Int J Adv Manuf Technol (2009) 44:1144–1153
DOI 10.1007/s00170-009-1927-9

ORIGINAL ARTICLE

Multicriteria optimization of cutting parameters in turning of UD-GFRP materials considering sensitivity

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Received: 2 June 2008 / Accepted: 6 January 2009 / Published online: 29 January 2009

Abstract In this paper, a new multicriteria optimization approach is proposed for the selection of the optimal values of cutting conditions in machining. This approach aims to handle the possible manufacturing errors in design stage. These errors are taken into consideration as change in design parameters and the design most robust to change is selected as the optimum design. Machining of a glass fiber composite material is chosen in case studies. Experiments on the unidirectional glass fiber reinforced composite material are performed to investigate the effect of cutting speed, feed, and cutting depth on the cutting forces. Also, material removal rate values are obtained. Minimizing cutting forces and maximizing the material removal are considered as objectives. It is believed that the used method provides a robust way of looking at the optimum parameter selection problems.

Keywords Machining optimization · UD-GFRP · Cutting force · Multicriteria optimization

1 Introduction

Fiber reinforced plastic composite (FRP) materials have been widely used in a variety of structures, such as aircraft, robots, and machines. An important aspect of production is machining. There is significant difference between the machining of conventional metals and their alloys and that of composite materials. This is because FRPs are anisotropic and inhomogeneous. Besides, the mechanism of material removal is different from that of single-phased material, such as metals. The material removal process is quite complex. Many variables such as the workpiece material, the cutting tool material, the rigidity of the machine and the set up, the cutting feed and speed, tool wear, and chip control must be considered. Kim and Ehmann [1] demonstrated that the knowledge of the cutting forces is one of the most fundamental requirements. This knowledge also gives very important information for cutter design, machine tool design, and detection of tool wear and breakage.

The anisotropic behavior of fiber reinforced composite can be brought into more efficient usage by arranging machining parameters. This provides the manufacturer a wide range of choice. The problem of finding optimum parameters for composites is to select the appropriate parameter values so as to achieve the highest performance for specified requirements. Işık [2] presented research results on the machining of unidirectional glass fiber reinforced composite (UD-GFRP) and recommended optimum cutting parameters to obtain better surface quality. Arul et al. [3] worked on optimization of machining of GFRP material. They analyzed data on the thrust force, torque, and tool life by using a group method data handling algorithm. Also, Palanikumar [4] worked on finding optimum cutting parameters for surface roughness using Taguchi's method. He mentioned the benefits of using Taguchi's method. It offers a simple and systematic approach to optimizing design. By applying this technique, one can significantly reduce the number of experiments and

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time required for experimentation. He also with his coworkers [5] investigated optimum machining parameters of GFRP materials. They considered the problem as a multiple performance optimization problem and this is the only study that multiobjective optimization is considered in machining of GFRP materials up to now.

This paper mainly focuses on finding optimum parameters considering the cutting forces and material removal rate of GFRP bars. Experiments were conducted through the established Taguchi's design method. Orthogonal cutting tests were carried out on UD-GFRP materials to ascertain the effect of tool–material interaction on tool wear and cutting forces. Then, the machining parameters are optimized by employing statistical techniques using a new approach based on sensitivity. The practical design properties can be different from what the manufacturers predict due to the uncertainties in material properties and the variations of parameters in manufacturing. Traditionally, these uncertainties are not taken care of but used method is taken care of via giving small deviations to parameters. From this perspective, this study is unique as an application to machining of GFRP materials. Suitability of the method is analyzed by finding optimum parameters for two cases.

2 Experimental design

By increase in application, study on machining of GFRP materials becomes popular in recent years. Sakuma and Seto [6, 7] performed turning tests on both glass fiber–epoxy composite materials and carbon fiber–epoxy composite materials that contained unidirectional fibers. Several kinds of tool materials such as sintered carbides, ceramics, and cermets are used and the wear patterns and the wear land growth are analyzed. The wear of sintered-carbide tools and high-speed steel tools is very severe. Hence, the cutting speed and feed of the machining operation should be selected carefully in the machining of carbon fiber–epoxy composite materials [8, 9]. Delamination, fiber pullout, fiber fragmentation, burring, and fuzzing are some of the types of damage caused by machining GFRP, as reported by Wang and Zhang [10].

Bağcı and Işık [11] investigated the turning of UD-GFRP material. In their study, an artificial neural network and response surface model based on experimental measurement data are developed to estimate surface roughness in orthogonal cutting of GFRP. Rao et al. [12] simulated orthogonal machining of unidirectional carbon fiber-reinforced polymer and glass fiber-reinforced polymer composites using finite element method. The cutting forces during orthogonal machining were studied both experimentally and numerically for a range of fiber orientations, depths of cut, and tool rake angles.

In this study, a new multicriteria optimization approach on experimental measurement data is developed to estimate cutting forces in orthogonal cutting of GFRPs. The approach include cutting speed, feed, and depth of cut as turning parameters.

2.1 Equipment and material

The cutting experiments were conducted turning in dry cutting conditions on a JOHNFORD TC-35 computer numerical control (CNC) lathe machine. The turning operation was carried out using TAEGUTEC TCMT 16T304 MT T3000 inserts and a SECO STGCL 2020K16 tool holder. Cemented carbide tools were chosen because of excellent wear resistance.

KISTLER TYPE 911 dynamometer measuring cutting forces was setup on the CNC lathe. The UD-GFRP rods were cut from 12 to 11 mm in diameter because of avoiding surface damage and balance problems and were machined with several cutting parameters. The obtained cutting forces were measured by the dynamometer as given in Fig. 1.

GFRP rods consist of unidirectional fibers that are pulled through a resin bath into the shape of the rod. The material was

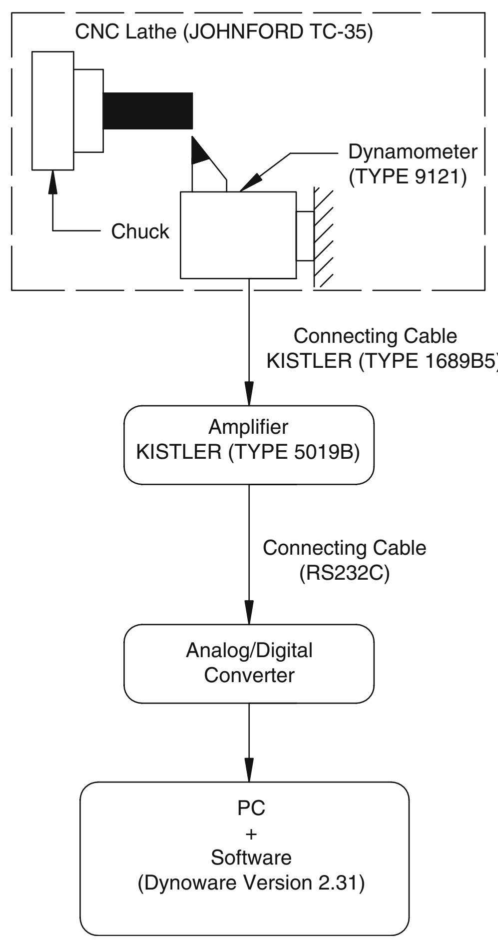

Fig. 1 Experimental setup

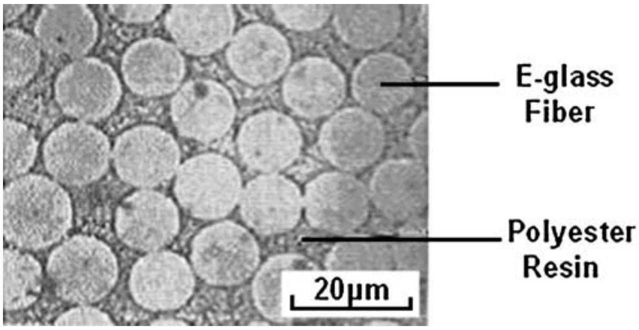

Fig. 2 A SEM photo of the GFRP

produced by pultrusion method with polyester and E-glass. It has 82.27% glass contents. Figure 2 shows a scanning electron microscope (SEM) photo of the GFRP material. The GFRP, of which the physical properties are listed in Table 1, was used as workpiece materials in this study.

2.2 Cutting conditions

Design of experiments is a powerful analysis tool for modeling and analyzing the effect of process variables over some specific variable, which is an unknown function of these process variables [13]. The experimental design method is an effective approach to optimize the various machining parameters. The selection of such points in the design space is commonly called *design of experiments* or *experimental design*. The choice of the experimental design can have a large influence on the accuracy and the construction cost of the approximations. Randomly chosen design points may cause breaks in part or tools or even prevent ability to machine a surface at all. Several experimental design techniques have been used to aid in the selection of appropriate design points. In a factorial design, a variable range is divided into levels between the lowest and the highest values [14]. A three-level full factorial design creates 3^n training data, where n is the number of variables. In this study, three independent variables, such as depth of cut (a_p), cutting speed (V_c), and feed rate (f) had total of 3^3=27 experimental runs. Experiments are done for two conditions (tool radii of 0.4 and 0.8 mm) with the same relief angle of 7°. To avoid thermal effects, lower cutting speeds were chosen. However, higher cutting speeds cause severe tool wear and the higher feeds cause a large deformation rate. Ranges for process parameters and the results obtained are shown in Table 2.

Table 1 Physical properties of unidirectional GFRP

Specific weight (g/cm^3)	2.03
Young modulus (N/mm^2)	65.8 (axial)
	7.2 (transverse)
Thermal coefficient of expansion (m/mK)	5
Thermal conductivity (°C)	0.8
Glass fiber	E-glass
Matrix material	Polyester resin

2.3 Experimental results

The design variable values considered in the experimental procedure are as follows:

$$V_c = \{75, 100, 125\}\ ;\ f = \{0.2, 0.3, 0.4\}\ ;$$
$$a_p = \{0.6, 0.9, 1.2\}.$$

Experimental force results obtained in the milling with above values are given in Table 2.

3 Methodology

Used approach has two main steps: obtaining Pareto optimum points and selecting the least sensitive point. In this study, design space is formed by the data obtained from the experiments. The Pareto points are found among these

Table 2 The experimental values obtained in machining (tool radii of 0.4/0.8 mm)

Data	a_p [mm]	f [mm/rev]	V_c [m/min]	F_z [N]	F_y [N]
1	1.2/1.2	0.2/0.2	125/125	5.3/9.7	2.2/4.9
2	1.2/1.2	0.2/0.2	100/100	7.1/7.9	2.3/4.8
3	1.2/1.2	0.2/0.2	75/75	10.4/7.3	4.1/4.2
4	1.2/1.2	0.3/0.3	125/125	6.2/14.7	3.2/6.8
5	1.2/1.2	0.3/0.3	100/100	10.3/13.8	4.3/5.9
6	1.2/1.2	0.3/0.3	75/75	12.4/14.2	5.3/7.2
7	1.2/1.2	0.4/0.4	125/125	8.5/19.5	4.2/7.8
8	1.2/1.2	0.4/0.4	100/100	12.1/17.7	6.1/7.7
9	1.2/1.2	0.4/0.4	75/75	17.4/20.4	8.2/10.2
10	0.9/0.9	0.2/0.2	125/125	3.4/7.8	2.1/2.9
11	0.9/0.9	0.2/0.2	100/100	5.1/5.9	2.2/2.8
12	0.9/0.9	0.2/0.2	75/75	7.2/5.3	3.2/3.1
13	0.9/0.9	0.3/0.3	125/125	5.5/13.8	2.1/5.8
14	0.9/0.9	0.3/0.3	100/100	8.1/10.9	3.2/4.9
15	0.9/0.9	0.3/0.3	75/75	11.3/12.8	5.1/6.1
16	0.9/0.9	0.4/0.4	125/125	6.4/15.7	3.1/6.8
17	0.9/0.9	0.4/0.4	100/100	10.9/13.9	5.1/5.9
18	0.9/0.9	0.4/0.4	75/75	13.4/17.2	6.2/8.2
19	0.6/0.6	0.2/0.2	125/125	3.1/4.9	1.2/1.9
20	0.6/0.6	0.2/0.2	100/100	3.9/3.9	1.1/1.9
21	0.6/0.6	0.2/0.2	75/75	6.2/4.1	2.2/2.1
22	0.6/0.6	0.3/0.3	125/125	4.2/7.7	2.1/2.8
23	0.6/0.6	0.3/0.3	100/100	5.9/6.8	2.1/2.9
24	0.6/0.6	0.3/0.3	75/75	7.3/6.2	3.2/4.1
25	0.6/0.6	0.4/0.4	125/125	4.4/12.8	2.2/4.8
26	0.6/0.6	0.4/0.4	100/100	7.1/8.8	2.2/3.9
27	0.6/0.6	0.4/0.4	75/75	11.2/9.2	4.2/5.2

data by using weighting function methodology and optimum point is selected among them according to new approach considering least sensitive point. This approach has proved to be very useful dealing with discrete variables defined on a population of cutting condition values obtained from experiments. Microsoft Excel is used when necessary.

3.1 Obtaining Pareto optimum set

Multicriteria optimization in last two decades has been acknowledged as an advanced design technique in optimization. The reason is that most of the real-world problems are multidisciplinary and complex, as there is always more than one important objective in each problem. To accommodate many conflicting design goals, one needs to formulate the optimization problem with multiple objectives. One important reason for the success of the multicriteria optimization approach is its natural property of allowing the designer to participate in the design selection process even after the formulation of the mathematical optimization model. In addition, usually, several competing objectives appear in a real-life application, and thus, the designer is faced with a decision-making problem in which the task is to find the best compromise solution between the conflicting requirements.

A multicriteria optimization problem can be formulated as follow:

$$\mathrm{Min}[f_1(x), f_2(x), \ldots, f_n(x)]$$

subject to

$$g_j(x) \geq 0 \quad j = 1, 2, \ldots, m \tag{1}$$

$$h_j(x) = 0 \quad j = 1, 2, \ldots, p < n$$

where x is n-dimensional design variable vector, $f_i(x)$ is objective function, and $g_j(x)$ and $h_j(x)$ are the inequality and equality constraints, respectively.

A variety of techniques and applications of multicriteria optimization have been developed over the past few years. The progress in the field of multicriteria optimization was summarized by Hwang and Masud [15] and later by Stadler [16]. Stadler inferred from his survey that if one has decided that an optimal design is to be based on the consideration of several criteria, then the multicriteria theory (Pareto theory) provides the necessary framework. In addition, if the minimization or maximization is the objective for each criterion, then an optimal solution should be a member of the corresponding Pareto set. Only then does any further improvement in one criterion require a clear tradeoff with at least one other criterion. Radford et al. [17] in their study has explored the role of Pareto optimization in computer-aided design. They used the weighting method, noninferior set estimation method, and constraint method for generating the Pareto optimal. The authors discussed the control and derivation of meaning from the Pareto sets. Also in this study, weighting method is used and brief explanation will be given at the following paragraphs.

Pareto optimality serves as the basic multicriteria optimization concept in virtually all of the previous literature [18]. A general multicriteria optimization problem is to find the vector of design variables $X=(x_1, x_2, \ldots, x_n)^T$ that minimize a vector objective function $F(X)$ over the feasible design space X. It is the determination of a set of nondominated solutions (Pareto optimum solutions or noninferior solutions) that achieves a compromise among several different, usually conflicting, objective functions. The Pareto optimal is stated in simple words as follows: A vector X^* is Pareto optimal if there exists no feasible vector X which would decrease some objective function without causing a simultaneous increase in at least one objective function. This definition can be explained graphically. An arbitrary collection of feasible solutions for a two-objective minimization problem is shown in Fig. 3. The area inside of the shape and its boundaries are feasible. The axes of this graph are the objectives F and Q. It can be seen from the graph that the noninferior solutions are found in the portion of the boundary between points A and B. Thus, here arises the decision-making problem from which a partial or complete ordering of the set of nondominated objectives is accomplished by considering the preferences of the decision maker. Most of the multicriteria optimization techniques are based on how to elicit the preferences and determine the best compromise solution. Used approach differs from other techniques from this perspective. This approach chooses the optimum point among the Pareto set points by considering the sensitivity to change in parameters.

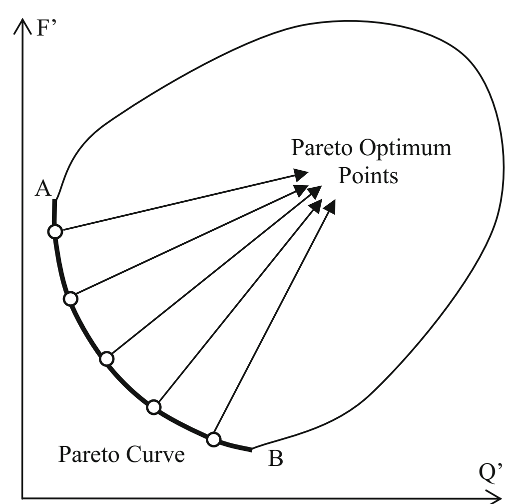

Fig. 3 Graphical interpretation of Pareto optimum

3.2 Weighting method

This technique is based on the preference techniques of the weights' prior assessment for each objective function. It transforms the multicriteria function to a single criterion function through a parameterization of the relative weighting of the criteria. With the variation of the weights, the entire Pareto set can be generated. Because the results of solving an optimization problem can vary significantly as the weighting coefficients change and very little is usually known about how to choose these coefficients, a necessary approach is to solve the same problem for many different values of weighting factors. However, because the shape and distribution characteristics of the Pareto set are unknown, it is difficult to determine beforehand the nature of the variations required in the weights so as to produce a new solution at each pass. The second important disadvantage of the method is that it will not identify the Pareto solutions in a nonconvex part of the set.

The idea of this technique consists in adding all the objective functions together using different coefficients for each. It means that we change our multicriteria optimization problem to a scalar optimization problem by creating one function of the form

$$f(x) = \sum_{i=1}^{k} w_i f_i(x) \tag{2}$$

where $w_i \geq 0$ are the weighting coefficients representing the relative importance of the criteria. It is usually assumed that

$$\sum_{i=1}^{k} w_i = 1 \tag{3}$$

since the results of solving an optimization model using Eq. 2 can vary significantly as the weighting coefficients change and since very little is usually known about how to choose these coefficients, a necessary approach is to solve the same problem for many different values of w_i.

Note that the weighting coefficients do not reflect proportionally the relative importance of the objectives but are only factors which when varied locate points in the domain. For the numerical methods of seeking the minimum of Eq. 3, this location depends not only on values of w_i but also on units in which the functions are expressed.

The best results are usually obtained if objective functions are normalized. In this case, the vector function is normalized to the following form

$$\widetilde{f}(x) = \left[\widetilde{f}_1(x), \widetilde{f}_2(x), \ldots.., \widetilde{f}_k(x)\right]^T \tag{4}$$

where

$$\widetilde{f}_i(x) = \frac{f_i(x)}{f_i^o} \quad i = 1, 2, \ldots, k. \tag{5}$$

Here, f_i^o is generally the maximum value of ith objective function. A condition $f_i^o \neq 0$ is assumed and if it is not satisfied which rarely happens, another value of normalizing function must be chosen by the decision maker.

In this study, total force and cutting flow of material were considered. Total force value is the resultant force of the forces obtained in experiments and cutting flow of material is obtained by using Eq. 6.

$$Q = \frac{a_p . a_e . f}{1,000} \left(\mathrm{m}^3/\mathrm{min}\right) \tag{6}$$

where a_p, a_e, and f represents cutting depth, cutting width (constant), and feed. Since the cutting flow of material Q is maximized and total force F is minimized, to minimize the composite weighted function, inverse of the force is maximized and the objective function is set as

$$J = \frac{1}{F} + w \cdot Q \tag{7}$$

where w is a weighting coefficient varied to obtain Pareto optimum points. For small values of w, objective function J leans toward ($1/F$) and for large values of w, J leans toward Q depending on the relative values of these functions. In order to bring the values in the same range, Q and ($1/F$) are normalized with their maximum values where the resulting functions are called Q' and F' and the relation in Eq. 8 is used to obtain Pareto optimum values

$$J = F' + w \cdot Q'. \tag{8}$$

The design space is related with the allowed maximal dimension of the controlled variable vectors used during the machining operation. The designed variables are the cutting speed (V_c), the feed (f), and the cutting depth (a_p). These variables can assume the following discrete values:

$$V_c = \{V_{c1},\ V_{c2},\ V_{c3}\};\ f = \{f_1, f_2, f_3, f_4, f_5\};\\ a_p = \{a_{p1},\ a_{p2},\ a_{p3}\}. \tag{9}$$

The design space is a typical discrete and nonconvex domain. The values of objectives (total cutting force and cutting flow of material) obtained for the different machin-

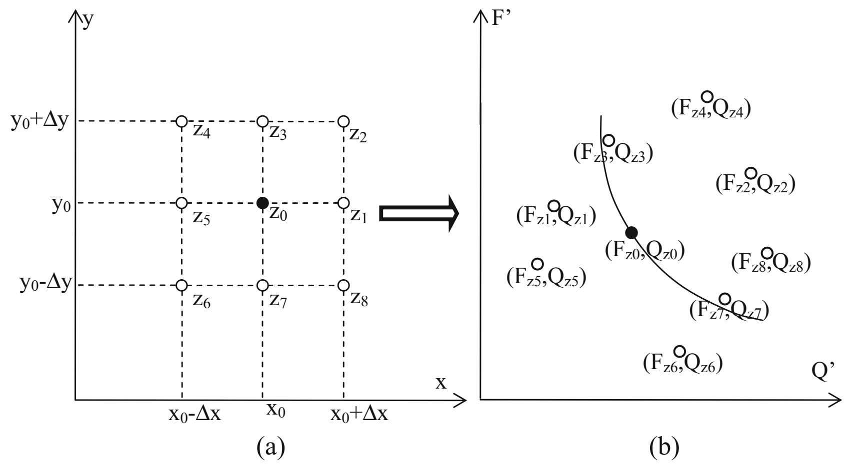

Fig. 4 Change in design parameters and objectives

ing parameters (V_c, f, a_p) are normalized with respect to the associated maximal values.

3.3 Selecting the least sensitive point

At the second step, among the Pareto optimum points, the optimum point is selected based on changes in objective function when small variations are permitted in design variables. In this work, equal contributions of each variable are considered. Based on ± variations in design variables, average changes in the objective function values are calculated at every Pareto optimum point. Figure 4 shows the change in parameter and objective values for two parameter case.

Table 3 Calculation of Pareto optimum points (tool radius of 0.4 mm)

Data	Inverse *F*	Norm (*F*)	Norm (*Q*)	*w* (0.1)	*w* (0.5)	*w* (1)	*w* (5)	*w* (10)
1	0.266667	0.573333	0.5	0.623333	0.823333	1.073333	3.073333	5.573333
2	0.212766	0.457447	0.5	0.507447	0.707447	0.957447	2.957447	5.457447
3	0.137931	0.296552	0.5	0.346552	0.546552	0.796552	2.796552	5.296552
4	0.212766	0.457447	0.75	0.532447	0.832447	1.207447	4.207447	7.957447
5	0.136986	0.294521	0.75	0.369521	0.669521	1.044521	4.044521	7.794521
6	0.112994	0.242938	0.75	0.317938	0.617938	0.992938	3.992938	7.742938
7	***0.15748***	***0.338583***	***1***	0.438583	0.838583	***1.338583***	***5.338583***	***10.33858***
8	0.10989	0.236264	1	0.336264	0.736264	1.236264	5.236264	10.23626
9	0.078125	0.167969	1	0.267969	0.667969	1.167969	5.167969	10.16797
10	0.363636	0.781818	0.375	0.819318	0.969318	1.156818	2.656818	4.531818
11	0.273973	0.589041	0.375	0.626541	0.776541	0.964041	2.464041	4.339041
12	0.192308	0.413462	0.375	0.450962	0.600962	0.788462	2.288462	4.163462
13	0.263158	0.565789	0.5625	0.622039	0.847039	1.128289	3.378289	6.190789
14	0.176991	0.380531	0.5625	0.436781	0.661781	0.943031	3.193031	6.005531
15	0.121951	0.262195	0.5625	0.318445	0.543445	0.824695	3.074695	5.887195
16	0.210526	0.452632	0.75	0.527632	0.827632	1.202632	4.202632	7.952632
17	0.125	0.26875	0.75	0.34375	0.64375	1.01875	4.01875	7.76875
18	0.102041	0.219388	0.75	0.294388	0.594388	0.969388	3.969388	7.719388
19	***0.465116***	***1***	***0.25***	***1.025***	***1.125***	1.25	2.25	3.5
20	0.4	0.86	0.25	0.885	0.985	1.11	2.11	3.36
21	0.238095	0.511905	0.25	0.536905	0.636905	0.761905	1.761905	3.011905
22	0.31746	0.68254	0.375	0.72004	0.87004	1.05754	2.55754	4.43254
23	0.25	0.5375	0.375	0.575	0.725	0.9125	2.4125	4.2875
24	0.190476	0.409524	0.375	0.447024	0.597024	0.784524	2.284524	4.159524
25	0.30303	0.651515	0.5	0.701515	0.901515	1.151515	3.151515	5.651515
26	0.215054	0.462366	0.5	0.512366	0.712366	0.962366	2.962366	5.462366
27	0.12987	0.279221	0.5	0.329221	0.529221	0.779221	2.779221	5.279221

Pareto optimum points are written as bold italics

Table 4 Calculation of Pareto optimum points (tool radius of 0.8 mm)

Data	Inverse *F*	Norm (*F*)	Norm (*Q*)	*w* (0.1)	*w* (0.5)	*w* (1)	*w* (5)	*w* (10)
1	0.136986	0.39726	0.5	0.44726	0.64726	0.89726	2.89726	5.39726
2	0.15748	0.456693	0.5	0.506693	0.706693	0.956693	2.956693	5.456693
3	0.173913	0.504348	0.5	0.554348	0.754348	1.004348	3.004348	5.504348
4	0.093023	0.269767	0.75	0.344767	0.644767	1.019767	4.019767	7.769767
5	0.101523	0.294416	0.75	0.369416	0.669416	1.044416	4.044416	7.794416
6	0.093458	0.271028	0.75	0.346028	0.646028	1.021028	4.021028	7.771028
7	0.07326	0.212454	1	0.312454	0.712454	1.212454	5.212454	10.21245
8	***0.07874***	***0.228346***	***1***	0.328346	0.728346	1.228346	***5.228346***	***10.22835***
9	0.065359	0.189542	1	0.289542	0.689542	1.189542	5.189542	10.18954
10	0.186916	0.542056	0.375	0.579556	0.729556	0.917056	2.417056	4.292056
11	0.229885	0.666667	0.375	0.704167	0.854167	1.041667	2.541667	4.416667
12	0.238095	0.690476	0.375	0.727976	0.877976	1.065476	2.565476	4.440476
13	0.102041	0.295918	0.5625	0.352168	0.577168	0.858418	3.108418	5.920918
14	0.126582	0.367089	0.5625	0.423339	0.648339	0.929589	3.179589	5.992089
15	0.10582	0.306878	0.5625	0.363128	0.588128	0.869378	3.119378	5.931878
16	0.088889	0.257778	0.75	0.332778	0.632778	1.007778	4.007778	7.757778
17	0.10101	0.292929	0.75	0.367929	0.667929	1.042929	4.042929	7.792929
18	0.07874	0.228346	0.75	0.303346	0.603346	0.978346	3.978346	7.728346
19	0.294118	0.852941	0.25	0.877941	0.977941	1.102941	2.102941	3.352941
20	***0.344828***	***1***	***0.25***	***1.025***	***1.125***	***1.25***	2.25	3.5
21	0.322581	0.935484	0.25	0.960484	1.060484	1.185484	2.185484	3.435484
22	0.190476	0.552381	0.375	0.589881	0.739881	0.927381	2.427381	4.302381
23	0.206186	0.597938	0.375	0.635438	0.785438	0.972938	2.472938	4.347938
24	0.194175	0.563107	0.375	0.600607	0.750607	0.938107	2.438107	4.313107
25	0.113636	0.329545	0.5	0.379545	0.579545	0.829545	2.829545	5.329545
26	0.15748	0.456693	0.5	0.506693	0.706693	0.956693	2.956693	5.456693
27	0.138889	0.402778	0.5	0.452778	0.652778	0.902778	2.902778	5.402778

Pareto optimum points are written as bold italics

The optimum point is selected as the one having the minimum changes on

$$\Delta V_j = \frac{1}{n}\sum_{i=1}^{n}\left\{\left[F'(z_i) - F'(z_0)\right]^2 + \left[Q'(z_i) - Q'(z_0)\right]^2\right\} \tag{10}$$

where n is the number of design variable change around every Pareto optimum point, $F'(z_o)$ and $Q'(z_o)$ are the objective function values at the Pareto optimum point, $F'(z_i)$ and $Q'(z_i)$ are the objective function values when a certain change is applied to a design parameter, and j is the index of the Pareto optimum point [19]. While calculating the change in objectives, objective values that are not in the feasible region are not taken into account. For example, change in objective values at points 1, 5, and 6 in Fig. 4 are not considered because they are not in the feasible region.

In this study, three parameters (the cutting speed, the feed, and the cutting depth) are concerned. Data given in Table 2 are used in calculation. Change in cutting depth is accepted as 0.3 mm, change in feed is accepted as 0.1 mm/rev, and change in cutting speed is accepted as 25 m/min.

4 Case study

The obtained results for two different cases (tool radii of 0.4 and 0.8 mm) will be shown in this section. The approach described above is applied to the experimental data given in Section 2.3. The objective functions are

Table 5 Pareto optimum points (tool radius of 0.4 mm)

Data	a_p	f	V_c	Weighting coefficients
7	1.2	0.4	125	1, 5, 10
19	0.6	0.2	125	0.1, 0.5

Table 6 Pareto optimum points (tool radius of 0.8 mm)

Data	a_p	f	V_c	Weighting coefficients
8	1.2	0.4	100	5, 10
20	0.6	0.2	100	0.1, 0.5, 1

Table 7 Change in objective values (tool radii of 0.4 mm)

Data	DF 1	DQ 1	SD 1	Data	DF 2	DQ 2	SD 2
4	0.118864	−0.25	0.076629	10	−0.21818	0.125	0.063228
5	−0.04406	−0.25	0.064441	11	−0.41096	0.125	0.184512
8	−0.10232	0	0.010469	13	−0.43421	0.3125	0.286195
13	0.227207	−0.4375	0.243029	14	−0.61947	0.3125	0.481398
14	0.041948	−0.4375	0.193166	20	−0.14	0	0.0196
16	0.114049	−0.25	0.075507	22	−0.31746	0.125	0.116406
17	−0.06983	−0.25	0.067377	23	−0.4625	0.125	0.229531

DF deviation of first objective, *DQ* deviation of second objective, *SD* Sum of deviation

evaluated using these values. Then, Pareto points are evaluated using weighting function. Even though weighting coefficient of objective function is changed from 10^{-6} to 10^{6}, it has been seen that it is enough to change from 10^{-1} to 10.

Firstly, objective values are calculated as mentioned in Section 3.2. Then, objectives are converted into same type (at column of inverse *F* in Tables 3 and 4) and their values are normalized (at column of norm (*F*) and norm (*Q*) in Tables 3 and 4). Pareto optimum points are written as bold italics in Tables 3 and 4. Found Pareto optimum points are given in Tables 5 and 6 for the case tool radii of 0.4 and 0.8 mm, respectively.

After Pareto optimum points are obtained, change is given to the design parameters and deviation of objectives (DF and DQ in Tables 7 and 8) and their sum (SD in Tables 7 and 8) is calculated. Each table have different amount of data since experimental data of previous study is used to compare. Numbers 1 and 2 in Tables 7 and 8 represents the first and second points in Tables 3 and 4.

As a last step, total average deviation is calculated by using Eq. 10. It is seen that first Pareto point has total average deviation as 0.104374 and second Pareto point has total average deviation as 0.197267 for the case tool radius of 4 mm. First Pareto point is the optimum point with having the minimum deviation. Found optimum cutting conditions are as a_p=1.2 mm, f=0.4 mm/rev, and V_c= 125 m/min. Same approach is applied for the case the tool radius of 0.8 mm. It is seen that first Pareto point has total average deviation as 0.124237 and second Pareto point has total average deviation as 0.250796. Again, first Pareto point is the optimum point with having the minimum deviation. Optimum cutting conditions are obtained as a_p =1.2 mm, f=0.4 mm/rev, and V_c=100 m/min. When surface roughness is considered [2], it is found that optimum cutting conditions are as a_p=1.2 mm, f=0.2 mm/rev, and V_c=100 m/min for tool radius of 0.4 mm and as a_p= 0.6 mm, f=0.2 mm/rev, and V_c=125 m/min for tool radius of 0.8 mm. As it is expected, slow feed value has given better surface roughness value.

5 Conclusions

Considering that cutting conditions regulate the machining process through the developed cutting forces, it becomes of high importance the optimization of machining parameters. Although there are several methods in literature for multi-criteria optimization of machining processes, a new approach is proposed in this work. Main advantage of this

Table 8 Change in objective values (tool radius of 0.8 mm)

Data	DF 1	DQ 1	SD 1	Data	DF 2	DQ 2	SD 2
4	0.041421	−0.25	0.064216	10	−0.45794	0.125	0.225338
5	0.06607	−0.25	0.066865	11	−0.33333	0.125	0.126736
6	0.042682	−0.25	0.064322	12	−0.30952	0.125	0.11143
7	−0.01589	0	0.000253	13	−0.70408	0.3125	0.593387
9	−0.0388	0	0.001506	14	−0.63291	0.3125	0.498233
13	0.067572	−0.4375	0.195972	15	−0.69312	0.3125	0.578074
14	0.138742	−0.4375	0.210656	21	−0.14706	0	0.021626
15	0.078532	−0.4375	0.197574	23	−0.06452	0	0.004162
				24	−0.44762	0.125	0.215988
				25	−0.40206	0.125	0.177279
				26	−0.43689	0.125	0.206501

DF deviation of first objective, *DQ* deviation of second objective, *SD* sum of deviation approach is that it is considering sensitivity of parameters while finding optimum points. In addition, there is no need to calculate complex modeling formulations or simulations of process, which takes a lot of time and hardware to find the optimum point. Instead, simple statistical calculations are enough to get results. Also, this approach gives much more reliable solutions because experimental data are used and these data are the exact values to represent the process. In a word, this approach is an easily applicable and more reliable method.

The proposed method is to evaluate average deviations from the Pareto optimum points due to imperfections during manufacturing processes, etc. and uses this criterion to select the optimum design point. This is an important concept in manufacturing. Uncontrollable variations are unavoidable, such as variations due to the quality of manufacturing tools, measurement tools, operators' mistakes, etc. The optimization based on sensitivity has proved to be very useful dealing with discrete variables defined on a population of cutting condition values obtained from experiments. It is believed that the used method provides a robust way of looking at the optimum parameter selection problem. It can easily handle the cases where each of the design variables has different uncertainty ranges.

Results of the case studies have showed the benefits of the new approach mentioned above. Optimum cutting conditions are determined for the machining of GFRP material at two different tool radii (0.4 and 0.8 mm). Optimum conditions are found as a_p=1.2 mm, f=0.4 mm/rev, and V_c=125 m/min for tool radius of 0.4 mm and a_p=1.2 mm, f=0.4 mm/rev, and V_c=100 m/min for tool radius of 0.8 mm. Also, it should be mentioned that tool radii are effective on optimum cutting speed more than other parameters and at higher speed and cutting depth values (in the feasible region) has given the least sensitive to change points.

When the results are compared with previous study, it is seen that feed values are different. Slow feed values gives better surface roughness values but higher feed values gives better results in this study since material removal rate is considered.

The results showed that this easy-to-apply technique has given the least sensitive to change optimum points. The experimental results for optimal settings show that there is a considerable improvement in the performance characteristics of machining process. Used technique will be more convenient and economical to predict the effects of different influential combinations of parameters.

Nomenclature

F_z	Tangential cutting force [N]
F_y	Feed cutting force [N]
α	Clearance angle [°]
γ	Rake angle [°]
θ	Fiber orientation angle [°]
a_p	Depth of cut [mm]
a_e	Width of cut [mm]
f	Feed [mm/rev]
V_c	Cutting speed [m/min]
r	Tool radius [mm]
NISE	Noninferior set estimation
DF	Deviation of the first objective
DQ	Deviation of the second objective
SD	Sum of deviations

Acknowledgments The author is very grateful to Dr. Erhan Altan of the Mechanical Engineering of Faculty, Yildiz Technical University for the support rendered. The author is also thankful to TUBITAK-BAYG for funding this research.

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