies k and $k+1$ of the i-th limb, and particularly, ${}_2\omega_3^i = \dot{q}_i$ is the i-th generalized speed.

On the other hand, in order to satisfy an algebraic requirement, the PPS-type limb is modeled as a R*PPS-type limb where the extra revolute joint R* is a fictitious kinematic pair and, therefore, its corresponding joint velocity rate is zero. With this consideration in mind, the velocity state can be written in screw form through the R*PPS-type limb as follows:

$$\mathrm{J}_4 \Omega_4 = \mathbf{V}_C \tag{28}$$

where

$\mathrm{J}_4 = \left[{}^0\$_4^1, {}^1\$_4^2, {}^2\$_4^3, {}^3\$_4^4, {}^4\$_4^5, {}^5\$_4^6\right]$ is the Jacobian of the R*PPS-type limb

$\Omega_4 = \left[{}_0\omega_1^4, {}_1\omega_2^4, {}_2\omega_3^4, {}_3\omega_4^4, {}_4\omega_5^4, {}_5\omega_6^4\right]^{\mathrm{T}}$ is a matrix containing the joint velocity rates of the R*PPS-type limb in which ${}_1\omega_2^4 = \dot{q}_4$ and ${}_2\omega_3^4 = \dot{q}_5$ are the generalized speeds of the limb.

The inverse velocity analysis consists of finding the joint velocity rates of the parallel manipulator for a given velocity state $\mathbf{V}_C$. This analysis is carried out by computing Ω_i $i = 1, \ldots, 4$ directly from expressions 27 and 28. In what follows, the forward velocity analysis is simplified by taking advantage of the properties of reciprocal screws via the definition of the Klein form.

Applying the Klein form of the i-th screw ${}^4\$_i^5$ to both sides of Eq. 27, it follows that

$$\left\{{}^0\$_i^1; {}^4\$_i^5\right\} {}_0\omega_1^i + \left\{{}^1\$_i^2; {}^4\$_i^5\right\} {}_1\omega_2^i + \left\{{}^2\$_i^3; {}^4\$_i^5\right\} \dot{q}_i + \left\{{}^3\$_i^4; {}^4\$_i^5\right\} {}_3\omega_4^i + \left\{{}^4\$_i^5; {}^4\$_i^5\right\} {}_4\omega_5^i + \left\{{}^5\$_i^6; {}^4\$_i^5\right\} {}_5\omega_6^i = \left\{{}^4\$^5{}_i; \mathbf{V}_C\right\} \quad (29)$$

Taking into account that the primal part of the i-th screw ${}^4\$_i^5$ is concurrent to the primal parts of the screws associated with the revolute joints in the same limb, then all the terms of the left side of Eq. 29 vanish excepting the term associated with the prismatic joint; in fact, $\left\{{}^2\$_i^3; {}^4\$_i^5\right\} = 1$, and therefore, the reductions of terms in Eq. 29 lead to

$$\left\{{}^4\$^5{}_i; \mathbf{V}_C\right\} = \dot{q}_i \quad i = 1, 2, 3 \quad (30)$$

Similarly, considering that ${}^0\$_4^1$ is a fictitious screw associated with a fictitious revolute joint, it is possible to restrict ${}_0\omega_1^4 = 0$, and taking into proper account that the screws associated with the spherical joint attached at the moving platform have concurrent primal parts, then the application of the Klein form of the screw ${}^3\$^4{}_4$, whose primal part is equal to the dual part of the screw ${}^1\$^2{}_4$ representing the lower prismatic pair of the central limb, to both sides of Eq. 28, the reduction of terms leads to

$$\left\{{}^3\$^4{}_4; \mathbf{V}_C\right\} = \dot{q}_4 \quad (31)$$

Moreover, since the primal part the screw ${}^4\$^5{}_4$ is orthogonal to the primal part of the screw ${}^1\$^2{}_4$, the application of the screw ${}^4\$^5{}_4$ to both sides of Eq. 28 yields the following expression:

$$\left\{{}^4\$^5{}_4; \mathbf{V}_C\right\} = \dot{q}_5 \quad (32)$$

Furthermore, it is straightforward to show that, applying the Klein form of the screw ${}^5\$^6{}_4$ to both sides of Eq. 28, all the terms of such equation vanish; in other words,

$$\left\{{}^5\$^6{}_4; \mathbf{V}_C\right\} = 0 \quad (33)$$

Finally, casting into a matrix–vector form Eqs. 30–33, it follows that

$$\mathrm{J}^{\mathrm{T}}\, \Delta\, \mathbf{V}_C = \mathrm{Q}_{\mathrm{vel}}, \quad (34)$$

wherein

$\mathrm{J} = \left[{}^4\$_1^5, {}^4\$_2^5, {}^4\$_3^5, {}^3\$_4^4, {}^4\$_4^5, {}^5\$_4^6\right]$ is the active Jacobian matrix of the parallel manipulator
$\mathrm{Q}_{\mathrm{vel}} = \left[\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5, 0\right]^{\mathrm{T}}$ is a matrix containing the active joint velocity rates of the parallel manipulator
$\Delta = \begin{bmatrix} 0_{3\times3} & \mathrm{I}_3 \\ \mathrm{I}_3 & 0_{3\times3} \end{bmatrix}$ is a 6×6 matrix called an operator of polarity that is defined by the identity matrix I_3 and the 3×3 zero matrix $0_{3\times3}$.

Expression 34 is simple, compact, and allows one to compute the velocity state $\mathbf{V}_C$ without the values of the passive joint velocity rates of the parallel manipulator. Naturally, such computation requires that the active Jacobian J must be invertible; otherwise, the parallel manipulator is at a singular configuration.

4.2 Acceleration analysis

Let $\mathbf{A}_C = [\dot{\omega}, \mathbf{a}_C - \omega \times \mathbf{v}_C]^{\mathrm{T}}$ be the reduced acceleration state, also known as accelerator, of the moving platform with respect to the fixed platform, in which $\dot{\omega}$ is the angular acceleration of the moving platform with respect to the fixed platform, whereas $\mathbf{a}_C$ is the linear acceleration of the center $\mathbf{S}_4$ of the moving platform. Then, the accelerator $\mathbf{A}_C$ can be written in screw form, see Rico and Duffy [37], through any of the UPS-type limbs of the parallel manipulator as follows:

$$\mathrm{J}_i \dot{\Omega}_i + \$_{\mathrm{Lie}_i} = \mathbf{A}_C \quad i = 1, 2, 3 \quad (35)$$

where

$\dot{\Omega}_i = \left[{}_0\dot{\omega}_1^i, {}_1\dot{\omega}_2^i, \ddot{q}_i, {}_3\dot{\omega}_4^i, {}_4\dot{\omega}_5^i, {}_5\dot{\omega}_6^i\right]^{\mathrm{T}}$ is a matrix containing the joint acceleration rates of the i-th limb
$\$_{\mathrm{Lie}_i}$ is the *Lie screw* of the i-th limb, which is computed using the composed Lie products as follows:

$$\$_{\mathrm{Lie}_i} = \left[{}_0\omega_1^i\,{}^0\$^1 \quad {}_1\omega_2^i\,{}^1\$^2 + \ldots + {}_5\omega_6^i\,{}^5\$^6\right] + \left[{}_1\omega_2^i\,{}^1\$^2 \quad {}_2\omega_3^i\,{}^2\$^3 + \ldots + {}_5\omega_6^i\,{}^5\$^6\right] + \ldots + \left[{}_4\omega_5^i\,{}^4\$^5 \quad {}_5\omega_6^i\,{}^5\$^6\right]. \quad (36)$$

Furthermore, the reduced acceleration state can be written in screw form through the R*PPS-type limb as follows:

$$\mathrm{J}_4 \dot{\Omega}_4 + \$_{\mathrm{Lie}_4} = \mathbf{A}_C, \quad (37)$$

where $\dot{\Omega}_4 = \left[{}_0\dot{\omega}_1^4, \ddot{q}_4, \ddot{q}_5, {}_3\dot{\omega}_4^4, {}_4\dot{\omega}_5^4, {}_5\dot{\omega}_6^4\right]^{\mathrm{T}}$ is a matrix containing the joint acceleration rates of the R*PPS-type limb.

Given the reduced acceleration state $\mathbf{A}_C$, the inverse acceleration analysis of the parallel manipulator, or in other words, the computation of the joint acceleration rates of the parallel manipulator, is computed directly from expressions 35 and 37.

On the other hand, the forward acceleration analysis of the parallel manipulator, indeed, the computation of the reduced acceleration state of the moving platform, with respect to the fixed platform, for a given set of active joint acceleration rates $\{\ddot{q}_1, \ddot{q}_2, \ldots, \ddot{q}_5\}$, is computed, following the trend indicated in Subsection 4.1, from

$$\mathrm{J}^{\mathrm{T}}\ \Delta\ \mathbf{A}_C = \mathrm{Q}_{\mathrm{accel}} \tag{38}$$

where

$$\mathrm{Q}_{\mathrm{accel}} = \begin{bmatrix} \ddot{q}_1 + \left\{{}^4\$_1^5;\ \$_{\mathrm{Lie}_1}\right\} \\ \ddot{q}_2 + \left\{{}^4\$_2^5;\ \$_{\mathrm{Lie}_2}\right\} \\ \ddot{q}_3 + \left\{{}^4\$_3^5;\ \$_{\mathrm{Lie}_3}\right\} \\ \ddot{q}_4 + \left\{{}^3\$_4^4;\ \$_{\mathrm{Lie}_4}\right\} \\ \ddot{q}_5 + \left\{{}^4\$_4^5;\ \$_{\mathrm{Lie}_4}\right\} \\ \left\{{}^5\$_4^6;\ \$_{\mathrm{Lie}_4}\right\} \end{bmatrix}$$

It is worth mentioning that Eq. 38 does not require the computation of the passive joint acceleration rates of the parallel manipulator; only the values of the active joints are required, of course, in addition to the data of the actual configuration of the spatial mechanism, and therefore, it is possible to affirm that the acceleration analysis we present in this study is computationally efficient.

Once the reduced acceleration state is computed by means of Eq. 38, the angular acceleration of the moving platform, with respect to the fixed platform, is obtained as the primal part of the accelerator $\mathbf{A}_C$, indeed $\dot{\omega} = \mathrm{P}(\mathbf{A}_C)$, while the linear acceleration of the center of the moving platform, vector $\mathbf{a}_C$, expressed in the reference frame XYZ using the dual part of $\mathbf{A}_C$ results in

$$\mathbf{a}_C = \mathrm{D}(\mathbf{A}_C) + \omega \times \mathbf{v}_C. \tag{39}$$

Finally, it is straightforward to show that, with slight modifications, the method of kinematic analyses here presented is available for other five-dof parallel manipulators with similar architectures, for example, the parallel manipulators shown in Fig. 3.

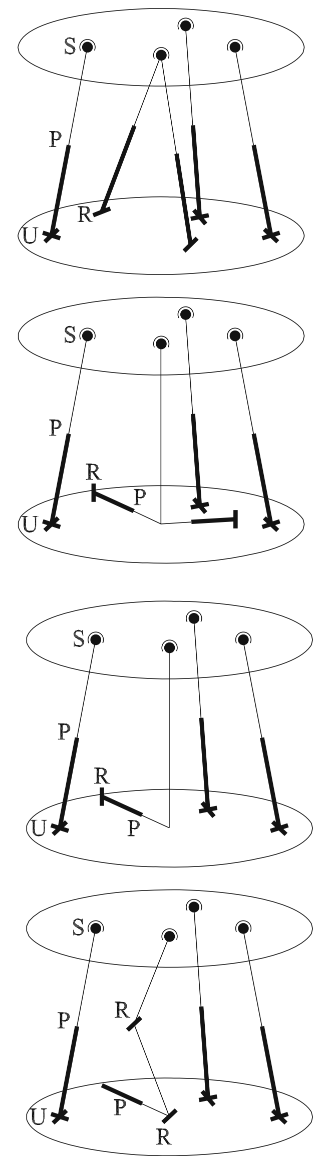


Fig. 3 Some 3R2T parallel manipulators

5 Case study

In this section, a numerical example is provided. The case study consists of solving the forward kinematics of a 3R2T parallel manipulator, and its validation using a different mathematical resource than the one proposed in this work.

Given the parameters and generalized coordinates listed in Table 1, it is required to determine the following kinematic properties of the parallel manipulator:

1. All the real feasible locations of the moving platform, with respect to the fixed platform, at the beginning of the motion
2. Select the home position of the parallel manipulator

Table 1 Parameters of the numerical example

$r = 0.25$, $a = 0.433$
$\mathbf{B}_1 = (-0.25, 0, 0.433)$, $\mathbf{B}_2 = (0.5, 0, 0)$,
$\mathbf{B}_3 = (-0.25, 0, -0.433)$, $\mathbf{B}_4 = (0, 0, 0)$
$q_1 = 0.559 + 0.1\sin(t)$, $q_2 = 0.579 + 0.025\sin(t)$,
$q_3 = 0.559 + 0.05\sin^2(t)$
$q_4 = 0.075\sin(t)$, $q_5 = 0.5 - 0.1\sin^2(t)$
$0 \le t \le 2\pi$

3. According to the periodical functions, see Table 1, assigned to the five generalized coordinates q_i, compute the time history of the linear and angular infinitesimal kinematic properties of the moving platform, with respect to the fixed platform, expressed in a global reference frame attached at the fixed platform

According to Eq. 11, a nonlinear system of three equations in the unknowns X_1, Y_2, and Z_3 is obtained at the beginning of the motion, or in other words, when the time t is equal to zero, as follows:

$$\left.\begin{array}{l} 1.25X_1^2 + 19.250Y_2^2 + 9.749Z_3^2 + .5X_1Y_2 \\ \quad - 25.115Y_2Z_3 - .866X_1Z_3 - .84X_1 - 25.66Y_2 \\ \quad + 18.147Z_3 + 8.696 = 0 \\ 1.25X_1^2 + 7.250Y_2^2 + 9.749Z_3^2 + 3.5X_1Y_2 \\ \quad - 9.526Y_2Z_3 - .866X_1Z_3 - .977X_1 - 6.841Y_2 \\ \quad + 7.676Z_3 + 1.844 = 0 \\ 5.X_1^2 + 8.Y_2^2 + 2.999Z_3^2 + 8.X_1Y_2 - 6.928Y_2Z_3 \\ \quad - 3.464X_1Z_3 - 3.636X_1 - 10.182Y_2 \\ \quad + 4.408Z_3 + 3.228 = 0 \end{array}\right\} \tag{40}$$

Applying the Sylvester dialytic elimination method with the purpose to solve Eq. 40, the resulting values of the variables X_1, Y_2, and Z_3 are listed in Table 2. Furthermore, ignoring imaginary and spurious solutions, only two real solutions are available for the forward position analysis: one solution yields the following

Table 2 Resulting values for X_1, Y_2 and Z_3

X_1	Y_2	Z_3
0.145e−1+.358i	.243−0.504e−10i	−.445+0.319e−1i
0.145e−1−.358i	.243+0.504e−10i	−.445−0.319e−1i
−0.069	.514	−.180
−.173	.514	−.240
−.375+.203i	.638−.138i	0.728e−2+0.389e−1i
−.375−.203i	.638+.138i	0.728e−2−0.389e−1i
0.928e−2−0.649e−1i	.638−.138i	−.212−.278i
0.928e−2+0.649e−1i	.638+.138i	−.212+.278i

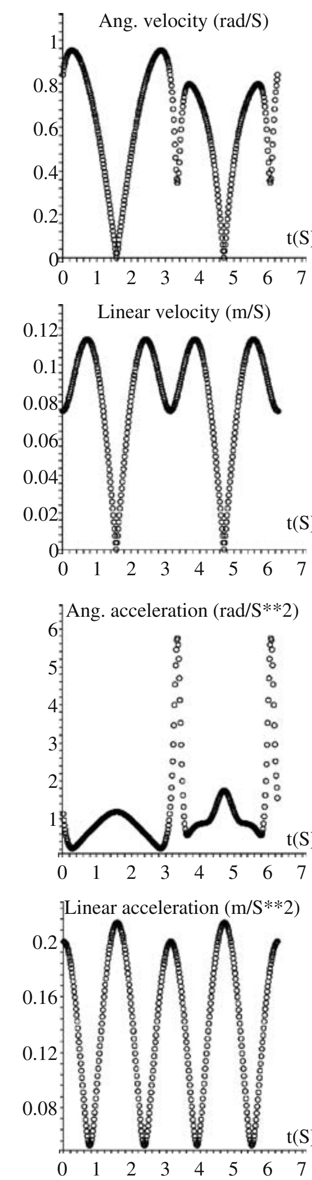
coordinates of the centers of the spherical joints attached at the moving platform:

$$\left.\begin{array}{l} \mathbf{S}_1 = (-0.069, 0.492, 0.240) \\ \mathbf{S}_2 = (0.242, 0.514, -0.060) \\ \mathbf{S}_3 = (-0.173, 0.492, -0.180) \\ \mathbf{S}_4 = (0, 0.5, 0) \end{array}\right\} \tag{41}$$

while the other solution indicates that

$$\left.\begin{array}{l} \mathbf{S}_1 = (-0.173, 0.492, 0.180) \\ \mathbf{S}_2 = (0.242, 0.514, 0.060) \\ \mathbf{S}_3 = (-0.069, 0.492, -0.240) \\ \mathbf{S}_4 = (0, 0.5, 0) \end{array}\right\} \tag{42}$$

Fig. 4 Forward kinematics of the moving platform using screw theory



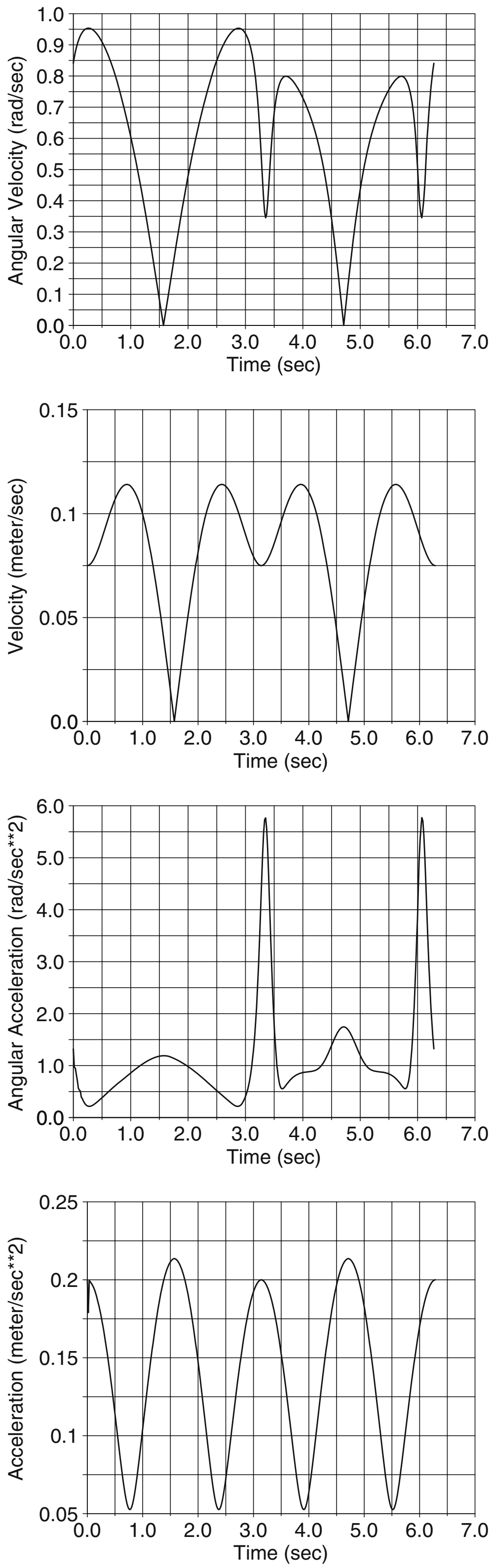


Fig. 5 Forward kinematics of the moving platform using ADAMS©

Due to the decoupled motion, the center of the moving platform, point $\mathbf{S}_4$, depends on the generalized coordinates q_4, associated to the lower prismatic joint, and on q_5, associated to the extendible vertical length of the central limb, and therefore, there is only a unique instantaneous position for this spherical joint. On the other hand, due to the in general inevitable presence of multiple solutions, the centers of the remaining spherical joints can reach two different positions, and therefore, the determination of the actual configuration of the mechanism is a mandatory task in order to approach the infinitesimal kinematics of the parallel manipulator. This task can be achieved using sensors. In this way, it is opportune to emphasize that this section reports an academic numerical example, and therefore, there are no particular reasons to select one of the two computed configurations as the actual configuration of the parallel manipulator. With this consideration in mind, taking Eq. 42 as the home position of the parallel manipulator, the relevant results of the angular and linear kinematic properties of the moving platform, with respect to the fixed platform, using the methodology of analysis introduced in Section 4, are provided in Fig. 4. Furthermore, in order to verify the numerical results obtained by means of screw theory, a simple model was implemented in the kinematic and dynamic simulation program ADAMS©, and the results generated with this commercial software are shown in Fig. 5. Finally, please note that the results generated using screw theory are in excellent agreement with those obtained with ADAMS©.

6 Conclusions

In this work, the position, velocity, and acceleration analyses of a five-dof decoupled parallel manipulator, belonging to the class known as 3R2T parallel manipulators, are successfully approached by means of simple geometric procedures and the theory of screws. The position and orientation of the moving platform are controlled, respectively, by means of a single PPS-type serial kinematic chain and a three-UPS spherical parallel manipulator. The moving platform is connected at the limbs of the parallel manipulator by means of four distinct spherical joints, and therefore, this mechanism can be considered as a nonoverconstrained parallel manipulator, a significative advantage from a manufacturing point of view. Furthermore, since all the limbs play the role of active legs, then the parallel manipulator under study brings the two main advantages of most parallel manipulators: accuracy and stiffness.

The architecture of the parallel manipulator is so simple that the forward finite kinematics, a challenging intensive task for most parallel manipulators, is shortened considerably by establishing simple geometric kinematic constraints. Consider, for instance, that the forward position analysis is presented in semiclosed form solution by applying recursively the Sylvester dialytic elimination method, which allows one to compute the 16 solutions, included reflected solutions, for the mechanism at hand. Certainly, this procedure is well-known; however, it is included in this contribution for the sake of completeness and as a necessary step to approach the infinitesimal kinematics of the parallel manipulator under study.

Simple and compact expressions are derived in this contribution by using the theory of screws for solving the velocity and acceleration analyses of the parallel manipulator. It is worth mentioning that the Klein form of the Lie algebra, *se*(3), allows one to solve the forward acceleration analysis without computing the passive joint acceleration rates of the parallel manipulator. A numerical example is included in order to illustrate the simplicity of the proposed method of infinitesimal kinematic analysis. Furthermore, the numerical results obtained via screw theory are verified with the aid of special software like ADAMS©.

Finally, as is pointed by Zhua et al. [32], the kinematics of 3R2T parallel manipulators, due to their short history, has not been studied very well. Therefore, in this work, the reader can find an option to systematically approach the position, velocity, and acceleration analyses of this class of limited-dof parallel manipulators.

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References

1. Stewart D (1965) A platform with six degrees of freedom. Inst Mech Eng Part I 180(15):371–386
2. Innocenti C, Wenger P (2004) Position analysis of the RRP-3(SS) multiloop spatial structure. In: Proc DETC'04 ASME 2004 design engineering technical conferences and computers and information in engineering conference, paper DETC2004-57382 CD-ROM format. Salt Lake City, Utah
3. Carbone G, Ceccarelli M (2005) A serial-parallel robotic architecture for surgical tasks. Robotica 23(03):345–354
4. Carbone G, Ceccarelli M (2005) A stiffness analysis for a hybrid parallel-serial manipulator. Robotica 22(05):567–576
5. Tanev TK (2000) Kinematics of a hybrid (parallel-serial) robot manipulator. Mech Mach Theory 35(9):1183–1196
6. Zheng XZ, Bin HZ, Luo YG (2004) Kinematic analysis of a hybrid serial-parallel manipulator. Int J Adv Manuf Technol 23(11–12):925–930
7. Gallardo-Alvarado J (2005) Kinematics of a hybrid manipulator by means of screw theory. Multibody Syst Dyn 14(3–4):345–366
8. Lu Y, Hu B (2006) Solving driving forces of 2(3-SPR) serial-parallel manipulator by CAD variation geometry approach. Trans ASME J Mech Des 128:1349–1351
9. Hunt KH, Primrose EJF (1993) Assembly configurations of some in-parallel-actuated manipulators. Mech Mach Theory 28(1):31–42
10. Zlatanov D, Dai MQ, Fenton EG, Benhabib B (1992) Mechanical design and kinematic analysis of a three-legged six degree-of-freedom parallel manipulator. In: Proc ASME robotics, spatial mechanisms, and mechanical systems conference, vol 45. ASME, New York, pp 529–536
11. Jin Y, Chen IM, Yang G (2006) Kinematic design of a 6-DOF parallel manipulator with decoupled translation and rotation. IEEE Trans Robot 22(3):545–551
12. Gallardo-Alvarado J, Alici G, Pérez-González L (2009) A new family of redundant parallel manipulators. Multibody Syst Dyn (in press)
13. Gallardo-Alvarado J, Rico-Martínez JM, Alici G (2006) Kinematics and singularity analyses of a 4-dof parallel manipulator using screw theory. Mech Mach Theory 41(9): 1048–1061
14. Gallardo-Alvarado J, Orozco-Mendoza H, Maeda-Sánchez A (2007) Acceleration and singularity analyses of a parallel manipulator with a particular Topology. Meccanica 42(3): 223–238
15. Gallardo-Alvarado J, Rodríguez-Castro R., Caudillo-Ramírez M, Rico-Martínez JM (2008) A family of spherical parallel manipulators with two legs. Mech Mach Theory 43(2): 201–216
16. Clavel R (1991) Conception d'un robot parallèle rapide à 4 degrés de liberté Thèse EPFL 925, EPFL
17. Weck M, Staimer M (2002) Parallel kinematic machine tools—current state and future potentials. Ann CIRP 51(2): 671–683
18. Zoppi M, Bruzzone LE, Molfino RM (2004) A novel 5-dof interconnected-chains PKM for manufacturing revolute surfaces. In: 4th Chemnitzer parallel kinematik Seminar. Chemnitz, 20–21 April 2004
19. Zhao JW, Fan KC, Chang TH, Li Z (2002) Error analysis of a serial-parallel type machine tool. Int J Adv Manuf Technol 19:174–179
20. Song J, Mu J (2005) A near-optimal part setup algorithm for 5-axis machining using a parallel kinematic machine. Int J Adv Manuf Technol 25:130–139
21. Gao F, Feng B, Zhao H, Li W (2006) A novel 5-DOF fully parallel kinematic machine tool. Int J Adv Manuf Technol 31:201–207
22. Lu Y, Xu JY (2008) Simulation of three-dimensional free-form surface normal machining by 3SPS+RRPU and 2SPS+RRPRR parallel machine tools. J Eng Manuf IMechE Part B 222(4):485–494
23. Chirikjian GS (1992) Theory and applications of hyper-redundant robotic manipulators. PhD dissertation, Department of Applied Mechanics, Division of Engineering and Applied Science, California Institute of Technology
24. Angerilli M, Frisoli A, Salsedo F, Marcheschi S, Bergamasco M (2001) Haptic simulation of an automotive manual gearshift. In: Proc of Romancy2001, international

workshop on robot-human communication. Bordeaux-Paris, 18–21 September 2001

25. Birglen L, Gosselin CM, Pouliot N, Monsarrat B, Lalibertè T (2002) SHaDe, a new 3-DOF haptic device. IEEE Trans Robot Autom 18(2):166–175
26. Merlet JP (2001) Micro parallel robot MIPS for medical applications. In: IEEE int. conf. on emerging technologies and factory automation. Antibes, 15–18 October 2001
27. Wu G, Li J, Fei R, Wang X, Liu D (2005) Analysis and design of a novel micro-dissection manipulator based on ultrasonic vibration. In: Proc 2005 IEEE ICRA. Barcelona, 18–22 April 2005, pp 468–473
28. Ng CC, Ong SK, Nee AYC (2006) Design and development of 3DOF modular micro parallel kinematic manipulator. Int J Adv Manuf Technol 31:188–200
29. Peirs J, Reynaerts D, Van Brussel H (1999) A miniature hydraulic parallel manipulator for integration in a self-propelling endoscope. In: EUROSENSORS XIII The 13th European conference on solid-state transducers, 22C4 medical applications. The Hague, pp 753–756
30. Zhu SJ, Huang Z, Zhao MY (2007) Kinematics of a 5-DOF 5-$\underline{R}$RR(RR) prototype. In: Proc of the ASME 2007 international design engineering technical conferences & computers and information in engineering conference IDETC/CIE 2007, paper DETC2007-35916 CD-ROM format. Las Vegas, Nevada
31. Zhu SJ, Huang Z, Zhao MY (2008) Kinematics of a partially decoupled 3R2T symmetrical parallel manipulator 3-RCRR. J Mech Eng Sci IMechE Part C 222(2):277–285
32. Zhu SJ, Huang Z, Zhao MY (2009) Singularity analysis for six practicable 5-DoF fully-symmetrical parallel manipulators. Mech Mach Theory 44:710–725
33. Tsai L-W (1999) Robot analysis. Wiley, New York
34. Innocenti C, Parenti-Castelli V (1990) Direct position analysis of the Stewart platform mechanism. Mech Mach Theory 35:611–621
35. Gallardo-Alvarado J, Orozco-Mendoza H, Rodríguez-Castro R, Rico-Martínez JM (2007) Kinematics of a class of parallel manipulators which generates structures with three limbs. Multibody Syst Dyn 17(1):27–46
36. Gallardo-Alvarado J, Aguilar-Nájera CR, Casique-Rosas L, Pérez-González L, Rico-Martínez JM (2008) Solving the kinematics and dynamics of a modular spatial hyper-redundant manipulator by means of screw theory. Multibody Syst Dyn 20(4):307–325
37. Rico JM, Duffy J (1996) An Application of screw algebra to the acceleration analysis of serial chains. Mech Mach Theory 31(4):445–457
38. Rico JM, Gallardo J, Duffy J (1999) Screw theory and higher order kinematic analysis of open serial and closed chains. Mech Mach Theory 34(4):559–586
39. Rico JM, Duffy J (2000) Forward and inverse accceleration analyses of in-parallel manipulators. Trans ASME J Mech Des 122(3):299–303
40. Gallardo J, Rico JM, Frisoli A, Checcacci D, Bergamasco M (2003) Dynamics of parallel manipulators by means of screw theory. Mech Mach Theory 38(11):1113–1131
41. Kong X, Gosselin CM (2004) Type synthesis of 3-DOF translational parallel manipulators based on screw theory. ASME J Mech Des 126(1):83–92
42. Kong X, Gosselin CM (2004) Type synthesis of 3-DOF spherical parallel manipulators based on screw theory. ASME J Mech Des 126(1):101–108
43. Kong X, Gosselin CM (2004) Type synthesis of 3T1R 4-DOF parallel manipulators based on screw theory. IEEE Trans Robot Autom 20(2):181–190