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ORIGINAL ARTICLE

A two-phase linear programming methodology for fuzzy multi-objective mixed-model assembly line problem

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Abstract We develop a fuzzy multi-objective linear programming (FMOLP) model for solving multi-objective mixed-model assembly line problem. In practice, vagueness and imprecision of the goals in this problem make the fuzzy decision-making complicated. The proposed model considers minimizing total utility work, total production rate variation, and total setup cost, using a two-phase linear programming approach. In the first phase, the problem is solved using a max–min approach. The max–min solution not being efficient, in general, we propose a new model in the second phase to maximize a composite satisfaction degree at least as good as the degrees obtained by phase one. To show the effectiveness of the proposed approach, a numerical example is solved and the results are compared with the ones obtained by the fuzzy mixed integer goal programming and weighted additive methods. The computational results show that the proposed FMOLP model achieves lower objective functions as well as higher satisfaction degrees.

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1 Introduction

Mixed-model assembly lines are a type of production line where a variety of product models similar in product characteristics are assembled. The effective utilization of a mixed-model assembly line requires solving two problems in a sequential manner as follows: (1) line design and balancing and (2) determination of the production sequence for different models. In this paper, we assume that the line has already been balanced and sequencing problem is only considered. Determining the sequence of introducing models to the mixed-model assembly line is of particular importance considering the crucial goals for the efficient implementation of just-in-time (JIT) systems.

Monden [1] defined two goals for the sequencing problems: (1) leveling the load (i.e., total assembly line) on each station on the line and (2) keeping a constant rate of usage of every model used by the line. To handle these problems, Goal chasing I and II (GC-I and GC-II) have been developed by Toyota Corporation. GC-I minimizes the one stage and assumes that the length of the unique workstation is equal to zero. GC-II solves GC-I under a special assumption regarding the product structure.

Miltenburg [2] developed a nonlinear programming for the second abovementioned goal. The time complexity function of the proposed program was exponential; therefore, he developed and solved the problem by applying two heuristic procedures.

Miltenburg et al. [3] solved the same problem with a dynamic programming algorithm. Inman and Bulfin [4] solved the problem proposed by Miltenburg [2] by converting it to a mathematically different approach. Other objectives have also been considered by a number of researchers. Yano and Rachamadugu [5] dealt with the problem of sequencing jobs with customer-specified options to minimize the total amount of incomplete works. Bard et al. [6] presented an analytical framework for a mixed-model assembly line sequencing problem in order to minimize the overall line length. Okamura and Yamashina [7] developed a sequencing method for mixed-model assembly lines to minimize the line stoppage. Kim and Jeong [8] proposed a generalized formulation of the product-sequencing problem in which the total unfinished work within the mixed-model assembly line's work station boundaries is to be minimized. They solved their model by using an optimal procedure using branch-and-bound technique and a heuristic procedure using lower bound and local search. Bautista and Cano [9] developed some useful procedure to solve the mixed-model assembly line sequencing problem proposed by Yano and Rachamadugu [5]. They also compared their implemented algorithms with others taken from literature using two computational experiments. Toksari et al. [10] introduced learning effect into assembly line sequencing problem. They showed that with the consideration of learning effects both the simple assembly line balancing and the U-type line balancing problems would remain polynomially solvable.

Sequencing mixed-model assembly lines have also been studied as a multi-objective problem. Hyun et al. [11] addressed three objectives minimizing total utility work, keeping a constant rate of model usage, and minimizing total setup cost. This problem was solved by proposing a new genetic evaluation and selection mechanism. McMullen [12] considered two objectives minimizing the number of setups and keeping a constant rate of part usage, and solved this problem by a TS method. Korkmazel and Meral [13] developed the weighted-sum approach for two goals introduced by Monden [1]. McMullen and Frazier [14] developed a simulated annealing (SA) method for the model used by McMullen [12] and compared it to the TS method. McMullen [15–17] also solved the same problem by using genetic algorithms (GAs), Kohonen self-organizing map, and ant colony optimization, respectively, and compared their performance with SA and TS methods. Mansouri [18] also solved the same problem with genetic algorithms. He introduced a new selection mechanism. Tavakkoli-Moghaddam and Rahimi-Vahed [19] solved the problem proposed by Hyun et al. [11] by using a new memetic algorithm. Rahimi-Vahed et al. [20] devised a multi-objective scatter search for a mixed-model assembly line sequencing problem to minimize the three objectives presented by Hyun et al. [11]. They empirically showed that their method outperforms three multi-objective genetic algorithms, i.e., PS-NC GA, NSGA-II, and SPEA-II, on a set of randomly generated problems.

Here, we consider three objectives simultaneously as follows: (1) total utility work, (2) total production rate variation, and (3) total setup cost. Our main purpose is to apply the two-phased fuzzy linear programming methodology where simultaneous minimization of the abovementioned objectives is desired. The structure of this paper is as follows: Section 2 presents a detailed description of the mixed-model assembly line. Section 3 proposes a fuzzy multi-objective linear programming algorithm. In Section 4, experimental and comparative results are given. Finally, we present our conclusions in Section 5. The computer codes implementing our proposed algorithm in the Lingo 8 software environment are provided in Appendices 1, 2, and 3.

2 The multi-objective mixed-model assembly line (MMAL) model

2.1 Mixed-model assembly line

A MMAL considered in this paper is a conveyor system moving at a constant speed (v_c). Similar products are launched onto the conveyor at a fixed rate. The line is partitioned into J stations. It is assumed that the stations are all closed types. A closed station has boundaries, which workers cannot cross. Such a closed station is often found in reality where the use of facilities is restricted within a certain boundary. The tasks allocated to each station are properly balanced and their operating times are deterministic. The worker moves downstream on the conveyor while performing his/her tasks to assemble a product. On completion of the job, the worker moves upstream to the next product. The worker's moving time is ignored.

The design of an MMAL involves several issues such as determining operator schedules, product mix, and launch intervals. Two types of operator schedules, early start schedule and late start schedule, are proposed in Bard et al. [6] (in the early start schedule, the operator starts the assembly process immediately after receiving the product at the workstation, while in the late start schedule the operator awaits assembling of the product until its latest possible starting time as specified in the pre-assigned scheduling program). An early start schedule is more common in practice and is used in this paper (Hyun et al. [11]). Second, minimum part (or model) set (MPS) production, a strategy widely accepted in mixed-model assembly lines, is also used in this paper. MPS is a vector representing a product mix, such that $(d_1,\ldots, d_M)=(D_1/h,\ldots, D_M/h)$, where M is the number of model types, D_m is the number of products of model type m that must be assembled during an entire planning horizon, and h is the greatest common divisor or highest common factor of D_1, D_2,…, D_M.This strategy operates in a cyclical manner. The number of products produced in one cycle is given by $I = \sum_{i=1}^{M} d_i$.Obviously, repeating the MPS, *h* time's products can meet the total demand in the planning horizon. Third, the launch interval (γ) is set to $T/(I \times J)$, where T is the total operation time required to produce one cycle of MPS products (Hyun et al. [11]).

Figure 1 represents the operations carried out in a closed station using an early start schedule strategy. It is noted that this example is similar to the example presented by Hyun et al. [11].

The vertical dotted lines describe station boundaries. Suppose that the MPS, (d_A, d_B, d_C), is (2, 2, 1). In Fig. 1, the models are sequenced as (A C B A B). The operation for the first model of every cycle is started at the left boundary of the station. A horizontal arrow represents the start and finish of an assembly operation, so that it shows the station length required for assembling the corresponding model. A dotted arrow represents the distance between two consecutive products. On the completion of an operation, the worker moves upstream. If the succeeding model is within the station boundary, the worker can begin to work on it. Otherwise, the worker has to wait until the model arrives at the left boundary of the station. A dotted arrow indicates that the worker is idle and its length amounts to the distance that the conveyor has moved during the idle time. The worker is also not allowed to cross over the boundaries. In this case, the incomplete operation, also called utility work, is turned over to utility workers.

2.2 Mathematical model

2.2.1 Notations

The following notations are used to describe the MMAL problem.

Indices and parameters:

- i Product i=1,…, I.
- m Model m=1,…, M.
- j Closed workstation j=1,…, J.
- L_j The fixed line length of station j.
- C_{jmr} The setup cost required when the model type is changed from m to r at station j.
- v_c The constant speed of the conveyor.
- γ The launch interval time.

Decision variable:

- X_{im} 1 if product i in a sequence is the mth model; otherwise 0.
- X_{imr} 1 if model type m and r are assigned, respectively, at positions i and i+1 in a sequence; otherwise 0.
- U_{ij} The amount of the utility work required for product i in a sequence at station j.
- Z_{ij} The starting positions of the work on product i in a sequence at station j.

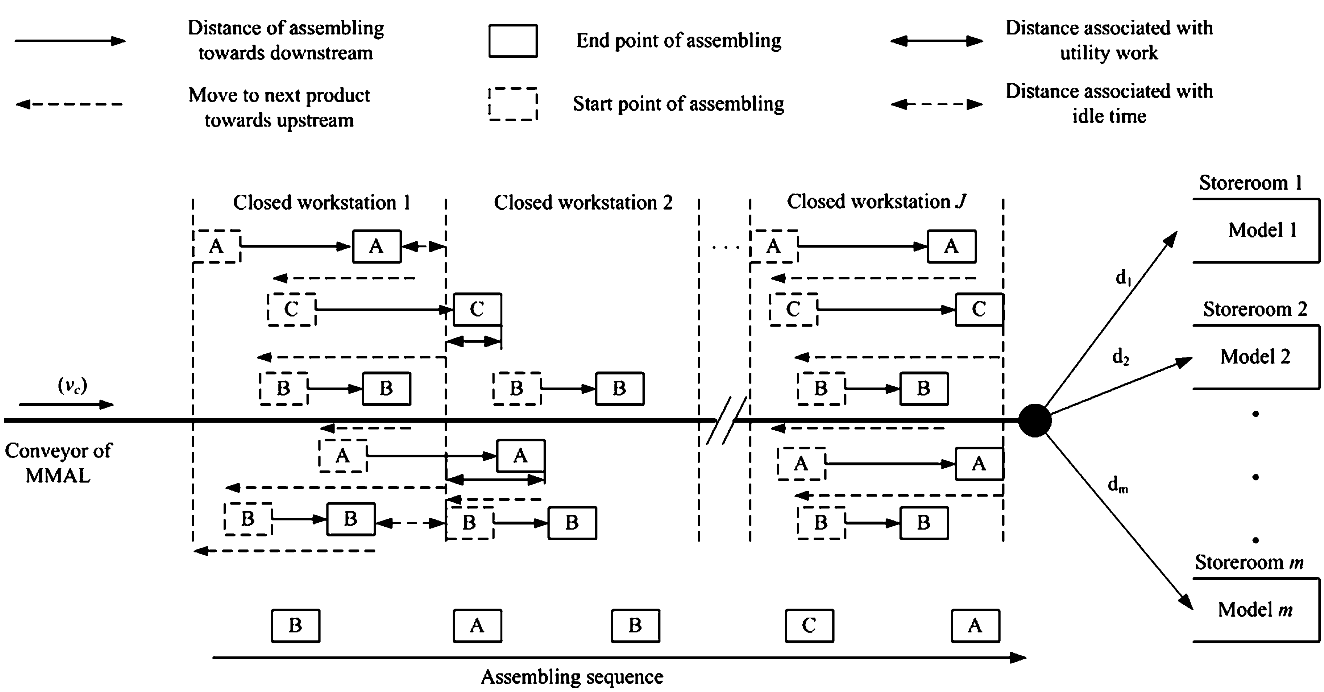

Fig. 1 The MMAL model

2.2.2 Objective functions

Minimizing total utility work The utility work is typically handled by the use of utility workers assisting the regular workers during the work overload. Let L_j be the fixed line length of station j and U_{ij} be the amount of the utility work required for product i in a sequence at station j. The following model is presented by Hyun et al. [11].

$$\text{Minimize} \sum_{j=1}^{J}\sum_{i=1}^{I}\left(U_{ij}+Z_{(i+1)j}/v_c\right) \tag{1}$$

s.t.

$$\sum_{m=1}^{M} x_{im}=1 \qquad \forall i \tag{1.1}$$

$$\sum_{i=1}^{I} x_{im}=d_m \qquad \forall m \tag{1.2}$$

$$Z_{(i+1)j}=\max\left[0,\min\left(Z_{ij}+v_c\sum_{m=1}^{M}x_{im}t_{jm}-(\gamma\times v_c),L_j-(\gamma\times v_c)\right)\right]\forall i,j \tag{1.3}$$

$$U_{ij}=\max\left[0,\left(Z_{ij}+v_c\sum_{m=1}^{M}x_{im}t_{jm}-L_j\right)\Big/v_c\right]\forall i,j \tag{1.4}$$

$$x_{im}=0 \text{ or } 1 \qquad \forall i,m \tag{1.5}$$

$$Z_{1j}=0, \qquad Z_{ij}\geq 0\forall i,j \tag{1.6}$$

$$U_{ij}\geq 0\forall i,j \tag{1.7}$$

The second term in the objective function takes into account for the utility work that may be required at the end of a cycle. Eq. (1.1) ensures that exactly one product is assigned to each position in a sequence. Eq. (1.2) guarantees that demand for each model is satisfied. Eq. (1.3) indicates the starting position of the worker at each station j on product $i+1$ in a sequence. Utility work U_{ij} for product i in a sequence at station j is determined by Eq. (1.4).

Minimizing total production rate variation One basic requirement of JIT systems is continual and stable part supply. Since this can be realized when the demand rate of model is constant over time, the objective is important to a successful operation of the system. Thus, the objective can be achieved by matching demand with actual production. The following model is suggested by Miltenberg [2].

$$\text{Minimize} \sum_{i=1}^{I}\sum_{m=1}^{M}\left(\left|\sum_{l=1}^{i}\frac{x_{lm}}{i}-\frac{d_m}{I}\right|\right) \tag{2}$$

s.t.

Constraints (1.1), (1.2), and (1.5).

The first term in the objective function is the production ratio of model m until product i is produced. The second term is the demand ratio of model m.

Minimizing total setup cost In many industries, sequence-dependent setups are considered as an important item in assembly operations. The model considering sequence-dependent setups developed by Hyun et al. [11] is considered in this paper.

$$\text{Minimize} \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{r=1}^{M} x_{imr}c_{jmr} \tag{3}$$

s.t.

$$\sum_{m=1}^{M}\sum_{r=1}^{M} x_{imr}=1 \qquad \forall i \tag{3.1}$$

$$\sum_{m=1}^{M} x_{imr}=\sum_{p=1}^{M} x_{(i+1)rp} \qquad i=1,\ldots,I-1,\forall r \tag{3.2}$$

$$\sum_{m=1}^{M} x_{\text{Im}r}=\sum_{p=1}^{M} x_{1rp} \qquad \forall r \tag{3.3}$$

$$\sum_{i=1}^{I}\sum_{r=1}^{M} x_{imr}=d_m \qquad \forall m \tag{3.4}$$

$$x_{imr}=0 \text{ or } 1 \qquad \forall i,m,r \tag{3.5}$$

Eq. (3.1) is a set of position constraints indicating that every position in a sequence is occupied by exactly one product. Eqs. (3.2) and (3.3) ensure that the sequence of products is maintained while repeating the cyclic production. Eq. (3.4) imposes the restriction that all the demands should be satisfied in terms of MPS.

3 Fuzzy multi-objective linear programming (FMOLP) methodology

In this study, a two-phase FMOLP methodology is employed.

3.1 Phase 1

First, the general multi-objective model for mixed-model assembly line is presented and then appropriate operators for this decision-making problem are discussed.

A general linear multi-objective model can be presented as:

Find a vector x written in the transformed $x^T=[x_1, x_2,\ldots, x_n]$ which minimizes objective function Z_k with

$$Z_k = \sum_{i=1}^{n} c_{ki}x_i \quad k = 1, 2, \ldots, p. \tag{4}$$

and constraints:

$$x \in X_d, \qquad X_d = \left\{ x \middle/ g(x) = \sum_{i=1}^{n} a_{ri}x_i \leq b_r, \qquad r = 1, 2, \ldots, m, \; x \geq 0 \right\} \tag{5}$$

where c_{ki}, a_{ri}, and b_r are crisp or fuzzy values.

Zimmermann [21] has solved problems (4)–(5) by using fuzzy linear programming. He formulated the fuzzy linear program by separating every objective function Z_j into its negative ideal solution $\left(Z_k^{NIS}\right)$ and positive ideal solution $\left(Z_k^{PIS}\right)$ by solving:

$$Z_k^{NIS} = \max Z_k, \quad x \in X_d, \quad Z_k^{PIS} = \min Z_k, \quad x \in X_d \tag{6}$$

Z_k^{PIS} is obtained through solving the multi-objective problem as a single objective using, each time, only one objective and $x \in X_d$ means that solutions must satisfy constraints.

Since for every objective function Z_k, its value changes linearly from Z_k^{PIS} to Z_k^{NIS} (Zimmermann [21]), it may be considered as a fuzzy number with the linear membership function $\mu_{z_k}(x)$ as shown in Fig. 2. It was shown that a linear programming problem (4)–(5) with fuzzy goal may be presented as follows:

Find a vector x to satisfy:

$$\widetilde{Z}_k = \sum_{i=1}^{n} c_{ki}x_i \underset{\sim}{\leq} Z_k^0 \qquad k = 1, 2, \ldots, p \tag{7}$$

s.t.

$$g_r(x) = \sum_{i=1}^{n} a_{pi}x_i \leq b_r \;\; r = 1, \ldots, m \tag{8}$$

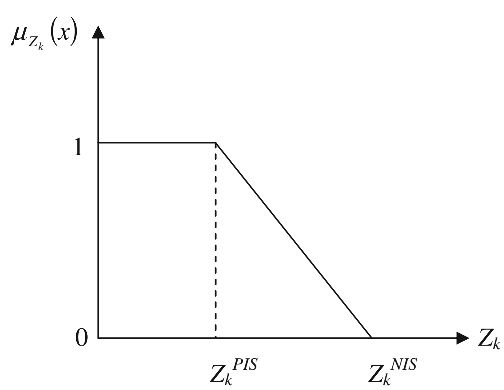

Fig. 2 Objective function Z_k as a fuzzy number

$$x_i \geq 0 \;\; i = 1, 2, \ldots, n \tag{9}$$

In this model, the sign ~ indicates the fuzzy environment. Z_k^0 is the satisfaction degree that the decision-maker wants to reach.

Assuming that membership function, based on preference or satisfaction, is the linear membership for minimization goals, (Z_k) is given as follows:

$$\mu_{Z_k}(x) = \begin{cases} 1 & \text{for } Z_k \leq Z_k^{PIS}, \\ \frac{\left(Z_k^{NIS} - Z_k(x)\right)}{\left(Z_k^{NIS} - Z_k^{PIS}\right)} & \text{for } Z_k^{PIS} \leq Z_k(x) \leq Z_k^{NIS}, \quad k = 1, 2, \ldots, p, \\ 0 & \text{for } Z_k \geq Z_k^{NIS}. \end{cases} \tag{10}$$

In fuzzy programming modeling, using Zimmermann's approach, a fuzzy solution is given by the intersection of all the fuzzy sets representing either fuzzy objective. The fuzzy solution for all fuzzy objectives may be given as:

$$\mu_D(x) = \left\{ \bigcap_{j=1}^{p} \mu_{Z_j}(x) \right\}. \tag{11}$$

The optimal solution x^* is given by Bellman and Zadeh [25]:

$$\mu_D(x^*) = \max_{x \in X_d} \mu_D(x) = \max_{x \in X_d} \left[\min_{j=1,\ldots,p} \mu_{Z_j}(x) \right]. \tag{12}$$

With the "*max–min*" operator and α satisfaction degree, the MOLP problem can be solved as a single objective problem:

$$\text{Minimize } \alpha \tag{13}$$

s.t.

$$\alpha \leq \frac{\left(Z_k^{NIS} - Z_k(x)\right)}{\left(Z_k^{NIS} - Z_k^{PIS}\right)} \qquad k = 1, 2, \ldots, p \tag{14}$$

$$g_r(x) \leq b_r \; r = 1, \ldots, m \tag{15}$$

$$x_i \geq 0 \qquad i = 1, 2, \ldots, n \quad \text{and} \quad \alpha \in [0, 1] \tag{16}$$

The "max–min" operator obtains a best solution (due to max operator) from among the set of worst objective values (due to min operator), each determined by a feasible solution. An alternative approach may decide a different operator. Indeed, quite reasonably, it may sometimes be desirable for a compensatory operator to be used instead of the min operator (see Lee and Li [22]).

3.2 Phase 2

Here, we make use of the result of phase 1 to overcome disadvantages of the one-phase approach.

Lee and Li [22], Guu and Wu [24], and Li and Li [23] used two-phased approaches to fix situations where the max–min operator is not efficient. The two-phase method uses the max–min operator in its first phase. It is well known that the optimal solution obtained by phase 1 may not be an efficient solution in the sense that there may exist another solution in the feasible space dominating the obtained solution by the max–min operator in phase 1 (a solution a is said to dominate solution b if (1) a is at least as good as b regarding all objectives and (2) a is strictly better than b for at least one objective; see Rahimi-Vahed et al. [20]). In the second phase, the solution is forced to improve upon and dominate the one obtained by the max–min operator, adding constraints and a new auxiliary objective function to phase 2 to achieve at least the satisfaction degree obtained in phase 1. An arithmetic average operator $\overline{\lambda}$ is proposed to obtain new satisfaction degrees that represent the MOLP objectives' satisfaction degrees. Thus, the proposed phase 2 problem is as follows:

$$\text{Maximize } \overline{\lambda} = \frac{1}{p}\sum_{k=1}^{p}(\lambda_k - \alpha)^2 \tag{17}$$

s.t.

$$\alpha \le \lambda_k \le \frac{\left(Z_k^{NIS} - Z_k(x)\right)}{\left(Z_k^{NIS} - Z_k^{PIS}\right)}, \qquad k = 1, 2, \ldots, p \tag{18}$$

$$g_r(x) \le b_r, \qquad r = 1, \ldots, m \tag{19}$$

$$x_i \ge 0, \qquad i = 1, 2, \ldots, n \quad \text{and} \quad \alpha, \lambda \in [0, 1]\ . \tag{20}$$

We note that the constraints (18) enforce a better solution in phase by the requirements $\alpha \le \lambda_k$, $k=1,\ldots, p$ while maximizing the mean squares of the improvements. A good starting point for solving the phase 2 problem is $\lambda_k=\alpha$, $k=1, \ldots, p$. Next, the general steps of our algorithm are outlined.

1. {Step 1} Compute the negative ideal solutions, i.e., the Z_k^{NIS}, as given in (6); use these as the initial point for the "*max–min*" model (13)–(16) to compute an optimal solution (the obtained value of α is to be used in phase 2).
2. {Step 2} Let $\lambda_k=\alpha$, $k=1,\ldots, p$ in phase 2, and solve (17)–(20) to get an optimal solution.

3.3 The LP model for fuzzy multi-objective MMAL sequencing problem

The fuzzy multi-objective linear programming for the proposed mixed-model sequencing problem formulation is presented as follows:

$$\text{Min } \widetilde{Z}_1 = \sum_{j=1}^{J}\left(\sum_{i=1}^{I} U_{ij} + Z_{(i+1)j}/v_c\right)\Big] \,\widetilde{\le}\, Z_1^0$$

$$\text{Min } \widetilde{Z}_2 = \sum_{i=1}^{I}\sum_{m=1}^{M}\left(\left|\sum_{l=1}^{i}\frac{x_{lm}}{i} - \frac{d_m}{I}\right|\right) \,\widetilde{\le}\, Z_2^0$$

$$\text{Min } \widetilde{Z}_3 = \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{r=1}^{M} x_{imr}c_{jmr} \,\widetilde{\le}\, Z_3^0$$

s.t.

Constraints (1.1) to (3.5).

The following solution procedure is employed to solve the FMOLP for the proposed mixed-model assembly line sequencing problem.

Algorithm: two-phase FMOLP method.

- Step 1 Construct the FMOLP for the multi-objective mixed-model assembly line sequencing formulation.
- Step 2 Solve the kth objective function with an optimization technique such as branch and bound (B&B) embedded in LINGO 8.0 software, and set Z_k^{PIS} to the objective function value of the found minimum solution (lower bound for the kth objective (Z_k)).
- Step 3 Determine the values of the other objective functions of the obtained sequence in the previous step and set Z_k^{NIS}=the maximum value among the obtained values (upper bound for the kth objective (Z_k)).
- Step 4 Repeat steps 2 and 3 for all the objective functions.
- Step 5 Define the membership function of each goal in the FMOLP model.
- Step 6 Construct the equivalent crisp formulation of FMOLP mixed-model assembly line sequencing problem according to Eqs. (13) to (16).
- Step 7 solve the equivalent crisp formulation in previous step; then calculate the relative membership α of each objective value's satisfaction degrees.
- Step 8 Set $\lambda_k=\alpha$, $k=1,\ldots, p$, and solve the problem (17)–(20) to get an optimal solution.

The model algorithm is illustrated by a numerical example.

4 Numerical example

In this paper, the problem is considered for four workstations and three product models. For this experiment, the following assumptions have to be considered:

1. Sequence-dependent setup costs occur only at the first station.
2. The velocity of conveyor, v_c, is set to 1 for the sake of computational convenience.

The assembly time and the length of each station are shown in Table 1. The MPS of this example is shown in Table 2.

Sequence-dependent setup costs are presented in Table 3. The multi-objective formulation of numerical example is presented as min Z_1, Z_2, and Z_3. Three objective functions Z_1, Z_2, and Z_3 are total utility work, total production rate variation, and total setup cost, respectively.

The linear membership function is used for fuzzifying the objective functions for the above problem. The data set for the values of the lower bounds and upper bounds of the objective functions and fuzzy number for the demands are given in Table 4 (steps 1 to 4).

The membership functions below for three objective functions are provided by which to minimize the total utility work, the total production rate variation, and the total setup cost (step 5).

$$\mu_{Z_1}(x) = \begin{cases} 1 & Z_1 \leq 9.80, \\ \frac{19.80 - Z_1}{10} & 9.80 \leq Z_1 < 19.80, \\ 0 & Z_1 \geq 19.80. \end{cases} \tag{21}$$

$$\mu_{Z_2}(x) = \begin{cases} 1 & Z_2 \leq 2.45 \\ \frac{4.45 - Z_2}{2} & 2.45 \leq Z_2 < 4.45, \\ 0 & Z_2 \geq 4.45. \end{cases} \tag{22}$$

$$\mu_{Z_3}(x) = \begin{cases} 1 & Z_3 \leq 5, \\ \frac{15 - Z_3}{10} & 5 \leq Z_3 < 15, \\ 0 & Z_3 \geq 15. \end{cases} \tag{23}$$

Table 1 Assembly time and workstation length

Workstation	Model			Workstation length
	1	2	3	
1	4	8	7	12
2	6	9	4	14
3	8	6	6	12
4	4	7	5	11

Table 2 Parameter setting

I	MPS	No. of feasible solutions	Launch interval
9	(4 3 2)	1,260	6.2

The crisp formulation of FMOLP mixed-model assembly line sequencing problem for the numerical example can be formulated as follows (step 6):

$$\text{Minimize}\,\alpha \tag{24}$$

s.t.

$$\alpha \leq \frac{(19.8 - Z_1)}{(19.8 - 9.8)} \tag{25}$$

$$\alpha \leq \frac{(4.45 - Z_2)}{(4.45 - 2.45)} \tag{26}$$

$$\alpha \leq \frac{(15 - Z_3)}{(15 - 5)} \tag{27}$$

$$\alpha \in [0, 1] \tag{28}$$

Constraints (1.1) to (3.5).

The LINGO software was used to run this FMOLP model, obtaining the results for the objectives as Z_1=15.6, Z_2=4.63, Z_3=5, and the overall degree of satisfaction with the DM's multiple fuzzy goals as 0.4.

After getting the optimal solution from the previous step, according to the optimal objective function values, the step 6 satisfaction degree, α, can be used in this step (step 7). The equation below represents the FMOLP mixed-model assembly sequencing problem that is transferred from the previous step.

$$\text{Minimize}\,\overline{\lambda} = \frac{1}{3}\sum_{k=1}^{3}(\lambda_k - 0.4)^2 \tag{29}$$

s.t.

$$0.4 \leq \lambda_1 \leq \frac{(19.8 - Z_1)}{(19.8 - 9.8)} \tag{30}$$

Table 3 Sequence-dependent setup cost

Model	Model		
	1	2	3
1	0	1	2
2	3	0	1
3	2	3	0

Table 4 The data set for membership functions

	μ=0	μ=1 (PIS)[a]	μ=0 (NIS)[b]
Z_1 (total utility work)	–	9.80	19.80
Z_2 (total production rate variation)	–	2.45	4.45
Z_3 (total setup cost)	–	5.00	15.00

[a] *PIS* positive ideal solution
[b] *NIS* negative ideal solution

$$0.4 \leq \lambda_2 \leq \frac{(4.45 - Z_2)}{(4.45 - 2.45)} \tag{31}$$

$$0.4 \leq \lambda_3 \leq \frac{(15 - Z_3)}{(15 - 5)} \tag{32}$$

$$\lambda_1, \lambda_2, \lambda_3 \in [0, 1] \tag{33}$$

Constraints (1.1) to (3.5).

This experiment is carried out with a branch-and-bound (B&B) method by using the LINGO 8.0 software, which is executed on a Pentium 4, 3 GHz Windows XP using 512 MB of RAM. The results of applying the two-phase method to the problem of mixed-model assembly sequencing line are shown in Table 5.

In the first phase of the solution procedure in this study, the acceptable DM satisfaction degree, α, is 0.4 in the fuzzy environment. Then, in phase 2, the DM satisfaction degree is improved by adding lower limits and a new auxiliary objective function. As a result of this modification, except for the second auxiliary objective function Z_2, all objective function values are decreased.

The proposed model was applied to the test problem and its performance compared with the Fuzzy Goal Programming (FGP) approach that proposed by Javadi et al. [26] for solving multi-objective mixed-model assembly line sequencing problem. This approach was constructed based on the desirability of the decision-maker (DM) and tolerances considered on goal values. The solution procedure was employed to solve the fuzzy mixed integer goal programming (f-MIGP) for the proposed mixed-model assembly line sequencing problem including the following steps (Javadi et al. [26]):

Step 1. Construct the f-MIGP for the multi-objective mixed-model assembly line sequencing formulation.
Step 2. Solve the kth objective function with an optimization technique such as B&B embedded in LINGO 8.0 software, and set g_k to the objective function value of the found minimum solution.
Step 3. Determine the values of the other objective functions of the obtained sequence in the previous step and set p_k=the maximum value among the obtained values—g_k.
Step 4. Repeat steps 2 and 3 for all the objective functions.
Step 5. Define the membership function of each fuzzy goal in the f-MIGP.
Step 6. Construct the equivalent crisp mixed integer goal programming (c-MIGP) formulation of the f-MIGP.
Step 7. Solve the equivalent c-MIGP formulation and obtain the satisfaction degree α.

The obtained results of fuzzy goal programming-based algorithm were applied to the mentioned problem represented in Table 6.

Furthermore, the proposed model compared with the weighted additive approach proposed by Tiwari et al. [27]. In the fuzzy multi-objective MMAL problem, fuzzy goals have unequal importance to DM, and the proper fuzzy DM operator should be considered. The weighted additive model can handle this problem, which is described as follows.

The weighted additive model is widely used in vector objective optimization problems; the basic concept is to use a single utility function to express the overall preference of DM to draw out the relative importance of criteria (Lai and Hwang [28]). In this case, a linear weighted utility function is obtained by multiplying each membership function of fuzzy goals by their corresponding weights and then adding the results together. Using the convex fuzzy model proposed by Bellman and Zadeh [25] and Sakawa [29],

Table 5 Obtained result of proposed model

Z_1^*	Z_2^*	Z_3^*	λ_1^*	λ_2^*	λ_3^*	O.F.V	Optimal sequence
9.8	3.53	7	1	0.45	0.8	0.17	BCAABBCAA

Table 6 Obtained results of fuzzy goal programming approach (Javadi et al. [26])

Z_1^*	Z_2^*	Z_3^*	O.F.V	Optimal sequence
15.6	4.63	5	0.4	BAAAABBCC

Table 7 Obtained results of weighted additive approach (case 1)

Z_1^*	Z_2^*	Z_3^*	λ_1^*	λ_2^*	λ_1^*	O.F.V	Optimal sequence
14.4	3.14	6	0	0.94	0.51	0.58	ABCCAAABB

the weighted additive model proposed by Tiwari et al. [27] is:

$$\mu_D(x) = \sum_{k=1}^{p} w_k \mu_{Z_k}(x) \tag{34}$$

Where the w_k, the given weight coefficients presenting the relative importance among the fuzzy goals, are so that $\sum_{k=1}^{p} w_k = 1\ ,\ 0 < w_k \leq 1$. The following crisp single objective programming of mixed-model assembly line sequencing problem is equivalent to the model below:

$$\text{Maximize} \sum_{k=1}^{3} w_k \lambda_k \tag{35}$$

s.t.

$$\lambda_1 \leq \frac{(19.8 - Z_1)}{(19.8 - 9.8)} \tag{36}$$

$$\lambda_2 \leq \frac{(4.45 - Z_2)}{(4.45 - 2.45)} \tag{37}$$

$$\lambda_3 \leq \frac{(15 - Z_3)}{(15 - 5)} \tag{38}$$

$$\lambda_1, \lambda_2, \lambda_3 \in [0, 1] \tag{39}$$

Constraints (1.1) to (3.5).

To specify the weight of objectives, there are some good approaches in the literature (Hwang and Yoon [30]).

Case 1. the DM's relative importance or weights of the fuzzy goals are given as w_1=0.15, w_2=0.6, and w_3=0.25 are weights of total utility work, total production rate variation, and total setup cost objective functions, respectively. Based on the convex fuzzy decision-making and the weights that are given by DM, the obtained results of the crisp single objective formulation for the numerical example are given in Table 7. These values of Table 7 represent that the satisfaction degree of Z_2 (total production rate variation) is more than Z_3 (total setup cost) and the satisfaction degree of Z_3 is more than Z_1 (total utility work)($\lambda_2 > \lambda_3 > \lambda_1$). It means that satisfaction degree of the objective functions is consistent with the DM's preferences ($w_2 > w_3 > w_1$).

Case 2. in this case, total utility work is the most important criterion for the DM in comparison with case 1; hence, the relative importance or weights of the fuzzy goals assumed as w_1=0.6, w_2=0.25, and w_3=0.15 are weights of objective functions, respectively.

The value of objectives and optimal sequence vary as follows in Table 8. Due to high weight on the total utility work criterion, the criterion performance is improved in comparison to case 1 from 14.4 to 10.81. Corresponding to the DM's preferences (w_1=0.6, w_2=0.25, w_3=0.15), in this solution, the satisfaction degree of the total utility work objective function is increased($\lambda_1 > \lambda_2 > \lambda_3$).

Case 3. If the DM's relative importance or weight of total utility work criterion changes from 0.6 to 0.15 and the total setup cost changes from 0.15 to 0.6 (weight of total production rate variation without any change), the optimal solution for this case is obtained as follows in Table 9.

Case 4. The DM's relative importance or weights of the fuzzy goals are same as given w_1=w_2=w_3=0.33. The optimal solution for this case is obtained as follows in Table 10.

Comparison of the results obtained using the f-MIGP approach and weighted additive method with the proposed FMOLP model is summarized in Table 11. This comparison shows the proposed FMOLP is superior to f-MIGP and weighted additive in the above test problem.

Table 8 Obtained results of weighted additive approach (case 2)

Z_1^*	Z_2^*	Z_3^*	λ_1^*	λ_2^*	λ_1^*	O.F.V	Optimal sequence
10.81	4.30	6	1	0	0	0.60	ABBBCAAAC

Table 9 Obtained results of weighted additive approach (case 3)

Z_1^*	Z_2^*	Z_3^*	λ_1^*	λ_2^*	λ_1^*	O.F.V	Optimal sequence
17.2	6.23	2	0	0	1	0.60	AAAABBBCC

Table 10 Obtained results of weighted additive approach (case 4)

Z_1^*	Z_2^*	Z_3^*	λ_1^*	λ_2^*	λ_3^*	O.F.V	Optimal sequence
10	2.86	8	0.98	0.79	0.7	0.82	ABCABBCAA

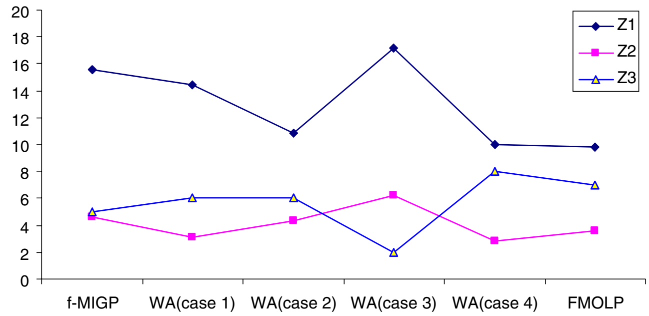

Fig. 3 Objective functions values

Figure 3 shows total utility work, total production rate variation, and total setup cost according to applied methods for solving proposed multi-objective MMAL sequencing problem.

5 Conclusions

A fuzzy multi-objective linear programming (FMOLP) model is developed for the mixed-model assembly line sequencing problem where three crucial objectives of the total utility work, the total production rate variation, and the total setup cost are considered for minimization. We propose a two-phase algorithm for the FMOLP. In the first phase, the problem is solved using a max–min approach. The max–min solution not being efficient, in general, we propose a new model in the second phase to maximize a composite satisfaction degree at least as good as the degrees obtained by phase one. To show the effectiveness of the proposed algorithm, a numerical example taken from the literature is solved to compare the performance of the FMOLP model with the fuzzy mixed integer goal programming and weighted additive methods. The numerical results show that the FMOLP model achieves lower objective functions as well as higher satisfaction degrees.

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Table 11 Solution comparisons

Item	f-MIGP	Weighted additive				FMOLP
	(Javadi et al. [26])	Case 1	Case 2	Case 3	Case 4	(proposed model)
Objective function	Max α		Max $\sum_{k=1}^{3} w_k\lambda_k$			Max $\bar{\lambda}$
Z_1^*	15.6	14.4	10.81	17.2	10	9.8
Z_2^*	4.63	3.148	4.30	6.23	2.86	3.53
Z_3^*	5	6	6	2	8	7
λ_1^*	–	0	1	0	0.98	1
λ_2^*	–	0.94	0	0	0.79	0.45
λ_3^*	–	0.51	0	1	0.7	0.8
O.F.V.	0.4	0.58	0.6	0.6	0.82	0.17

Appendix 1. Lingo formulation for objective function 1

```
MODEL:
SETS:
     ICROM /1..9/;
     JSTAT /1..4/:L;
     MPROD /1..3 /:D;
     IJlink (ICROM, JSTAT): U,Z;
     IMlink (ICROM, MPROD): X;
     JMlink (JSTST, MPROD): T;

ENDSETS
!------------------------------------------------------------------;
DATA:
    !import the data from excel;
    L,D,T=@OLE('G:\all dirs\papers\mixed model\data13.xls');
    !export the data back to excel;
    @OLE('G:\all dirs\papers\mixed model\data13.xls')=U,Z,X;
ENDDATA
!------------------------------------------------------------------;
! OBJECTIVE 1: total utility work;

MIN=@SUM(JSTAT(J):@SUM(ICROM(I):U(I,J))+@SMAX(0,@SMIN((Z(9,J)+@SUM(MPRO
D(M):X(9,M)*T(J,M))-6.2),(L(J)-6.2))));
!------------------------------------------------------------------;
! Constraints;
@FOR (ICROM (I):@SUM (MPROD (M): X (I,M))= 1);
@FOR (MPROD (M):@SUM (ICROM (I): X (I,M))= D(M));
@FOR (JSTAT (J): Z (L,J)=0);
@FOR(ICROM(I)|J #GE#2:@FOR(JSTAT(J):Z(I,J)=@SMAX(0,@SMIN((Z(I-
1,J)+@SUM(MPROD(M):X(I-1,M)*T(J,M))-6.2),(L(J)-6.2)))));
@FOR(ICROM(I):@FOR(JSTAT(J):U(I,J)>=Z(I,J)+@SUM(MPROD(M):X(I,M)*T(J,M))
-L(j)));
@FOR(ICROM(I):@FOR(MPROD(M):@BIN(X(I,M)));


END
```

Appendix 2. Lingo formulation for objective function 2

```
MODEL:
SETS:
     ICROM /1..9/;
     JSTAT /1..4/:L;
     MPROD /1..3/:D;
     RPROD /1..3/;
     LCOMU /1..9/;
     IJlink (ICROM, JSTAT): U, Z;
     IMlink (ICROM, MPROD): X;
     JMlink (JSTST, MPROD): T;
     MRlink (MPROD, RPROD): C;
ENDSETS
!------------------------------------------------------------;
DATA:
    !import the data from excel;
    L,D,C,T=@OLE('G:\all dirs\papers\mixed model\data14.xls');
    !export the data back to excel;
    @OLE('G:\all dirs\papers\mixed model\data14.xls')=U,Z,X;
ENDDATA
!-----------------------------------------------------------;
! OBJECTIVE 2: total production rate variation;
MIN=@SUM(ICROM(I):@SUM(MPROD(M):@ABS(@SUM(LCOMU(L)|L#LE#I:(X(L,M)
/I))-(D(M)/9)))));
!-----------------------------------------------------------;
! Constraints;
@ FOR(ICROM(I):@SUM(MPROD(M):X(I,M))=1);
@FOR(MPROD(M):@SUM(ICROM(I):X(I,M))=D(M));
@FOR(ICROM(I):@FOR(MPROD(M):@BIN(X(I,M))));

END
```

Appendix 3. Lingo formulation for objective function 3

```
MODEL:
SETS:
     ICROM /1..9/;
     JSTAT /1..4/:L;
     MPROD /1..3/:D;
     RPROD /1..3/;
     PPROD /1..3/;
     LCOMU /1..9/;
     IJlink (ICROM, JSTAT): U, Z;
     IMlink (ICROM, MPROD): X;
     JMlink (JSTST, MPROD): T;
     MRlink (MPROD, RPROD): C;
ENDSETS
!-------------------------------------------------------------;
DATA:
    !import the data from excel;
    L,D,C,T=@OLE(‘G:\all dirs\papers\mixed model\data15.xls‘);
    !export the data back to excel;
    @OLE(‘G:\all dirs\papers\mixed model\data15.xls‘)=U,Z,X;
ENDDATA
!-------------------------------------------------------------;
! OBJECTIVE 3: total setup cost;
MIN=@SUM(ICROM(I)|I#LE#8:@SUM(MPROD(M):@SUM(RPROD(R):X(I,M)*X(I+1
,R)*C(M,R))));
!-------------------------------------------------------------;
! Constraints;

@FOR(ICROM(I):@SUM(MPROD(M):X(I,M))= 1);
@FOR(MPROD(M):@SUM(ICROM(I):X(I,M))= D(M));
@FOR(ICROM(I):@FOR(MPROD(M):@BIN(X(I,M))));

END
```

References

1. Monden Y (1983) Toyota production system. Institute of Industrial Engineers Press, Atlanta
2. Miltenburg J (1989) Level schedules for mixed-model assembly lines in just-in-time production systems. Manage Sci 35(2):192–207 doi:10.1287/mnsc.35.2.192
3. Miltenburg J, Steiner G, Yeomans S (1990) A dynamic programming algorithm for scheduling mixed-model just-in-time production systems. Math Comput Model 13:57–66 doi:10.1016/0895-7177(90)90370-3
4. Inman PR, Bulfin RL (1991) Note on sequencing JIT mixed model assembly lines. Manage Sci 37(7):904–910 doi:10.1287/mnsc.37.7.901
5. Yano CA, Rachamadugu R (1991) Sequencing to minimize work overload in assembly lines with product options. Manage Sci 37:572–586 doi:10.1287/mnsc.37.5.572
6. Bard JF, Dar-El EM, Shtub A (1992) An analytic framework for sequencing mixed-model assembly lines. Int J Prod Res 30:35–48 doi:10.1080/00207549208942876
7. Okamura K, Yamshina H (1979) A heuristic algorithm for the assembly line model-mix sequencing problem to minimize the risk of stopping the conveyor. Int J Prod Res 17:233–247 doi:10.1080/00207547908919611
8. Kim S, Jeong B (2007) Product sequencing problem in mixed model assembly line to minimize unfinished works. Comput Ind Eng 53(2): 206–214 doi:10.1016/j.cie.2007.06.011
9. Bautista J, Cano J (2008) Minimizing work overload in mixed model assembly lines. Int J Prod Econ 112(1):177–191 doi:10.1016/j.ijpe.2006.08.019
10. Toksari MD, Isleyen SK, Guner E, Baykoc OF (2008) Simple and U-type assembly line balancing problems with a learning effect. Appl Math Model 32(12):2954–2961 doi:10.1016/j.apm. 2007.10.007
11. Hyun CJ, Kim Y, Kim YK (1998) A genetic algorithm for multiple objective sequencing problems in mixed model assembly lines. Comput Oper Res 25(7–8):675–690 doi:10.1016/S0305-0548(98)00026-4
12. McMullen PR (1998) JIT sequencing for mixed-model assembly lines with setups using tabu search. Prod Plann Contr 9(5):504–510 doi:10.1080/095372898233984
13. Korkmazel T, Meral S (2001) Bicriteria sequencing methods for the mixed-model assembly line in just-in-time production systems. Eur J Oper Res 131:188–207 doi:10.1016/S0377-2217(00)00061-8
14. McMullen PR, Frazier GV (2000) A simulated annealing approach to mixed-model sequencing with multiple objectives on a JIT line. IIE Trans 32(8):679–686
15. McMullen PR (2001a) An efficient frontier approach to addressing JIT sequencing problems with setups via search heuristics. Comput Ind Eng 41:335–353 doi:10.1016/S0360-8352(01)00059-6
16. McMullen PR (2001b) A Kohonen self-organizing map approach to addressing a multiple objective, mixed-model JIT sequencing problem. Int J Prod Econ 72:59–71 doi:10.1016/S0925-5273(00)00091-8
17. McMullen PR (2001c) An ant colony optimization approaches to addressing a JIT sequencing problem with multiple objectives. Artif Intell Eng 15:309–317 doi:10.1016/S0954-1810(01)00004-8
18. Mansouri SA (2005) A multi-objective genetic algorithm for mixed-model sequencing on JIT assembly lines. Eur J Oper Res 167(3):696–716
19. Tavakkoli-Moghaddam R, Rahimi-Vahed AR (2006) Multi-criteria sequencing problem for a mixed-model assembly line in a JIT production system. Appl Math Comput 181(2):1471–1481
20. Rahimi-Vahed AR, Rabbani M, Tavakkoli-Moghaddam R, Torabi SA, Jolai F (2007) A multi-objective scatter search for a mixed model assembly line sequencing problem. Adv Eng Inform 21:85–99 doi:10.1016/j.aei.2006.09.007
21. Zimmermann HJ (1978) Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst 1:45–56 doi:10.1016/0165-0114(78)90031-3
22. Lee S, Li RJ (1993) Fuzzy multiple objective programming and compromise programming with Pareto optimum. Fuzzy Sets Syst 53:275–288 doi:10.1016/0165-0114(93)90399-3
23. Li X, Li B (2006) Computing efficient solutions to fuzzy multiple objective linear programming problems. Fuzzy Sets Syst 157:1328–1332 doi:10.1016/j.fss.2005.12.003
24. Guu M, Wu Y (1999) Two-phase approach for solving the fuzzy linear programming problems. Fuzzy Sets Syst 107:191–195 doi:10.1016/S0165-0114(97)00304-7
25. Bellman RE, Zadeh LA (1970) Decision making in a fuzzy environment. Manage Sci 17:141–164 doi:10.1287/mnsc.17.4.B141
26. Javadi B, Rahimi-Vahed A, Rabbani M, Dangchi M (2007) Solving a multi-objective mixed-model assembly line sequencing problem by a fuzzy goal programming approach. Int J Adv Manuf Technol 39(9–10):975–982
27. Tiwari RN, Dharmahr S, Rao JR (1987) Fuzzy goal programming—an additive model. Fuzzy Sets Syst 24:27–34 doi:10.1016/0165-0114(87)90111-4
28. Lai YJ, Hwang CL (1994) Fuzzy multiple objective decision making, methods and applications. Springer, Berlin
29. Sakawa M (1993) Fuzzy sets and interactive multiobjective optimization. Plenum, New York
30. Hwang CL, Yoon K (1987) Multiple attribute decision making: methods and applications. Springer, Heidelberg